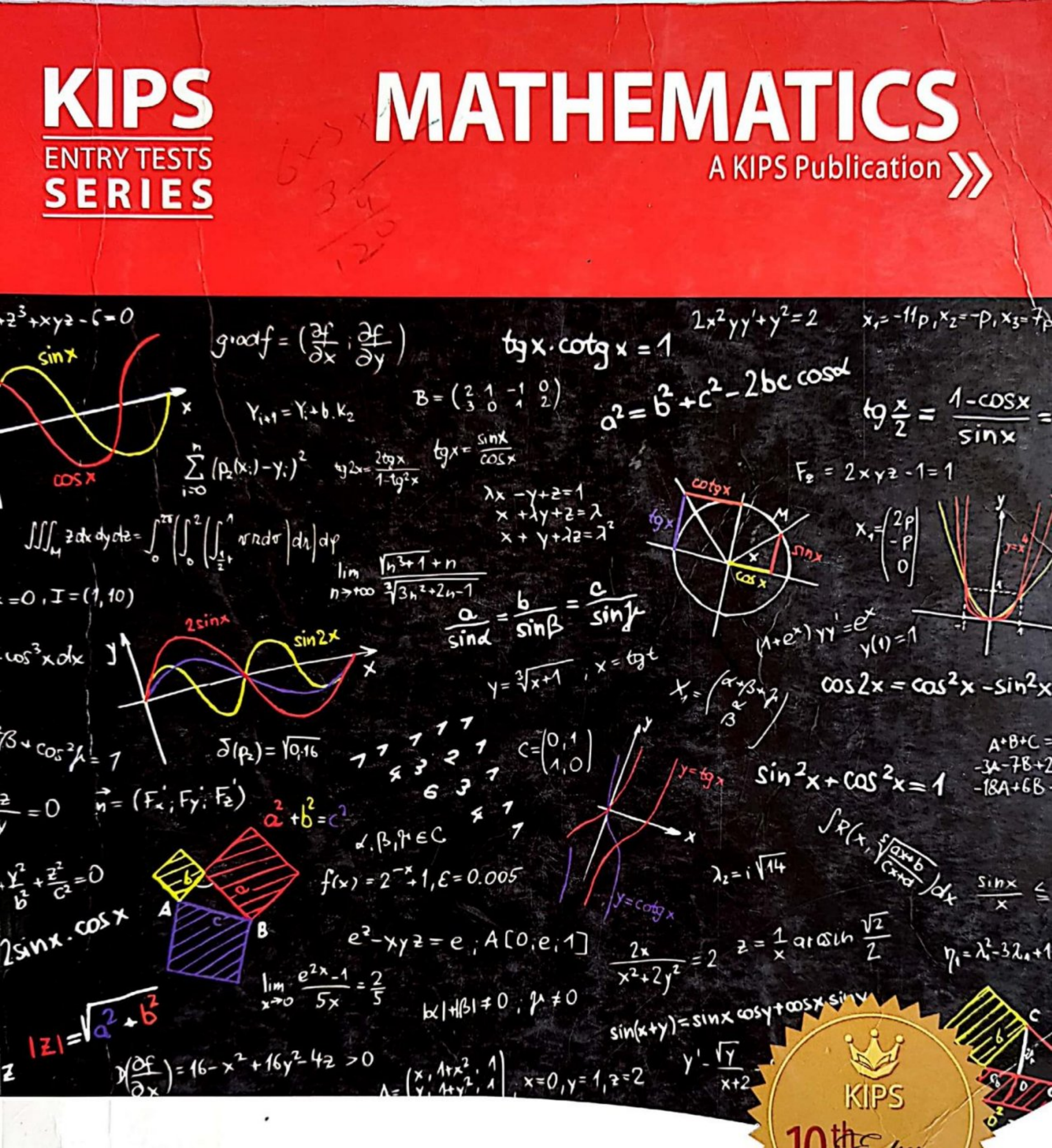
KIPS
ENTRY TESTS
SERIES

MATHEMATICS

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- Quick Review of the Syllabus
- Highlighted Points
- Hundreds of Practice Questions
- Diagnostic Test
- Sample Paper for Entry Test
- Chapter-Wise Exercises with Answer Keys

KIPS
PREPARATIONS

KIPS MATHEMATICS ENTRY TEST SERIES

For All Kinds of Entry & Aptitude Tests)

Editors

Prof. Taimoor Iqbal

Prof. Ali Butt

Contributors

Prof. Rauf Ahmed

Prof. Abdus Salam Ch.

Prof. Sajjad Ahmad

Prof. Faisal Aftab

Prof. Khurram Gill

Prof. Javed Ali Waseer

Prof. Abdul Manaf

Prof. Waqas Bhatti

Prof. Shahbaz

Prof. Maqsood Tufail

Prof. Muzaffar Subhani

Prof. Arshad Nazir

Chief Editor

Abid Wazir Khan

Chief Coordinator

Salman Maqsood

Coordinators

Akbar Ali Mughal

Dr. Ali Shahzad

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KIPS ENTRY TEST SERIES

Part - I

KIPS PUBLICATION

Symbols and Notations

✦ Arithmetic and Algebra

Equal to	$=$
Not equal to	$\neq$
Identity	$\equiv$
Approximately equal to	$\approx$
Approaches	$\rightarrow$
Less than	$<$
Greater than	$>$
Less than or equal to	$\leq$
Greater than or equal to	$\geq$
Factorial of $n \in Z^+$	$\lfloor \underline{n}$ or $n!$
Common logarithm	$\log_{10} a$
Natural logarithm	$\log_e a$
Logarithm (to base b)	$\log_b a$
Summation	$\sum$

✦ Sets and Logic

Belongs to	$\in$
Does not belong to	$\notin$
Such that	:
Such that	∋
Implies that	$\Rightarrow$
Number of elements in a set	$n(S)$
Union	$\cup$
Intersection	$\cap$

Universal set	X or U
Empty set	ϕ
Conjunction	$\wedge$
Disjunction	$\vee$
Negation	$\neg p$ or $\sim$p
Implication	$\rightarrow$
Biconditional	$\leftrightarrow$

✦ Geometry and Trigonometry

Angle	$\angle$
Triangle	$\triangle$
Square	$\square$
Circle	$\bigcirc$
Parallel to	$\parallel$
Perpendicular to	$\perp$
Congruent to	$\equiv$
Similar to	$\sim$

✦ Calculus

Increment of x	Δx or ∂x or dx
Limit of a function	$\lim_{x \to a} f(x)$
Derivative of $f(x)$	$\frac{d}{dx} f(x)$
Indefinite integral	$\int f(x)\,dx$
Definite integral with limits **a** and **b**	$\int_a^b f(x)\,dx$

- This diagnostic test is designed to help you to pinpoint the weak areas in your background. Sit aside 1 hour to take this test. Check your answers with those at the end of the test. Then evaluate yourself.

KIPS DIAGNOSTIC TEST

(1) $\left(\frac{1+i}{1-i}\right)^{104} = ?$

(a) i (b) $-i$
(c) 1 (d) -1

(2) The cube roots of -64 are;

(a) $-4, -4\omega^2, -4\omega^8$ (b) $-4\omega^3, -4\omega, -4\omega^2$
(c) $-4\omega^8, 4(1+\omega), 4(1+\omega^2)$ (d) $-3, -3+\omega, -3+\omega^2$

(3) The value of "K" for which $1-\sqrt{2}$ is one root of the equation $x^2-2x+k=0$, is?

(a) 1 (b) $\sqrt{2}$
(c) -1 (d) $-\sqrt{2}$

(4) If $1+2+3+\cdots+n=25$, then $1^3+2^3+3^3+\cdots+n^3=?$

(a) 165 (b) 385
(c) 255 (d) 625

(5) $\frac{{}^{25}C_{11}}{{}^{25}C_{10}} = ?$

(a) $\frac{5}{11}$ (b) $\frac{15}{11}$
(c) $\frac{5}{10}$ (d) $\frac{15}{10}$

(6) Number containing three digits, repetition NOT allowed are.

(a) 720 (b) 999
(c) 648 (d) 600

(7) Domain of $\sqrt{16x-16x^2}$ is;

(a) $[0,1]$ (b) $(0,1)$
(c) $\mathbb{R}-[0,1]$ (d) $\mathbb{R}-(0,1)$

(8) If $\sin\theta+\cos\theta=0, \theta=?$

(a) -45 (b) 45
(c) 180 (d) 225

(9) If $\theta=30$, then $\frac{1+\tan^2\theta}{2\tan\theta}=?$

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$
(c) $-\sqrt{3}$ (d) $-\frac{\sqrt{3}}{2}$

(10) Slope of a line which is parallel to the line joining the points $(2,5)$ and $(3,8)$ is;

(a) 1 (b) 3

(c) -3 (d) $-\frac{1}{3}$

(11) The point of intersection of the lines $3x+2y=8$ and $5x-11y+1=0$ is;

(a) $(1,2)$ (b) $(-1,2)$

(c) $(2,1)$ (d) $(2,-1)$

(12) The length of tangent from the point $(2,5)$ to the circle $x^2+y^2-2x-3y-1=0$ is;

(a) 5 (b) 4

(c) $\sqrt{5}$ (d) 3

(13) $\hat{i}.\hat{j}\times\hat{k}=?$

(a) 0 (b) 1

(c) -1 (d) none of these

(14) $\begin{vmatrix} co\sec x & -1 \\ -1 & co\sec x \end{vmatrix}=?$

(a) $2\sec x$ (b) $\cot^2 x$

(c) $-\cot^2 x$ (d) $-2co\sec x$

(15) Let $f(x)=\begin{cases} x-4, x>2 \\ m, x=2 \\ 2-x^2, x<2 \end{cases}$,then find the value of "m" such that the function is continuous at $x=2$.

(a) 0 (b) 1

(c) 2 (d) -2

(16) $\sin\left(\tan^{-1}(1)\right)=?$

(a) $-\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$

(c) 1 (d) -1

(17) $\frac{d}{dx}(\log_7 x)=?$

(a) $7^x \ln 7$ (b) $\frac{1}{x\ln 7}$

(c) $\frac{1}{x}$ (d) $\frac{1}{x\ln 10}$

(18) Solution of the differential equation $\frac{dy}{dx}=\frac{x}{x^2+1}$ is;

(a) $y=\frac{1}{2}\ln\left(x^2+1\right)+c$ (b) $y=2\ln\left(x^2+1\right)+c$

(c) $y=\ln\left(x^2+1\right)+c$ (d) $y=x^2+1+c$

(19) Two identical dice are rolled. The probability that the same number will appear on the both is;

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$

(c) $\frac{1}{18}$ (d) $\frac{1}{9}$

(20) The fifth term of the sequence $(-1)^n + 1$ is;

(a) 1 (b) 0

(c) -1 d) -2

(21) One root of the equation $2^{2x} - 10.2^x + 16 = 0$ is;

(a) 1 (b) 0

(c) -1 (d) 2

(22) If $\alpha = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ then $\alpha^{-1} = ?$

(a) $\begin{pmatrix} -0.5 & 0.5 \\ 0.5 & -0.5 \end{pmatrix}$ (b) $\begin{pmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (d)none of these

(23) $\sin 38 - \cos 52 = ?$

(a) $\sin 90$ (b) $\cos 90$

(c) $\tan 90$ (d) $\sec 90$

(24) The term free from t in $\left(4t + \frac{5}{t}\right)^8$ is;

(a) 8^{th} (b) 9^{th}

(c) 5^{th} (d) 4^{th}

(25) If $\sin\theta = \frac{2}{3}$ then $\cos 2\theta = ?$

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$

(c) $\frac{2}{9}$ (d) $\frac{4}{9}$

(26) The line $y = 2x + L$ is tangent to the circle $x^2 + y^2 = 9$,then $L = ?$

(a) $4\sqrt{5}$ (b) $3\sqrt{5}$

(c) Less than $3\sqrt{5}$ (d) greater than $3\sqrt{5}$

(27) $\tan\left(\cos^{-1}\left(\frac{2}{3}\right) + \sin^{-1}\left(\frac{2}{3}\right)\right) = ?$

(a) 0 (b) 1

(c) -1 (d) ∞

(28) Domain of the function $f=\{(1,3),(3,5),(2,6)\}$ is;

(a) $\{1,2,5\}$ (b) $\{1,3,6\}$

(c) $\{1,2,3\}$ (d) $\{1,5,6\}$

(29) Which of the following matrices is not invertible?

(a) $\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$ (b) $\begin{pmatrix}-1 & -1\\ -1 & 2\end{pmatrix}$

(c) $\begin{pmatrix}2 & 3\\ 4 & 6\end{pmatrix}$ (d) $\begin{pmatrix}2 & 1\\ 2 & 2\end{pmatrix}$

(30) The minimum value of $\sin x.\cos x$ is;

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{-1}{2}$ (d) $-\frac{1}{4}$

(31) The value of "α" for which $4x^2+4(\alpha+1)x+\alpha^2=0$, has equal roots, is;

(a) $-\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $-\frac{1}{4}$

(32) If given that $g(0)=2, g'(0)=1$ and $f(x)=xg(x)$, then $f'(0)=?$

(a) -2 (b) 0

(c) 2 (d) $\frac{1}{2}$

(33) The area between $y=4\sin x$ from 0 to π is;

(a) 1 (b) 2

(c) 4 (d) 8

(34) A coin is tossed 8 times, the probability of getting a head 5 times is;

(a) $\frac{7}{16}$ (b) $\frac{7}{64}$

(c) $\frac{7}{128}$ (d) $\frac{7}{32}$

(35) Focus of the conic $x^2-6x-8y=7$ is;

(a) $(0,3)$ (b) $(0,-3)$

(c) $(3,0)$ (d) $(-3,0)$

(36) If the line $y=x+\frac{c}{2}$ is tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ then the value of c is;

(a) 10 (b) -10

(c) ± 10 (d) $\pm\sqrt{10}$

(37) $\int_3^5 [x]dx = ?$, where $[x]$ is greatest integer function.

(a) 8 (b) 7

(c) 6 (d) 0

(38) If $f'(x) = g(x)$ then $\int f(x)g(x)dx = ?$

(a) $[f(x)]^2 + c$ (b) $[g(x)]^2 + c$

(c) $\frac{1}{2}[f(x)]^2 + c$ (d) $\frac{1}{2}[g(x)]^2 + c$

(39) Center of the sphere $4x^2 + 4y^2 + 4z^2 - 4x - 4y + 4z = 1$ is;

(a) $\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$

(c) $\left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ (d) $(-2, -2, 2)$

(40) If one of the root of the equation $x^2 - px + q = 0$ is twice of the other then;

(a) $p^2 = q$ (b) $2q = p^3$

(c) $2p^2 = 9q$ (d) $2q^2 = 9p$

NUMBER SYSTEM

Table of Sets

Name	Definition
Set of natural numbers	$N=\{1,2,3,.....\}$
Set of whole numbers	$W=\{0,1,2,3,\cdots\}$
Set of integers	$Z=\{0,\pm1,\pm2,\pm3,.....\}$
Set of positive integers	$Z^{+}=\{1,2,3,....\}$
Set of negative integers	$Z^{-}=\{-1,-2,-3,.....\}$
Set of odd numbers	$O=\{1,3,5,....\}$
Set of even numbers	$E=\{0,2,4,.....\}$
Set of prime numbers	$P=\{2,3,5,7,11,13,17,19,.....\}$
Set of rational numbers	$Q=\left\{x/x=\frac{p}{q},p,q\in Z,q\neq 0\right\}$
Set of irrational numbers	$Q'=\left\{x/x\neq\frac{p}{q},p,q\in Z,q\neq 0\right\}$
Set of real numbers	$\mathbb{R}=Q\cup Q'$

Decimal Representation of Rational and Irrational Numbers

Name	Definition	Examples
Terminating decimals	A decimal having finite number of digits in its decimal part.	434.77779 , 684.765003 0.000000000001
Non terminating decimals	A decimal having infinite number of digits in its decimal part.	3.1452..... 76.9000123.....
Recurring decimals or Periodic decimals	A decimal in which one or more digits repeat indefinitely.	$3.13131313\cdots=3.\overline{13}$ 1.121121121.....
Irrational numbers	Non terminating and non recurring numbers.	2.31415162..... $3.141592654.....=\pi$ $\sqrt{n}$,where n is prime
Rational numbers	Terminating, Recurring	0.78 $4.\overline{424}=4.424424...$

Operations

Name of operation	Definition	Examples
Unary operations	An operation which converts one number into other.	Square root, trigonometric operations, derivative, integration etc
Binary operations	(i). An operation which converts two numbers into third. (ii).A rule denoted by * in a set S that assigns to any pair of elements of S taken in a definite order to another element of S.	Following are some binary operations. Addition(+) Multiplication($\times$) Division($\div$) Subtraction($-$) Union, intersection and difference of sets. The most important binary operations are $+$ and $\times$

Real Number System

Properties of Real Numbers

Property / binary operation	Addition	Multiplication
1.Closure law	$\forall x, y \in \mathbb{R}, x+y \in \mathbb{R}$	$\forall x, y \in \mathbb{R}, x.y \in \mathbb{R}$
2.Associative law	$\forall x, y, z \in \mathbb{R}, x+(y+z)=(x+y)+z$	$\forall x, y, z \in \mathbb{R}$ $x.(y.z)=(x.y).z$
3.Identity law	$\forall x \in \mathbb{R}, \exists 0 \in \mathbb{R}$ such that $x+0=0+x=x$,0 is identity element of addition	$\forall x \in \mathbb{R}$, $\exists 1 \in \mathbb{R}$ such that $x.1=1.x=x$,1 is identity element of multiplication
4.Inverse Property	$\forall x \in \mathbb{R}, \exists(-x) \in \mathbb{R}$ such that $x+(-x)=(-x)+x=0$	$\forall x \in \mathbb{R}(x \neq 0), \exists \frac{1}{x} \in \mathbb{R}$ such that $x.\frac{1}{x}=\frac{1}{x}.x=1$
5.Commutative law	$\forall x, y \in \mathbb{R}$, $x+y=y+x$	$\forall x, y \in \mathbb{R}$, $x.y=y.x$
6.Distributive law	$\forall x, y, z \in \mathbb{R}$ Distributivity of multiplication over addition $(i)x(y+z)=xy+xz$ $(ii)(x+y)z=xz+yz$	

Properties of Equality of Real Numbers

The following properties hold for the equality sign "="

Name of property	Definition
1.Reflexive property	$\forall x \in \mathbb{R}, x = x$
2.Symmetric property	$\forall x, y \in \mathbb{R}, x = y \Rightarrow y = x$
3.Transitive property	$\forall x, y, z \in \mathbb{R}$ $x = y \wedge y = z \Rightarrow x = z$
4.Additive property	$\forall x, y, z \in \mathbb{R}\ , x = y \Rightarrow x + z = y + z$
5.Multiplicative property	$\forall x, y, z \in \mathbb{R}, x = y \Rightarrow x.z = y.z$
6.Cancellation property w.r.t. addition	$\forall x, y, z \in \mathbb{R}, x + z = y + z \Rightarrow x = y$
7.Cancellation property w.r.t. multiplication	$\forall x, y, z \in \mathbb{R}, x.z = y.z \Rightarrow x = y, z \neq 0$

Properties of Inequalities of Real Numbers (Order Properties)

The following properties hold for the signs of inequality *i.e.* <(less than) and>(greater than).

Name of the property	Definition
1.Trichotomy property	$\forall x, y \in \mathbb{R}$ either $x = y$ or $x < y$ or $x > y$
2.Transitive property	$\forall x, y, z \in \mathbb{R}$ $(i) x > y \wedge y > z \Rightarrow x > z$ $(ii) x < y \wedge y < z \Rightarrow x < z$
3.Additive property	$\forall x, y, z \in \mathbb{R}$ $(i) x > y \Rightarrow x + z > y + z$ $(ii) x < y \Rightarrow x + z < y + z$
4.Multiplicative property	$\forall x, y, z \in \mathbb{R}$ $(i) x > y \Rightarrow xz > yz (z > 0)$ $(ii) x > y \Rightarrow xz < yz (z < 0)$

Properties of Fractions of Real Numbers

Name of the property	Definition
1.Principle for equality of fractions	$\frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc$
2.Rule for product of fractions	$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
3.Golden rule of fractions	$\frac{a}{b} = \frac{ka}{kb} (k \neq 0)$
4.Rule for quotient of fractions	$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$

Chapter–1

Complex Number System

- The numbers of the form $x+iy$ where $x,y\in\mathbb{R}$ are called complex numbers. The complex numbers are denoted by $\mathbb{C}$.
- $x+iy=(x,y)$, Where x is called real part and y is called imaginary part of the complex number.

Powers of *i*:

- $i=\sqrt{-1}$
- $i^2=-1$
- $i^3=-i$
- $i^4=1$
- $i^{4n}=1$ where $n\in\mathbb{Z}$
- $i^n+i^{n+1}+i^{n+2}+i^{n+3}=0$, $n\in\mathbb{Z}$

Operations on Complex Numbers

Name of operation	Definition
1.Equality of complex numbers	$\left.\begin{array}{l}a+ib=c+id\\(a,b)=(c,d)\end{array}\right\}\Leftrightarrow a=c\wedge b=d$
2.Addition of complex numbers	$a+ib+c+id=(a+c)+i(b+d)$ or $(a,b)+(c,d)=(a+c,b+d)$
3.Scalar multiplication of complex numbers	$k(a+ib)=ka+ikb$ $k(a,b)=(ka,kb)$
4.Multiplication of complex numbers	$(a+ib)(c+id)=(ac-bd)+(ad+bc)i$ $(a,b)(c,d)=(ac-bd,ad+bc)$
5.Division of complex numbers	$\frac{(a,b)}{(c,d)}=\left(\frac{ac+bd}{c^2+d^2},\frac{bc-ad}{c^2+d^2}\right)$

Properties of Fundamental Operations on Complex Numbers

Property/ operation	Addition	Multiplication
1. Identity property	$(0,0)=0+0.i$	$(1,0)=1+0.i$
2. Inverse property	$\forall(a,b)\in\mathbb{C}$ has additive inverse $(-a,-b)$ *i.e.* $(a,b)+(-a,-b)=(0,0)$	Every non zero complex number (a,b) has an inverse $\left(\frac{a}{a^2+b^2},\frac{-b}{a^2+b^2}\right)$ *i.e.* $(a,b)\left(\frac{a}{a^2+b^2},\frac{-b}{a^2+b^2}\right)=(1,0)$
3.Distributive property	Multiplication is distributive over addition in $\mathbb{C}$ $(a,b)[(c,d)\pm(e,f)]=(a,b)(c,d)\pm(a,b)(e,f)$	

Conjugate of a Complex Number:

□ Let Z denotes a complex number then $Z = x + iy = (x, y)$;the conjugate of Z is denoted by $\overline{Z}$ and is defined as $\overline{Z} = x - iy = (x, -y)$.

Properties of Conjugate:

$\forall z, z_1, z_2 \in \mathbb{C}$

(i). $\overline{(\overline{z})} = z$

(ii). $z.\overline{z} = |z|^2$

(iii). $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

(iv). $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}} (z_2 \neq 0)$

(v). $\frac{1}{z} = \frac{\overline{z}}{|z|^2} (z \neq 0)$

Modulus and Argument of a Complex Number:

□ Let $Z = x + iy = (x, y)$ then

- Modulus of $Z = |Z| = \sqrt{x^2 + y^2}$
- Argument of Z = $\arg Z = \tan^{-1}\left(\frac{y}{x}\right)$

Properties of argument:

(1) $\arg(z_1.z_2) = \arg(z_1) + \arg(z_2)$

(2) $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

(3) $\arg(\overline{z}) = -\arg(z) = \arg\left(\frac{1}{z}\right)$

(4) $\arg(z.\overline{z}) = 0$

(5) $\arg\left(\frac{z}{\overline{z}}\right) = 2\arg z$

Properties of Modulus of a Complex Number:

$\forall z, z_1, z_2 \in \mathbb{C}$

(i). $|Z| = |-Z| = |\overline{Z}| = |-\overline{Z}|$

(ii). $|Z_1.Z_2| = |Z_1|.|Z_2|$

(iii). $|Z_1| - |Z_2| \leq |Z_1 + Z_2| \leq |Z_1| + |Z_2|$ This is called triangle inequality.

Chapter–1

Graphical Representation of Complex Numbers:

❑ Since a complex number is considered as an order pair of real numbers, so we can represent such numbers by points in a xy-plane (Cartesian plane) which is called **complex plane** or **Argand Diagram.**

Principal Argument of a Complex Number

Let θ be the principal argument of a complex number $x+iy$ defined in the range $-\pi<\theta\leq\pi$, such that

- In 1st Quadrant $\theta=\tan^{-1}\left(\frac{y}{x}\right)$
- In 2nd Quadrant $\theta=\pi-\tan^{-1}\left(\frac{y}{|x|}\right)$
- In 3rd Quadrant $\theta=\tan^{-1}\left(\frac{|y|}{|x|}\right)-\pi$
- In 4th Quadrant $\theta=-\tan^{-1}\left(\frac{|y|}{x}\right)$

Polar Form of a Complex Number

❑ Since every complex number $x+iy$ can represented in the form of a point (x,y) ,so we can express every complex number in the form of polar coordinates (r,θ). Where $r=\sqrt{x^2+y^2}$ and $\theta=\tan^{-1}\left(\frac{y}{x}\right)$.

- $x+iy=r\cos\theta+ir\sin\theta$ **or**
- $x+iy=r(\cos\theta+i\sin\theta)$ **or**
- $x+iy=rcis\theta$
- To convert polar coordinates into Cartesian coordinates replace (r,θ) into $(r\cos\theta,r\sin\theta)$

De Moivre's Theorem:

❑ Let $Z=x+iy=r\cos\theta+ir\sin\theta$ then $(Z)^n=r^n(\cos n\theta+i\sin n\theta)=r^n cisn\theta$ is called De Moivre's theorem. $\forall n\in Z$

Applications of De Moivre's Theorem:

(i). $(Z)^{\frac{1}{n}}=r^{\frac{1}{n}}\left(\cos\frac{\theta+2\pi k}{n}+i\sin\frac{\theta+2\pi k}{n}\right)$ or $(Z)^{\frac{1}{n}}=r^{\frac{1}{n}}cis\frac{\theta+2\pi k}{n}$ where $k=0,1,2,3,\ldots\ldots(n-1)$. This formula is used to find the *n*th root of a complex number.

(ii). $Z_1.Z_2=r_1.r_2\left(\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\right)$ or $Z_1.Z_2=r_1.r_2cis(\theta_1+\theta_2)$

(iii). $\frac{Z_1}{Z_2}=\frac{r_1}{r_2}\left(\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)\right)$ or $\frac{Z_1}{Z_2}=\frac{r_1}{r_2}cis(\theta_1-\theta_2)$

- The proof of all the theorems is beyond the scope of this book.
- Complex numbers do not hold order axioms.

* **After the name of French mathematician Abraham De Moivre (1667 – 1754).**

PRACTICE EXERCISE

(1) If 'a' and 'b' are real numbers then $a + b$ is a real no. This law is called
(a) Closure law
(b) commutative law
(c) associative law
(d) Distributive law

(2) If $a \in \mathbb{R}$ then multiplicative inverse of a is
(a) 0
(b) a
(c) $\frac{1}{a}$
(d) $-a$

(3) Which of the following is field?
(a) $\mathbb{Q}$
(b) $\mathbb{N}$
(c) W
(d) Q'

(4) $1 > -1 \Rightarrow -3 > -5$, this property is called
(a) Additive property
(b) transitive property
(c) multiplicative property
(d) closure property

(5) Let $a,b,c,d \in \mathbb{R}$ then $a=b$ and $c=d \Rightarrow$
(a) $a+b=c+d$
(b) $a-b=c+d$
(c) $a+c=b+d$
(d) $a-c=b+d$

(6) If $Z_1 = (x_1, y_1)$, $Z_2 = (x_2,y_2)$ where $Z_1, Z_2 \in \mathbb{C}$ then Z_1+Z_2=?
(a) (x_1+x_2, y_1+y_2)
(b) (x_1+y_2, x_2+y_1)
(c) (x_1+y_1, x_2+y_2)
(d) (x_1-x_2, y_1-y_2)

(7) If $Z_1 = 3-6i$ and $Z_2=4+5i$ then Z_1Z_2=?
(a) $42-9i$
(b) $9-42i$
(c) $42+9i$
(d) $4+i$

(8) If $Z_1 = (1, 0)$, $Z_2 = (2, 3)$ then Z_1Z_2=?
(a) $-2+3i$
(b) $2-3i$
(c) $2+3i$
(d) $-2-3i$

(9) $(3, 5) + (0, 4)$ =?
(a) (3, 5)
(b) (0, 9)
(c) (0, 4)
(d) (3, 9)

(10) Additive inverse of (3,3) in C is
(a) (3, 0)
(b) (0, 3)
(c) (–3, 3)
(d) (-3,-3)

(11) If Z =(a,b) then multiplicative inverse of Z is
(a) $\left(\frac{a}{a^2+b^2}, \frac{b}{a^2+b^2}\right)$
(b) $\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$
(c) $\left(\frac{-a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$
(d) $\left(\frac{-a}{a^2+b^2}, \frac{b}{a^2+b^2}\right)$

(12) If $Z = a+b$ then $|Z|$=?
(a) $a+b$
(b) $\sqrt{a^2+b^2}$
(c) a^2+b^2
(d) a^2+b^2+2ab

(13) If $Z = a+b$, then $\overline{Z}$ = ?
(a) $a-b$
(b) $-(a+b)$
(c) $-a+b$
(d) $a+b$

(14) i^{15} =?
(a) 1 (b) $-i$
(c) i (d) –1

(15) The value of i^{25} =?
(a) i (b) $-i$
(c) 1 (d) –1

(16) Conjugate of (-3, 4) is
(a) (3,4) (b) (3,-4)
(c) (-3,-4) (d) (-3,4)

(17) $(-i)^{31}$
(a) i (b) $-i$
(c) 1 (d) –1

(18) The multiplicative inverse of $1-3i$ is
(a) $3i$ (b) $\frac{1}{3i}$
(c) $-\frac{1}{3}$ (d) none

(19) The value of i^{-29} is
(a) i (b) $-i$
(c) 1 (d) –1

(20) If $Z \in \mathbb{C}$ then $Z.\overline{Z}$ =?
(a) Z (b) $-Z$
(c) Z^2 (d) $|Z|^2$

(21) If $Z_1, Z_2 \in \mathbb{C}$ then $Z_1 + Z_2$ =?
(a) $\overline{Z_1} + \overline{Z_2}$ (b) Z_1-Z_2
(c) $Z_1 + Z_2$ (d) $\overline{Z_1 - Z_2}$

(22) The value of $(3+2i)^3$ is
(a) $62+9i$ (b) $46-9i$
(c) $46i-9$ (d) $46i+9$

(23) C has no identity with respect to + other than _____.
(a) $1+0i$ (b) $0+1i$
(c) $0+0i$ (d) $1+1i$

(24) The conjugate of $\frac{2+3i}{-i+1}$ =?
(a) $\frac{2-3i}{1+i}$ (b) $\frac{2-3i}{1-i}$
(c) $\frac{2+3i}{1+i}$ (d) $\frac{2+3i}{-1-i}$

(25) $\left(-1+\sqrt{-3}\right)^4 + \left(-1-\sqrt{-3}\right)^4$ =?
(a) -2 (b) 4
(c) -4 (d) -16 +16 ω^2

(26) The smallest positive integer "k" for which $\left(\frac{1+i}{1-i}\right)^k = 1$ is;

(a) 2 (b) 4
(c) 8 (d) 16

(27) Set of even prime natural numbers is;

(a) Finite (b) Empty
(c) Singleton (d) infinite

(28) In n is any positive integer then the value of $\frac{i^{4n+1} - i^{4n-1}}{2} = ?$

(a) 1 (b) -1
(c) i (d) $-i$

(29) $\frac{\sqrt{18}}{\sqrt{72}}$ is ;

(a) Rational number (b) irrational
(c) imaginary (d) none of these

(30) $2\sqrt{-9} \times \sqrt{-16} = ?$

(a) 24 (b) -24
(c) 48 (d) -48

(31) $\left|\frac{1+i}{1+\frac{1}{i}}\right| = ?$

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{3}{\sqrt{2}}$ (d) 1

(32) $\left(\frac{1-i}{1+i}\right)^{100} = x + iy$ then;

(a) $x = 2, y = -1$ (b) $x = 1, y = 0$
(c) $x = 0, y = 1$ (d) $x = -1, y = 2$

(33) $\{1, -1\}$ is closed with respect to;

(a) Addition (b) multiplication
(c) division (d) both b & c

(34) $(\cos 20 + i \sin 20)^5 \div (\cos 30 + i \sin 30)^3 = ?$

(a) $\cos 20 + i \sin 20$ (b) $\cos 30 + i \sin 30$
(c) $\cos 10 + i \sin 10$ (d) $\cos 20 - i \sin 20$

(35) The polar coordinates of a point are $(2, 319)$ then the Cartesian coordinates of the point are;

(a) $(1.509, -1.312)$ (b) $(-1.509, -1.312)$
(c) $(1.509, 1.312)$ (d) $(-1.509, 1.312)$

(36) The set of rational numbers between two real numbers is _____
(a) Finite (b) Empty
(c) Infinite (d) Singleton

(37) Multiplicative inverse of a non-zero element $a \in Z$ is _____ As $Z = \{0, \pm1, \pm2, \pm3, \ldots\}$ so all have the multiplicative inverse except 0 as $\frac{1}{0} = \infty$ and also the inverse of 2 is $\frac{1}{2}$ which is not Dom of Z.
(a) 1 (b) –a
(c) $\frac{1}{a}$ (d) Non – existent

(38) The $\{x \in R, x^2 + 10 = 0\}$ is _____
(a) {10} (b) {–10}
(c) Null set (d) R

(39) Additive identity on N is _____
(a) 0 (b) 1
(c) No identity (d) None of these

(40) Additive inverse of $2+\sqrt{3}$ is _____
(a) $2-\sqrt{3}$ (b) $-2+\sqrt{3}$
(c) $-2-\sqrt{3}$ (d) 0

(41) The multiplicative inverse of $\sqrt{7}$ is _____
(a) $-\frac{1}{\sqrt{7}}$ (b) $\frac{\sqrt{7}}{7} \Rightarrow \frac{1}{\sqrt{7}}$
(c) $\frac{1}{7}$ (d) 1

(42) $0.\overline{3575}$ is _____
(a) Rational (b) Irrational
(c) Integer (d) Non-recurring decimal.

(43) If $|x-3| < 5$, then x lies between _____
(a) -2 and 8 (b) -6 and -2
(c) -2 and 4 (d) -4 and 2

(44) $|x^2| =$ As square of any no. is +ve
(a) x^2 (b) $-x^2$
(c) x (d) Both a and b

(45) If Z is a complex number then the minimum value of $|Z|+|Z-1|$ is _____ $|z| - |z-1|$ $1-1=1$
(a) 1 (b) 2
(c) 0 (d) -2

(46) If $\frac{Z_1}{Z_2}$ is purely imaginary then $\left|\frac{Z_1+Z_2}{Z_1-Z_2}\right| = ?$ $\frac{|z|}{|z|} = 1 \Rightarrow |z| = |z|$
(a) 0 (b) 1
(c) -1 (d) Cannot be determine

(47) $\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^6 =$ _____

(a) ω (b) ω^2
(c) 1 (d) 0

(48) If the square of a number x is "i" then x = ?

(a) $\left(\frac{1+i}{\sqrt{2}}\right)$ (b) $\frac{1-i}{\sqrt{2}}$
(c) $\frac{i}{\sqrt{2}}$ (d) $1+i$

(49) If $Z = x+iy$, then $Z^2 = |Z|^2$ if

(a) x = 0 (b) y = 0
(c) $x \neq 0, y \neq 0$ (d) $x = 0, y \neq 0$

(50) $\arg\left(\frac{Z_1}{Z_2}\right) = ?$

(a) $\arg Z_1 \div \arg Z_2$ (b) $\arg Z_1 + \arg Z_2$
(c) $\arg Z_1 - \arg Z_2$ (d) None of these

(51) The modulus of the complex number $1+i\tan\alpha = ?$

(a) 1 (b) $\tan\alpha$
(c) $\sec\alpha$ (d) $\operatorname{cosec}\alpha$

(52) If $(\cos\theta+i\sin\theta)^2 = x-iy$ then $x^2+y^2 = ?$

(a) 1 (b) 0.
(c) 2 (d) -1

(53) If $Z = 4+6i^2$ then $|Z| = ?$

(a) $\sqrt{52}$ (b) 16
(c) 36 (d) 2

(54) If $Z = \frac{2+7i}{2-8i}$, then $\frac{2-7i}{2+8i} = ?$

(a) Z^{-1} (b) $\bar{Z}$
(c) $\bar{\bar{Z}}$ (d) $-Z$

(55) If $Z = \cos\theta+i\sin\theta$, then $\frac{1}{Z} = ?$

(a) 1 (b) -1
(c) $\cos\theta - i\sin\theta$ (d) None of these

Chapter-2

SETS, FUNCTIONS AND GROUPS

Set*

Collection of well defined distinct objects is called set.

Ways of Describing a Set

There are three ways to describe a set.

Way of description	Definition	Example
1.The Descriptive Method	In this method the set can be described in words.	The set of natural numbers
2.The Tabular Method	In this method the set is described by listing its elements with in brackets.	Set of natural numbers $=\{1,2,3,\cdots\}$
3.Set Builder Method	In this method a symbol or letter is used for an arbitrary member of the set and stating the property common to all the members of the set.	Set of natural numbers $=\{x / x \in N\}$

Order of a Set

The number of elements present in a set is called order of the set.

Types of Sets (w.r.t. the order)

Name of the Set	Definition	Example
1.Finite Set	If a set has definite number of elements present in it.	$\{1,2,3,\cdots,10\}$
2.Infinite Set	If a set has indefinite number of elements present in it.	$N=\{1,2,3,\cdots\}$ $Z=\{0,\pm1,\pm2,\cdots\}$
3.Null Set or Empty Set or Void Set	A set having no elements. It is denoted by ϕ.	$\phi=\{\ \}$
4.Singleton Set	A set having only one element.	$\{7\}$

* The theory of sets is attributed to the German Mathematician George Cantor (1845-1918).

Name of the Set	Definition	Example
Equal sets	Two sets A & B are equal iff they have same elements. Order of the elements of the sets does not matter.	If $A=\{c,a,t\}$ & $B=\{a,c,t\}$ then $A=B$
Equivalent or similar sets	Two sets A & B are equivalent iff a one to one correspondence exists between the elements of the sets. It is denoted by $A\sim B$ or $A\equiv B$	If $A=\{1,2,3,4,5\}$ & $B=\{a,e,i,o,u\}$ then $A\sim B$
Subset	If every element of a set A is an element of a set B ,then A is a subset of B. written as $A\subseteq B$	Let $A=\{1,2,3\}$ & $B=\{1,2,3,\cdots,10\}$ then $A\subseteq B$ If a set has n-elements then it has 2^n subsets. 2^n-1 → proper subsets
Superset	If $A\subseteq B$ then B is called superset of A. it is denoted by $B\supset A$.	In above example $B\supset A$
Proper Subset	If $A\subseteq B$ and B contains at least one element which is not an element of the set A then A is called proper subset of B.	In the above example A is proper subset of B.
Improper subset	Every set itself is called improper subset.	Note: The empty set has no proper subsets.
Power set	The collection of all proper and improper subsets of a set A is called power set of A. denoted by $P(A)$	If $A=\{1,2\}$ then $P(A)=\{\phi,\{1\},\{2\},\{1,2\}\}$ If a set has n-elements then there are 2^nelements in its power set.

Name of the Set	Definition	Example
Universal set or Universe of Discourse	The super set of all the sets under a particular discussion.	It is denoted by U, E or X
Remember that $N \subset Z \subset Q \subset \mathbb{R} \subset \mathbb{C}$		

Operations on Sets

Name of Operation	Definition
1.Union of Two Sets	$A \cup B = \{x / x \in A \vee x \in B\}$
2.Intersection of Two Sets	$A \cap B = \{x / x \in A \wedge x \in B\}$
3.Difference of Two Sets	$A - B = \{x / x \in A \wedge x \notin B\}$

Some Sets Based on the Operations

Name of the Set	Definition
Disjoint Sets	For two sets A & B if $A \cap B = \phi$.
Overlapping Sets	For two sets A & B if $A \cap B \neq \phi$ but neither $A \subseteq B$ nor $B \subseteq A$
Complement of a Set	$A' = A^c = \{x / x \in U \wedge x \notin A\}$ where U is universal set and $A \subseteq U$

Properties and Laws of Sets

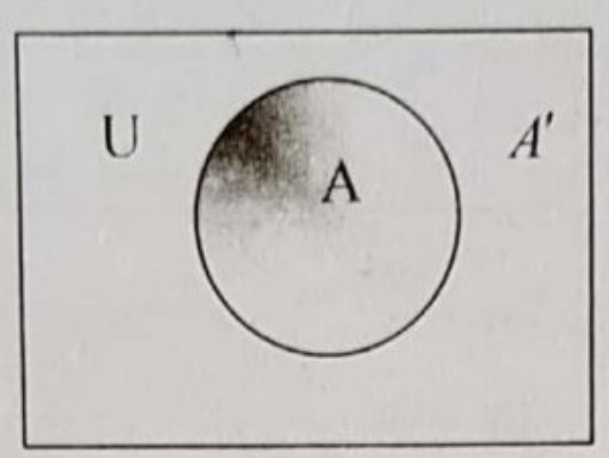
Property/Law	Description
Commutative Property	(i). $A \cup B = B \cup A$ (ii). $A \cap B = B \cap A$ Remember that $A - B \neq B - A$ in general
Associative Property	(i). $A \cup (B \cup C) = (A \cup B) \cup C$ (ii). $A \cap (B \cap C) = (A \cap B) \cap C$
Distributivity of Union over Intersection	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Distributivity of Intersection over Union	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
De Morgan's Laws	(i). $(A \cup B)' = A' \cap B'$ (ii). $(A \cap B)' = A' \cup B'$
The absorption Laws	(i). $A \cap (A \cup B) = A$ (ii). $A \cup (A \cap B) = A$
Idempotent Laws	(i). $A \cup A = A$ (ii). $A \cap A = A$

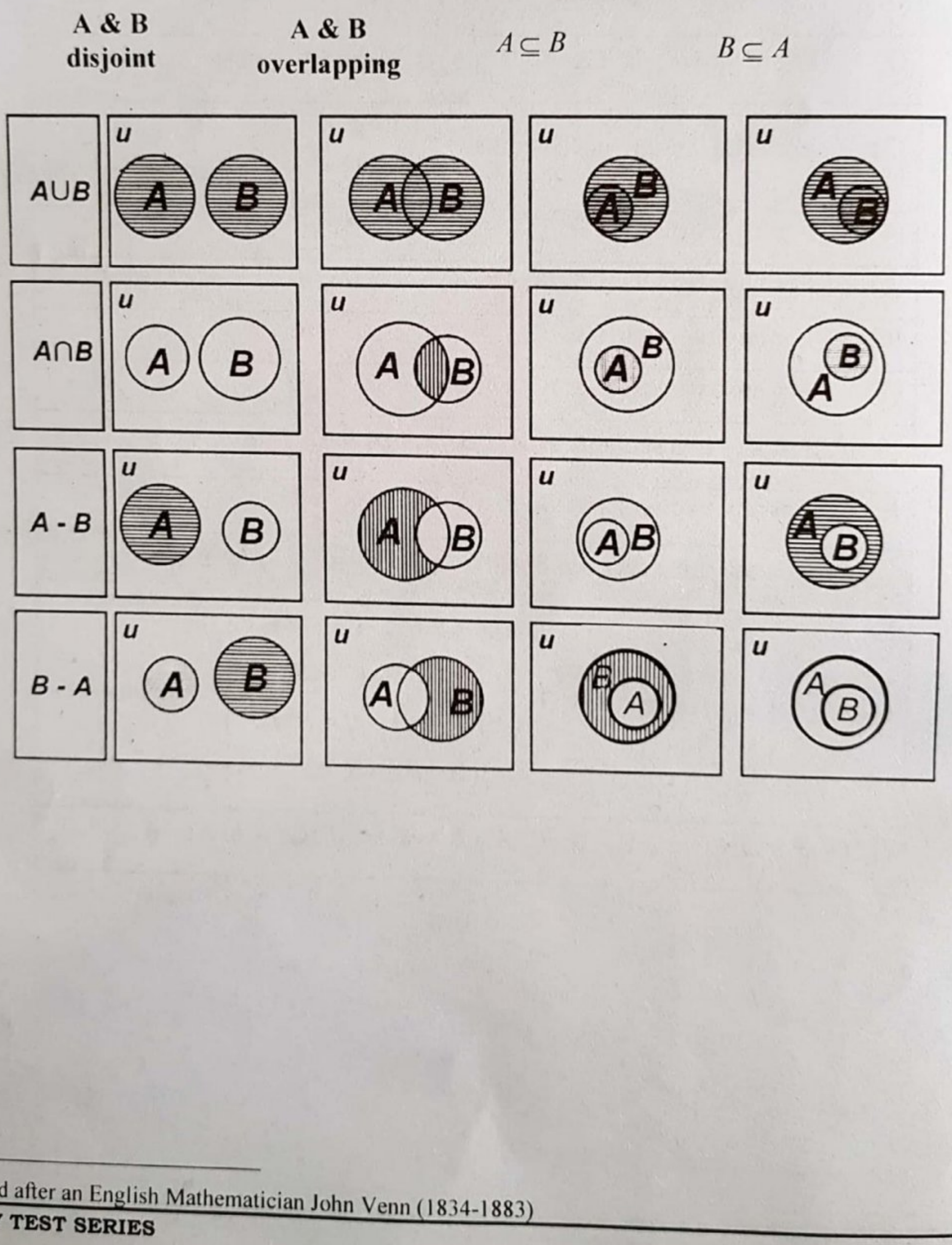
The Complement Laws	(i). $A \cap A' = \phi$ (ii). $A \cup A' = U$
The Double Complement Law	$(A')' = A$
Identity Laws	(i). $A \cup \phi = A$ (ii). $A \cap \phi = \phi$

Theorems

(1)	For any two sets A & B $A \subset (A \cup B)$ and $B \subset (A \cup B)$
(2)	For any two sets A & B $(A \cap B) \subset A$ and $(A \cap B) \subset B$
(3)	Let U be the universal set then $(i) U' = \phi$, $(ii) \phi' = U$
(4)	For any two sets A & B, $A - B = A \cap B'$
(5)	For any two sets A & B, $A \subset B \Rightarrow A \cap B = A$
(6)	For any two sets A & B, $A \subset B \Rightarrow A \cup B = B$
(7)	For any two sets A & B, $A \subset B \Rightarrow B' \subset A'$
(8)	For any two sets A & B, $A \subset B \Rightarrow A \cup (B - A) = B$
(9)	For any two sets A & B, $A \cap B' = A \Leftrightarrow A \cap B = \phi$
(10)	For any three sets A,B & C, $A - (B \cup C) = (A - B) \cap (A - C)$
(11)	For any two sets A & B, $(i) A - B = A - (A \cap B)$, $(ii) (A \cap B) \cup (A - B) = A$, $(iii) (A - B) \cup B = A \cup B$, $(iv) (A - B) \cap B = \phi$
(12)	For any two sets A & B, $A = B \Leftrightarrow (A \cap B') \cup (A' \cap B) = \phi$

Venn Diagrams*





* Named after an English Mathematician John Venn (1834-1883)

Some Use Full Results

Relation Between Set A & B	Result Suggested
1. If $A \cap B = \phi$	(i). $n(A \cup B) = n(A) + n(B)$ (ii). $n(A \cap B) = 0$
2. If $A \cap B \neq \phi$	$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
3. If $A \subseteq B$	(i). $n(A \cup B) = n(B)$ (ii). $n(A \cap B) = n(A)$ (iii). $n(A - B) = 0$ (iv). $n(B - A) = n(B) - n(A)$
4. If $B \subseteq A$	(i). $n(A \cup B) = n(A)$ (ii). $n(A \cap B) = n(B)$ (iii). $n(A - B) = n(A) - n(B)$ (iv). $n(B - A) = 0$
5.If A & B are disjoint.	(i). $n(A - B) = n(A)$ (ii). $n(B - A) = n(B)$
6.If A & B are overlapping.	(i). $n(A - B) = n(A) - n(A \cap B)$ (ii). $n(B - A) = n(B) - n(A \cap B)$

Logic[1]

Introduction	(a) The word logical means correctly reasoned[2]. (b) The rules of logic give precise meaning to mathematical statements.
Inductive Logic	In inductive logic we can draw conclusions from limited number of observations or experiences.
Deductive Logic	In deductive logic the conclusions are drawn from the premises.
Aristotelian Logic	The logic based on the propositions.*i.e.* Only true or false possibility of a statement.
Non-Aristotelian Logic	A logic in which there is more chances other than true or false.

[1] The first person who used logic is Aristotle (384-322 BC).
[2] From *Oxford Illustrated* dictionary.

Proposition	A statement which is either true or false but not both.	*e.g.* Birds fly in the air. The sun rises in the east. $4 < 5$ are propositions. Where as Open the door. Go to Lahore, etc are not propositions.
Truth value	The true of falsity of a proposition is called its truth value.	

❑ The following are the major statements in logic.

1. **Negation** $(\sim or \neg)$:

If **P** is any proposition then negation of P is denoted by $\sim P$ or $\neg P$ and is defined as if **P** is true then $\neg P$ is false and vice versa.

P	$\neg P$
T	F
F	T

The above table is called truth table of negation of **P**.

2. **Conjunction** $(\wedge)$:

The conjunction $p \wedge q$ (read as p and q) is true when both p and q are true otherwise always false.

P	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The above table is the truth table of $p \wedge q$.

3.**Disjunction** $(\vee)$:

The disjunction $p \vee q$ (read as p or q) is false when both p and q are false otherwise always true.

P	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The above table is the truth table of $p \vee q$.

4.**Implication or Conditional** $(\rightarrow)$:

The implication $p \rightarrow q$ (read as p implies q) is false when p is true and q is false otherwise always true.

- **p** is called **antecedent** or **hypothesis** or **premise**.
- **q** is called **consequent** or **conclusion**.

❑ The following statements are equivalent to $p \rightarrow q$.

If p, then q
p implies q
p only if q
q, if p
p is sufficient for q
q is necessary for p

Associated with every implication there are three statements:

(i). The **converse** of $p \rightarrow q$ is $q \rightarrow p$

(ii). The **inverse** of $p \rightarrow q$ is $\neg p \rightarrow \neg q$

(iii). The **contra positive** of $p \rightarrow q$ is $\neg q \rightarrow \neg p$

Remember that:
The truth value of original implication and its contra positive is always equal.

P	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The above table is the truth table of $p \rightarrow q$.

5.**Biconditional or Double Implication** $(\leftrightarrow)$:

The double implication $p \leftrightarrow q$ (read as p iff q) is true when both p and q are true or both are false otherwise always false.

The following statements are equivalent to $p \leftrightarrow q$

p iff q

If p then q and if q then p

p if and only if q

P	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

The above table is the truth table of $p \leftrightarrow q$.

Some Important Terms related to Logic:

Term	Definition
Tautology	A statement which is always true is called tautology.
Absurdity or Contradiction	A statement which is always false, is called absurdity.
Contingency	A statement which can be true or false depending upon the truth values of the variables involved.
Quantifiers	The words or the symbols which give the idea of a quantity are called quantifiers.
Universal Quantifier	$\forall$ (For all) is called universal quantifier.
Existential Quantifier	$\exists$ (There exist) is called existential quantifier.

Logical Equivalences:

Compound propositions that have same truth values are called logically equivalent.

$\Leftrightarrow$ Stands for logically equivalent.

Some useful equivalences
$p \to q \Leftrightarrow (\neg p \vee q)$
$p \to q \Leftrightarrow (\neg q \to \neg p)$

A Link between Set Theory and Logic:

Let $p \equiv P$ and $q \equiv Q$

Logical Statement	Equivalence in Set Theory
$\neg p$	Compliment of P
$p \wedge q$	$P \cap Q$
$p \vee q$	$P \cup Q$
$p \rightarrow q$	$P' \cup Q$
$p \leftrightarrow q$	$P = Q$
(i). $\neg(p \wedge q)$ (ii). $\neg(p \vee q)$	$(i) P' \cup Q'$ $(ii) P' \cap Q'$ De Morgan's laws

Relations

Name	Definition	Example /Remark
Cartesian Product	The Cartesian product of any two sets A & B is defined as $A \times B = \{(x, y) / x \in A \wedge y \in B\}$	Let $A = \{a, b\}$ & $B = \{1, 2\}$ then $A \times B = \{(a,1), (a,2), (b,1), (b,2)\}$
Binary Relation/ Relation	Any subset of the Cartesian product $A \times B$ is called a relation from A to B.	$r_1 = \{(a,1), (b,2)\}$
Domain of a Relation	The set of the first elements of the ordered pairs forming a relation.	Dom of $r_1 = \{a, b\}$
Range of a Relation	The set of the second elements of the ordered pairs forming a relation.	Range of $r_1 = \{1, 2\}$
Inverse relation	If we interchange the domain and the range of every ordered pair of a relation then the new relation formed is the inverse of the given.	$r_1^{-1} = \{(1, a), (2, b)\}$

Chapter-2

Functions

Name	Definition	Example/Remark
Function	Let A & B are two non-empty sets then f is a function from A to B if, (i). $f \subseteq A \times B$ (ii). $Domf = A$ (iii). First elements of no two pairs of f are equal.	In the above example r_1 is a function. ▪ A function f from A to B is denoted by $f : A \to B$
Into function	A function $f : A \to B$ is called into if $ranf \subseteq B$ but $ranf \neq B$	
Onto function (surjective)	A function $f : A \to B$ is called onto if $ranf = B$.	
One-one function	A function $f : A \to B$ is called one-one if second elements of no two of its ordered pairs are equal.	
Injective function	A function which is both 1-1 and into is called injective.	
Bijective function	A function which is both 1-1 and onto is called bijective.	
Linear function	A function of the form $\{(x, y) / y = mx + c\}$ is called linear.	The graph of a linear function is always a straight line.
Quadratic function	The function of the form $\{(x, y) / y = ax^2 + bx + c\}$ is called quadratic.	The graph of a quadratic function is always a parabola.
Inverse function	The inverse of a function f is denoted by f^{-1} and can be obtained by interchanging the elements of the ordered pairs involved in the function.	▪ Inverse of a function may or may not be a function. ▪ Inverse of a line is line.

Properties of binary* operations

Let S be a non-empty set and * is a defined binary operation for S.

Name of the Property	Definition
Closure Property	A non-empty set S is called closed w.r.t. * if $\forall a,b \in S \Rightarrow a*b \in S$
Commutative Property	$\forall a,b \in S \Rightarrow a*b = b*a$
Associative Property	$\forall a,b,c \in S \Rightarrow a*(b*c) = (a*b)*c$
Identity Element	An element $e \in S$ is called identity w.r.t. * if $a*e = e*a = a$
Inverse Property	For any element $a \in S$, $\exists a' \in S$ such that $a*a' = a'*a = e$, then a and a' are called inverse of each other w.r.t. *.

Residue Classes Of Modulo n:

Let $n \in N$, then 0, 1, 2,……,n – 1 are called residue classes of modulo n.

e.g. $\{\bar{0},\bar{1},\bar{2},\bar{3}\}$ are the residue classes of modulo 4.

- The following table* represents the multiplication of residue classes of 5.

$\otimes$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

- The following table represents the addition of the residue classes of 7.

$\oplus$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

- Following is the table of multiplication of cube roots of unity.

$\otimes$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

* In chapter 1 we have discussed about binary and unary operations.
* Table is called Cayley's table named for the British mathematician Arthur Cayley(1821-1895)

❑ The following table describe that which properties are satisfied by the specified set of numbers.

Set of numbers /Property		Natural ($\mathbb{N}$)	Whole (W)	Integers ($\mathbb{Z}$)	Rational ($\mathbb{Q}$)	Real ($\mathbb{R}$)
Closure	$\oplus$	✓	✓	✓	✓	✓
	$\otimes$	✓	✓	✓	✓	✓
Associative	$\oplus$	✓	✓	✓	✓	✓
	$\otimes$	✓	✓	✓	✓	✓
Identity	$\oplus$	×	✓	✓	✓	✓
	$\otimes$	✓	✓	✓	✓	✓
Inverse	$\oplus$	×	×	✓	✓	✓
	$\otimes$	×	×	×	×	×
Commutative	$\oplus$	✓	✓	✓	✓	✓
	$\otimes$	✓	✓	✓	✓	✓

Groups

Let S be a non-empty set and * is a defined binary operation for S.

Name	Definition	Examples
Groupoid	If S is closed w.r.t. *.	$(\mathbb{N},+),(\mathbb{Z},+)$ $,(\mathbb{R},+),(\mathbb{Q},+)$
Semi Group	(i).S is closed w.r.t. *. (ii).* is an associative binary operation.	$(\mathbb{N},+),(\mathbb{Z},+),(\mathbb{R},+)$ $(\mathbb{Q},+)$
Monoid	(i).S is closed w.r.t. *. (ii).* is an associative binary operation. (iii).S contains identity w.r.t. *.	$(\mathbb{Z},+),(\mathbb{R},+)$ $(\mathbb{Q},+),(\mathbb{Z},\otimes),(\mathbb{R},\otimes)$ $(\mathbb{Q},\otimes)$
Group	(i).S is closed w.r.t. *. (ii).* is an associative binary operation. (iii).S contains identity w.r.t. *. (iv).Inverse of each element of S w.r.t. * must belongs to S.	$(\mathbb{Z},+),(\mathbb{R},+)$ $(\mathbb{Q},+)$ $(M_{2\cdot2},+)$.
Abelian Group*	A group (S,*) is called Abelian if * is commutative binary operation.	$(M_{2\cdot2},+),(\mathbb{Z},+),(\mathbb{R},+)$ $(\mathbb{Q},+),(\mathbb{R}-\{0\},\otimes)$

* Named after a Norwegian mathematician N.H. Abal (1802-29).

Some Concept Related To Groups

Order of a group	The number of elements of a group is called order of the group.
Finite & infinite groups	A group is called finite if its order is finite otherwise it is called infinite group.
Uniqueness of identity	In a group the identity element is unique.
Uniqueness of inverse	In a group inverse of each element w.r.t. * is unique.
Reversal law of inverses	$\forall a,b \in S$ where S is a group then $(ab)^{-1} = b^{-1}a^{-1}$
Idempotent element	Let S be a group & $a \in S$ then a is called idempotent if $a * a = a$.

PRACTICE EXERCISE

(1) Which one is not a set
(a) A = {1,2,3}
(b) A={3,4,7}
(c) A={3,4,5,4}
(d) B={9,7}

(2) An element 'b' of a set B can be written as
(a) $b \subseteq B$
(b) $b < B$
(c) $b \in B$
(d) $B \in b$

(3) The union of all even integers and odd integers is
(a) Z, the set of all integers
(b) N, the set of natural numbers
(c) Z^+, positive whole numbers
(d) $\overline{Z}$, -ve whole numbers

(4) If A = {1, 3, 4}, B = {c, d, f} then A∩B=?
(a) {0}
(b) $\{\phi\}$
(c) ϕ
(d) {1, 3, 4, c, d, f}

(5) Two sets X and Y are disjoint if
(a) X =Y
(b) $X \bigcup Y = Z$ (where Z is universal set)
(c) X∩Y=ϕ
(d) X≠Y

(6) If A = {4,3}. Then P(A) is
(a) $\{\phi,\{4\},\{3\},\{4,3\}\}$
(b) {{3},{4},{4,3}}
(c) $\{\phi,\{3\},\{4\}\}$
(d) $\{\phi,\{3,4\}\}$

(7) For any three sets X, Y and Z, $X \cap (Y \cup Z)$=?
(a) $(X \cup Y) \cap (X \cap Z)$
(b) $(X \cap Y) \cup (X \cap Z)$
(c) $X \cap (Y \cap Z)$
(d) $(X \cup Y) \cap (X \cup Z)$

(8) For any two sets X and Y, $X \cup (X \cap Y)$=?
(a) X
(b) Y
(c) ϕ
(d) $X \cap Y$

(9) For any subset B of X, $B \cup B$=?
(a) ϕ
(b) X
(c) $B \bigcup X$
(d) $B \cap B$

(10) $A \cup A' = ?$
(a) $A \cup \phi$
(b) $A \cap X$
(c) $A \cup (A - X)$
(d) $A \cup (X - A)$

(11) A/X=?
(a) A'
(b) A
(c) ϕ
(d) X∪A

(12) Which binary operation is not defined in the set of natural numbers?
(a) +
(b) –
(c) ×
(d) none

(13) To each element of a group, there correspond how many inverse elements?
(a) Only one
(b) at least one
(c) more than one
(d) two

(14) The identity element of N, w-r-t "+" is
(a) 1
(b) 0
(c) 2
(d) none

(15) If $A \subseteq B$ then which of the following must be true?
(a) $A-B=0$
(b) $A-B=\phi$
(c) $B-A=0$
(d) $B-A=\phi$

(16) $n(A \cup B)=n(A)+n(B)$ only if
(a) $A \cup B=\phi$
(b) $A \cup B=A \cap B$
(c) $A \cap B=\phi$
(d) $A=B$

(17) The truth value of the proposition 3 is a +ve number or 2+2=7 is
(a) True
(b) false
(c) contingency
(d) none

(18) The truth value of the statement $\left[(p \to q) \wedge p\right] \to q$ is false, when
(a) p is true, q is false
(b) p is true, q is true
(c) p is false, q is true
(d) none of these

(19) The inverse of A→B is
(a) B→A
(b) ~A→~B
(c) ~B→~A
(d) A→~B

(20) The contra positive of ~B→~A is
(a) A→B
(b) B→A
(c) A→~B
(d) ~A→B

(21) ~B→~A is converse of
(a) A→B
(b) B→A
(c) ~A→B
(d) ~A→~B

(22) The inverse of the function {(1,3),(2,1),(4,5)} is
(a) {(3,1),(2,1),(4,5)}
(b) {(1,3),(2,1),(5,4)}
(c) {(3,1),(1,2),(5,4)}
(d) {(3,1),(2,1),(5,4)}

(23) Which of the following statement cannot be true?
(I) inverse of a function is always function
(II) inverse of a line is line
(III) inverse of injective function is function
(a) I only
(b) II only
(c) I and II only
(d) I and III only

(24) The set $\{1,-1,i,-i\}$ is group w.r.t
(a) +
(b) ×
(c) ÷
(d) -

(25) A semi group is monoid if
(a) It has identity
(b) it is closed
(c) it holds associative property
(d) it has inverse property

(26) Which of the following is not monoid w.r.t addition?
(a) Z
(b) W
(c) N
(d) R

(27) The order of the group (A, +) is _____ if A= {0, +1, +2, +3 …}
(a) 4
(b) 3
(c) 8
(d) none

(28) Which of the following is true?
(a) $\{1,2\}=\{2,1\}$
(b) $a \in \{\{a\}\}$
(c) $a \subseteq \{a\}$
(d) $\{a\} \subseteq \{\{a\}\}$

(29) Let S be a non empty set. Then what is the identity element of $(p(S), \cap)$?

(a) $P(S)$ (b) empty set
(c) set S itself (d) no identity

(30) $(A-B)\cup B = ?$

(a) A (b) B
(c) $A-B$ (d) $A\cup B$

(31) Let $U=\{1,2,3,\cdots,10\}, A=\{1,2,5\}, B=\{6,7\}$ then $A\cap B' = ?$

(a) B' (b) B
(c) A (d) A'

(32) If A & B are any two sets then which of the following is always true?

(a) $A\cup B\subseteq A\cap B$ (b) $A\cap B\subset A\cup B$
(c) $A\cup B = A\cap B$ (d) all of these

(33) Let $A=\{1,2,3,4,5\}, B=\{1,2\}$ then $A\setminus(A\setminus B) = ?$

(a) ϕ (b) A
(c) B (d) $A\cup B$

(34) If A & B are any two sets then $A\cap(B\setminus A) = ?$

(a) ϕ (b) A
(c) B (d) $A\cup B$

(35) If A & B are any two sets then $A\cup B = A\cap B$ iff

(a) $A\subset B$ (b) $B\subset A$
(c) $A=B$ (d) All of these

(36) If $A=\{a,b,c\}, B=\{2,6,10\}$ then $n(A\times B) = ?$

(a) 6 (b) 9
(c) 12 (d) 18

(37) $A\cap(A\cup B)' = ?$

(a) A (b) B
(c) ϕ (d) $A\cap A'$

(38) Which of the following is smallest set A such that $A\cup\{1,2\}=\{1,2,3,5,9\}$

(a) {2,3,5} (b) {3,5,9}
(c) {1,2,5,9} (d) {1,2,3,5,9}

(39) If $n(A)=3, n(B)=6$ and $A\subset B$ then $n(A\cup B) = ?$

(a) 3 (b) 9
(c) 6 (d) 8

(40) In a group of people 60% like chocolate and 70% like strawberry then percentage of people who like both chocolate and strawberry is:

(a) 30% (b) 60%
(c) 10% (d) 40%

(41) B – A is a subset of _____

(a) A (b) B
(c) $A-B$ (d) $A\cap B$

(42) If $x\in A\cup B$, then

(a) $x\in A\wedge x\in B$ (b) $x\in A$ *but* $x\notin B$
(c) $x\notin A$ *but* $x\in B$ (d) $x\in A\vee x\in B$

(43) $n(A) = 20, n(B) = 16, n(A \cap B) = 10$ then $n(A \cup B) = ?$
(a) 16 (b) 26
(c) 36 (d) 6

(44) $A - B = B - A$, if and only if _____
(a) $A \subseteq B$ (b) $B \subseteq A$
(c) A = B (d) $A = \varphi$

(45) $B - A$ _____ A'
(a) = . (b) $\supset$
(c) $\subseteq$ (d) None of these

(46) If $A = \{1,2\}, B = \{3,4\}$ then $1 \notin$ _____
(a) A (b) B
(c) $A \cup B$ (d) $A - B$

(47) $A \times B = B \times A$ iff _____
(a) $A \subset B$ (b) $B \subset A$
(c) A = B (d) None of these

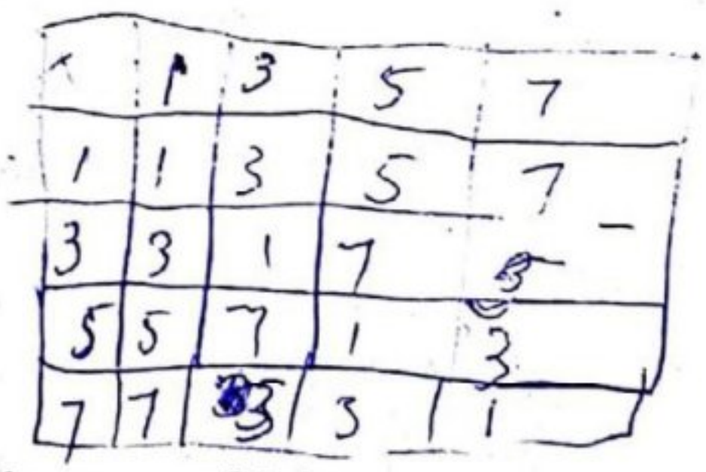
(48) If a set A has 3 elements and B has 6 elements such that $A \subseteq B$ then the number of elements in $A \cap B = ?$
(a) 3 (b) 9
(c) 6 (d) 12

(49) Which of the following operation is NOT commutative?
(a) x*y = 2xy (b) x * y = x + y
(c) x * y = 2x (d) x * y = 2x + 2y

(50) If x * y = 3xy then the identity element w.r.t * is _____
(a) 1 (b) $\frac{1}{3}$
(c) $\frac{1}{9}$ (d) None of these

(51) If x * y = 5xy then inverse of 2 w.r.t * is _____
(a) $\frac{1}{2}$ (b) $\frac{1}{10}$
(c) $\frac{1}{50}$ (d) $\frac{1}{500}$

(52) Which of the following is NOT a group w.r.t multiplication?
(a) Z (b) R
(c) Q (d) All of these



(53) If $G = \{1,3,5,7\}$ is group under multiplication modulo 8 then inverse of 5 is _____
(a) $\frac{1}{5}$ (b) 1
(c) 5 (d) None of these

(54) Which of the following is square root function?
(a) $y = \sqrt{x}$ (b) $y^2 = x$
(c) $y = \sqrt{x}, x \geq 0$ (d) $y^2 = x, x \in R$

(55) Truth set of $P \wedge q$ is _____
(a) $P \cup Q$ (b) $P \cap Q$
(c) $P' \cap Q$ (d) $P \cup Q'$

MATRICES AND DETERMINANTS

Introduction

Name of mathematician	Work in matrices
James Sylvester (1814-1897)	Used the word matrix first time
Arthur Cayley (1821-1895)	Used in linear transformations
Seki Kowa (1642-1708)	Concept of determinants
Gottfried Wilhelm Leibniz (1646-1716)	Invention of determinants
Gabriel Crammer (1704-1752)	Used determinants for solving system of linear equations

Matrix

- A rectangular array of numbers enclosed by a pair of brackets is called a matrix.

OR

- Arrangement of numbers in rows and columns is called a matrix.

Rows

- The horizontal lines of numbers are called rows of a matrix.

Columns

- The vertical lines of numbers are called columns of a matrix

Entries or Elements

- The numbers used in rows or columns are said to be the entries or elements of the matrix.

Order of a matrix

- Order of a matrix is defined as:
  Number of rows × **Number of columns** (read as number of rows by number of columns)

General definition of a matrix

- Generally a bracketed rectangular array of m × n elements

a_{ij} ($i = 1, 2, 3, \ldots m, j = 1, 2, 3, 4 \ldots n$), arranged in m rows and n columns is called an m by n matrix (written as m × n matrix).

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Types of matrices

Name of matrix	Definition	Example
Row matrix (row vector)	A matrix, which has only one row (*i.e.* 1 × n matrix) is said to be a row matrix or row vector.	$[a_{11} \quad a_{12} \quad a_{13} \quad \cdots \quad a_{1n}]_{1\times n}$ $[1\ 2\ 0\ 5]_{1\times 4}$
Column matrix (column vector)	A matrix, which has only one column (*i.e.* an m × 1 matrix) is said to be a column matrix or column vector.	$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{m1} \end{bmatrix}_{m\times 1}, \quad \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}_{4\times 1}$
Square matrix	The matrix, which has the same number of rows and columns, is called a square matrix. *OR* If m = n then the matrix of order m × n is said to be a square matrix of order n or m.	$[1]_{1\times 1}$ $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}_{2\times 2}$ $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}_{3\times 3}$
Rectangular matrix	A matrix, whose number of rows is not equal to number of columns, is called a rectangular matrix. *OR* If m ≠ n, then the matrix is called rectangular matrix of order m×n.	$\begin{bmatrix} 2 & -1 & 7 \\ 9 & 5 & -2 \end{bmatrix}_{2\times 3}$ $[a_{11} \quad a_{12}]_{1\times 2}$

Diagonal matrix	Let A = $\left[a_{ij}\right]_{n\times n}$ be a square matrix of order n. If $a_{ij}=0$ for all $i\neq j$ and at least one $a_{ij}\neq 0$ for $i=j$ Where $a_{11}, a_{22}, a_{33}\ldots\ldots\ldots, a_{nn}$ is called main, leading or principal diagonal.	$\begin{pmatrix}2&0&0\\0&0&0\\0&0&0\end{pmatrix}, \begin{bmatrix}1&0&0\\0&9&0\\0&0&7\end{bmatrix}$ $\begin{bmatrix}-2&0&0\\0&7&0\\0&0&0\end{bmatrix}$
Scalar matrix	Let A = $\left[a_{ij}\right]_{n\times n}$ be a square matrix of order n. If $a_{ij}=0$ for all $i\neq j$ and $a_{ij}=k$ for $i=j$, where k is any non zero scalar.	$\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix}$
Unit matrix *or* Identity matrix	Let A = $\left[a_{ij}\right]_{n\times n}$ be a square matrix of order n. If $a_{ij}=0$ for all $i\neq j$ and $a_{ij}=1$ for $i=j$. Unit matrix is both diagonal & scalar.	$I_2=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $I_3=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$
Null matrix *or* Zero matrix	A square or rectangular matrix whose each element is zero is called a null or zero matrix.	$\begin{pmatrix}0&0\\0&0\end{pmatrix}, \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$ $\begin{bmatrix}0&0&0\end{bmatrix}$
Equal matrices	Two matrices of the same order are said to be equal if their corresponding entries are equal.	$\begin{pmatrix}1&7&2\\3&4&8\end{pmatrix}=\begin{pmatrix}2-1&4+3&2\\2+1&2+2&8\end{pmatrix}$
Sub matrix	Matrix obtained by deleting some rows or columns from an existing matrix are said to be sub-matrix of the original matrix.	

Classification of matrices w.r.t. operations

Matrix operation/Name	Definition	Example
Transpose of a matrix	If A is a matrix of order $m \times n$ then an $n \times m$ matrix obtained by interchanging the rows and columns of A, is called the transpose of A. Denoted by A^t or A'.	$A=\begin{bmatrix}2 & 7\\ 1 & 5\\ 5 & 4\end{bmatrix} \Rightarrow A'=\begin{bmatrix}2 & 1 & 5\\ 7 & 5 & 4\end{bmatrix}$ ▪ $\left(A'\right)'=A$ ▪ $(AB)'=B'A'$
Symmetric matrix	For a square matrix A if $A^t=A$, then A is called symmetric matrix	$\begin{pmatrix}a & s & d\\ s & g & b\\ d & b & t\end{pmatrix}, \begin{pmatrix}1 & 3 & 5\\ 3 & 9 & 6\\ 5 & 6 & 8\end{pmatrix}$
Skew symmetric or anti symmetric	For a square matrix A if $A^t=-A$, then A is called skew symmetric matrix	$\begin{pmatrix}0 & 1 & -3\\ -1 & 0 & -7\\ 3 & 7 & 0\end{pmatrix}$
Hermitian matrix	A square matrix A with complex entries is called hermitian if $\left(\overline{A}\right)'=A$.	$\begin{pmatrix}1 & 1+i\\ 1-i & 4\end{pmatrix}$
Skew hermitian matrix	A square matrix A with complex entries is called skew hermitian if $\left(\overline{A}\right)'=-A$	$\begin{pmatrix}0 & 2+5i\\ -2+5i & 0\end{pmatrix}$
Upper triangular matrix	A square matrix $A=\left[a_{ij}\right]_{m\times n}$ is called an upper triangular if $a_{ij}=0 \ \forall \ i>j$	$U=\begin{pmatrix}1 & 7 & 2\\ 0 & -9 & 7\\ 0 & 0 & 0\end{pmatrix}$
Lower triangular matrix	A square matrix $A=\left[a_{ij}\right]_{n\times n}$ is said to be a lower triangular matrix if $a_{ij}=0 \ \forall \ i<j$	$L=\begin{pmatrix}2 & 0 & 0\\ 2 & -3 & 0\\ -1 & 6 & 9\end{pmatrix}$
Triangular matrix	A square matrix A is named as triangular whether it is upper triangular or lower triangular.	In above examples U & L are triangular matrices.

Matrix operations & their properties

Operation	Definition	Properties
Addition of matrices	Two matrices are conformable for addition if they are of the same order	1.A+B=B+A(commutative property) 2.A+(B+C)=(A+B)+C(associative property) 3.A+O=O+A=A(O is additive identity)
negative of a matrix	If A is a given matrix, then the matrix obtained by writing the negative of each entry in place of the each entry of A is called the negative of A.	
Scalar multiplication	If $A = [a_{ij}]$ is an $m \times n$ matrix and k is a scalar, then the product of k and A denoted by kA is the matrix formed by multiplying each entry of A by k, that is: $kA = [ka_{ij}]$	$(a)c(A+B) = cA + cB$ $(b)(c+d)A = cA + dA$ $(c)c(AB) = (cA)B = A(cB)$ } (Distributive properties w.r.t. scalar multiplication)
Subtraction of matrices	If $A = [a_{ij}]$ and $B = [b_{ij}]$ are matrices of order $m \times n$ then we define subtraction of B from A as: $A - B = A + (-B)$	
Multiplication of two matrices	Two matrices A and B are said to be conformable for the product AB if the number of columns of A is equal to the number of rows of B.	1. $A(BC) = (AB)C$ (Associative property) 2. $AB \neq BA$ (not commutative in general) 3. $IA = AI = A$ (I is multiplicative identity) 4.A(B+C)=AB+AC (distributative property of multiplication w.r.t. +) 5. $A_{m\times n}B_{n\times p} = (AB)_{m\times p}$
Trace of a matrix	For a square matrix of order $n \times n$,the trace of the matrix $a_{11} + a_{22} + a_{33} + \cdots + a_{nn}$	The trace of $\begin{pmatrix} 1 & 9 & 2 \\ 3 & 8 & 5 \\ 4 & 6 & 0 \end{pmatrix}$ is 9.

Elementary row and column operations on a matrix

Usually a given system of linear equations is reduced to a simple equivalent system by applying in turn a finite number of elementary operations, which are stated as below:

1. Interchanging two equations.
2. Multiplying an equation by a non-zero number.
3. Adding a multiple of one equation to another equation.

a). **Elementary row operation on a matrix**

The following elementary row operations are applied to matrices to obtain equivalent matrix.

i). Interchanging two rows. $R_i \leftrightarrow R_j$

ii). Multiplying a row by a non-zero number. $kR_i \leftrightarrow R_i'$

iii) Adding a multiple of one row to another row. $R_i + kR_j = R_i'$

b). **Elementary column operation on a matrix**

Following are the elementary column operations that are used to obtain equivalent matrices.

i). Interchanging two columns. $C_i \leftrightarrow C_i'$

ii). Multiplying a column by a non-zero number. $kC_i \leftrightarrow C_i'$

iii) Adding a multiple of one column to another column. $C_i + kC_j = C_i'$

Echelon and Reduced Echelon Forms of Matrices:

In any non-zero row of a matrix the first non-zero entry is called the leading entry of that row. The zeros before the leading entry of a row are named as the leading zero entries of the row.

(a) Echelon Form of a Matrix

An m × n matrix A is called in (row) echelon form if:

i). In each successive non-zero row, the number of zeros before the leading entry greater than the number of such zeros in the preceding row.

ii). Every non-zero row in A precedes every zero row (if any).

iii). The first non zero entry (or leading entry) in each row is 1.

(b) Reduced echelon form of a matrix

An m × n matrix A is said to be in reduced (row) echelon form if it is in (row) echelon form and if the first non-zero entry (or leading entry) in R_i lies in C_j then all other entries of C_j are zero.

Rank of a matrix

Let A be a non-zero matrix. If 'r' is the number of non-zero rows when it is reduced to reduced echelon form, then 'r' is called the (row) rank of the matrix A.

Determinants

1. **The determinant of a 1 × 1 matrix** A=[a_{11}] is defined as $|A| = |a_{11}| = a_{11}$

2. **Determinant of a 2 × 2 matrix:**

The determinant of a matrix is denoted by enclosing its square array between vertical bars instead of brackets. The number of elements in any row or column is called the order of determinant.

- If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $|A| = ad - bc$
- If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $adjA = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

3. Determinant of a square matrix of order n ≥ 3

Before defining the determinant of a square matrix of order ≥ 3 we define some important terms.

- **Minor of an element of a square matrix:**

  Given an n×n matrix $A=\left(a_{ij}\right)_{n\times n}$ if we delete ith row and jth column of A, we get an (n−1)×(n−1) matrix. The determinant of this matrix is called the minor of the element a_{ij} and is denoted by M_{ij}.

- **Cofactor of an element of a square matrix:**

  The cofactor of an element a_{ij} of a square matrix A is denoted by A_{ij} and is defined as

  $$A_{ij}=(-1)^{i+j}M_{ij}$$

- The **determinant of a square matrix of order** 'n' is the sum of the products of each element of row (or column) and its cofactor. *i.e.*

$$\left.\begin{aligned}|A| &= a_{11}A_{11}+a_{12}A_{12}+.....+a_{1n}A_{1n}\\ &= a_{21}A_{21}+a_{22}A_{22}+.....+a_{2n}A_{2n}\\ &= a_{31}A_{31}+a_{32}A_{32}+.....+a_{3n}A_{3n}\end{aligned}\right\}$$

Adjoint of a matrix of order n ≥ 3

If A is a square matrix, then the matrix obtained by taking the cofactor of each element of A, replacing them correspondingly and then taking transpose of that, is called adjoint of A.

Invèrse of a Matrix

- $A^{-1}=\dfrac{1}{|A|}adjA$ if $|A|\neq 0$
- **Singular matrix**

  A square matrix A is said to be singular if $|A|=0$

- **Non singular matrix**

  A square matrix is said to be non-singular if $|A|\neq 0$

Properties of determinants

1. If the rows and columns of a determinant are interchanged the value of the determinant does not change.
2. If two rows (or columns) of a determinant are interchanged, the value of determinant is multiply by –1.
3. If all the entries in any row (or column) are zero, the value of the determinant is zero.
4. If any two rows (or columns) of a determinant are identical, the value of the determinant is zero.
5. If any row (or column) of a determinant is multiplied by a non-zero number k, the value of the new determinant becomes equal to k times the value of original determinant.
6. If any row (or column) of a determinant consists of two terms, it can be written as the sum of two determinants.
7. If any row (or column) of a determinant is multiplied by a non-zero number k and the result is added to the corresponding entries of another row (or column), the value of the determinant does not change.

8. $|A^t| = |A|$

9. The determinant of a triangular matrix (upper, lower, or diagonal) is equal to the product of the elements of the main diagonal.

System of Linear Equations

Non-homogeneous system	Homogeneous system
▪ An equation of the form $ax+by+cz=d$ $a\neq 0, b\neq 0, c\neq 0, d\neq 0$ is called non-homogeneous equation in three variables.	▪ An equation of the form $ax+by+cz=0$ is called homogeneous equation in three variables.
▪ $\left.\begin{aligned} a_1x+b_1y+c_1z&=d_1\\ a_2x+b_2y+c_2z&=d_2\\ a_3x+b_3y+c_3z&=d_3 \end{aligned}\right\}$ is called a system of non-homogeneous equations in three variables where d_1, d_2 and d_3 cannot zero simultaneously.	▪ $\left.\begin{aligned} a_1x+b_1y+c_1z&=0\\ a_2x+b_2y+c_2z&=0\\ a_3x+b_3y+c_3z&=0 \end{aligned}\right\}$ is called system of homogeneous equations in three variables.
▪ The matrix form of the system is $\begin{pmatrix} a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3 \end{pmatrix}\begin{pmatrix} x\\ y\\ z \end{pmatrix}=\begin{pmatrix} d_1\\ d_2\\ d_3 \end{pmatrix}$ can be written as $AX=D$ where $A=\begin{pmatrix} a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3 \end{pmatrix}$, $X=\begin{pmatrix} x\\ y\\ z \end{pmatrix}$, $D=\begin{pmatrix} d_1\\ d_2\\ d_3 \end{pmatrix}$	▪ The matrix form of the system is $\begin{pmatrix} a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3 \end{pmatrix}\begin{pmatrix} x\\ y\\ z \end{pmatrix}=\begin{pmatrix} 0\\ 0\\ 0 \end{pmatrix}$ can be written as $AX=O$ where $A=\begin{pmatrix} a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3 \end{pmatrix}$, $X=\begin{pmatrix} x\\ y\\ z \end{pmatrix}, O=\begin{pmatrix} 0\\ 0\\ 0 \end{pmatrix}$.

Solutions of systems of linear equations

Non-homogeneous system	Homogeneous system
▪ If $\|A\| \neq 0$ then the solution of the system exits and is unique so the system is called **consistent**. ▪ If $\|A\| = 0$ then the solution of the system does not exists and the system is called **inconsistent**. ▪ The system can be solved by using any one of the following methods. (i). Using matrices $AX=D$ (ii). Using crammer's rule. (iii). Using echelon & reduced echelon forms.	▪ This system has two types of solutions. (i). **Trivial solution**: $x=0, y=0, z=0$ this solution of the system always exists. (ii). **Non-trivial solutions**: Any solution other than $x=0, y=0, z=0$ is called non trivial solution. The non trivial solution exists only if $\|A\| = 0$ or rank of A<3. The non trivial solutions are infinite and in the case if non trivial solutions exist the system is called **consistent** ▪ The system can be solved by using the echelon and reduced echelon forms.

Crammer's Rule

Consider the system of equations,

$$\left.\begin{aligned} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \\ a_3x+b_3y+c_3z=d_3 \end{aligned}\right\}$$ By using crammer's rule the solution of the system is defined as,

$$x=\frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{|A|}, y=\frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{|A|}, z=\frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{|A|}; |A| \neq 0$$

PRACTICE EXERCISE

(1) $\begin{vmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{vmatrix} = ?$

(a) b
(b) b^2
(c) b^3
(d) 0

(2) If C and D are two matrices of order pxq & qxp respectively then C - D is a matrix of order…..?.

(a) pxq
(b) qxp
(c) pxp
(d) qxq
(e) none

(3) If $\begin{pmatrix} 5 & 0 & 9 \\ 8 & \lambda & -8 \\ 2 & 2 & 4 \end{pmatrix}$,is a singular matrix then λ=?

(a) 112
(b) –111
(c) 111
(d) 11
(e) -112

(4) If $X = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & -1 & 1 \end{pmatrix}$,then $X^{-1} = ?$

(a) $\begin{pmatrix} -3 & 2 & 4 \\ -1 & 1 & 1 \\ 2 & -1 & -2 \end{pmatrix}$
(b) $\begin{pmatrix} -3 & 2 & 4 \\ -1 & 1 & 0 \\ 2 & -1 & -2 \end{pmatrix}$
(c) $\begin{pmatrix} 3 & 2 & 4 \\ -1 & 1 & 0 \\ 2 & -1 & -2 \end{pmatrix}$
(d) $\begin{pmatrix} 3 & 2 & 4 \\ 1 & 1 & 0 \\ 2 & -1 & -2 \end{pmatrix}$

(5) If $B = \begin{pmatrix} -1 & 6 \\ 0 & 3 \end{pmatrix}$,then ad joint of B =?

(a) $\begin{pmatrix} 1 & 0 \\ 6 & -3 \end{pmatrix}$
(b) $\begin{pmatrix} 3 & -6 \\ 0 & -1 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 6 \\ 0 & 3 \end{pmatrix}$
(d) $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$

(6) If $B=\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$, then adjoint of B =?

(a) $\begin{pmatrix}0&0&-1\\0&1&0\\-1&0&0\end{pmatrix}$ (b) $\begin{pmatrix}0&0&1\\0&-1&0\\-1&0&0\end{pmatrix}$

(c) $\begin{pmatrix}1&0&1\\0&-1&0\\-1&0&0\end{pmatrix}$ (d) $\begin{pmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix}$

(7) Which of the following is a diagonal matrix?

(I) $\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ (II) $\begin{pmatrix}3&0&0\\0&9&0\\0&0&0\end{pmatrix}$

(III) $\begin{pmatrix}4&0&0\\0&4&0\\0&0&4\end{pmatrix}$

(a) II only (b) II & III

(c) III only (d) all

(8) Suppose X is an lxm matrix and Y is an mxn matrix. Then the product YX is of order…?

(a) lxn matrix (b) lxm matrix

(c) mxn matrix (d) Cannot be multiplied

(9) $\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix}$

(a) Scalar matrix (b) rectangular matrix

(c) identity matrix (d) none

(10) $3X+\begin{pmatrix}1&0&2\\2&1&3\\1&1&5\end{pmatrix}=\begin{pmatrix}4&0&2\\-1&-2&0\\0&1&7\end{pmatrix}, X=?$

(a) $\begin{pmatrix}1&0&0\\-1&-1&-1\\3&0&4\end{pmatrix}$ (b) $\begin{pmatrix}1&0&0\\1&-1&1\\3&0&4\end{pmatrix}$

(c) $\begin{pmatrix}1&0&0\\-1&-1&-1\\\frac{-1}{3}&0&\frac{2}{3}\end{pmatrix}$ (d) $\begin{pmatrix}1&0&8\\1&-1&1\\3&0&4\end{pmatrix}$

(11) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{pmatrix}$ is .....

(a) Scalar matrix
(b) diagonal matrix
(c) identity matrix
(d) rectangular matrix

(12) Which one is generally not true if A,B and C are matrices
(a) A+B=B+A
(b) A (BC) = (AB)C
(c) A+(B+C) =(A+B)+C
(d) AB=BA

(13) Which matrix is symmetric
(a) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 6 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 2 & 3 \\ 5 & 5 & 4 \\ 3 & 4 & 6 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 2 & 3 \\ 5 & 5 & 4 \\ 7 & 4 & 6 \end{pmatrix}$
(d) all of these

(14) If $A=\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 3 \\ 0 & 2 & 3 \end{pmatrix}$, then $|A| = ?$
(a) –1
(b) 3
(c) 0
(d) 1

(15) Value of the determinant $\begin{vmatrix} 7 & 2 & -3 & 1 & 9 \\ 2 & 3 & -2 & 0 & 2 \\ 6 & 4 & 5 & 3 & 3 \\ 6 & 7 & 1 & 2 & 4 \\ 7 & 2 & -3 & 1 & 9 \end{vmatrix} = ?$
(a) 7
(b) 50
(c) –50
(d) 0

(16) $A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, then minor of a_{21}=?
(a) $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$
(b) $\begin{pmatrix} a_{21} & a_{12} \\ a_{31} & a_{22} \end{pmatrix}$
(c) $\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$
(d) $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

(17) If $AB = BA = I$ and $AC = CA = I$, then what about B and C.

(a) They are equal
(b) They are not equal
(c) Each is equal to A
(d) None

(18) If A and B are non singular matrices then which one is true

(a) $(AB)^{-1} = A^{-1}B^{-1}$
(b) $(AB)^{-1} = (BA)^{-1}$
(c) $(AB)^{-1} = BA^{-1}$
(d) $(AB)^{-1} = (B^{-1}A^{-1})$

(19) If $AB = BA = I$, then which one is true

(a) A is a unit matrix
(b) B is a unit matrix
(c) A and B are inverse of each other
(d) none

(20) $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = ?$

(a) 0
(b) (a-b) (b-c)(c-a)
(c) abc
(d) 1

(21) $\begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix} = ?$

(a) 1
(b) 2
(c) 0
(d) 4

(22) $\begin{vmatrix} -1 & 1 & 0 \\ 2 & 3 & -1 \\ 2 & 1 & 2 \end{vmatrix} = ?$

(a) $\begin{vmatrix} 3 & 3 & 0 \\ 5 & 3 & -1 \\ 3 & 1 & 2 \end{vmatrix}$
(b) $\begin{vmatrix} 0 & 1 & 0 \\ 5 & 3 & -1 \\ 3 & 1 & 2 \end{vmatrix}$
(c) $\begin{vmatrix} 3 & 3 & 7 \\ 5 & 3 & -1 \\ 3 & 1 & 5 \end{vmatrix}$.
(d) none

(23) $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, then $|A| = ?$

(a) $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$
(b) $a_{12}A_{21} + a_{22}A_{22} + a_{32}A_{32}$
(c) $a_{13}A_{31} + a_{23}A_{32} + a_{13}A_{33}$
(d) none

(24) If for a square matrix A, $A^p = 0$ then A is called
(a) square matrix
(b) nil potent matrix
(c) idempotent matrix
(d) periodic matrix

(25) For a square matrix A, if $A^2 = A$ then A is called
(a) symmetric matrix
(b) periodic matrix
(c) idempotent matrix
(d) nilpotent matrix

(26) If $A = \left[a_{ij}\right]$ is *nxm*, then transpose of A is of the order
(a) *mxm*
(b) *nxn*
(c) *mxn*
(d) *nxm*

(27) For a square matrix A, if $A = A^t$ then A is called _____ matrix
(a) symmetric
(b) transpose
(c) non-symmetric
(d) singular

(28) Given an mxm matrix $A = \left[a_{ij}\right]$ if we delete the *ith* row and *jth* column of A, we get an *(m-1) x (m-1)* matrix. The determinant of this matrix is called the _____ of the element a_{ij}
(a) cofactor
(b) minor
(c) modulus
(d) none

(29) The condition for the system of linear homogenous equations to have a trivial solution is
(a) $|A| = 0$
(b) $|A| \neq 0$
(c) $|A| > 0$
(d) $|A| < 0$

(30) $(CD)^t = ?$
(a) C^tD^t
(b) D^tC^t
(c) $(DC)^t$
(d) none

(31) If any two rows in a square matrix are identical then $|A| = ?$
(a) 0
(b) 1
(c) –1
(d) can be any value

(32) $5\begin{pmatrix} 4 & -3 & 2 \\ 7 & 1 & 5 \\ 0 & 3 & 2 \end{pmatrix} = ?$

(a) $\begin{pmatrix} 20 & -15 & 10 \\ 7 & 2 & 2 \\ 0 & 3 & 3 \end{pmatrix}$
(b) $\begin{pmatrix} 20 & -15 & 10 \\ 35 & 2 & 2 \\ 0 & 3 & 3 \end{pmatrix}$
(c) $\begin{pmatrix} 2 & -15 & 10 \\ 3 & 2 & 2 \\ 0 & 3 & 3 \end{pmatrix}$
(d) $\begin{pmatrix} 20 & -15 & 10 \\ 35 & 5 & 25 \\ 0 & 15 & 10 \end{pmatrix}$

(33) A matrix of order 1xm is called

(a) Row matrix (b) column matrix

(c) null matrix (d) diagonal matrix

(34) The sum of the products of the elements of a row (or column) by the cofactors of the corresponding elements of different row (or column) is

(a) 0 (b) 1

(c) –1 (d) equal to matrix

(35) A matrix of order nx1 is called

(a) Row matrix (b) column matrix

(c) null matrix (d) diagonal matrix

(36) A square matrix C = $[c_{ij}]$ is called a _____ matrix if $c_{ij} = 0$ for all $i \neq j$ and at least one $c_{ij} \neq 0$ for all $i=j$.

(a) Identity matrix (b) diagonal matrix

(c) rectangular (d) null

(37) If A is a square matrix then AI=IA = ___

(a) 0 (b) 1

(c) A (d) none

(38) If each element in any row of a square matrix is 0, then value of its determinant is

(a) 0 (b) 1

(c) –1 (d) 5

(39) [-1 4 7 2] is a

(a) diagonal matrix (b) scalar matrix

(c) row matrix (d) column matrix

(40) If any two columns of a square matrix A are interchanged then |A| =?

(a) 0

(b) multiplicative inverse of the determinant of the original matrix

(c) additive inverse of the determinant of the original matrix

(d) 1

(41) The rank of the matrix $\begin{pmatrix} 1 & -1 & 2 & -3 \\ 2 & 0 & 7 & -7 \\ 3 & 1 & 12 & -11 \end{pmatrix}$ is……?

(a) 4 (b) 3

(c) 1 (d) 2

(42) If A is a non singular matrix, then $(A^{-1})^{-1}$ =?

(a) A
(b) –A
(c) I
(d) A^t

(43) Let A = $[a_{ij}]$ be a square matrix. Then the cofactor of a_{ij} i.e, A_{ij} =?

(a) $(-1)M_{ij}$
(b) $(-1)^{i+j}$
(c) $(-1)^{i-j} M_{ij}$
(d) $(-1)^{j-i} M_{ij}$

(44) A square matrix A is called non singular if $|A| \neq$?

(a) 0
(b) 1
(c) $\frac{1}{|A|}$
(d) $|A^{-1}|$

(45) Which one of the following statement is true

(a) The additive inverse of the matrix A is denoted by –A
(b) The additive inverse of A is denoted by A^{-1}
(c) The additive inverse of A is A
(d) The additive inverse of A is [A]

(46) Solution set of the system 4x-3y+2z-3=0, x+y-3z - 13=0, $\frac{1}{2}x+y+\frac{1}{4}z-\frac{9}{2}=0$, is

(a) {(1,-1,0)}
(b) {(4,3,-2)}
(c) {(-3,0,5)}
(d) {(2,10,43)}

(47) If A,B,C are three matrices such that $AB=C$ then which is true?

(I) $B=\frac{C}{A}$ (II) $B=CA^{-1}$ (III) $B=A^{-1}C$

(a) I and III
(b) II and III
(c) III only
(d) I only

(48) A matrix A is said to be a skew symmetric matrix if A^t is equal to

(a) A
(b) –A
(c) A^2
(d) $-A^2$

(49) If A is a square matrix of order 4, and |A|=5 the |2A|=?

(a) 10
(b) 20
(c) 40
(d) 80

(50) If $X\begin{bmatrix}5 & 2\\ -2 & 1\end{bmatrix}=\begin{bmatrix}-1 & 5\\ 12 & 3\end{bmatrix}$, then $X=?$

(a) $\begin{bmatrix}2 & 4\\ 3 & 3\end{bmatrix}$

(b) $\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

(d) $\begin{pmatrix}2 & 2\\ 22 & 2\end{pmatrix}$

(51) If A and B are two invertible matrices of order 3 each then $(BA)^{-1}=?$

(a) $B^{-1}A^{-1}$

(b) $A^{-1}B^{-1}$

(c) $A^{-1}B$

(b) AB^{-1}

(52) If A is a square matrix of order 2 having a row of zeros the determinant of A = ?

(a) 1

(b) 0

(c) Any real number

(d) -2

(53) $\Delta=\begin{vmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{vmatrix}$, where ω is cube root of unity, then $\Delta=?$

(a) 1

(b) -1

(c) 0

(d) 3

(54) If A is a square matrix of order 3 then $\det(2A)=?$

(a) 2 det A

(b) 4 det A

(c) 0

(d) 8 det A

(55) $\begin{vmatrix}1 & 0 & 0\\ 2 & \cos x & \sin x\\ 3 & \sin x & \cos x\end{vmatrix}=?$

(a) 1

(b) 0

(c) cos 2 x

(d) sin 2 x

(56) The system of the equations $\begin{matrix}x+2y=11\\ -3x-6y=33\end{matrix}$ has

(a) Only one solution

(b) Finite many solutions

(c) No solution

(d) Infinite solutions

(57) The value of "λ" for which the system of equation does not have a solution

$x+y+z=6$

$4x+\lambda y-\lambda z=0$

$3x+2y-4z=-5$

(a) 3

(b) -3

(c) 0

(d) 1

(58) If A and B are square matrices of same order then $(A+B)^2=A^2+2AB+B^2$ if

(a) AB = BA

(b) A = -B

(c) $A = B^{-1}$

(d) A = B

(59) If $\begin{vmatrix} 3 & 2 \\ x & 4 \end{vmatrix} = 0$ then x = ?

(a) 0

(b) 6

(c) -6

(d) -5

(60) $\begin{vmatrix} 42 & 1 & 6 \\ 28 & 7 & 4 \\ 14 & 3 & 2 \end{vmatrix} = ?$

(a) 2

(b) -2

(c) 0

(d) 1

QUADRATIC EQUATIONS

Polynomial Function:	A function of the form $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, $a_n \neq 0$, where **n** is a non negative integer and the coefficients a_n, a_{n-1} ….a_1 and a_0 are real numbers. It can be considered as a polynomial function of x. The highest power of x in a polynomial in x is called the **degree of the polynomial**. So, the expression is a polynomial of degree n.

Some Important Theorems on Polynomials

The proofs of the theorems are beyond the scope of this book.

Remainder Theorem:	▪ When a polynomial $P(x)$ is divided by $x - a$, then the remainder is a constant $P(a)$. ▪ When a polynomial $P(x)$ is divided by $ax - b$, then the remainder is a constant $P\left(\frac{b}{a}\right)$
Factor Theorem:	▪ Let $P(x)$ be a polynomial ,then a) If $x - a$ is a factor of $P(x)$, then $P(a) = 0$. b) If P(a) = 0 , then x-a is a factor of P(x). ▪ Let P(x) be a polynomial ,then a) If $ax - b$ is a factor of $P(x)$, then $P\left(\frac{b}{a}\right) = 0$ b) If $P\left(\frac{b}{a}\right) = 0$, then ax-b is a factor of $P(x)$.

Rational Root Theorem:	If $P(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$, $a_n\neq 0$, with the integral coefficients then if $P(x)=0$, has any rational root(s), that must be of the form $\frac{p}{q}$, where p is a factor of $\mathbf{a_o}$**(constant term)** and q is a factor of $\mathbf{a_n}$**(leading coefficient)**.
Root Location Theorem:	If $P(x)$ is a polynomial over real numbers then for any two real numbers **a** and **b** there will be a root of $P(x)$ if $P(a).P(b)<0$
Sum of Roots of a Polynomial:	Let $P(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$, $a_n\neq 0$, Sum of roots of P(x) = $-\frac{a_{n-1}}{a_n}$
Product of Roots of a Polynomial:	Let $P(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$, $a_n\neq 0$ then Product of roots of $P(x)=(-1)^k\frac{a_0}{a_n}$, Where k is the number of roots of P(x).
Descarte's Sign Rule:	▪ For +ve real roots, if the sign varies **n** times in $P(x)$ then there are ***n or n ±2k*** +ve real roots of $P(x)$. Where $n,k\in\mathbb{Z}$ ▪ For –ve real roots, if the sign varies **n** times in $P(-x)$ then there are ***n or n ±2k*** –ve real roots of $P(x)$. Where $n,k\in\mathbb{Z}$.

NOTES

Polynomials with low degree have **special names.**

- Degree 0, **constant polynomial.**
- Degree 1, **linear polynomial.**
- Degree 2, **quadratic polynomial** or **a quadratic.**
- Degree 3, **cubic polynomial** or **a cubic.**
- Degree 4, **quartic polynomial** or **quartic.**
- 0 is a polynomial of **degree zero.**
- Any **constant** is a **polynomial of degree 0.**
- A polynomial having **one term** is called **monomial.**
- A polynomial having **two terms** is called **binomial.**
- A polynomial having **three terms** is called **trinomial.**
- To obtain Remainder when $P(x)$ is divided by $x-a$ find the value of the polynomial $P(x)$ at $x = a$.
- To determine, if a given linear polynomial $x - a$ is a factor of $P(x)$ or not, all we need is to check whether $P(a) = 0$ or not.
- Dividend = (Divisor) (Quotient) + Remainder.

Synthetic Division

Outline of the Method

i). Write down the coefficients of the dividend $f(x)$ from left to right in decreasing order of powers of x. insert 0 for any missing term(s).

ii). To the left of the first line, write the value of **a** from the divisor (x-a).

iii). Use the following patterns to write the 2^{nd} & 3^{rd} line:

Vertical pattern (↓) add terms

Diagonal pattern (↗) multiply by a.

Quadratic Equation:

A 2^{nd} degree polynomial is called quadratic.

Or

A quadratic equation is an equation that can be written in the form of $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.

Relation between the Roots and the Coefficients of a Quadratic Equation:

Sum of the Roots = S = $-\frac{b}{a} = -\frac{coeffiecient \text{ of } x}{coeffiecient \text{ of } x^2}$

Product Of The Roots=p = $\frac{c}{a} = \frac{\text{constant term}}{coefficient \text{ of } x^2}$

Difference Of Roots = $\sqrt{S^2 - 4P}$

Formation of an Equation Whose Roots Are Given:

Let S = sum of the roots and P = product of the roots of a quadratic equation then the required equation will be;

$x^2 - Sx + P = 0$

Nature of the Roots of a Quadratic Equation:

The nature of the roots of an equation depends on the value of the expression $b^2 - 4ac$, which is called discriminate. (Denoted by Disc or Δ)

Real ,equal and repeated roots	If disc$= b^2 - 4ac = 0$, then the roots are real, equal and repeated.
Real and unequal roots	If disc $= b^2 - 4ac > 0$, then the roots are real and unequal.
Complex(imaginary) roots	If disc $= b^2 - 4ac < 0$, then the roots are complex. The complex roots of a quadratic equation always appear in the form of conjugate pair.
Rational /irrational roots	If **Disc** $= b^2 - 4ac$ is a **perfect square**. Then the roots are **rational**, otherwise irrational. (May or may not be same)

Solution of Quadratic Equations:

There are **three basic techniques** to solve a quadratic equation:

i). By **factorization**

ii). By **completing squares**, extracting square roots.

iii). By applying the **quadratic formula**.

NOTE:

- The graph of a quadratic equation is a **parabola**.
- The solution of an equation is also called its **roots** or **zeros**.
- **Quadratic formula** is given by: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Types of Equations Reducible To the Quadratic Equation

There are certain types of equations, which do not look to be of degree 2, but they can be reduced to the quadratic form. Some of those forms are given below:

Type I	$ax^{2n}+bx^{n}+c=0, a\neq 0$ Put $x^{n}=y$ and get the given equation reduced to quadratic equation in y.
Type II	$(x+a)(x+b)(x+c)(x+d)=k$ Where $a+b=c+d$
Type III	Equations in which the variable occurs in exponent are called **exponential** equations. $P^{x}+q.P^{2x}=y$.
Type IV	An equation, which remains unchanged when x is replaced by $\frac{1}{x}$ is called a **reciprocal** equation. In such an equation the coefficients of the terms equidistant form the beginning and end are equal in magnitude. $ax^{4}+bx^{3}+cx^{2}+bx+a=0$
Type V	Equations involving radical expression of the variable are called **radical** equations. To solve a radical equation we first obtain an equation free of radical. Every solution of radical equation is a solution of the radical free equation but the new equation has solutions that are not solutions of the original radical equation. Such extra solutions (roots) are called **extraneous roots**. Following are some of the types of radical equations. (a) $\left(ax^{2}+bx\right)+m\sqrt{ax^{2}+bx+c}=n$ (b) $\sqrt{ax+b}+\sqrt{cx+d}=\sqrt{ex+f}$
Type VI	$\frac{a}{x+b}+\frac{c}{x+d}=\frac{e}{x+f}$ $x\neq -b, -d, -f$
Type VII	$ay+\frac{b}{y}=c, y\neq 0$

System of Two Equations Involving Two Variables;

To determine the value of two variables, we need a pair of equations. Such a pair of equation is called a **system of simultaneous equations**.

No specific rule can be laid down for the solution of such equation. Simply, one of the variables is eliminated and the resulting equation in one variable is solved.

Homogeneous Equation:

An equation is said to be homogenous if every term in the equation is of the same degree.

An equation of the form $ax^2 + 2hxy + by^2 = 0$ is said to be **homogeneous equation of degree 2**.

- The solution of simultaneous equations gives the point(s) of intersection of their graphs.

Three Cube Roots of Unity:

The three cube roots of unity are

$1, \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$. Denoted by $1, \omega, \omega^2$ respectively.

Properties of Cube Roots of Unity:

i). Each complex cube root of unity is square of the other.

If $\omega = \frac{-1+\sqrt{3}i}{2}$ then $\omega^2 = \frac{-1-\sqrt{3}i}{2}$.

ii). The sum of all the three cube roots of unity is zero *i.e.*

$1+\omega+\omega^2=0$

iii). The product of all the three cube roots of unity is unity i.e.

$1.\omega.\omega^2=1$

iv). For any $n \in \mathbb{Z}$, ω^n is equivalent to one of the cube roots of unity.

Four Fourth Roots of Unity:

Four fourth roots of unity are **1, -1, *i*, -*i***

Properties of Four Fourth Roots of Unity:

i). Sum of all the four fourth roots of unity is zero. *i.e.* $1+(-1)+i+(-i)=0$
ii). The real fourth roots of unity are additive inverses of each other. +1 and –1 are the real fourth roots of unity and $+1+(-1)=0=(-1)+1$
iii). Both the complex/ imaginary fourth roots of unity are conjugate of each other. i and $-i$ are complex imaginary fourth roots of unity, which are obviously conjugate of each other.
iv). Product of all the fourth roots of unity is – 1. *i.e.* $(1)(-1)(i)(-i)=-1$

NOTES

❑ $\omega^n+\omega^{n+1}+\omega^{n+2}=0$
❑ $\omega^n.\omega^{n+1}.\omega^{n+2}=\omega^{3n}=1$, where $n\in\mathbb{Z}$
❑ Sum of cube roots of any real number is zero.
❑ Product of cube roots of any real number is that real number.
❑ Sum of four fourth roots of any +ve integer is zero.
❑ Product of four fourth roots of any +ve integer is the additive inverse of that integer.

PRACTICE EXERCISE

(1) $3x^4 + \sqrt{5}x^2 + 3x + 1$ is a polynomial of degree_____.
(a) 3 (b) 2
(c) 4 (d) 7

(2) The product of four fourth roots of 64 =?
(a) 1 (b) -1
(c) 4 (d) -64

(3) $7y^2 + 5\sqrt{y} + 3$ is a polynomial over.
(a) Natural numbers (b) Integers
(c) Rational Numbers (d) None

(4) $3x^2 + 9x + 7$ is a
(a) Binomial (b) Monomial
(c) Trinomial (d) None

(5) If $y^4 + y^3 - 2y^2 - 8y + 2$ is divided by $(y+1)$ then the remainder is....
(a) 8 (b) 0
(c) -1 (d) -8

(6) Which is the root of the equation $x^4 - 9x^3 + 6x^2 + 2$?
(a) 1 (b) 2
(c) -2 (d) -1

(7) The value of the polynomial $4x^4 - 3x^3 + 2x^2 - x + 5$ at $x = 3$ is
(a) 150 (b) 21
(c) 313 (d) 263

(8) Quadratic equation is an equation of degree
(a) 0 (b) 1
(c) 2 (d) 3

(9) If $(\alpha + \beta) = 2$, $\alpha\beta = 1$, then value of $\alpha^3 + \beta^3$ =
(a) 1 (b) 2
(c) -2 (d) 0

(10) If S_1 is the solution set of equation f (x,y) = 0 and S_2 is the solution set of g (x,y) = 0 then the solution set of the system f (x,y) = 0 and g (x,y) = 0 is
(a) $S_1 \cap S_2$ (b) $S_1 \cup S_2$
(c) $S_1 - S_2$ (d) $S_1 + S_2$

(11) $\left(\frac{1+\sqrt{-3}}{2}\right)^9 + \left(\frac{1-\sqrt{-3}}{2}\right)^9 = ?$
(a) 2 (b) 3
(c) 0 (d) -2

(12) If the discriminant of $ax^2 + bx + c = 0$ is not a perfect square then its roots are:

(a) Equal (b) Irrational

(c) Rational (d) Real

(13) The product of the roots of the equation $9x^2-5x-27=0$ is

(a) $\frac{5}{27}$ (b) $\frac{-5}{9}$

(c) $\frac{-1}{3}$ (d) -3

(14) The sum of the roots of the equation $12x^2-7x+4=0$ is

(a) $\frac{-7}{12}$ (b) $\frac{12}{7}$

(c) $\frac{7}{12}$ (d) $\frac{1}{3}$

(15) If S = the sum of the roots of the equation and P=Product of the roots of the equation, then their equation can be written as:

(a) $x^2+Sx-P=0$ (b) $x^2+Sx+P=0$

(c) $x^2-Sx+P=0$ (d) $x^2-Sx-P=0$

(16) If the discriminant of the quadratic equation is +ve then the roots are:

(a) Complex (b) Rational & equal

(c) Real & equal (d) Real and unequal

(17) Sum of the roots of equation $ax^2+bx+c=0$ is

(a) $\frac{c}{a}$ (b) $\frac{b}{a}$

(c) $\frac{-b}{a}$ (d) $\frac{-c}{a}$

(18) If the roots of the equation are $\frac{-7}{4}, \frac{4}{7}$ then the equation will be

(a) $28x^2+33x-28=0$ (b) $28x^2+49x+16=0$

(c) $28x^2-49x+16=0$ (d) $28x^2-33x+28=0$

(19) Value of ω (cube root of unity) is

(a) $-1+i\sqrt{3}$ (b) $-1-i\sqrt{3}$

(c) $\frac{1+i\sqrt{3}}{2}$ (d) $\frac{-1+i\sqrt{3}}{2}$

(20) $1+\omega+\omega^2=?$

(a) 1 (b) 0

(c) -1 (d) ω^3

(21) $1.\omega.\omega^2=?$

(a) 1 (b) 0

(c) -1 (d) ω

(22) The equation $5x^2+7x+8=0$ has
(a) Real roots
(b) Rational roots
(c) Irrational roots
(d) Imaginary roots

(23) Which statement is true about the discriminant of the quadratic equation $cx^2+dx+e=0$ with $c\neq0$
(a) $d+4ce$
(b) $4ce$
(c) d^2-4ce
(d) b^2-4ac

(24) The nature of the roots of equation $9x^2-5x+3=0$ is
(a) Real & equal
(b) Real & unequal
(c) complex
(d) Irrational

(25) The product of the roots of equation $px^2-tx+n=0$ is
(a) $\frac{n}{p}$
(b) $\frac{t}{p}$
(c) $\frac{-t}{p}$
(d) $\frac{p}{n}$

(26) What should be added to $9x^4+2x^2+25$ to make it a perfect square?
(a) $32x^2$
(b) $30x^2$
(c) $-32x^2$
(d) $14x^2$

$(3x^2)^2+2x^2+(5)^2$
$(3x^2)^2+2(3x)(5)x^2+(5)^2$
$30x^2$ given $2x^2$
So $-32x^2$

(27) Cube roots of -64 are
(a) $4,\omega,\omega^2$
(b) $-4,\omega,\omega^2$
(c) $-4,-4\omega,-4\omega^2$
(d) $4,4\omega,4\omega^2$

(28) If discriminant of a quadratic equation is –ve then the roots are:
(a) real and equal
(b) Rational & unequal
(c) irrational & unequal
(d) Complex

(29) If roots of a quadratic equation are equal then discriminant
(a) $=0$
(b) >0
(c) <0
(d) >0 and is a perfect square

(30) $\omega^{10}+\omega^{11}+\omega^{12}=$ $\omega^9.\omega+\omega^9.\omega^2+\omega^9.\omega^3$
$\omega+\omega^2+1=0$
(a) 1
(b) 0
(c) -1
(d) ω^{33}

(31) If discriminant of a quadratic equation is a perfect square than the roots are
(a) equal
(b) Irrational
(c) Rational
(d) Complex

(32) Which one is true?
(a) $\omega=\frac{1}{\omega^2}$
(b) $\omega^2=\frac{1}{\omega}$
(c) $\omega^3=1$
(d) All

(33) Value of ω^2 is:

(a) $-1+i\sqrt{3}$ (b) $-1-i\sqrt{3}$

(c) $\left(\frac{1+i\sqrt{3}}{2}\right)$ (d) $\left(\frac{1+i\sqrt{3}}{2}\right)$

(34) $\omega^{109} = ?$

(a) 1 (b) 3

(c) $\left(\frac{-1-i\sqrt{3}}{2}\right)$ (d) $\left(\frac{-1+i\sqrt{3}}{2}\right)$

(35) $9^{1+x} + 9^{1-x} = 15$ is called ..... equation.

(a) Exponential (b) Radical

(c) Reciprocal (d) Simultaneous

(36) $ax^4 + bx^3 + cx^2 + bx + a = 0$ is called ..... equation.

(a) exponential (b) Quadratic

(c) Radical (d) Reciprocal

(37) A polynomial having one term is called .....

(a) monomial (b) Binomial

(c) Trinomial (d) None

(38) What is the type of equation $14(x^4+1)-7(x^3+x)+8x^2=0$

(a) Rational (b) Irrational

(c) Reciprocal (d) Radical

(39) Name the type of equation $\sqrt{x+a}+\sqrt{x+b}=e\sqrt{cx+d}$.

(a) Rational (b) Irrational

(c) Reciprocal (d) None

(40) The first degree functions are called ..... Functions

(a) Linear (b) Quadratic

(c) Cubic (d) Quartic

(41) For what value of m, the roots of the equation.

$(m+1)x^2+2(m+3)x+2m+3=0, m\neq -1$ are equal?

(a) 1 (b) 3

(c) 7 (d) 17

(42) Let $p(x)$ be a polynomial of degree n>1 then $x-r$ is a factor of $p(x)$ if and only if $p(r)$:

(a) =0 (b) >0

(c) <0 (d) 1

(43) Which one of the following is the solution set of the system of equations $x^2+y^2+5xy=43$, $y^2+xy-5x=5$

(a) {(1,3)} (b) {(2,3}

(c) {(1,2)} (d) {(3,1)}

(44) For what value of 'm' and 'n' -1 and 2 are the roots of the polynomial x^3+mx^2+nx+6?

(a) 1,3
(b) -4,9
(c) 2,7
(d) -4,1

(45) Cube roots of –1 are:

(a) w, w^2
(b) $1, w, w^2$
(c) $1, \frac{1}{\omega}, \frac{1}{\omega^2}$
(d) $-1, -w, -w^2$

(46) Find the condition that one root of the equation $ax^2+bx+c=0\ (a\neq 0)$ is cube of the other.

(a) $a^{\frac{1}{4}}c^{\frac{1}{4}}\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)+b=0$
(b) $a^{\frac{1}{4}}c^{\frac{1}{4}}\left(a^{\frac{1}{2}}+c^{\frac{1}{2}}\right)+b=0$
(c) $abc(a+b)+c=0$
(d) $a^{\frac{1}{4}}b^{\frac{1}{4}}c^{\frac{1}{4}}(a+b+c)=0$

(47) If x-a is one of the factors of the polynomial ax^2+bx+c, then one of the roots of $ax^2+bx+c=0$ is

(a) –a
(b) b
(c) c
(d) a

(48) If $x^4-500=0$, then the sum of the roots of the equation is

(a) 500
(b) -500
(c) 0
(d) 1

(49) To complete a job A & B take 4 days together. A alone takes twice as long as B alone to finish the same job. How long would each one alone take to do job?

(a) A 12 –days, B 5-days
(b) A 6 days, B 7-days
(c) A 12-days, B 6-days
(d) A 6-days, B 3-days

(50) If α,β are the roots of the equation $5x^2-x-2=0$,then find the equation with roots $2\alpha,2\beta$.

(a) $10x^2-3x-2=0$
(b) $x^2-3x-2=0$
(c) $x^2-3x-1=0$
(d) $5x^2-2x-8=0$

(51) If one root of the equation $(b-c)x^2+(c-a)x+(a-b)=0$ is double of the other then

(a) $2(a-c)^2=9(a-b)(b-c)$
(b) $(a-c)^2=(a-b)(b-c)$
(c) $(a-c)=(a-b)(b-c)$
(d) $(a-c)=(a-b)^2(b-c)^2$

(52) If the roots of the quadratic equation $5x^2 - Px + 1 = 0$ are real and distinct then

(a) $-2 < P < 2$
(b) $-2\sqrt{5} > P > 2\sqrt{5}$
(c) $P < -2\sqrt{5}$ and $P > \sqrt{5}$
(d) $-2\sqrt{5} < P < 2\sqrt{5}$

(53) If the roots of the equation $ax^2 + bx + a = 0$ are complex then the roots of $x^2 + 4ax + b^2 = 0$ are _____

(a) Real (b) Complex
(c) Rational (d) Equal

(54) $x = \sqrt{2+\sqrt{2+\sqrt{2}+...\infty}}$ then x = ?

(a) 2 (b) – 7
(c) Both a and b (d) None of these

(55) If $\left(4+\sqrt{15}\right)^x + \left(4-\sqrt{15}\right)^x = 62$ then x = ?

(a) 2 (b) -2
(c) 4 (d) Both a and b

(56) If one root of the equation $Px^2 + qx + r = 0$ is twice of the other then $2q^2$ =?

(a) Pr (b) 9Pr
(c) P^2r (d) Pr^2

(57) The quadratic equation whose one root is $3+\sqrt{2}$ is

(a) $x^2+6x+7=0$ (b) $x^2+6x-7=0$
(c) $x^2-6x+7=0$ (d) $x^2+6x+5=0$

(58) If one root of the equation $3x^2 + 13x + k = 0$ is reciprocal of the other then k =?

(a) 13 (b) 3
(c) -5 (d) $\frac{1}{5}$

(59) If $(x+1)$ is a factor of $x^4+(P-3)x^3-(3P-5)x^2+(2P-9)x+6$ then P =?

(a) 4 (b) – 4
(c) 1 (d) 0

(60) If $2+i\sqrt{3}$ is a root of $x^2 + Px + q = 0$ where P, q are real then P + q =?

(a) 5 (b) 4
(c) 3 (d) 20

PARTIAL FRACTIONS

Partial Fraction

To change a given rational fraction into a number of simpler partial fractions having the numerator is of lower degree than the denominator is called **partial fraction resolution** (or simply partial fraction).

Some Important Terms

Term	Definition	Example
Conditional Equation	A type of equation in which two algebraic expressions are equal for particular value(s) of the variable(s).	(i). $4x=5$ (ii). $2x^2+3x+4=0$
Identity	A type of equation which holds for all values of the variable involved.	(i). $(a-b)x=ax-bx$ (ii). $(x+2)(x-2)=x^2-4$
Rational fraction	The quotient of two polynomials $\frac{P(x)}{Q(x)}, Q(x)\neq 0$ with no common factors, is called a rational fraction.	$\frac{x^3-x^2+x+1}{x^2+5}$

- Rational fractions are of two types;

Proper Rational Fraction	The rational fraction $\frac{P(x)}{Q(x)}$ is called proper rational fraction if the degree of $P(x)$ is less than the degree of $Q(x)$.	(i). $\frac{2}{x-1}$ (ii). $\frac{2x}{x^2+1}$ (iii). $\frac{7x^2}{x^3-3}$
Improper Rational Fraction	The rational fraction $\frac{P(x)}{Q(x)}$ is called improper rational fraction if the degree of $P(x)$ is equal to or greater than the degree of $Q(x)$.	(i). $\frac{x}{3x+2}$ (ii). $\frac{x^2+1}{4x+1}$ ❑ Every improper rational fraction can be changed into proper by using long division.

Resolution of a Rational Fraction into Partial Fractions

Following are the main points of resolving a rational fraction $\frac{P(x)}{Q(x)}$ into partial fractions:

1. The degree of $P(x)$ must be less than $Q(x)$ .if not, divide and work with remainder theorem.
2. Clear the given equation of fractions.
3. Equate the coefficients of like terms (powers of x).
4. Solve the resulting equations for the coefficients.

Here we shall discuss the methods to break up a rational fraction into partial fractions.

Case I	*Resolution of* $\frac{P(x)}{Q(x)}$ *into partial fractions when* $Q(x)$ *has only non-repeated linear factors (***i.e.** *denominator contains factors real ,linear but not repeated)*

If $Q(x)=(x-a_1)(x-a_2)....(x-a_n)$ where $a_1 \neq a_2 \neq ..... \neq a_n$

$$\therefore \frac{P(x)}{Q(x)}=\frac{A_1}{x-a_1}+\frac{A_2}{x-a_2}+....+\frac{A_n}{x-a_n}$$

Where the coefficients $A_1, A_2, ...A_n$ are the numbers to be found.

Case II	*When* $Q(x)$ *has repeated linear factors.(***i.e.** *denominator contains factors real, linear but repeated)*

To each *n* times linear factor of the form $(x-a)^n$ there will correspond the sum of *n* partial fractions as follows:

If $Q(x)=(x-a)^n, n \geq 2$

$$\therefore \frac{P(x)}{Q(x)}=\frac{A_1}{x-a}+\frac{A_2}{(x-a)^2}+....+\frac{A_n}{(x-a)^n}$$

Where the coefficients $A_1, A_2, ...A_n$ are the numbers to be found.

Case III	*When* $Q(x)$ *has non-repeated irreducible quadratic factors(***i.e.** *denominator contains factors ,real quadratic ,but not repeated)*

If the polynomial $Q(x)$ contains non-repeated irreducible quadratic factor then $\frac{P(x)}{Q(x)}$ may be written in the form of partial fraction as;

$\frac{Ax+B}{ax^2+bx+c}$. where *A* & *B* are the numbers independent of *x* to be found.

Case IV	*When* $Q(x)$ *has repeated irreducible quadratic factors (***i.e.** *denominator contains factors, real quadratic & repeated).*

If the polynomial $Q(x)$ contains repeated irreducible quadratic factors of the form $(ax^2+bx+c)^n, n \geq 2$

$$\therefore \frac{P(x)}{Q(x)}=\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+....+\frac{A_nx+B_n}{(ax^2+bx+c)^n}$$

Where $A_1, B_1, A_2, B_2, ...A_n, B_n$ are numbers to be found.

PRACTICE EXERCISE

(1) $4x=5$ is a(an).....

(a) Conditional Equation
(b)identity
(c) Equation
(d) fraction

(2) $(x-2)(x+2)=x^2-4$ is a(an).....

(a) Equation
(b) identity
(c) Conditional
(d) fraction

(3) If $px^3+qx^2-ax+b=2x^3-3x^2-4x+5$, then values of p, q, a and b are respectively

(a) 2, 3, 5, 4
(b) 2, 3, 5, -4
(c) 2,- 3, 4, 5
(d) -2, 3, 4, -5

(4) The algebraic equations can be divided into

(a)one kind
(b)two kinds
(c) Three kinds
(d) four kinds

(5) An equation in which two algebraic expressions are equal for particular values of the variable is called;

(a) identity
(b)equation
(c) conditional equation
(d) fraction

(6) Rational fractions are of ;

(a) Two types
(b) three types
(c) Four types
(d) one type

(7) The process of partial decomposition results in;

(a) An identity
(b) partial fraction
(c) equation
(d) Conditional Equation

(8) $\frac{2x-5}{x^2+4}$ is _____ rational fraction.

(a) proper
(b) improper
(c) conditional
(d) none

(9) If $\frac{7x+25}{(x+3)(x+4)}=\frac{A}{x+3}+\frac{B}{x+4}$,then find the values of A & B.

(a) $A=2, B=3$
(b) $A=1, B=5$
(c) $A=4, B=3$
(d) $A=1, B=3$

(10) The partial fraction of $\frac{x^2}{(x^2+1)(x-1)}$ is of the form;

(a) $\frac{Ax+B}{(x^2+1)^2}+\frac{C}{x-1}$

(b) $\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{E}{x-1}$

(c) $\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}$

(d) $\frac{Ax+B}{(x^2+1)}+\frac{C}{x-1}$

(11) The partial fraction of $\frac{4x^2+8x}{x^4+2x^2+9}=?$

(a) $\frac{x+2}{x^2-2x+3}+\frac{-2-x}{x^2+2x+3}$

(b) $\frac{x+2}{x^2+2x+3}+\frac{2-x}{x^2-2x+3}$

(c) $\frac{x+2}{x^2+2x+3}+\frac{2+x}{x^2-2x+3}$

(d) $\frac{x+2}{x^2+2x+3}+\frac{x}{x^2-2x-3}$

(12) $-1+\frac{1}{4(1-x)}+\frac{1}{4(1+x)}+\frac{1}{2(1+x^2)}$ is the partial fraction of ;

(a) $\frac{x^2}{1-x^2}$

(b) $\frac{x^3}{1-x^3}$

(c) $\frac{x^4}{1-x^4}$

(d) $\frac{x^4}{1+x^4}$

(13) An improper rational fraction can be reduced by division to a;

(a) Rational fraction

(b) irrational fraction

(c) Polynomial and rational form

(d) Polynomial + Proper Rational fraction

(14) The rational fraction $\frac{P(x)}{Q(x)}$ is called a proper fraction if the degree of $P(x)$ is _____ than the degree of $Q(x)$.

(a) Less

(b) greater

(c) Equal

(d) none of these

(15) $\frac{x}{2x-3}$ is a(an)_____rational fraction.

(a) Proper

(b) improper

(c) Conditional

(d) none

SEQUENCES AND SERIES

Sequence

i.	A **Sequence** is an arrangement of numbers **subject to some definite rule.**
ii.	A **Sequence** is a function whose **domain is a subset of the set of natural numbers.** $f : N \to \mathbb{R} / C$

NOTES

❑ Notation used for sequence is $\{a_n\}$.
❑ Each number in the sequence is called a **term.**
❑ The **n^{th} term** or **general term** of a sequence is denoted by a_n.
❑ So a_1, a_2, a_3, a_4 will be the **first, second, third, fourth** terms respectively of a sequence.
❑ If all members of a sequence are real numbers, then it is called a **real sequence.**
❑ If the domain of a sequence is a finite set then the sequence is called a **finite sequence** otherwise, an infinite sequence.
❑ An **infinite sequence** has no last term.

Arithmetic Sequence or Arithmetic Progression (A.P.):

i.	A sequence every term of which after the first is obtained from the preceding term by adding a fixed number in it is called an **arithmetic sequence** or **arithmetic progression.**
ii.	A sequence of quantities in which the algebraic difference between any term and the preceding term is a constant(*i.e.* $a_n - a_{n-1}$)
iii.	The general form of an *A.P.* is $a, a+d, a+2d, \cdots\cdots, a+(n-1)d$
iv.	The value of **n** is always a +ve integer

Common Difference (d)	The difference of two consecutive terms of an A.P. is called the **common difference.** Common difference may be +ve or – ve.
General Term or n^{th} Term of an A.P	The **general** or **n^{th}** form of an A.P. is, $a_n = a+(n-1)d$

Arithmetic Mean (A.M.):

i. If three quantities are in A.P. then middle term is known as **arithmetic mean (A.M.)** of the other two.

ii. If a, A, b are in A.P. then $A = \frac{a+b}{2}$

iii. If α, β, γ are in A.P. then their A.M. is β.

iv. 'n' Arithmetic Means between Two Given Numbers:

If $a, A_1, A_2, A_3, \ldots\ldots A_n, b$ are in A.P then the numbers $A_1, A_2, A_3, \ldots\ldots A_n$ are called **n arithmetic means** between 'a' and 'b'.

v. **n^{th} arithmetic mean** between ***a*** & ***b*** is given by; $A_n = \frac{a+nb}{n+1}$

vi. The **sum of n A.M's** between any two numbers ***a*** & ***b*** is equal to **n-times the A.M. between a & b.*i.e.*** $A_1 + A_2 + A_3 + \cdots + A_n = n\left(\frac{a+b}{2}\right)$.

Series:	The sum of an indicated number of terms of a sequence is called a **series**.
Arithmetic Series:	If $a, a+d, a+2d, a+3d, \ldots\ldots\ldots, a+(n-1)d$ is an A.P. Then series $a + (a + d) + (a + 2d) + \ldots\ldots + [a + (n-1)d]$ is called an **arithmetic series.**
Sum of First 'n' Terms of an Arithmetic Series	**(i).**If any three of ***a***, ***n***, ***d*** and $\mathbf{S_n}$ are given use **formula,** $S_n = \frac{n}{2}\{2a+(n-1)d\}$. **(ii).**If a_1=**first term**, and a_n= **general term** is given then use **formula,** $S_n = \frac{n}{2}\{a_1 + a_n\}$. **iii).**If a=**first term** and l= **last term** is given then use **formula,** $S_n = \frac{n}{2}\{a+l\}$.

❑ If we have to take **three numbers in A.P.**, we take these to be, $a-d,\ a,\ a+d$.
❑ If we have to take **four numbers in A.P.**, we take these to be, $a-3d,\ a-d,\ a+d;\ a+3d$.
❑ If we have to take **five numbers in A.P.**, we take these to be, $a-2d, a-d,\ a,\ a+d,\ a+2d$.

Geometric Sequence or Geometric Progression (G.P.)

i.	A sequence of numbers in which every term after the first is obtained from the preceding term by multiplying it with a constant number is called a **geometric sequence** or **geometric progression**.
ii.	**Geometric progression** is a sequence of numbers (terms) when the ratio of any term (except the 1^{st}) to the preceding one is constant.
iii.	The general form a G.P. is $a, ar, ar^2, ar^3, \cdots\cdots, ar^{n-1}$

Common Ratio(r):	i. The **constant ratio** of any two constant terms of a G.P. is called its **Common Ratio**. **ii.** The quotient $\frac{a_n}{a_n - 1}$ is called **common ratio** of the G.P. The **common ratio** cannot be **zero** and **one**.
General term or nth term of G.P.	$a_n = ar^{n-1}\ (a_n \neq 0)$

❑ If we have to take **three numbers in G.P.**, we take these to be, $\frac{a}{r}, a, ar$.
❑ If we have to take **four numbers in G.P.**, we take these to be, $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.
❑ If we have to take **five numbers in G.P.**, we take these to be, $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.

Geometric Mean (G.M.):
i. If three quantities are in G.P. then middle term is known as **geometric mean (G.M.)** of the other two.
ii. If a, G, b are in G.P. then $G = \pm\sqrt{ab}$.
iii. nth geometric mean between two numbers; If a, G_1, G_2, G_3, ……G_n, b are in G.P then the numbers G_1, G_2, G_3, ……G_n are called **n geometric means** between 'a' and 'b'.
iv. **n^{th} G.M.** between a & b is given by; $G_n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$
If $a = b$ then A.M. = G.M.

Geometric Series	If $a, ar, ar^2, ar^3, \cdots\cdots, ar^{n-1}$ is a G.P. then $a+ar+ar^2+ar^3+\cdots\cdots+ar^{n-1}$ is called **geometric series.**
Sum of 'n' Terms of a Geometric Series	(a) $S_n = \frac{a(1-r^n)}{1-r}, \lvert r\rvert < 1$. (b) $S_n = \frac{a-lr}{1-r}, \lvert r\rvert < 1$ (c) $S_n = \frac{a(r^n-1)}{r-1}, \lvert r\rvert > 1$ (d) $S_n = \frac{lr-a}{r-1}, \lvert r\rvert > 1$
The Infinite Geometric Series:	The series $a+ar+ar^2+ar^3+\cdots\cdots+ar^{n-1}+\cdots\infty$ is called infinite geometric series.
Sum of an Infinite Geometric Series	$S_\infty = \frac{a}{1-r}, \lvert r\rvert < 1$
Convergent series	If for an infinite geometric series $\lvert r\rvert < 1$,the series is said to be convergent.
Divergent series	If for an infinite geometric series $\lvert r\rvert > 1$ the series is said to be divergent.
Recurring Decimals:	A type of decimals in which the **decimals repeats endlessly** is called **recurring** or **repeated decimals**. *e.g.* $0.\overline{31} = 0.31313131\ldots$ Every **recurring decimal** can be converted into a fraction called **common fraction (vulgar fraction)** by using the formula for S_∞.

Harmonic Progression (H.P.)

A sequence of numbers is called a **harmonic sequence** or **harmonic progression** if the reciprocals of its terms are in arithmetic progression.

Harmonic Mean (H.M.)

i. If three quantities are in H.P. then the middle is called **harmonic mean (H.M.)** of the other two.

ii. If a, H, b are in H.P. then $H = \frac{2ab}{a+b}$

iii. **'n' Harmonic Means between Two Numbers**

If $a, H_1, H_2, H_3 \ldots\ldots\ldots\ldots, H_n, b$ are in H.P then the numbers $H_1, H_2, H_3 \ldots\ldots\ldots\ldots, H_n$ are called **n harmonic means** between a & b.

n^{th} harmonic mean between a & b is given by,

$n^{th}\ H.M. = \frac{(n+1)ab}{na+b}$.

NOTES

- No term of a H.P. can be zero.
- Every H.P. has a corresponding A.P.
- There is no general method to find the sum of a H.P.
- $\mathbf{G^2 = A.H}$
- **A>G>H,** if a & b are two distinct +ve real numbers and G>0.
- **A<G<H,** if a & b are two distinct –ve real numbers and G<0.

Chapter–6

Sigma Notation *or* Summation Notation

(i) $1+2+3+\cdots+n=\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$
(ii) $1^2+2^2+3^2+\cdots+n^2=\sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$
(iii) $1^3+2^3+3^3+\cdots+n^3=\sum_{k=1}^{n}k^3=\left(\frac{n(n+1)}{2}\right)^2$
(iv) $\sum_{k=1}^{n}\left[k^m-(k-1)^m\right]=n^m$

NOTES

❑ $\sum$ **(Sigma)** is a Greek letter used to represent sum of numbers.
❑ For the **Sum of 1st n–natural numbers** = 1+2+3+……+n use **formula (i).**
❑ For the **Sum of squares of 1st n–natural numbers** = $1^2+2^2+3^2+\ldots\ldots\ldots+n^2$ use **formula (ii).**
❑ For the **Sum of cubes of 1st n–natural numbers** = $1^3+2^3+3^3+\ldots\ldots\ldots+n^3$ use **formula (iii).**

PRACTICE EXERCISE

(1) If $a_4 = 4$ of an A.P. then find the sum of first 7-terms of the corresponding arithmetic series;

(a) 4
(b) 16
(c) 28
(d) 40

(2) Which term of the sequence $5, \frac{7}{2}, 2, \ldots\ldots$ is -19?

(a) 14^{th}
(b) 15^{th}
(c) 18^{th}
(d) 17^{th}

(3) For $\frac{1}{4}, \frac{2}{5}, 1, \cdots, 6^{th}$ term is

(a) $\frac{-2}{7}$
(b) -2
(c) $\frac{1}{9}$
(d) $\frac{-5}{6}$

(4) $9^{\frac{1}{3}}.9^{\frac{1}{9}}.9^{\frac{1}{27}}.\cdots.\infty = ?$

(a) 1
(b) 6
(c) 9
(d) 3

(5) A sequence of numbers in which every term after the first is obtained from the preceding term by multiplying it with a constant number is called a geometric sequence or G.P. The constant number is called;

(a) Term
(b) common difference
(c) Common ratio
(d) none

(6) G.M between two numbers 4 and -81 is

(a) 324
(b) -324
(c) -18
(d) none

(7) If the first term of an infinite geometric series is equal to twice of the sum of all the terms that follows it, then the value of r is;

(a) $\frac{1}{3}$
(b) 1
(c) $\frac{1}{2}$
(d) 2

(8) Which one is true? $(a > 0, b > 0)$

(a) G>A>H
(b) A>G>H
(c) H>G>A
(d) H>A>G

(9) The n^{th} term of an A.P is ..... when a=-7, d=13

(a) $20-5n$
(b) $13n-20$
(c) $13n-7$
(d) $7-13n$

Chapter-6

(10) H.M. between $3+\sqrt{27}$ and $3-3\sqrt{3}$ is

(a) -6

(b) $\sqrt{45}$

(c) $\frac{\sqrt{27}}{3}$

(d) $\frac{-4}{\sqrt{27}}$

(11) $x+x^2+x^3+\cdots n=?$

(a) $\frac{x(x-1)}{x+1}$

(b) $\frac{x^n-1}{x-1}$

(c) $\frac{x(x^n-1)}{x-1}$

(d) $\frac{x^n+1}{x-1}$

(12) Which one is true?

(a) $A^2 = GH$

(b) $G^2 = AH$

(c) $H^2 = AG$

(d) None

(13) S_n for geometric series if $|r| < 1$ is given by

(a) $\frac{n}{2}\{2a+(n-1)d\}$

(b) ar^{n-1}

(c) $\frac{a(r^n-1)}{r-1}$

(d) $\frac{a(1-r^n)}{1-r}$

(14) a_{27} of $7, \frac{23}{2}, \frac{32}{2}, \cdots$ is

(a) 117

(b) 124

(c) $\frac{119}{2}$

(d) $\frac{146}{2}$

(15) For $a+ar+ar^2+ \ldots\ldots\ldots \infty$, $S = ?$

(a) $a(1-r^2)$

(b) $a(r^2-1)$

(c) $\frac{n}{2}\{2a+(n-1)d\}$

(d) $\frac{a}{1-r}$

(16) Sum of all the terms in the sequence $1, \frac{5}{6}, \frac{25}{36}, \cdots \infty$ is

(a) 6

(b) $\frac{1}{3}$

(c) $\frac{2}{4}$

(d) $\frac{5}{6}$

(17) The constant ratio of G.P may be any number not equal to:

(a) 2

(b) 0

(c) -1

(d) none

(18) 4^{th} term of the sequence whose general term is $\frac{64}{25}\left(\frac{-5}{2}\right)^{n-1} = ?$

(a) 6120
(b) -321
(c) 190
(d) -40

(19) 9^{th} term of the sequence $\frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \cdots$ is

(a) $\frac{1}{128}$
(b) $\frac{1}{256}$
(c) $\frac{1}{384}$
(d) $\frac{1}{192}$

(20) $\frac{5}{27} = ?$

(a) $0.\overline{222}$
(b) $0.\overline{102}$
(c) $0.\overline{185}$
(d) $0.\overline{227}$

(21) For $S_n = \frac{n}{2}\{2a+(n-1)d\}$, d is called

(a) common ratio
(b) common difference
(c) common multiple
(d) none

(22) The general term of the sequence a, ar, ar^2 ..... is

(a) $a+(n-1)d$
(b) $\frac{a(1-r^n)}{1-r}$
(c) $\frac{n}{2}\{2a+(n-1)d\}$
(d) ar^{n-1}

(23) Common fraction of $7.7\overline{55}$ is:

(a) $\frac{3139}{95}$
(b) $\frac{495}{3839}$
(c) $\frac{290}{61}$
(d) $\frac{349}{45}$

(24) Geometric mean between two numbers a and b is given by:

(a) $\frac{a+b}{2}$
(b) $\pm\sqrt{ab}$
(c) $\frac{2ab}{a+b}$
(d) $\frac{a+b}{2ab}$

(25) For an H.P. $\frac{3}{11}, \frac{6}{31}, \frac{3}{20}, \cdots$, the common difference of the corresponding A.P is

(a) $\frac{1}{4}$
(b) $\frac{2}{3}$
(c) $\frac{3}{2}$
(d) $\frac{5}{7}$

(26) The first term of a G.P is -36 and common ratio is $-\frac{1}{3}$ then $a_5 = ?$

(a) $\frac{5}{3}$ (b) $\frac{6}{5}$

(c) $-\frac{3}{5}$ (d) $-\frac{4}{9}$

(27) The arithmetic mean between c and e is given by:

(a) $\frac{c+e}{2}$ (b) $\frac{c-e}{2}$

(c) $\frac{e-c}{2}$ (d) $c+e$

(28) The general term of a G.P is , $\left(\frac{2}{3}\right)^{n-3}$ the sequence is:

(a) 1,2,3,….. (b) $-\frac{2}{3}, 1, \frac{7}{3}, \cdots$

(c) $-\frac{7}{3}, -\frac{5}{3}, -1, \cdots$ (d) none

(29) For $a+(a+d)+(a+2d)+ \ldots\ldots +[a+(n-1)d]$, $S_n = ?$

(a) $a+(n-1)d$ (b) $\frac{n}{2}\{2a+(n-1)d\}$

(c) $\frac{a(1-r^n)}{1-r}$ (d) $\frac{a(r^n-1)}{r-1}$

(30) If A, G and H have usual meanings, then $G>?$ $(a>0, b>0)$

(a) H (b) A

(c) $\frac{1}{A}$ (d) $\frac{1}{G}$

(31) If a, b, c are in A.P., then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$, are in

(a) A.P (b) G.P

(c) H.P (d) None

(32) Formula for the n^{th} term of an arithmetic sequence is given by:

(a) $a_n = a+d$ (b) $a_n = a+nd$

(c) $a_n = a-nd$ (d) $a_n = a+(n-1)d$

(33) H.M between two numbers 'a' and 'b' is given by

(a) $\frac{a+b}{2}$ (b) $\frac{ab}{a+b}$

(c) $\frac{a+b}{2ab}$ (d) $\frac{2ab}{a+b}$

(34) S_n of geometric series if $|r| > 1$ is

(a) $\frac{n}{2}\{2a+(n-1)d\}$

(b) $\frac{a(r^n-1)}{r-1}$

(c) $\frac{a(1-r^n)}{1-r}$

(d) none

(35) A.M. between two numbers a and b is given by:

(a) $a+b$

(b) $a-b$

(c) $\frac{a-b}{2}$

(d) $\frac{a+b}{2}$

(36) For what value of 'n' is $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ the G.M between a and b ?

(a) $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) -1

(d)1

(37) Three G.M between 5 & 0.008 are

(a) 02,0.5.0.6

(b) 0.02,0.05,0.06

(c) 1,0.2,0.04

(d) none

(38) Two A.M between $\frac{3}{2}$ and $\frac{27}{14}$ are

(a) $\frac{1}{7},\frac{2}{3}$

(b) $\frac{23}{14},\frac{25}{14}$

(c) $\frac{16}{3},\frac{14}{5}$

(d) $\frac{19}{14},\frac{21}{14}$

(39) 7+77+777+..... n terms

(a) $\frac{7}{9}\left[\frac{10}{9}(10^n-1)-n\right]$

(b) $\frac{1}{9}\left[\frac{10}{9}(10^n-1)-n\right]$

(c) $\frac{2}{9}\left[\frac{10}{9}(10^n-1)-n\right]$

(d) $\frac{2}{7}\left[\frac{10}{9}(10^n-1)-n\right]$

(40) When $1.\overline{53}$ is written in the form of vulgar fraction then the difference between the numerator and denominator is

(a) 153

(b) 152

(c) 53

(d) 52

(41) If $y=1+2x+4x^2+8x^3+\ldots\ldots\ldots\infty$ then x=?

(a) $y+1$

(b) $y-1$

(c) $\frac{y-1}{y}$

(d) $\frac{y-1}{2y}$

(42) The H.M and A.M between two numbers are 4 and $\frac{9}{2}$ respectively the numbers are

(a) 5,4
(b) 7,2
(c) 6,3
(d) 12,-3

(43) $2+(2+5)+(2+5+8)+\ldots\ldots$ to n-terms =?

(a) $\frac{n(n+1)}{2}$
(b) $\frac{n(n+1)^2}{2}$
(c) $\frac{n^2(n+1)}{2}$
(d) none

Back solve

(44) A clock strikes twice when its hour hand is at one, four time when it is at 2 and so on. How many times does the clock strike in 12 hours?

(a) 78
(b) 128
(c) 156
(d) 96

(45) $\sum_{k=1}^{5}\left(k^2-(k-1)^2\right)=?$

(a) 5
(b) 25
(c) 125
(d) 625

(46) If $t, 2t+2, \ldots 3$ are in G.P. then the fourth term is;

(a) 27
(b) -27
(c) $\frac{27}{2}$
(d) $-\frac{27}{2}$

(47) $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+\cdots to\ 9-terms = ?$

(a) $-\frac{5}{6}$
(b) $-\frac{1}{2}$
(c) 1
(d) $-\frac{3}{2}$

(48) If $2+5+8+11+\cdots+a_n = 60100$ then n=?

(a) 100
(b) 200
(c) 150
(d) 250

(49) Which term of the sequence $3, \sqrt{3}, 1, \cdots$ is $\frac{1}{243}$?

(a) 12
(b) 13
(c) 14
(d) 15

(50) $11^3+12^3+13^3+\cdots+20^3$ is;

(a) odd integer divisible by 5
(b) An even integer
(c) Multiple of 10
(d) odd integer

(51) The quadratic equation in x such the arithmetic mean of its roots is 4 and its geometric mean is 9, is

(a) $x^2 - 8x + 81 = 0$ (b) $x^2 + 8x - 81 = 0$

(c) $x^2 + 81x + 8 = 0$ (d) $x^2 + 81x - 8 = 0$

(52) If the pth, qth, and rth terms of an A.P are in G.P then the common ratio of the G.P is _____

(a) $\frac{r-q}{q-P}$ (b) $\frac{q-r}{q-P}$

(c) $\frac{P-q}{r-P}$ (d) $\frac{p-q}{p-r}$

(53) If a, b, c form a G.P with common ratio r $(0 < r < 1)$. If a, 2b, 3c form an A.P then r = ?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{5}$ (d) $\frac{1}{4}$

(54) Three numbers form a G.P. If we double the middle number we get an A.P the common ratio of the G.P is _____

(a) $2\sqrt{3}$ (b) $2+\sqrt{3}$

(c) $2-\sqrt{3}$ (d) Both b and c

$\frac{a}{r}, a, ar$ ↓ $\frac{a}{r}, 2a, ar$ ↓ $2a - \frac{a}{r} = ar - 2a$

(55) If pth term of an A.P is $\frac{3P-1}{6}$ then the sum of first n-terms of the A.P is

Back Solve

(a) $\frac{n(3n+1)}{12}$ (b) $\frac{n(n+1)}{2}$

(c) $\frac{n(n+3)}{4}$ (d) $\frac{3n(n+1)}{4}$

$a\left(2 - \frac{1}{r}\right) = a(r-2)$

$2 - \frac{1}{r} = r - 2$

$4r = r^2 + 1$

$r^2 - 4r + 1 = 0$

(56) If pth term of an A.P is q and qth term is p, then mth term = ?

(a) p + q (b) p + q + m

(c) p + q – m (d) p + m – q

(57) The sum of 15 terms of an A.P is 600 their common difference is 5 then a_1 = ?

(a) 8 (b) 5

(c) 3 (d) 1

(58) The sum of three numbers in A.P is 15 and their product is 80 the largest number is _____

$a-d, a, a+d$ $a - d \times a \times a + d = 80$

$3a = 15$ $(5-d) \times 5 \times (5+d) = 8$

$a = 5$ $(25 - 5d)(5+d) = 8$

$125 + 25d - 25d - 5d^2 = 80$

$45 = 5d^2$

$9 = d^2$

$d = \pm 3$

(a) 2 (b) 5

(c) 8 (d) 11

(59) The sum of integers from 1 to 100 that are divisible by 2 or 5 = ?

(a) 3000 (b) 3020

(c) 3050 (d) 3090

$2+4+5+6+8+10+12+14+15+16+18+20$

125 80

45 77

(60) The ratio of sum of first 3 terms of a G.P to the sum of first 6 terms is 125 : 152 then r = ?

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$

(c) $\frac{1}{5}$ (d) $\frac{1}{3}$

(61) If $0.\overline{125} = \frac{p}{q}$ then p + q = ? $\frac{p}{q} = \frac{125}{999}$ Adding ratio $p+q = 999+125$ $= 1124$

(a) 125 (b) $\frac{125}{999}$

(c) 1124 (d) 9925

(62) The value of $0.4\overline{23}$ = ?

(a) $\frac{423}{999}$ (b) $\frac{419}{990}$

(c) $\frac{416}{999}$ (d) None of these

(63) The sum of three numbers in G.P is 19 and their product is 216 then r = ?

(a) 1.5 (b) 0.5

(c) 1.25 (d) 2.25

(64) The sum of first 6 – terms of a G.P is 9 times the sum of first 3 terms then r = ?

(a) 0 (b) 1

(c) -1 (d) 2 $G_6 = 9(a+ar+ar^2)$

$a+ar+ar^2+ar^3+ar^4+ar^5 = 9(a+ar+ar^2)$

$1+r+r^2+r^3+r^4+r^5 = 9(1+r+r^2)$

put r = 1

(65) If a, b, c are in H.P then $\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right)\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)$ = ?

(a) $\frac{4}{ac}-\frac{3}{b^2}$ (b) $\frac{ac}{4}-\frac{b^2}{3}$

(c) $\frac{acb}{4-c^2}$ (d) $\frac{1}{ab}-\frac{4}{ac}$

PERMUTATIONS, COMBINATIONS AND PROBABILITY

Factorial Notation

Let n be a positive integer. Then the product n(n-1) (n-2) ….3.2.1 is called **n** factorial.

Symbol

In above case n factorial is denoted by $n!$ and it can also be written as $\lfloor n$.

Examples

0! =1(by definition)
1! = 1
2! = 2.1 = 2 = 2! = 2.1!
3! = 3.2.1 = 6
4! = 4.3.2.1 = 24 = 4! = 4.3!
5! = 5.4.3.2.1 = 120 = 5! = 5.4!
6! = 6.5.4.3.2.1 = 720 = 6! = 6.5!
7! =5040
8! =40320
9! =362880
10! =3628800

Note

- $(a+b)! \neq a!+b!$
- $(a.b)! \neq a!.b!$

Principles of Counting

(i).Fundamental Principle of Addition
If an operation can be performed in **m** ways and another operation can be performed in **n** ways, independent of first operation then either of the two operations can be performed in **m+n** ways. This is called fundamental principal of addition.
(ii).Fundamental Principle Of Multiplication
If an operation can be performed in **m** ways and afterwards a second operation can be performed in **n** ways then both these operations can be performed in **m x n** ways. This is called fundamental principle of multiplication.

Note

- These principles can be extended up to finite number of operations.

Permutations

The different arrangements which can be made out of a given number, taken one or more at a time, are called permutations.

OR

An arrangement of a finite number of objects one or more at a time is called a permutation of these objects (w.r.t. their order).

Rules of Permutations

(i). The number of permutations of n-distinct things taken all at a time is n!

(ii). The n umber of permutations of "n" distinct objects taken "r" at a time is given by: ${}^{n}p_{r} = \frac{n!}{(n-r)!}\ 0 \le r \le n$

(iii). ${}^{n}p_{n} = n!$

(iv). ${}^{n}p_{0} = 1$

(v). If out of "n" given objects "p" are alike and of one kind,"q" are alike and of second kind ,"r" are alike and of third kind ,and so on then the number of permutations of these n-objects taken all at a time is given by: $\frac{n!}{p!.q!.r!}$,the formula can also be written as, $\binom{n}{p,\ q,\ r}$

(vi). The number of circular permutations of "n" different objects taken all at a time is $(n-1)!$.

(vii). Number of arrangements of n-beads (keys) etc, can be arranged on a circular wire (key ring) in $\frac{(n-1)!}{2}$

Combinations

The different groups or selections which can be made out of a given number of objects ,taken one or more at a time (irrespective of their arrangements) are called combinations.

Rules of Combinations

(i). The number of combinations of "**n**" distinct objects, taken all at a time is 1.

(ii). The number of combinations of "**n**" distinct objects taken "***r***" at a time is given by: ${}^{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

(iii). The number of combinations of "**n**" distinct objects taken "***r***" at a time when "***p***" particular objects are always included is given by: ${}^{n-p}C_{r-p}$

(iv). The number of combinations of "**n**" distinct objects taken "***r***" at a time when "***p***" particular objects are never included is given by: ${}^{n-p}C_{r}$

(v). ${}^{n}C_{0} = {}^{n}C_{n} = 1$

(vi). ${}^{n}C_{r} = {}^{n}C_{r-1}$

(vii). ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

If ${}^{n}C_{r} = {}^{n}C_{s}$,the either $r = s$ or $r + s = n$

(viii). $\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{n-r+1}{r}$

(ix). ${}^{n}C_{r}.r! = {}^{n}P_{r}$

(x). ${}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + \ldots\ldots\ldots + {}^{n}C_{n} = 2^{n}$

(xi). ${}^{n}C_{0} + {}^{n}C_{2} + {}^{n}C_{4} + \ldots\ldots\ldots = {}^{n}C_{1} + {}^{n}C_{3} + \ldots\ldots\ldots = 2^{n-1}$

(xii). The number of ways of selecting one or more objects out of "n" objects is $2^{n} - 1$

Notes

- In most cases, given problem cannot be determined whether it belongs to permutation or combination. On the basis of few words or on the assumption in expression of language given in the problem, we can identify that problem according to the following table.

Permutation	Combination
Arrangements, Standing or sitting in a row or in a circle Problem regarding digits, letters(A,B,C….etc) Formation of words, numbers etc	Selection,Choice,Draw ,etc Distributation, formation of a group, committee, team etc Problem regarding geometry

Probability

Probability is the numerical evaluation of a chance that a particular event would occur.

Before defining the probability classically we shall discuss some important terms related to the probability.

Random experiment	An experiment in which all possible outcomes are known in advance.
Sample space	The set of all possible outcomes of an experiment is called the sample space of the experiment. It is denoted by S.
Event	A subset of the sample space is called an event.
Mutually exclusive or disjoint events	Two events A and B are said to be mutually exclusive or disjoint if and only if they cannot both occur at the same time i.e. they have no point in common. $A \cap B = \phi$

Equally likely events	Two events A and B are said to be equally likely when one event is as likely to occur as the other.
Independent events/dependent events	Two or more events such that occurrence of any one does not depend on the occurrence of any other, they are said to be independent events. Other wise dependent events.

Definition & Results on Probability

Probability	The probability of happening of an event E is defined as; $P(E)=\frac{n(E)}{n(S)}$.
Probability of not happening of an event	$P(\overline{E})=1-P(E)$
Results: If E is an event and S is the sample space;	
(i). $0\le P(E)\le 1$	
(ii). $P(E)+P(\overline{E})=1$	
(iii). If E is an impossible event then $P(E)=0$.	
(iv). If E is a certain event then $P(E)=1$.	
(v). Addition theorem: for any two events A & B $P(A\cup B)=P(A)+P(B)-P(A\cap B)$.	
(vi). If A & B are mutually exclusive events then $P(A\cup B)=P(A)+P(B)$.	
(vii). Multiplication law :if A & B are any two independent events ,then the probability that both the events occur simultaneously in any order is given as; $P(A\cap B)=P(A)\times P(B)$	

PRACTICE EXERCISE

(1) $0! = ?$

(a) 0
(b) 1
(c) 2
(d) ∞

(2) $4! + 0! =$

(a) 24
(b) 20
(c) 25
(d) 23

(3) $\dfrac{(n+3)!}{(n+1)!} = ?$

(a) $(n+2)(n+1)$
(b) $(n+3)(n+2)$
(c) $(n+3)(n+2)(n+1)$
(d) $(n+1)!$

(4) ${}^{n}P_{r} = ?$

(a) $\dfrac{n!}{(n+r)!}$
(b) $\dfrac{n}{n-r}$
(c) $\dfrac{n!}{(n+1)!}$
(d) $\dfrac{n!}{(n-r)!}$

(5) ${}^{7}P_{7} = ?$

(a) 720
(b) 360
(c) 5040
(d) 120

(6) ${}^{15}P_{5} = ?$

(a) 243058
(b) 342040
(c) 360360
(d) 542002

(7) How many signals can be given with three different flags when any number of them may be used?

(a) 12
(b) 15
(c) 17
(d) 19

(8) In how many ways can four books of Urdu and three books of Islamiat be placed on a shelf so that the books on the same subject are always together?

(a) 288
(b) 32
(c) 144
(d) 270

(7−2+1)! × 2!
6! × 2!
4! × 3! × 2!
24 × 6 × 2

(9) ${}^{n}C_{r} = ?$

(a) $\dfrac{n}{r(n-r)}$
(b) $\dfrac{n!}{(n-r)!}$
(c) $\dfrac{n!}{r!(n-1)!}$
(d) $\dfrac{n!}{r!(n-r)!}$

(10) ${}^{4}C_{4} = ?$

(a) 24
(b) 120
(c) 1
(d) 0

(11) How many even numbers of four-digit number can be formed from the digits 1,2,3,4,5, if repetition is permitted.
(a) 24 (b) 40
(c) 250 (d) 124

(12) $^{17}C_{10} + {}^{17}C_{12} = ?$
(a) $^{17}C_2$ (b) $^{16}C_2$
(c) $^{16}C_{12}$ (d) none

(13) The number of diagonals of a 12 sided figure is:
(a) 66 (b) 54
(c) 30 (d) 48

(14) ${}^nC_r + {}^nC_{r-1} = ?$
(a) ^nC_r (b) $^{n-1}C_r$
(c) $^{n+1}C_{r-1}$ (d) $^{n+1}C_r$

(15) The probability that a 3-digit number chosen at random divisible by 5 is
(a) $\frac{1}{3}$ (b) $\frac{1}{5}$
(c) $\frac{1}{7}$ (d) $\frac{1}{10}$

(16) If A and B are disjoint then $P(A \cup B) = ?$
(a) $P(A)+P(B)-P(A \cap B)$ (b) $P(A).\ P(B)$
(c) $1-P(E)$ (d) $P(A)+P(B)$

(17) If a probability of a bulb being defective is 0.2. Estimate how many defective bulbs there would be in 600 bulbs.
(a) 30 (b) 120
(c) 50 (d) 60

(18) If A & B are mutually exclusive events then $P(A \cup B) = ?$
(a) $P(A)+P(B)$ (b) $P(A)+P(B)-P(A \cap B)$
(c) $P(A)+P(B)-P(AUB)$ (d) $P(A)+P(B)+P(AUB)$

(19) The probability of getting 5, when one die is rolled is
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{5}$ (d) $\frac{1}{6}$

(20) If $P(B) = \frac{7}{12}$ then $P(\overline{B}) = ?$
(a) $\frac{1}{12}$ (b) $\frac{5}{12}$
(c) $\frac{8}{7}$ (d) $\frac{7}{5}$

(21) If A & B are independent events then $P(A \cap B)$ is
(a) $P(A) + P(B)$ (b) $P\left(\frac{A}{B}\right)$
(c) $P(A)\ .\ P(B)$ (d) $P(A) - P(B)$

(22) Factorial of -9 =?
(a) -362880
(b) 362880
(c) 40320
(d) Can not be calculated

(23) If A is an impossible event then $P(A)$ =?
(a) 1
(b) -1
(c) 0
(d) ∞

(24) Two people are selected at random, from the group of 10 people in which 60% have blue eyes. The probability that none of them will have blue eyes?
(a) 0.11
(b) 0.6
(c) 0.13
(d) 0.20

(25) $\dfrac{19!}{19!(19-19)!}=?$
(a) 0
(b) 1
(c) undefined
(d) 19

(26) 52=?
(a) $\dfrac{52!}{51!}$
(b) $\dfrac{52!}{53!}$
(c) $52.51!$
(d) $\dfrac{51!}{52!}$

(27) The factorial form of $(n+2)(n+1)n$ is
(a) $(n+2)!$
(b) $\dfrac{(n+2)!}{(n+1)!}$
(c) $\dfrac{(n+2)!}{(n-1)!}$
(d) $\dfrac{(n+2)!}{n!}$

(28) In how many ways can 7 keys be arranged in a circular key ring?
(a) 5040
(b) 720
(c) 360
(d) 1440

(29) If ${}^nC_{11}={}^nC_9$ then n =?
(a) 11
(b) 9
(c) 10
(d) 20

(30) Two dice are thrown simultaneously find the probability that the sum of the dots of the dice is a prime no?
(a) $\dfrac{4}{9}$
(b) $\dfrac{5}{9}$
(c) $\dfrac{5}{12}$
(d) $\dfrac{7}{36}$

(31) If $n(E_1)=18$ and $n(E_2)=6$ and $n(S)=36$ also E_1 and E_2 are independent then $P(E_1\cap E_2)$=?
(a) 0
(b) $\dfrac{22}{36}$
(c) $\dfrac{1}{12}$
(d) $\dfrac{1}{36}$

(32) Three dice are thrown simultaneously find the probability to get greatest sum of dots?

(a) $\frac{18}{216}$ (b) $\frac{1}{18}$

(c) $\frac{1}{216}$ (d) $\frac{6}{216}$

(33) A number is selected at random from the three digit numbers, the probability that the number is divisible by 4 is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{16}$ (d) $\frac{1}{8}$

(34) What is the probability to get a black jack from a well shuffled pack of playing cards?

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$

(c) $\frac{3}{13}$ (d) $\frac{1}{26}$

(35) If $\binom{n}{5}=\binom{n}{7}$,then n=?

(a) 12 (b) 5

(c) 7 (d) 0

(36) If ${}^{n}P_{6}=3.{}^{n}P_{5}$,then n=?

(a) 6 (b) 7

(c) 8 (d) 10

(37) There are 10 bulbs in a hall. Each one of them can be switched on independently. In how many ways the hall can be illuminated?

(a) 10^2 (b) 1023

(c) 2^{10} (d) 10!

(38) $\binom{13}{9}-\binom{12}{8}=?$

(a) ${}^{12}C_{7}$ (b) ${}^{13}C_{7}$

(c) ${}^{12}C_{9}$ (d) ${}^{13}C_{8}$

(39) There are 13-points in a plane no three of which are collinear expect 5. How many total lines can be drawn in the plane?

(a) 78 (b) 68

(c) 69 (d) 80

(40) If $\binom{15}{3x}=\binom{15}{x+3}$,then x=?

(a) 3 (b) 4

(c) 5 (d) 9

(41) If ${}^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{x}$ then x = ?

(a) 4 (b) r – 1

(c) r + 1 (d) n

(42) Number of the arrangements of the letters of the word "EAMCET" so that no two vowels are together is _____

(a) 360 (b) 144

(c) 72 (d) 54

(43) How many 10-digit numbers can be written by using the digits 1 and 2?

(a) ${}^{10}C_{1} + {}^{9}C_{2}$ (b) 2^{10}

(c) ${}^{10}C_{2}$ (d) 10!

(44) A cricket team of 11 players is to be selected from 13 players 4 of which are bowlers and 2 are wicket keepers, team consisting one wicket keeper and at least 3 bowlers is _____

(a) 8 (b) 22

(c) 112 (d) 220

(45) The number of 5-digits telephone numbers having at least one of their digit repeated is _____

(a) 90,000 (b) 10, 000

(c) 30240 (d) 69760

(46) The number of ways in which 5 identical rings can be worn on the four fingers of a hand = ?

(a) 256 (b) 512

(c) 1024 (d) 2048

(47) The number of ways of arranging 21 white balls and 3 black balls in a row so that no two black balls are placed together?

(a) 1440 (b) 1540

(c) 1640 (d) 1740

(48) A box contains 2 white, 3black and 4 red balls. The number of ways in which 3 balls be drawn from the box if at least one black ball is to be included in each draw is _____

(a) 32 (b) 48

(c) 64 (d) 96

(49) If ${}^{n+2}C_{8} : {}^{n-2}P_{4} = 57 : 16$ then n =?

(a) 19 (b) 21

(c) 23 (d) 27

(50) The number of rectangles that you can find on a chess board = ?

(a) 1196 (b) 1096

(c) 1296 (d) 1000

MATHEMATICAL INDUCTION AND BINOMIAL THEOREM

Binomial Theorem

An algebraic expression consisting of two terms such as $a+x$, $x-2y$, $ax+b$ etc is called a binomial or a binomial expression.

Binomial Formula

The following expansions are valid for any $n \in z^+$.(n is called index)

1. $(a+b)^n = a^n + \frac{n}{1!}a^{n-1}b^1 + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-2}b^3 + \ldots\ldots\ldots + b^n$

2. $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b^1 + {}^nC_2a^{n-1}b^2 + \ldots\ldots + {}^nC_ra^{n-r}b^r + \ldots\ldots + {}^nC_na^{n-n}b^n$

3. $(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$

Characteristics of the Binomial Expansion

The following points can be observed in the expansion of $(a+b)^n$.

1. The number of terms in the expansion is one greater than its index. (i.e. number of terms $= n+1$)
2. The sum of exponents of **a** and **b** in each term of the expansion is equal to its index.
3. The exponent of **a** decrease from index to zero.
4. The exponent of **b** increases for zero to index.
5. The coefficients of the term equidistant form beginning and end of the expansion are equal as ${}^nC_r = {}^nC_{n-r}$.
6. The T_{r+1} is the r^{th} term in the expansion of $(a+b)^n$ is given by $T_{r+1} = \binom{n}{r}a^{n-r}b^r$.
7. **Middle term(s)** in the expansion of $(a+b)^n$

   a. **If n is even**, then there is **only one** middle term *i.e.* $\left(\frac{n+2}{2}\right)th$ term.

   b. **If n is odd**, then there will be **two** middle terms. *i.e.* $\left(\frac{n+1}{2}\right)th$ term and $\left(\frac{n+3}{2}\right)th$ term.

i. In the expansion

$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b^1 + {}^nC_2a^{n-1}b^2 + \ldots\ldots + {}^nC_ra^{n-r}b^r + \ldots\ldots + {}^nC_na^{n-n}b^n$

${}^nC_0, {}^nC_1, {}^nC_2, \ldots\ldots, {}^nC_n$ are called binomial coefficients.

ii. Properties of binomial coefficients:

In the expansion of $(1+x)^n$ where $n \in N$

(i). sum of binomial coefficients is 2^n *i.e.* ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n$

(ii). Sum of binomial coefficients of odd terms = sum of binomial coefficients of even terms = 2^{n-1} *i.e.* ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots\ldots {}^nC_n = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots\ldots {}^nC_{n-1} = 2^{n-1}$ *(n is even)*

The binomial theorem when the index n is a negative integer or a fraction

1. When n is negative integer or a fraction, and $|x<1|(i.e, -1<x<1)$ then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots$$

$$\ldots\ldots + \frac{n(n-1)(n-2)\ldots.(n-r+1)}{r!}x^r + \ldots\ldots\ldots$$

2. The general term in the expansion is $T_{r+1} = \frac{n(n-1)(n-2)\ldots.(n-r+1)}{r!}x^r$

3. The number of terms in the expansion is always infinite.

Some particular cases of the expansion of $(1+x)^n$,when $n<0$

Binomial expansion	$(r+1)^{th}$ *Term*
$(1+x)^{-1}$	$(-1)^r x^r$
$(1+x)^{-2}$	$(-1)^r (r+1)x^r$
$(1+x)^{-3}$	$(-1)^r \frac{(r+1)(r+2)}{2}x^r$
$(1-x)^{-1}$	x^r
$(1-x)^{-2}$	$(r+1)x^r$
$(1-x)^{-3}$	$\frac{(r+1)(r+2)}{2}x^r$

PRACTICE EXERCISE

(1) The no of terms in the expansion of $(a-b)^{17}$ is

(a) 2 (b) 17

(c) 18 (d) 20

(2) The middle term in the expansion of $(x+y)^{30}$ is

(a) 13^{th} (b) 14^{th}

(c) 15^{th} (d) 16^{th}

(3) The binomial coefficient of 21st term in the expansion of $(a+b)^{23}$ is

(a) 1771 (b) 2891

(c) 3421 (d) 1563

(4) The sum of coefficients in the expansion of $(1+x)^7$ is

(a) 64 (b) 32

(c) 100 (d) 128

(5) Sum of even coefficients is equal to ..... in the binomial expansion of $(1+x)^n$ where $n \in Z^+$

(a) 2 n (b) 2n-1

(c) 2^{n-1} (d) 2^n

(6) If 'x' be so small that its square and higher powers can be neglected then $\frac{(1+x)^2}{(1+2x)^3} = ?$

(a) 1+8x (b) 1-4x

(c) 2+7x (d) 1+4x

(7) The general term of $\left(\frac{a}{2}-\frac{2}{a}\right)^6 = ?$

(a) $(-1)^r \binom{6}{r}\left(\frac{a}{2}\right)^{6-2r}$ (b) $\binom{6}{r}\left(\frac{a}{2}\right)^{6-2r}$

(c) $2^r \binom{6}{r}\left(\frac{a}{2}\right)^{6-2r}$ (d) $(-1)\binom{6}{r}\left(\frac{a}{2}\right)^{6-2r}$

(8) The 6^{th} term from the end in the expansion of $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11}$ is

(a) $16x$ (b) $77x$

(c) $\frac{16}{77}x$ (d) $\frac{77}{16x}$

(9) $\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\cdots+n\binom{n}{n}=?$

(a) 2^{n-1}

(b) 2.2^{n-1}

(c) $n.2^{n-1}$

(d) None

(10) In the expansion of $(a+b)^{16}$, in every term the sum of the exponents of a and b is:

(a) 15

(b) 7

(c) 1

(d) 16

(11) The number of terms in the expansion of $(a+b)^n$ where $n \in Z^+$ is

(a) n-1

(b) n

(c) n+1

(d) none

(12) The general term in the expansion of $(a+b)^n$ is denoted by T_{r+1} so T_{r+1}=?

(a) $\binom{n}{r}a^{n-r}b^r$

(b) $\binom{n}{r}a^nb^r$

(c) $\binom{n}{r}a^rb^n$

(d) none of these

(13) The index in $(a-b)^{17}$ is

(a) 18

(b) 17

(c) a

(d) b

(14) Sum of odd coefficients in the binomial expansion of $(a+b)^n$ where $n \in Z^+$ is equal to

(a) 2^n

(b) 2n

(c) 2^{n-1}

(d) 2 (n-1)

(15) The 3^{rd} term in the expansion of $\left(1+\frac{6}{x}\right)^{\frac{1}{3}}=?$

(a) $-\frac{4}{x^2}$

(b) $-\frac{27}{1024x}$

(c) -1024

(d) 27

(16) The value of $(8.2)^{1/3}$ correct to 3 decimal places is

(a) 2.000

(b) 2.0021

(c) 2.00156

(d) 2.016

(17) If 'x' be so small that its square and higher powers may be neglected then $\frac{(1-x)^{\frac{1}{2}}(9-4x)^{\frac{1}{2}}}{(8+3x)^{\frac{1}{3}}}=?$

(a) $\frac{3}{2}-\frac{48x}{61}$

(b) $\frac{3}{2}-\frac{61x}{48}$

(c) $\frac{2}{3}-\frac{61x}{48}$

(d) $\frac{1}{2}-\frac{x}{48}$

(18) The coefficient of the term involving x^5 in the expansion of $\left(x^2-\frac{3}{2x}\right)^{10}=?$

(a) 15309
(b) -15309
(c) $-\frac{15309}{8}$
(d) $\frac{15309}{9}$

(19) $\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\cdots+\binom{n}{n-1}=?$

(a) 2^{n-1}
(b) 2^n
(c) 2^n-1
(d) 2^n+1

(20) $2+2^2+2^3+\ldots\ldots+2^n=?$

(a) 2^{n-1}
(b) $2(2n)$
(c) $2(2^{n+1})$
(d) $2(2^n-1)$

(21) $\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+\cdots+\binom{k+2}{3}=?$

(a) $\binom{k+2}{4}$
(b) $\binom{k+3}{3}$
(c) $\binom{k}{3}$
(d) $\binom{k+3}{4}$

(22) Which one is false?

(a) In the binomial expansion the binomial coefficients equidistant from beginning and end are equal
(b) The binomial coefficient of general term T_{r+1} is ${}^nC_r$.
(c) $n^2 > n+4$ is true for $n \geq 3$.
(d) The index in $(x-y)^8$ is 9

(23) If 'n' is even then the middle term in the expansion of $(a+b)^n$ will be

(a) $(n+1)^{th}$ term
(b) $\frac{n+2}{3}$ th term
(c) n^{th} term
(d) $\left(\frac{n}{2}+1\right)^{th}$ term

(24) For $(1+x)^n$ the 2^{nd} term is

(a) $\frac{n(n-1)}{2!}x^2$
(b) nx
(c) $\frac{n(n-1)(n-2)}{3!}x^3$
(d) 0

(25) The sum of coefficients in the expansion of $(1+b)^n$ is

(a) n
(b) 2^n
(c) 2^{n-1}
(d) x^{2n}

(26) The values of "x" for which the expansion $(1-4x)^{-\frac{1}{2}}$ is valid?

(a) $|x|<1$
(b) $|x|<\frac{1}{4}$
(c) $|x|>1$
(d) $|x|>\frac{1}{4}$

(27) The middle term in the expansion of $\left(x+\frac{1}{x}\right)^{10}$ is.

(a) $\frac{1}{x}\binom{10}{4}$
(b) ${}^{10}C_5$
(c) $\binom{10}{7}x^4$
(d) ${}^{10}C_6$

(28) The term free from Z in the expansion of $\left(\sqrt{\frac{Z}{3}}+\frac{3}{2Z^2}\right)^{10}$ is;

(a) $\frac{5}{4}$
(b) $\frac{1}{4}$
(c) $\frac{9}{4}$
(d) $\frac{4}{5}$

(29) Total number of terms in the expansion of $(x+z)^{50}+(x-z)^{50}$ are;

(a) 100
(b) 102
(c) 52
(d) 26

(30) The coefficient of x^{100} in the expansion of $(1-x)^{-3}$ is;

(a) −1001
(b) 10001
(c) 5151
(d) −5151

(31) If the co-efficient of (2r +4)th and (r – 2)th term in the expansion of $(1+x)^{18}$ are equal then r = ?

(a) 6
(b) 8
(c) 11
(d) 16

(32) The co-efficient of x^5 in the expansion of $(1+x^2)(1+x)^4 = ?$

(b) 30 (b) 4

(c) 90 (d) 60

(33) The value of "m" such that in the binomial expansion of $(1-x)^m$ co-efficient of x^2 is 3 are _____

$3 = {}^mC_2 (1)^{m-2} x^2$

$3 = \frac{m(m-1)(m-2)!}{(m-2)!\,2!}$

$6 = m^2 - m$

(a) m = 3 (b) m = -2

(c) m = 1 (d) both a and b

(34) The greatest positive integer which divides $11^n - 10n - 1$ is _____

$0 = m^2 - m - 6$

$(m=3, m=-2)$

(a) 100 (b) 200

(c) 300 (d) 1000000

(35) If x^{18} occurs in the rth term in the expansion $(2x^4 - 3x)^9$ then the value of r = ?

$(2x^4)^3 (3x)^6$

$-(2x^{12} \cdot 3x^6)$

(a) 6 (b) 7

(c) 8 (d) 9

(36) The co-efficient of x^{-2} in the expansion of $\left(3x - \frac{7}{x}\right)^8 = ?$

(a) -25412184 (b) 2423922

(c) 15421 (d) 31624620

(37) The term independent of x in the expansion of $\left(\sqrt{x} - \frac{n}{x^2}\right)^{10}$ is 405, then n = ?

(a) 3 (b) 4

(c) 5 (d) -1

(38) If the co-efficient of x^2 and x^3 in the expansion of $(3+ax)^9$ are same then a = ?

(a) 8 (b) 7

(c) $\frac{9}{7}$ (d) $\frac{8}{7}$

(39) If the middle term in the expansion of $\left(\frac{k}{2} + 2\right)^8$ is 1120 then k = ?

(a) 2 (b) -3

(c) 1 (d) 0

(40) The sum of co-efficients of last 3-terms in the expansion of $(8-3x)^{\frac{1}{2}} = ?$

(a) 8 (b) $\frac{8}{3}$

(c) $-\frac{8}{3}$ (d) Cannot be determine

FUNDAMENTALS OF TRIGONOMETRY

Trigonometry

The branch of Mathematics, which deals with the study of measurements of triangles is called **trigonometry**.

Types of Trigonometry

There are two types of trigonometry.

i). Plane Trigonometry
ii). Spherical Trigonometry

Plane Trigonometry

In this type we are concerned with angles, triangles and other figures, which lie in a plane.

Spherical Trigonometry

In this type we deal with spherical triangles, that is, triangles, which lie on a sphere, and therefore sides of which are circular arcs.

Angles

Two rays with a common starting point form an angle. One of the rays of angle is called **initial side** and the other as **terminal side**. Showing the direction of rotation from the initial side to the terminal side identifies the angle.

OR

In elementary geometry an angle is defined as the union of two non-collinear rays, which have a common end point.

- An angle is said to be positive/negative if the rotation is anti-clock wise/clock wise.

Systems of Measures of Angles

There are two different systems of units for measurement of angles in trigonometry.

i). Sexagesimal System (English System)
ii). Circular System

i). Sexagesimal System

In this system of measurement of angles the angles are measured in degrees, minutes and seconds.

Degree

If the circumference of a circle is divided into 360 congruent (equal in length) parts, the angle subtended by one part at the center of the circle is called a degree.

1 degree (1^0) is divided into 60 minutes (60′) and 1 minute (1′) is divided into 60 seconds (60″).

As this system of measurement of angles owes its origin to the English and because 90, 60 are multiples of 6 and 10, so it is known as **English system** or **sexagesimal system**.

II). Circular System

There is another system of angular measurement, called the circular system. In this system, the unit of measurement of angles is the ''radian''.

Radian

Radian is the measure of the angle subtended at the center of the circle by an arc, whose length is equal to the radius of the circle.

Relation between the Length of an Arc of a Circle and the Circular Measure of Its Central Angle

- $l = r\theta$ OR $\theta = \frac{l}{r}$, if θ is in radians.
- $l = r\frac{\pi}{180}\theta$, if θ is in degrees.

Where '**r**' is the radius of the circle, **ℓ** is the length of the arc and θ is the circular measure of the central angle.

Note

Remember that '**r**' and '**ℓ**' are measured in terms of the same unit and the angle θ is always measured in radians.

Area Of A Sector Of A Circle Whose Central Angle Is θ:

- $A = \frac{1}{2}r^2\theta$, when the radius and the central angle (in radians) is given
- $A = \frac{1}{2}rl$, when radius of the circle and the value of the arc length is given.

Relationship between Degree and Radians

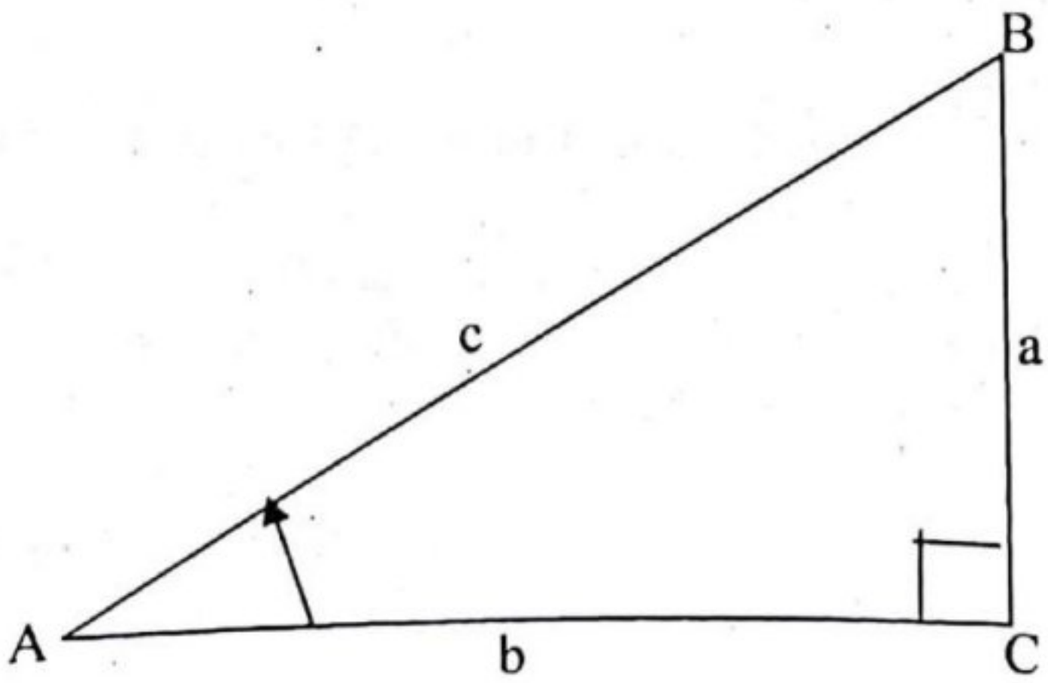
(i). 1 radian= $\frac{180°}{\pi}$ =57.29577°app

(ii). 1°= $\frac{\pi}{180°}$ = 0.01745 radians

Remember that:

- one rotation = $360°$
- $\frac{1}{2}rotations\,(anticlockwise) = 180°$ (Straight Angle)
- $\frac{1}{4}rotations\,(anticlockwise) = 90°$ (Right Angle)

Definition of Trigonometric Functions for a Right Triangle



(i). $\sin A = \frac{a}{c} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$

(ii). $\cos A = \frac{b}{c} = \frac{\textit{adjacent} \text{ side}}{\text{Hypotenuse}}$

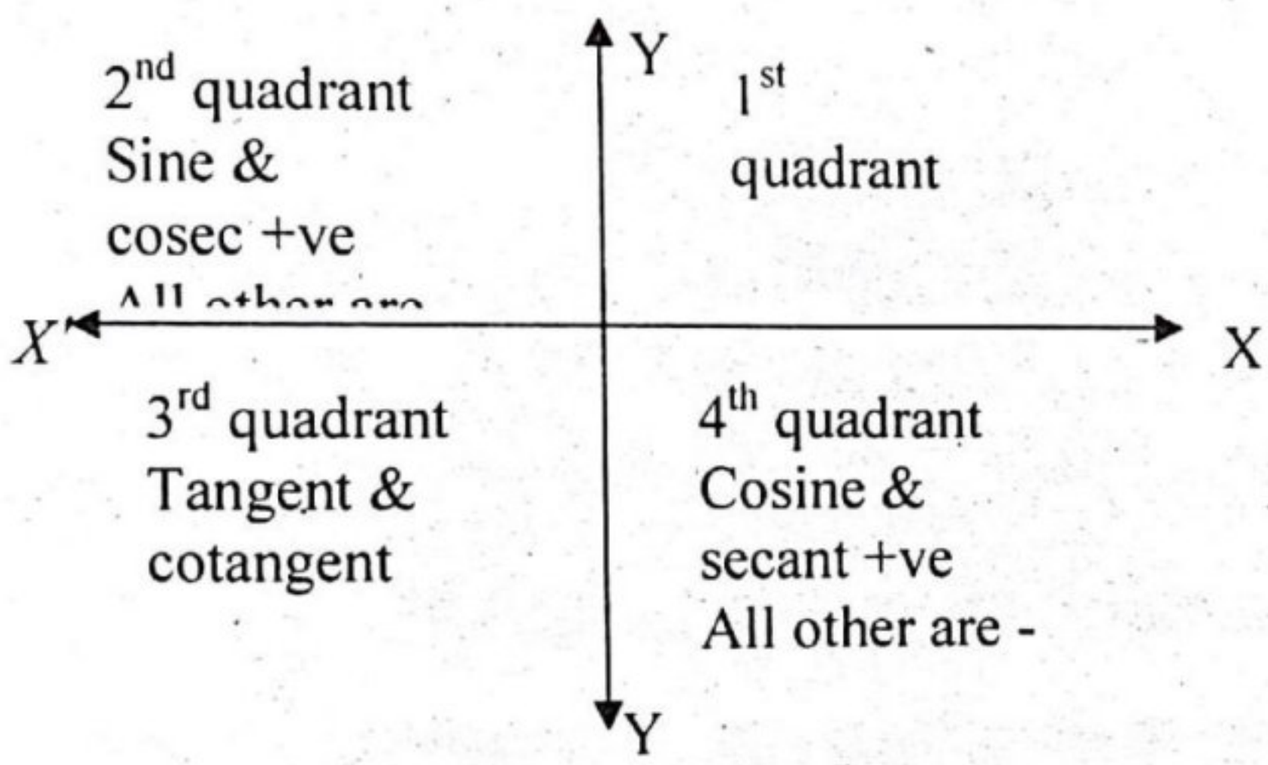
(iii). $\tan A = \frac{a}{b} = \frac{\text{Opposite side}}{\textit{adjacent} \text{ side}}$

(iv). $\cot A = \frac{b}{a} = \frac{\textit{adjacent} \text{ side}}{\text{Opposite side}}$

(v). $\sec A = \frac{c}{b} = \frac{\text{Hypotenuse}}{\textit{adjacent} \text{ side}}$

(vi). $\text{cos}\acute{e}cA = \frac{c}{a} = \frac{\text{Hypotenuse}}{\textit{opposite} \text{ side}}$

Signs Of The Trigonometric Ratios



Some Standard Interconvertions between Degrees and Radians:

ANGLE IN DEGREES	ANGLE IN RADIANS	ANGLE IN DEGREES	ANGLE IN RADIANS
0°	0	210°	$7\pi/6$
30°	$\frac{\pi}{6}$	225°	$\frac{5\pi}{4}$
45°	$\frac{\pi}{4}$	240°	$\frac{4\pi}{3}$
60°	$\frac{\pi}{3}$	270°	$\frac{3\pi}{2}$
90°	$\frac{\pi}{2}$	300°	$\frac{5\pi}{3}$
120°	$\frac{2\pi}{3}$	315°	$\frac{7\pi}{4}$
135°	$\frac{3\pi}{4}$	330°	$\frac{11\pi}{6}$
150°	$\frac{5\pi}{6}$	360°	2π
180°	π		

Relationship among Trigonometric Functions

(i). $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(ii). $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$

(iii). $\sec\theta = \dfrac{1}{\cos\theta}$

(iv). $\sin^2\theta + \cos^2\theta = 1$

(v). $1 + \tan^2\theta = \sec^2\theta$

(vi). $\cos ec^2\theta - \cot^2\theta = 1$

Functions of negative angles:

(i). $\sin(-\theta) = -\sin\theta$

(ii). $\cos(-\theta) = \cos\theta$

(iii). $\tan(-\theta) = -\tan\theta$

(iv). $\cot(-\theta) = -\cot\theta$.

(v). $\sec(-\theta) = \sec\theta$

(vi). $\cos ec(-\theta) = -\cos ec\theta$

Domains, Ranges and Periods of Trigonometric Functions

Trigonometric Function	Domain	Range	Period
$Y = \sin x$	$x \in \mathbb{R}$	$-1 \le Y \le 1$	2π
$Y = \cos x$	$x \in \mathbb{R}$	$-1 \le Y \le 1$	2π
$Y = \tan x$	$x \in \mathbb{R}$ But $x \neq (2k+1)\dfrac{\pi}{2}, k \in \mathbb{Z}$	$Y \in \mathbb{R}$	π
$Y = \cot x$	$x \in \mathbb{R}$ but $x \neq k\pi, k \in \mathbb{Z}$	$Y \in \mathbb{R}$	π
$Y = \sec x$	$x \in \mathbb{R}$ But $x \neq (2k+1)\dfrac{\pi}{2}, k \in \mathbb{Z}$	$Y \ge 1$ OR $Y \le -1$	2π
$Y = \cos ecx$	$x \in \mathbb{R}$ But $x \neq k\pi, k \in \mathbb{Z}$	$Y \ge 1$ OR $Y \le -1$	2π

Trigonometric Functions In Terms Of Other Trigonometric Functions

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\cos ec\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\cos ec\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{sec\theta}$	$\frac{\sqrt{co\sec^2\theta-1}}{co\sec\theta}$
$\tan\theta$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\cos ec^2\theta-1}}$
$\cot\theta$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\cos ec^2\theta-1}$
$\sec\theta$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1-\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\cos ec\theta}{\sqrt{\cos ec^2\theta-1}}$
$\cos ec\theta$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\cos ec\theta$

PRACTICE EXERCISE

(1) The 3600^{th} part of the degree is called_____
(a) Degree (b) Minute
(c) Second (d) None

(2) Basic unit of measurement of angle is
(a) Degree (b) Radian
(c) Second (d) Minute

(3) The ratio between the area of the sector to the length of arc of a circle is
(a) $2r:1$ (b) $2:r$
(c) $1:2r$ (d) $r:2$

(4) Sign of the $\cos(-885)$ is
(a) -ve (b) +ve
(c) 0 (d) Both (a) & (b)

(5) $(1-\cos^2\phi)(1+\cot^2\phi)=?$
(a) 0 (b) 1
(c) -1 (d) 2

(6) $\dfrac{1}{1-\sin\phi}+\dfrac{1}{1+\sin\phi}=?$
(a) $\sec^2\phi$ (b) $2\cos ec^2\phi$
(c) $2\sec^2\phi$ (d) $\tan^2\phi$

(7) If $\sin\phi<0$ and $\tan\phi>0$ then terminal side lies in which quadrant
(a) I (b) II
(c) III (d) IV

(8) $\cos 30^{\circ}\cos 60^{\circ}-\sin 30^{\circ}\sin 60^{\circ}=?$
(a) 0 (b) 0.866
(c) 1 (d) None

(9) 1 radian = ... Degree.
(a) $57\,17'45''$ (b) $57\,18'48''$
(c) $57\,18'32''$ (d) $57\,19'43''$

(10) 1° = ... radians
(a) 0.1745 (b) 0.01745
(c) 0.001745 (d) 0.0001745

(11) $(\cot^2\phi-1)(1-\cos^2\phi)=?$
(a) $1-\sin^2\phi$ (b) $1+\sin^2\phi$
(c) $\cos^2\phi-\sin^2\phi$ (d) All

(12) The coterminal angle of -45^0 is
(a) 45^0 (b) 315^0
(c) -315^0 (d) All

(13) Measure of the central angle of an arc of a circle whose length is equal to the radius of the circle is known as .....
(a) 1 degree (b) 1 radian
(c) 1 right Angle (d) 1 reflex angle

(14) In the 2nd quadrant $\cos\phi$ is
(a) +ve (b) –ve
(c) both (d) 0

(15) $\sin\frac{3\pi}{2} = ?$
(a) 0 (b) 1
(c) -1 (d) none

(16) $60' = ?$
(a) $1''$ (b) $60''$
(c) $3600''$ (d) $600''$

(17) Through how many radians does the minute hand of a clock turn in 20 minutes?
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$
(c) 2π (d) $\frac{2\pi}{3}$

(18) $\tan^2\theta = ?$
(a) $\sec^2\theta - 1$ (b) $1 - \sec^2\theta$
(c) $\sec^2\theta + 1$ (d) $\frac{\cos^2\theta}{\sin^2\theta}$

(19) The terminal side of $-\frac{\pi}{2}$ lies along:
(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$
(c) $\overrightarrow{OY}$ (d) $\overrightarrow{OY'}$

(20) When $\theta = \frac{\pi}{3}$ radians $r = 6m$ then area of sector = ?
(a) $\frac{3\pi}{7}$ m^2 (b) 6π m^2
(c) $19\frac{\pi}{2}$ m^2 (d) $\frac{13\pi}{11}$ m^2

(21) The radian measure of the angle at center of a circle, of radius 8cm which cuts off an arc 216cm long is
(a) 15 radians (b) 16.92 radians
(c) 27 radians (d) 29.71 radians

(22) The length of the arc cut off on a circle of radius $11cm$ by a central angle of 6π radians is
(a) $\frac{6\pi}{11}$ cm (b) 66π cm
(c) $\frac{11\pi}{6}$ cm (d) 23π cm

(23) The area of a sector with a central angle of 3.5 radians in a circular region whose radius $8m$ is

(a) 112 m^2 (b) 66 m^2
(c) 14 m^2 (d) 70 m^2

(24) A planet subtends an angle of 1.8°as observed from the earth. Its distance from the earth is 542300km, length of the diameter of the planet is

(a) 12340π km (b) 27650π km
(c) 5721 πkm (d) 5423π km

(25) The terminal side of $-\frac{13\pi}{2}$ lies along

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OY}$
(c) $\overrightarrow{OX'}$ (d) $\overrightarrow{OY'}$

(26) The terminal side of –1710 lies along

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OY}$
(c) $\overrightarrow{OX'}$ (d) $\overrightarrow{OY'}$

(27) The sign of tan 2140 is

(a) +ve (b) –ve
(c) may be any (d) none

(28) $\sin\left(4.09\frac{\pi}{2}\right)=?$

(a) 0 (b) 1
(c) –1 (d) ∞

(29) $\cos\left(-150\frac{\pi}{2}\right)=?$

(a) 0 (b) 1
(c) –1 (d) ∞

(30) If $\tan\theta = a$ then $\sin\theta=?$

(a) $\frac{1}{a}$ (b) $\frac{a}{1+a^2}$
(c) $\frac{\sqrt{a}}{1-a^2}$ (d) $\frac{a}{\sqrt{1+a^2}}$

(31) Which one is true?

(I) $\sin(-\theta)=\sin\theta$ (II) $\cos(-\theta)=\cos\theta$ (III) $\tan(-\theta)=\tan\theta$

(a) I only (b) II only
(c) I & II only (d) II & III only

(32) 105 =?

(a) $\frac{7\pi}{12}$ radians (b) $\frac{\pi}{3}$ radians
(c) $\frac{5\pi}{4}$ radians (d) $\frac{11\pi}{6}$ radians

(33) If $\cot\theta = \frac{15}{8}$, then find the value of $\cos\theta$ where θ is not in the first quadrant.

(a) $\frac{15}{17}$ (b) $-\frac{15}{17}$

(c) $\frac{17}{15}$ (d) $-\frac{17}{15}$

(34) If $\tan\theta = \frac{1}{\sqrt{7}}$ then $\frac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta} = ?$

(a) $\frac{4}{3}$ (b) $\frac{5}{7}$

(c) $\frac{3}{4}$ (d) $\frac{1}{2}$

(35) If $\sec^2\varphi + \csc^2\varphi = \sec^2\varphi \csc^2\varphi$, then state the domain.

(a) $\varphi \neq \frac{n\pi}{2}, n \in \mathbb{Z}$ (b) $\varphi \neq \frac{n\pi}{3}, n \in \mathbb{Z}$

(c) $\varphi \neq \frac{n\pi}{6}, n \in \mathbb{Z}$ (d) $\varphi \neq n\pi, n \in \mathbb{Z}$

(36) If $\cos\theta = \frac{12}{13}$ then $\tan\theta = ?$

(a) $\frac{5}{12}$ (b) $\frac{-5}{12}$

(c) $\frac{12}{5}$ (d) Both a and b

(37) If $\cot\theta = \frac{15}{8}$, then $\csc\theta \times \cos\theta = ?$

(a) $-\frac{15}{8}$ (b) $\frac{15}{8}$

(c) $\frac{8}{15}$ (d) $\frac{-8}{15}$

(38) If $\tan\theta = 2$ then $\frac{3\sin\theta + 4\cos\theta}{\cos\theta + \sin\theta} = ?$

(a) $\frac{10}{3}$ (b) $\frac{10}{7}$

(c) $\frac{7}{3}$ (d) $\frac{17}{3}$

(39) The trigonometric ratio of $\frac{13\pi}{3}$ are same as that of

(a) $\frac{\pi}{3}$ (b) π

(c) $\frac{\pi}{6}$ (d) $\frac{11\pi}{3}$

(40) $\sin^2 30^o + \sin^2 60^o + \tan^2 45^o = ?$

(a) 1
(b) 2
(c) 3
(d) 4

(41) If $2\sin 45^o + \frac{1}{2}\cos ec 45^o = 3x$ then x = ?

(a) 1
(b) $\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$
(d) 2

(42) $\sec\theta \cos ec\theta \sin\theta \cos\theta = 1$, if

(a) $\forall \theta \in R$
(b) $\theta \neq (2n+1)\frac{\pi}{2}, n \in Z$
(c) $\theta \neq \frac{n\pi}{2}, n \in Z$
(d) $\theta \neq n\pi, n \in Z$

(43) $\frac{7\pi}{12} = ?$

(a) 95°
(b) 105°
(c) 115°
(d) 125°

(44) $\sec(-216^o).\cos ec(-15^o)$ _____

(a) > 0
(b) < 0
(c) < 1
(d) = 1

(45) $\cot\left(\frac{19\pi}{3}\right) = ?$

(a) $\sqrt{3}$
(b) $-\sqrt{3}$
(c) $\frac{1}{\sqrt{3}}$
(d) $-\frac{1}{\sqrt{3}}$

TRIGONOMETRIC IDENTITIES

The Fundamental Law of Trigonometry

□ For any two angles α and β (real numbers)

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$ is called **fundamental law of trigonometry.**

Addition Formulas:

(i). $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

(ii). $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

(iii). $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

(iv). $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

(v). $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

(vi). $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

(vii). $\cot(\alpha+\beta)=\dfrac{\cot\alpha\cot\beta-1}{\cot\beta+\cot\alpha}$

(viii). $\cot(\alpha-\beta)=\dfrac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$

Double Angle Formulas

(i). $\sin 2\alpha=2\sin\alpha\cos\alpha$

(ii). $\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$

(iii). $\tan 2\alpha=\dfrac{2\tan\alpha}{1-\tan^2\alpha}$

Half Angle Formulas

(i). $\sin\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1-\cos\alpha}{2}}$

(ii). $\cos\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1+\cos\alpha}{2}}$

(iii). $\tan\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

Multiple Angle Formulas

(i). $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

(ii). $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

(iii). $\tan 3\alpha = \dfrac{3\tan\alpha - \tan^3\alpha}{1-3\tan^2\alpha}$

(iv). $\sin 4\alpha = 4\sin\alpha\cos\alpha - 8\sin^3\alpha\cos\alpha$

(v). $\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$

(vi). $\tan 4\alpha = \dfrac{4\tan\alpha - 4\tan^3\alpha}{1-6\tan^2\alpha+\tan^4\alpha}$

Powers of Trigonometric Functions

(i). $\sin^2\alpha = \dfrac{1-\cos 2\alpha}{2}$

(ii). $\cos^2\alpha = \dfrac{1+\cos 2\alpha}{2}$

(iii). $\sin^3\alpha = \dfrac{3}{4}\sin\alpha - \dfrac{1}{4}\sin 3\alpha$

(iv). $\cos^3\alpha = \dfrac{3}{4}\cos\alpha + \dfrac{1}{4}\cos 3\alpha$

(v). $\sin^4\alpha = \dfrac{3}{8} - \dfrac{1}{2}\cos 2\alpha + \dfrac{1}{8}\cos 4\alpha$

(vi). $\cos^4\alpha = \dfrac{3}{8} + \dfrac{1}{2}\cos 2\alpha + \dfrac{1}{8}\cos 4\alpha$

Sum/Differences to Products Formulas

(i). $\sin\alpha + \sin\beta = 2\sin\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

(ii). $\sin\alpha - \sin\beta = 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

(iii). $\cos\alpha + \cos\beta = 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

(iv). $\cos\alpha - \cos\beta = -2\sin\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

The ∠ b/w minute & hours hand

$\theta = (11/2)M - 30H$

hands of clock in straight line but opposite direction

↳ 22 times a day

The hand of clock are straight 44 time a day.

$\theta = (11/2)20 - 30\cdot4$

$= 110 - 120$

$(11/2)30 - 30(8)$

30 / 4 / 120

15 / 11 / 15 / 15× / 165

30 / 8 / 240 / 165 / 085

Products to Sum/Differences Formulas

(i). $\sin\alpha\sin\beta=\frac{1}{2}\{\cos(\alpha-\beta)-\cos(\alpha+\beta)\}$

(ii). $\cos\alpha\cos\beta=\frac{1}{2}\{\cos(\alpha-\beta)+\cos(\alpha+\beta)\}$

(iii). $\sin\alpha\cos\beta=\frac{1}{2}\{\sin(\alpha-\beta)-\sin(\alpha+\beta)\}$

(iv). $\cos\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$

Trigonometric Ratios of Allied Angles

The angles associated with basic angles of measure θ to a right angle or its multiples are called **allied angles**.

Sine	Cosine	Tangent
$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$	$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$	$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$
$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$	$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$	$\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$
$\sin(\pi-\theta)=\sin\theta$	$\cos(\pi-\theta)=-\cos\theta$	$\tan(\pi-\theta)=-\tan\theta$
$\sin(\pi+\theta)=-\sin\theta$	$\cos(\pi+\theta)=-\cos\theta$	$\tan(\pi+\theta)=\tan\theta$
$\sin\left(\frac{3\pi}{2}-\theta\right)=-\cos\theta$	$\cos\left(\frac{3\pi}{2}-\theta\right)=-\sin\theta$	$\tan\left(\frac{3\pi}{2}-\theta\right)=\cot\theta$
$\sin\left(\frac{3\pi}{2}+\theta\right)=-\cos\theta$	$\cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta$	$\tan\left(\frac{3\pi}{2}+\theta\right)=-\cot\theta$
$\sin(2\pi-\theta)=-\sin\theta$	$\cos(2\pi-\theta)=\cos\theta$	$\tan(2\pi-\theta)=-\tan\theta$
$\sin(2\pi+\theta)=\sin\theta$	$\cos(2\pi+\theta)=\cos\theta$	$\tan(2\pi+\theta)=\tan\theta$

PRACTICE EXERCISE

(1) The distance 'r' of the point (x_1, y_1) from the origin is given by:

(a) $\sqrt{x_1^2 + y_1^2}$ (b) $\sqrt{(x_1 + y_1)^2}$

(c) $\sqrt{x_1^2 - y_1^2}$ (d) $x_1^2 - y_1^2$

(2) $\dfrac{\sin 30 + \sin 60 + \sin 90}{\cos 30 + \cos 60 + \cos 90} = ?$

(a) *Tan 30°* (b) *Tan 45°*

(c) *Tan 60°* (d) *sin 0°*

(3) *Cos (α+β) Cos (α−β) = ?*

(a) *2 sin α cos β* (b) *−2 cos α cos β*

(c) *−2 sin α sin β* (d) *2 cos α sin β*

(4) $\tan 135\ 45' = ?$

(a) $\tan 44\ 25'$ (b) $-\tan 44\ 15'$

(c) $\cot 44\ 25'$ (d) $-\cot 44\ 25'$

(5) Which one is false?

(a) *sin 2q = 2 cosq sinq* (b) $\cos 2q = \cos^2 q - \sin^2 q$

(c) *cos2q = sin²q-1* (d) $\tan 2q = \dfrac{2\tan q}{1-\tan^2 q}$

(6) $\cos^4 q - \sin^4 q = ?$

(a) *cosec 2q* (b) *sec 2q*

(c) $\dfrac{1}{\sec 2q}$ (d) $\dfrac{1}{\cos 2q}$

(7) Which one is not true?

(a) $\cos(a\text{-}b) = \cos a \cos b + \sin a \sin b$ (b) $\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

(c) $\cot(a+b) = \dfrac{\cot a \cot b - 1}{\cot a + \cot b}$ (d) $\tan(270 + \theta) = \cot\theta$

(8) Fundamental law is:

(a) $\cos(a+b) = \cos a \cos b + \sin a \sin b$ (b) $\cos(a\text{-}b) = \cos a \cos b + \sin a \sin b$

(c) $\sin(a\text{-}b) = \cos a \cos b + \sin a \sin b$ (d) $\sin(a\text{-}b) = \sin a \sin b + \cos a \cos b$

(9) Which one is true?

I) $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

II) $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \beta}{1 - \tan\alpha \tan\beta}$

III) $\cos(\alpha - \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

(a) I only (b) II & III only

(c) I & II only (d) II only

(10) $\tan(a+b) = ?$

(a) $\dfrac{\tan a + \tan b}{1 + \tan a \tan b}$

(b) $\dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

(c) $\dfrac{\tan a + \tan b}{1 - \tan a \tan b}$

(d) $\dfrac{\tan a - \tan b}{1 - \tan a \tan b}$

(11) $\cos\left(\dfrac{\pi}{2} - \theta\right) = ?$

(a) $\cos\theta$

(b) $\sin\theta$

(c) $-\cos\theta$

(d) $-\sin\theta$

(12) $\cos a - \cos b = ?$

(a) $2\cos(a+b)\cos(a-b)$

(b) $2\sin(a+b)\sin(a-b)$

(c) $2\sin\left(\dfrac{a+b}{2}\right)\sin\left(\dfrac{a-b}{2}\right)$

(d) $-2\sin\left(\dfrac{a+b}{2}\right)\sin\left(\dfrac{a-b}{2}\right)$

(13) $\cos a = ?$

I) $-\cos^2\dfrac{a}{2} - \sin^2\dfrac{a}{2}$ II) $2\cos^2\dfrac{a}{2} - 1$ III) $1 - 2\sin^2\dfrac{a}{2}$

(a) I only

(b) II and III only

(c) I and II only

(d) all

(14) $\sin\theta - \sin\phi = ?$

(a) $2\cos\left(\dfrac{\theta+\phi}{2}\right)\sin\left(\dfrac{\theta-\phi}{2}\right)$

(b) $2\sin\left(\dfrac{\theta+\phi}{2}\right)\cos\left(\dfrac{\theta-\phi}{2}\right)$

(c) $2\cos\theta\sin\phi$

(d) none

(15) $\sin(a+b)\sin(a-b) = ?$

(a) $\cos^2 a - \cos^2 b$

(b) $\cos^2 b - \cos^2 a$

(c) $\sin^2 a + \sin^2 b$

(d) $\sin^2 b - \sin^2 a$

(16) $\tan\dfrac{\theta}{2} = ?$

(a) $\sqrt{\dfrac{1-\cos\theta}{2}}$

(b) $\sqrt{\dfrac{1+\cos\theta}{2}}$

(c) $\pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

(d) None

(17) $cosec\ 2\theta - cot\ 2\theta = ?$

(a) $tan\ 2\theta$

(b) $cos\ 2\theta$

(c) $tan\ \theta$

(d) $cos\ \theta$

(18) $\cos\alpha + \cos\beta = ?$

(a) $2\cos\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

(b) $2\cos\left(\dfrac{\alpha-\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

(c) $2\cos\left(\dfrac{\alpha-\beta}{2}\right)\sin\left(\dfrac{\alpha+\beta}{2}\right)$

(d) $2\cos\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

(19) $Tan1110^0 =$

(a) $\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) ∞

(d) 0

(20) $\frac{1-\tan^2\theta}{1+\tan^2\theta} = ?$

(a) $tan2\theta$

(b) $cos2\theta$

(c) $tan\theta$

(d) $\sin 2\theta$

(21) $\sin(\theta + 270) = ?$

(a) $sin\theta$

(b) $cos\theta$

(c) $-cos\theta$

(d) $-sin\theta$

(22) $Sin(\alpha + \beta)Sin(\alpha - \beta) = ?$

(a) $Sin^2\alpha + Sin^2\beta$

(b) $Cos^2\alpha - Cos^2\beta$

(c) $Sin^2\beta - Sin^2\alpha$

(d) $Cos^2\beta - Cos^2\alpha$ or $\sin^2\alpha - \sin^2\beta$

(23) $\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = ?$

(a) $tan\alpha + tan\beta$

(b) $cos\alpha + cos\beta$

(c) $cos(\alpha+\beta)$

(d) $sin\alpha + sin\beta$

(24) If $\sin\alpha = \frac{2}{3}$ and $\cos\beta = \frac{4}{5}$ where α, β are in first quadrant then $\sin(\alpha - \beta) = ?$

(a) $\frac{15\sqrt{5}}{4}$

(b) $\frac{8-3\sqrt{5}}{15}$

(c) $16 - \frac{11\sqrt{7}}{2}$

(d) $7 - \frac{5\sqrt{2}}{19}$

(25) The value of $\tan 225^\circ = ?$ $2|25$ (45) $\tan(270-45)$ $\cot 45$

(a) 0

(b) $\frac{1}{\sqrt{3}}$

(c) 1

(d) ∞

(26) If $\cos\alpha = -\frac{2}{7}$ and terminal ray of α is in 2^{nd} quadrant then $\sin\frac{\alpha}{2} = ?$

(a) $\pm\frac{\sqrt{5}}{2}$

(b) $\frac{3}{\sqrt{14}}$

(c) $\pm\frac{\sqrt{7}}{2}$

(d) $\pm\frac{5}{\sqrt{2}}$

(27) $cos^4\alpha$ when expressed in terms of trigonometric functions of multiples of α with exponent as unity equals to;

(a) $\frac{1}{8}\{3+4\cos 2\alpha+\cos 4\alpha\}$

(b) $\{3+4\cos 2\alpha+\cos 4\alpha\}$

(c) $\frac{1}{6}\{3+4\cos 2\alpha+\cos 4\alpha\}$

(d) $\frac{1}{6}\{5+4\cos 2\alpha+\cos 4\alpha\}$

(28) $\frac{\sin 3\theta}{\sin\theta}-\frac{\cos 3\theta}{\cos\theta}=?$

(a) 0

(b) $sin^2\theta$

(c) $2tan\theta$

(d) 2

(29) $\frac{\sin 2\alpha}{1-\cos 2\alpha}=?$

(a) $sin\alpha$

(b) $cot\alpha$

(c) $cosec\alpha$

(d) $sec\alpha$

(30) $\frac{\cos^3\alpha-\sin^3\alpha}{\cos\alpha-\sin\alpha}=?$

(a) $1+sin\alpha$

(b) $1-sin\alpha$

(c) $\frac{2+\sin 2\alpha}{2}$

(d) $\frac{1+\cos 2\alpha}{2}$

(31) When $sin3\theta cos5\theta$ is expressed as sum or difference, we get;

(a) $\frac{1}{2}(\cos 2\theta-\sin 4\theta)$

(b) $\frac{1}{2}(\sin 8\theta-\sin 2\theta)$

(c) $\frac{1}{4}(\tan 2\theta-\sin 4\theta)$

(d) $\frac{1}{2}(9\cos 2\theta-\cos 4\theta)$

(32) $\cos\frac{3\alpha}{4}+\cos\frac{4\alpha}{3}$ can be written in product form as

(a) $2sin6\alpha cos2\alpha$

(b) $2sin\alpha cos\alpha$

(c) $2\cos\frac{25\alpha}{24}\cos\frac{7\alpha}{24}$

(d) $2\cos\frac{52\alpha}{2}\cos\frac{5\alpha}{12}$

(33) $\tan\left(\frac{\theta-\phi}{2}\right)=?$

(a) $\sin\left(\frac{\theta-\phi}{2}\right)$

(b) $\cos\left(\frac{\theta-\phi}{2}\right)$

(c) $-\tan\left(\frac{\phi-\theta}{2}\right)$

(d) $2\sin(\theta-\phi)\cos(\theta-\phi)$

(34) $\sin 15^\circ=?$

(a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(b) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(c) $\frac{\sqrt{3}+1}{2}$

(d) $\frac{\sqrt{3}-1}{2}$

(35) $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4\pi}{9}=?$

(a) $\frac{3}{4}$ (b) $\frac{3}{5}$

(c) $\frac{3}{17}$ (d) $\frac{3}{16}$

(36) The maximum value of $\cos^2\theta-\sin^2\theta=?$

(a) 2 (b) 1

(c) 3 (d) 0

(37) If $\tan\alpha+\tan\beta=3$ and $\tan\alpha.\tan\beta=2$ then $\tan(\alpha+\beta)=?$

(a) 2 (b) 3

(c) -3 (d) -2

(38) If $\tan\theta+\cot\theta=2$ then $\tan^2\theta+\cot^2\theta=?$

(a) 1 (b) 2

(c) 3 (d) 4

(39) If $\sin\theta+\cos ec\theta=\sqrt{3}$ then $\sin^3\theta+\cos ec^3\theta=?$

(a) 0 (b) 1

(c) 2 (d) $3\sqrt{3}$

(40) $\frac{\sin 9^\circ\sin 81^\circ}{\cos 81^\circ\cos 9^\circ}=?$

(a) 0 (b) -1

(c) 1 (d) 2

(41) $\tan 18^\circ+\tan 27^\circ+\tan 18^\circ.\tan 27^\circ=?$

(a) 1 (b) -1

(c) $\frac{\pi}{4}$ (d) 3

(42) $\sin 12^\circ\sin 48^\circ\sin 54^\circ=?$

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

(43) $\cos^2(45^\circ+x)-\sin^2(45^\circ-x)=?$

(a) 0 (b) 1

(c) -1 (d) -2

(44) Maximum value of $\cos x-\sin x=?$

(a) 1 (b) 0

(c) $\sqrt{2}$ (d) 2

(45) $\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{3\pi}{5}\cos\frac{4\pi}{5}=$

(a) $\frac{1}{64}$ (b) $\frac{1}{32}$ (c) $\frac{1}{48}$ (d) $\frac{1}{16}$

TRIGONOMETRIC FUNCTIONS AND THEIR GRAPHS

The General Form of a Trigonometric Function

- $y = A.f(Bx + C)$

The following terms are related to the trigonometric functions.

(i).Amplitude:

The amplitude is the one-half the vertical distance between the lowest and the highest point of the graph. The amplitude of a function = $|A|$

- The amplitude is associated with only the sine and cosine functions.

(ii).Period:

- Since the values of all the trigonometric functions repeat themselves at regular intervals. which is called period of the function.
- For some number p , $f(x) = f(x + p)$ for all numbers x, then p is called period of that function.
- Period of any function $= \frac{p}{B}$, where p is the normal period of the function.
- This behavior of trigonometric functions is called **periodicity**.

(iii).Phase Shift:

The phase shift is the distance to the right or left that the graph is changed from its normal position.

- The phase shift of a function $= -\frac{C}{B}$

(iv).Frequency:

- The frequency is equal to $\frac{1}{period}$

Domains, Ranges and Periods of Trigonometric Functions

Trigonometric Function	Domain	Range	Period
$Y = \sin x$	$x \in \mathbb{R}$	$-1 \le Y \le 1$	2π
$Y = \cos x$	$x \in \mathbb{R}$	$-1 \le Y \le 1$	2π
$Y = \tan x$	$x \in \mathbb{R}$ But $x \neq (2k+1)\frac{\pi}{2}, k \in \mathbb{Z}$	$Y \in \mathbb{R}$	π
$Y = \cot x$	$x \in \mathbb{R}$ But $x \neq k\pi, k \in \mathbb{Z}$	$Y \in \mathbb{R}$	π
$Y = \sec x$	$x \in \mathbb{R}$ But $x \neq (2k+1)\frac{\pi}{2}, k \in \mathbb{Z}$	$Y \ge 1$ OR $Y \le -1$	2π
$Y = \operatorname{cosec} x$	$x \in \mathbb{R}$ But $x \neq k\pi, k \in \mathbb{Z}$	$Y \ge 1$ OR $Y \le -1$	2π

PRACTICE EXERCISE

(1) Domain of sine function is

(a) R

(b) $R-\{x/x=(2k+1)\frac{\pi}{2},\ k\in Z\}$

(c) Z

(d) $R-\{x/-1<x<1\}$

(2) Range of cosine function is

(a) $\{x/x\in\mathbb{R}\wedge -1\le x\le 1\}$

(b) $\{-1,1\}$

(c) R

(d) $R-\{x/-1<x<1\}$

(3) Domain of tangent function is

(a) R

(b) $R-\left\{x/x=(2k+1)\frac{\pi}{2},k\in\mathbb{Z}\right\}$

(c) $R-\{x/x=k\pi,k\in\mathbb{Z}\}$

(d) none

(4) Period of cotangent function is

(a) π

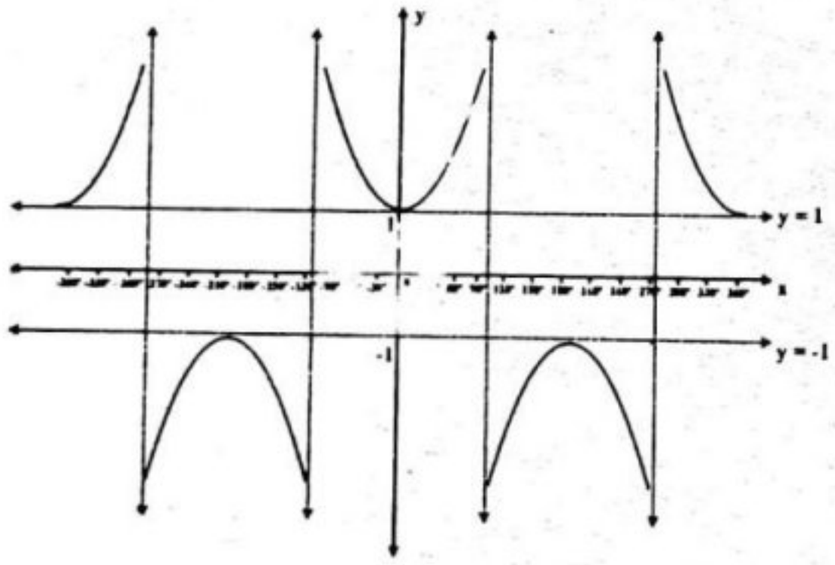
(b) 2π

(c) 0

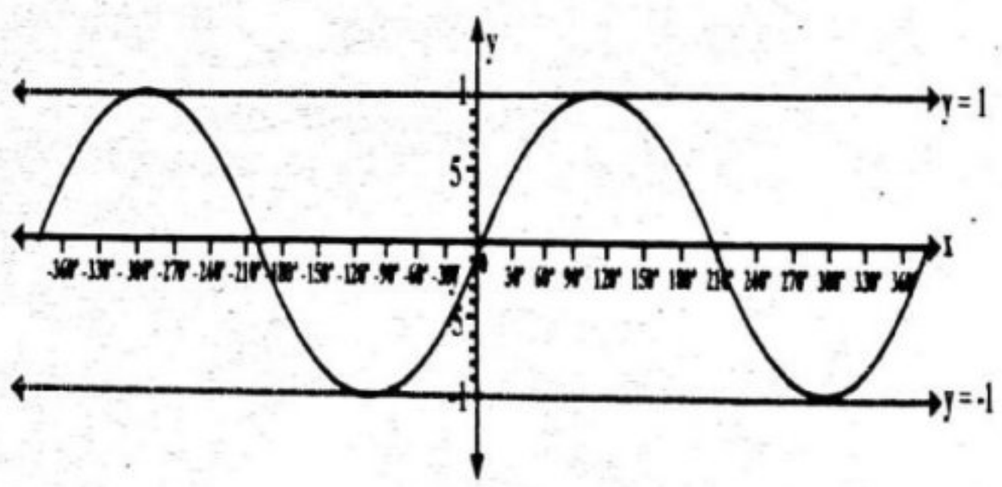
(d) 3π

(5) Which graph is of sine function?

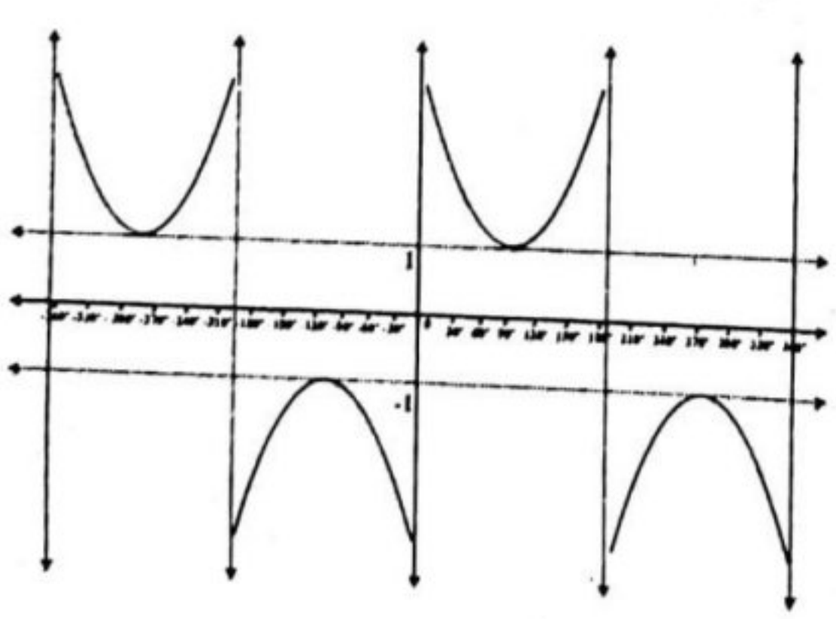
(a)



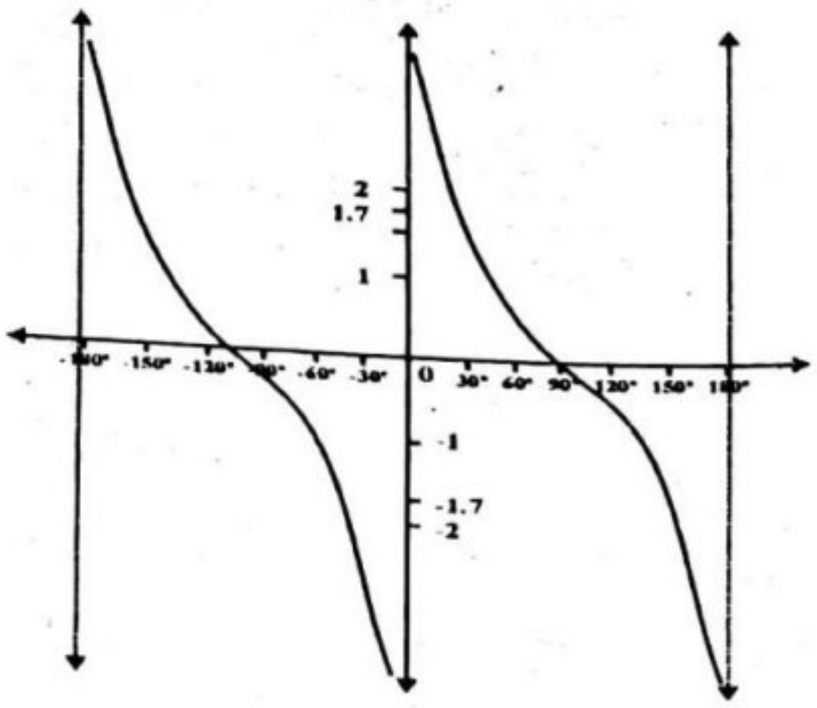
(b)



(c)

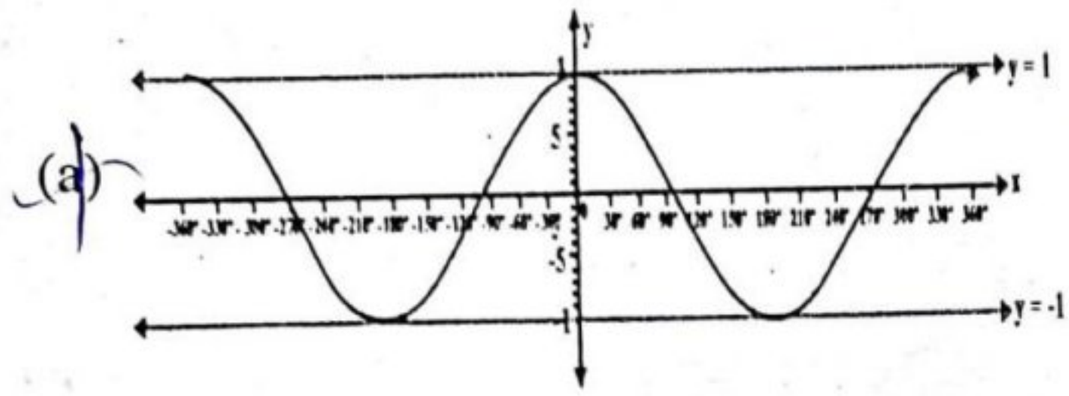


(d)

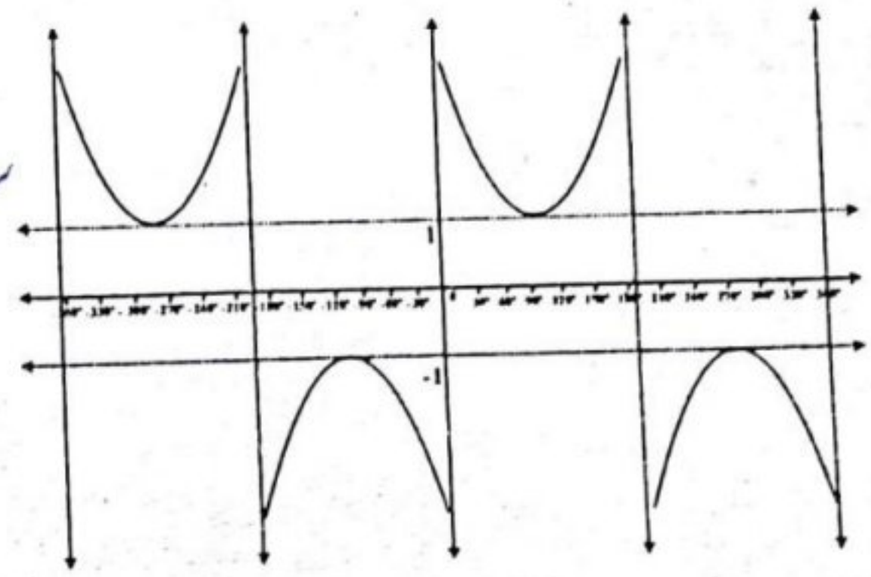


(6) Which graph is of cosecant function?

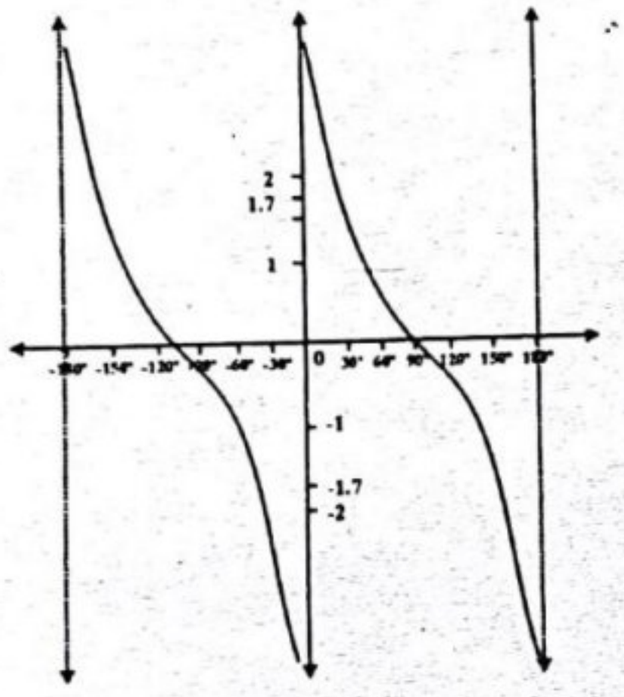
(a)



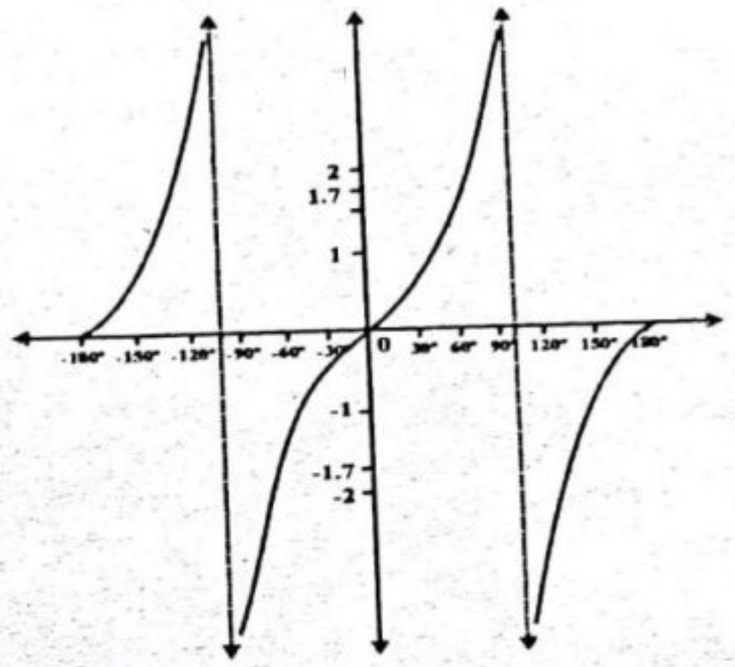
(b)



(c)



(d)



(7) Domain of cosine function is

(a) R

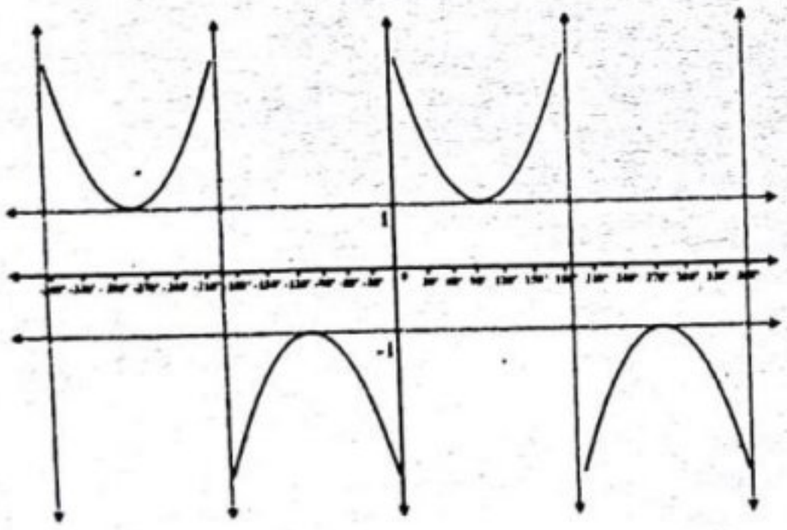
(b) $\{x / x \in \mathbb{R} \wedge -1 < x < 1\}$

(c) $R - \{x / x = k\pi, k \in \mathbb{Z}\}$

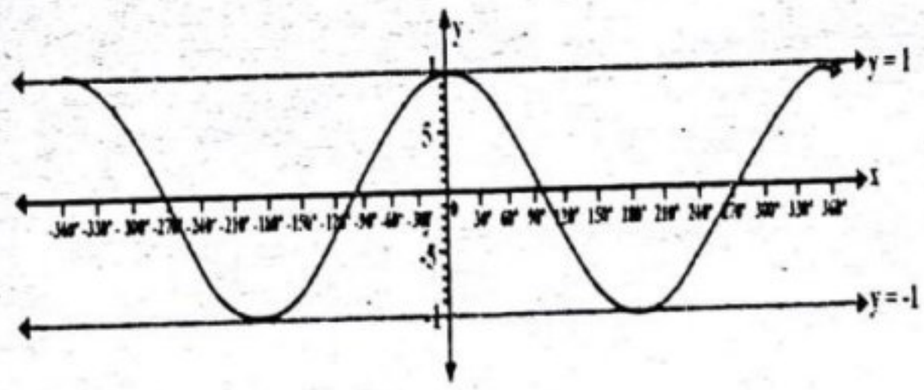
(d) $R - \{-1 < x < 1\}$

(8) Graph of cosine function is

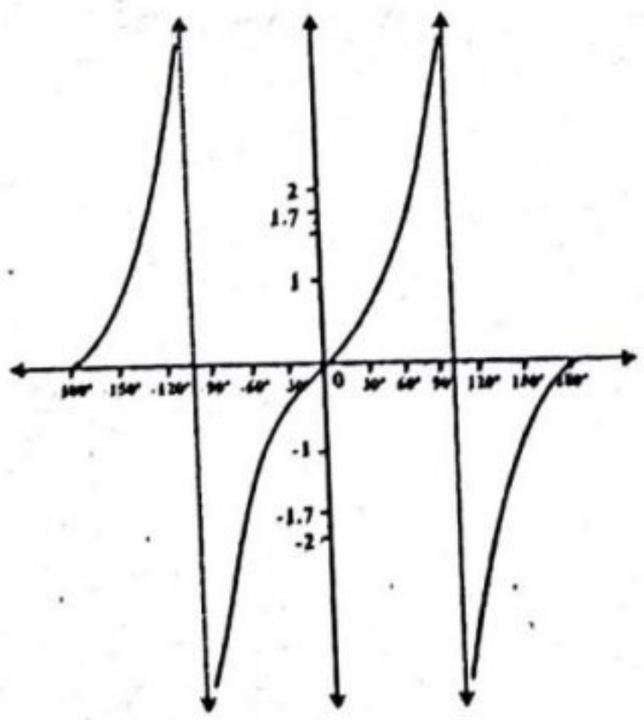
(a)



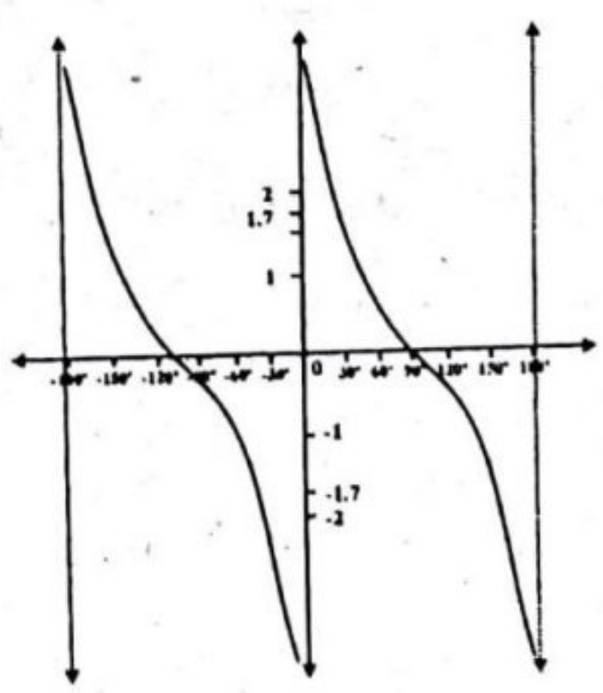
(b)



(c)



(d)



(9) Range of cosecant function is

(a) R

(c) $R-\{y/-1<y<1\}$

(b) $\{y/y\in\mathbb{R}\wedge-1\le y\le 1\}$

(d) $R-\{y/y=k\pi, k\in\mathbb{Z}\}$

(10) Period of secant function is

(a) 0

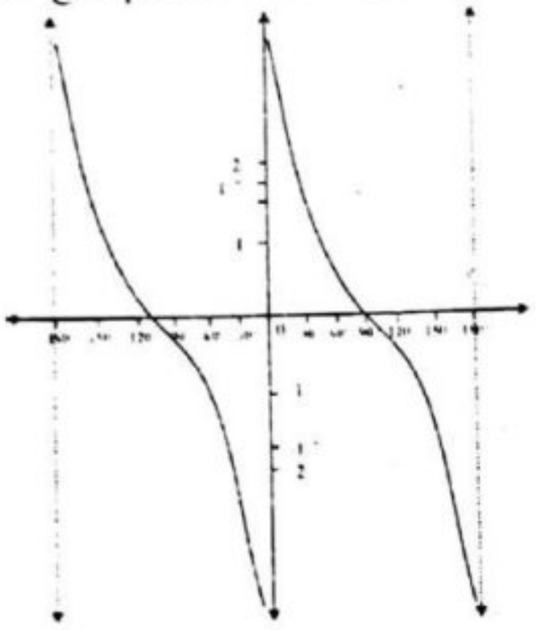
(b) $\frac{\pi}{2}$

(c) π

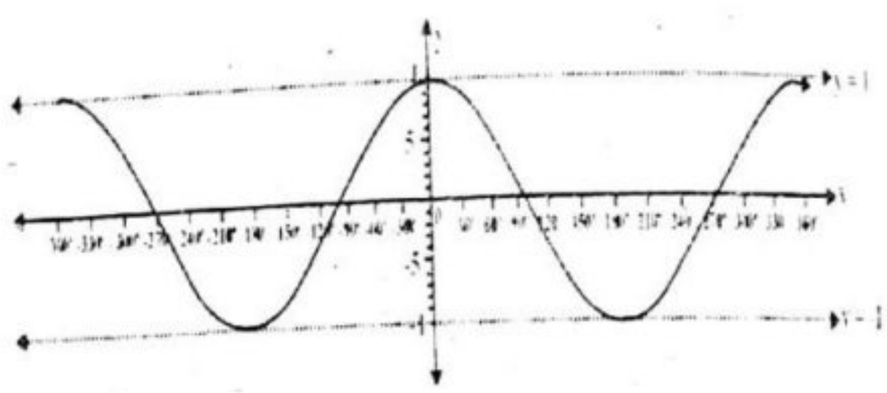
(d) 2π

(11) Which graph is of tangent function?

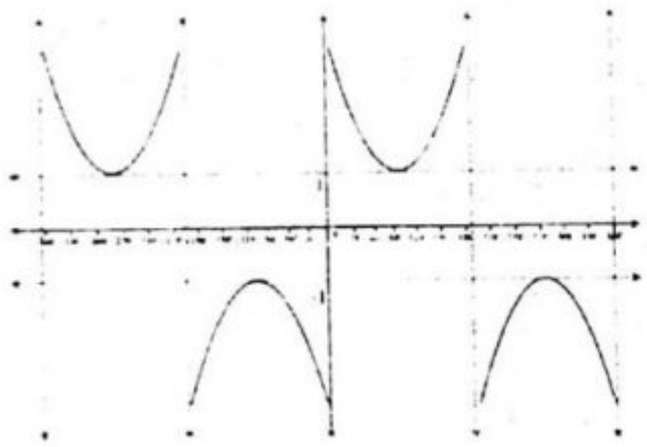
(a)



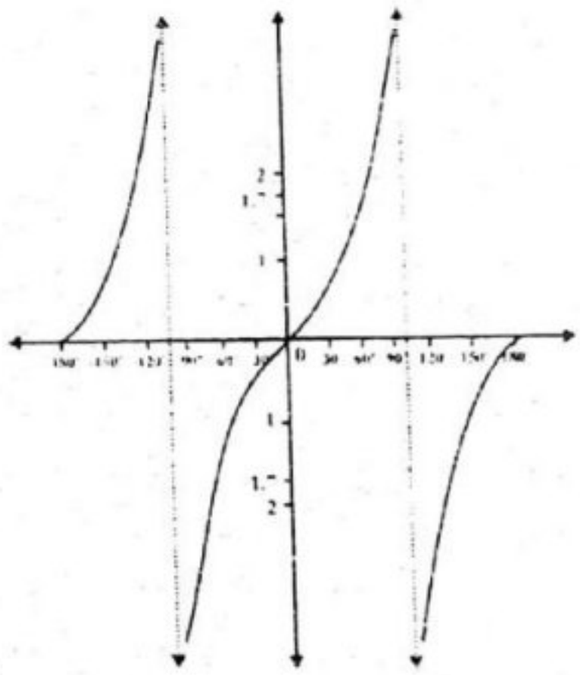
(b)



(c)



(d)



(12) Range of sine function is

(a) R

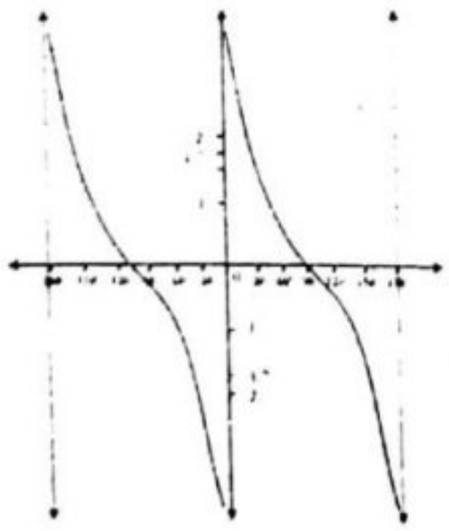
(c) $\{x/x\in\mathbb{R}\wedge-1<x<1\}$

(b) $\{x/x\in\mathbb{R}\wedge-1\le x\le 1\}$

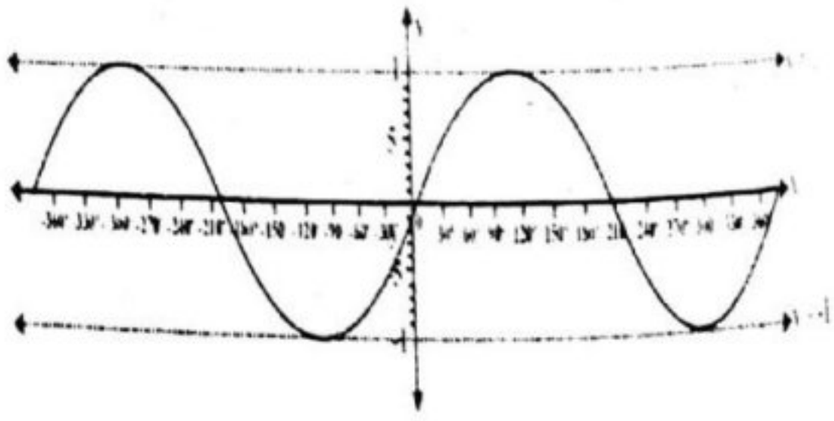
(d) $R-\{x/-1<x<1\}$

(13) Graph of secant function is

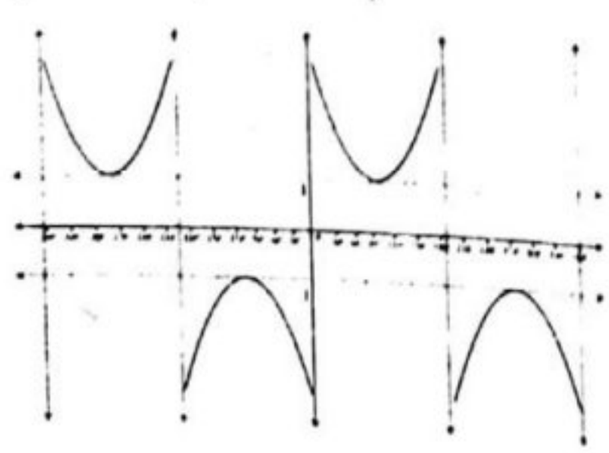
(a)



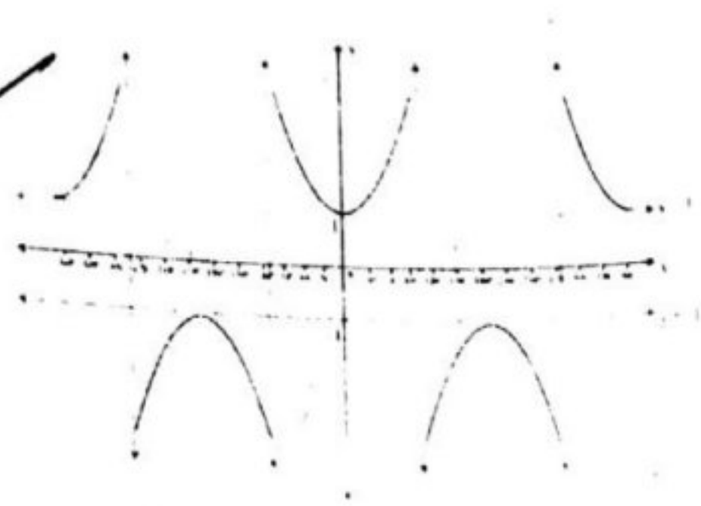
(b)



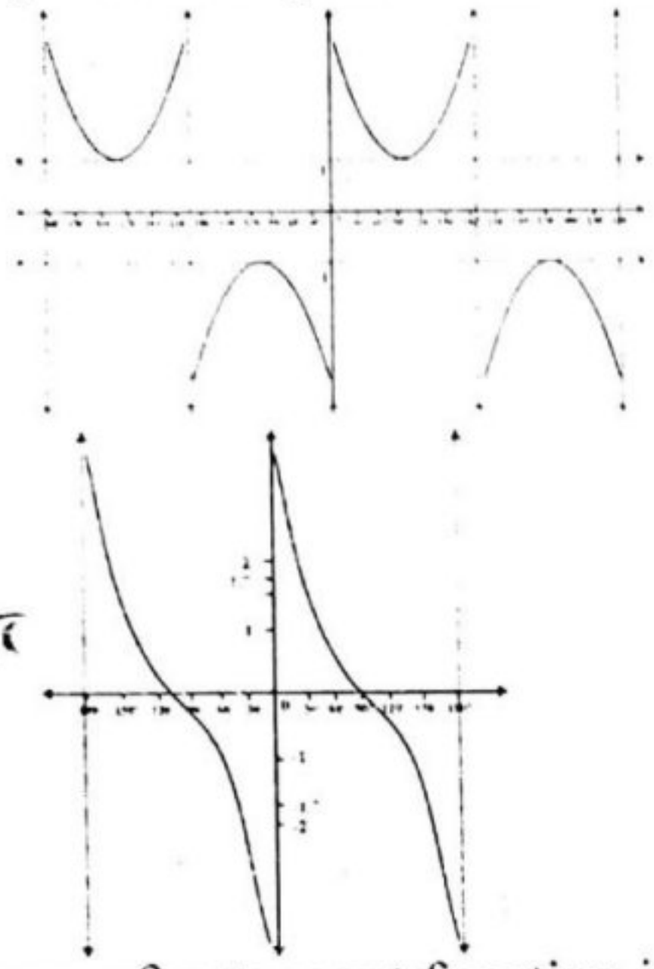
(c)

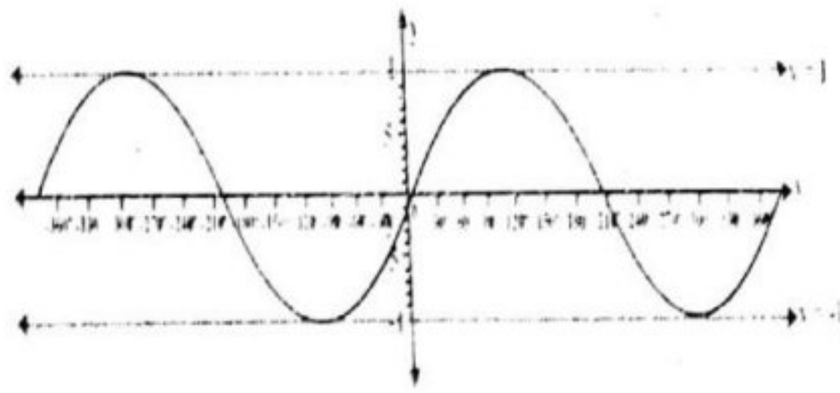


(d)

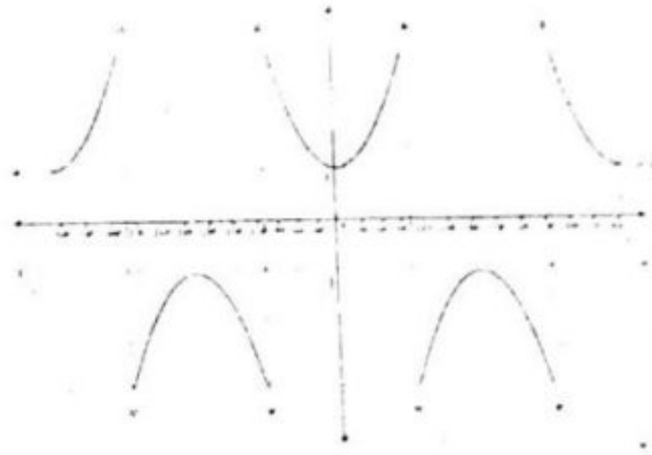


(14) Graph of cotangent function is

(a) 

(b) 

(c)

(d) 

(15) Range of cotangent function is

(a) R

(b) $R-\left\{x/x=(2k+1)\frac{\pi}{2},k\in\mathbb{Z}\right\}$

(c) $R-\{x/x=k\pi,k\in\mathbb{Z}\}$

(d) none of these

(16) The range of the function $2\sin 7x$ is;

(a) $[-1,1]$

(b) $[-7,7]$

(c) $[-2,2]$

(d) $\left[\frac{-7}{2},\frac{7}{2}\right]$

(17) The domain of $\tan 3x=?$

(a) $R-\left\{x/x=(2n+1)\frac{\pi}{2},n\in\mathbb{Z}\right\}$

(b) $R-\left\{x/x=(2n+1)\frac{3\pi}{2},n\in\mathbb{Z}\right\}$

(c) $R-\left\{x/x=(2n+1)\frac{\pi}{3},n\in\mathbb{Z}\right\}$

(d) $R-\left\{x/x=(2n+1)\frac{\pi}{6},n\in\mathbb{Z}\right\}$

(18) Domain of $2\cot 3x=?$

(a) $R-\{x/x=n\pi,n\in\mathbb{Z}\}$

(b) $R-\left\{x/x=\frac{n\pi}{3},n\in\mathbb{Z}\right\}$

(c) $R-\{x/x=2n\pi,n\in\mathbb{Z}\}$

(d) None of these

(19) Amplitude of $-17\sin 14x=?$

(a) 14

(b)-14

(c) 17

(d)-17

(20) Period of $\cos(3x+7)=?$

(a) 2π

(b) $\frac{2\pi}{3}$

(c) $\frac{2\pi}{7}$

(d) $\frac{\pi}{3}$

(21) Period of $\tan\frac{x}{7} = ?$

(a) 7π (b) $\frac{\pi}{7}$

(c) $\frac{7\pi}{2}$ (d) $\frac{2\pi}{7}$

(22) If $\sin x = x$, then the value of x, in radians=?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) 0 (d) $\frac{\pi}{3}$

(23) If $\sin x = \cos x$, then the value of x , in radians=?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

(24) Range of $3\cot\frac{x}{7} = ?$

(a) $-3 \le y \le 3$ (b) $-7 \le y \le 7$

(c) Real numbers (d) none of these

(25) The graph of the sine function is symmetrical about;

(a) The x-axis (b) the y-axis

(c) Origin (d) both a & b

$y = \sin x$

$y = \sin(-x)$

$y = -\sin x \rightarrow$ odd

APPLICATIONS OF TRIGONOMETRY

Elements of the Triangle ABC

- For a given triangle ABC, we shall use *a, b, c* to denote the measure of sides and A, B, C to denote the measures of angles.
- The quantities *a, b, c,* A, B and C are called the **elements of the triangle** ABC.

Types of Triangles

Name of triangle	Sides	Angles
Scalene	3 sides are different	3 angles are different
Isosceles	2 sides are same	2 angles are same
Equilateral	3 sides are same	3 angles are same(*i.e.* of 60°)
Acute	May be scalene, isosceles or equilateral	3 angles are acute(less than 90°)
Obtuse	May be scalene or isosceles but not equilateral	1 angle is obtuse(greater than 90° but less than 180°)
Right	May be scalene or isosceles but not equilateral	1 angle is of 90°
Oblique	May be scalene, isosceles or equilateral	No angle is of 90°

Solution of the Triangle

- If any three out of six elements out of which at least one side, are given the remaining three elements can be determined. This process of finding the unknown elements is called the **solution of the triangle**.

Solution of Right Triangles

In order to solve a right triangle, we have to find:

(i) The measures of two acute angles and

(ii) The lengths of the three sides.

a. Any right triangle can be solved by using **trigonometric ratios** or **Pythagoras theorem.**

Solution of Oblique Triangles

CASE I: When measures of **one side** and **two angles** are given, triangle can be solved by using **the law of sines.**

CASE II: When measures of **two sides** and **their included angle** are given in this case, we can use any one of the following methods.

(i) First law of cosine and then law of sines

(ii) First law of tangents and then law of sines

CASE III: When measures of **three sides** are given

In this case, we can take help of the following formulas:

(i) the law of cosine

(ii) The half angle formulas.

The law of sines	The law of cosine	The law of tangents
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{a}{\sin C}$	$a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = a^2 + c^2 - 2ac\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$ **or** $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$	$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$, $\frac{b-c}{b+c} = \frac{\tan\left(\frac{B-C}{2}\right)}{\tan\left(\frac{B+C}{2}\right)}$, $\frac{c-a}{c+a} = \frac{\tan\left(\frac{C-A}{2}\right)}{\tan\left(\frac{C+A}{2}\right)}$

Half Angles Formulas

Sine	Cosine	Tangent
$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ $\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$ $\sin\frac{C}{2} = \sqrt{\frac{(s-b)(s-a)}{ab}}$	$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$ $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$	$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ $\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$ $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

- In all above cases $s = \frac{a+b+c}{2}$

Area of Triangles

Two sides & their included angle is given	One side and two angles are given	Three sides are given
$\Delta = \frac{1}{2}bc\sin A$ $\Delta = \frac{1}{2}ac\sin B$ $\Delta = \frac{1}{2}ab\sin C$	$\Delta = \frac{a^2\sin B\sin C}{2\sin A}$ $\Delta = \frac{b^2\sin A\sin C}{2\sin B}$ $\Delta = \frac{c^2\sin A\sin B}{2\sin C}$	$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ This formula is called **Hero's formula.**

Circles Connected To Triangles

Circum circle	In circle	Ex-circle
▪ The circle passing through the vertices of a triangle is called a **circum circle**. ▪ The centre of circum circle is called **circum centre**. It is the point of intersection of the right bisectors of the sides of the triangle. ▪ The radius of circum circle is called **circum radius** ▪ It is denoted by "**R**" $R = \frac{abc}{4\Delta}$ $R = \frac{a}{2\sin A}$ $R = \frac{b}{2\sin B}$ $R = \frac{c}{2\sin C}$	▪ The circle drawn inside a triangle touching its three sides is called its **inscribed circle** or **in circle**. ▪ The centre of in-circle is called **in-centre**. It is the point of intersection of the bisectors of angles of the triangle. ▪ Its radius is called **in-radius** ▪ It is denoted by "**r**" $r = \frac{\Delta}{s}$	▪ A circle, which touches one side of the triangle externally and the other two produced sides, is called an **escribed circle** or **ex-circle** or **e-circle**. ▪ The centers of these circles (are called **ex-centers**) are the points where the internal bisector of one and the external bisectors of the other two angles of the triangle meet. ▪ The radii of ex-circles opposite to the vertex A, B and C are denoted by $\mathbf{r_1}$, $\mathbf{r_2}$ and $\mathbf{r_3}$ respectively. $r_1 = \frac{\Delta}{s-a}$ $r_2 = \frac{\Delta}{s-b}$ $r_3 = \frac{\Delta}{s-c}$

Relations Between r, R, r_1, r_2, r_3, s and Δ:

(i). $r_1r_2 + r_2r_3 + r_3r_1 = s^2$

(ii). $rr_1r_2r_3 = \Delta^2$

(iii). $r_1 + r_2 + r_3 = 4R + r$

(iv). $rr_1r_2r_3 = rs^2$

(v). $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

These two relations are only for equilateral triangle.

(i). $r : R : r_1 = 1 : 2 : 3$

(ii). $r : R : r_1 : r_2 : r_3 = 1 : 2 : 3 : 3 : 3$

Formation of Triangles

It is possible that two triangles, one triangle, or no triangle can be constructed with the data. This is called an ambiguous case.

Case I: If $m\angle A > 90$ and $a \le b$ then no triangle can be formed.

Case II: If $m\angle A > 90$ and $a > b$ then only one triangle can be formed.

CaseIII: If $m\angle A < 90$ and $a < b.\sin A$ then no triangle can be formed.

CaseIV: If $m\angle A < 90$ and $a = b.\sin A$ then only one triangle can be formed.

CaseV: If $m\angle A < 90$ and $a > b$ with $b.\sin A < a < b$ then two triangles can be formed.

PRACTICE EXERCISE

(1) The measure of the angle of depression of an airport as observed by a pilot while flying at a height of 3000m is 30°. How far is the plane from a point direct over the airport?

(a) $\frac{3000}{\sqrt{3}}$ (b) $3000\sqrt{3}$

(c) $\frac{\sqrt{3}}{3000}$ (d) $\frac{1}{3000\sqrt{3}}$

(2) In a triangle ABC, $a = 11$, $b = 9$, $c = 19$, which angle is the smallest?

(a) α (b) β

(c) γ (d) none

(3) Find the distance of a man from the foot of a $200m$ high tower if the angle of elevation of its top as observed by the man is 60^o.

(a) $115.47m$ (b) $346.41m$

(c) $240.23m$ (d) $150m$

(4) $\sin^2\left(\frac{\theta}{2}\right) = ?$

(a) $1+cos\theta$ (b) $\frac{1-\cos\theta}{2}$

(c) $1-cos\theta$ (d) $\frac{1+\cos\theta}{2}$

(5) In a right angled triangle ABC with right angle at B, which one is true

(a) $a^2+b^2=c^2$ (b) $a^2-c^2=b^2$

(c) $a^2+c^2=b^2$ (d) $a^2+b^2=b^2$

(6) Which one is true?

(a) $a^2+b^2-c^2=2abCos\alpha$ (b) $b^2=a^2+c^2+2acCos\beta$

(c) $\cos\alpha = \frac{a^2+b^2+c^2}{2ab}$ (d) $a^2=b^2+c^2-2bcCos\alpha$

(7) The measure of the angle of elevation of a kite is 30^o. The string of the kite is $250m$ long. If the say in the string is $5m$, find the height of the kite.

(a) $115.5m$ (b) $270.3m$

(c) $219.1m$ (d) $122.5m$

(8) Which one is known as Hero's formula?

(a) $\triangle = \frac{1}{2}ab\sin\alpha$ (b) $\triangle = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$

(c) $\triangle = \sqrt{s(s-a)(s-b)(s-c)}$ (d) $\triangle = \frac{abc}{4R}$

(9) In a triangle $a = 17$, $b=19$, $c=15$. The largest angle is

(a) α (b) β

(c) γ (d) none

(10) A circle which passes through all three vertices of a triangle is called;
(a) Circum circle (b) in circle
(c) Escribed circle (d) semi circle

(11) Which one is true?
(a) A circum circle touches all three sides of a Δ
(b) The in center is the point of intersection of $\perp$ bisectors of sides of a Δ.
(c) The radius of escribed circle opposite to side C is $\frac{\Delta}{s-a}$.
(d) All three sides of a Δ are tangent to in circle

(12) Which one is false?
(a) $\sin\alpha = \frac{2\Delta}{ab}$ (b) $r_1 = s\tan\frac{\alpha}{2}$
(c) $r = (s-a)\tan\frac{\alpha}{2}$ (d) $\Delta = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$

(13) There are _____ elements in a triangle.
(a) 3 (b) 4
(c) 5 (d) 6

(14) The circum centre of a triangle is a point
(a) At which the angle bisectors meet
(b) At which perpendicular bisectors of sides meet
(c) At which medians meet
(d) At which altitudes meet

(15) Circum radius of a triangle is given by
(I) $R = \frac{a}{2\sin\alpha}$
(II) $R = \frac{abc}{2\Delta}$
(III) $R = \frac{b}{\sin\beta}$
(a) I only (b) I and II only
(c) II and III only (d) II only

(16) If $a = 315$, $b=346$, $c=120$ then product of the three sides=?
(a) 721325 (b) 12895
(c) 1001 (d) 13078800

(17) According to laws of sine;
(a) $\frac{c}{\sin\alpha} = \frac{b}{\sin\beta}$ (b) $\frac{a}{\sin\beta} = \frac{b}{\sin\alpha}$
(c) $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$ (d) all of these

(18) Law of cosine is

(a) $c^2=a^2+b^2-2abCosy$

(b) $a^2+2bcCos\alpha=b^2+c^2$

(c) $\cos\alpha=\dfrac{b^2+c^2-a^2}{2bc}$

(d) all of these

(19) Which one is not true?

(a) $\cos\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{s(s-a)}{bc}}$

(b) $\sin\dfrac{\beta}{2}=\pm\sqrt{\dfrac{s(s-b)}{ac}}$

(c) $\cos\dfrac{\gamma}{2}=\pm\sqrt{\dfrac{s(s-c)}{ab}}$

(d) $\tan\dfrac{\beta}{2}=\pm\sqrt{\dfrac{(s-a)(s-c)}{s(s-b)}}$

(20) The ex-centre with respect to the vertex B is the point of intersection of the;

(a) External bisector of A and B and internal bisector of C

(b) External bisectors of B and C and internal bisector of A

(c) External bisectors of A and C and internal bisector of B

(d) Internal bisectors of A, B and C

(21) $r_1r_2+r_2r_3+r_3r_1=?$

(a) Δ^2

(b) s^2

(c) $\dfrac{\Delta}{s}$

(d) $\dfrac{s}{\Delta}$

(22) $(s-b)\tan\dfrac{\beta}{2}=?$

(a) r_1

(b) r

(c) r_2

(d) r_3

(23) $\cot\dfrac{\alpha}{2}\cot\dfrac{\beta}{2}\cot\dfrac{\gamma}{2}=?$

(a) Δ

(b) s^2

(c) $\dfrac{\Delta}{s}$

(d) $\dfrac{\Delta}{r^2}$

(24) From the two successive points on a straight road 1000meters apart a man observes that the angles of elevation of the top of a tower directly ahead of him are of 30° & 60°. How high is the tower above the road?

(a) $500m$

(b) $1000m$

(c) $500\sqrt{3}$ m

(d) $1956m$

(25) In a ΔABC if $A=120^0, b=3$ and $C=30^0$, then a = ?

(a) $2\sqrt{3}$

(b) $3\sqrt{3}$

(c) 2

(d) $\dfrac{1}{2}$

(26) $\cos\gamma = ?$

(a) $\dfrac{a^2+b^2+c^2}{2ab}$ (b) $\dfrac{c^2+b^2-a^2}{abc}$

(c) $\dfrac{2bc-b^2}{a^2}$ (d) $\dfrac{a^2+b^2-c^2}{2ab}$

(27) In a ΔABC if $a=5, B=45^0$ and $c=2\sqrt{2}$, then b = ?

(a) $\sqrt{3}$ (b) $\sqrt{13}$

(c) $2\sqrt{13}$ (d) 6

(28) For a ΔABC $\dfrac{c+a}{c-a} = ?$

(a) $\dfrac{\tan(\gamma-\alpha)}{\tan(\gamma+\alpha)}$ (b) $\dfrac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}$

(c) $\dfrac{\tan\left(\dfrac{\gamma+\alpha}{2}\right)}{\tan\left(\dfrac{\gamma-\alpha}{2}\right)}$ (d) all of these

(29) If for a ΔABC, a = 7, b=9, c=7 then γ=?

(a) 40° (b) 45°

(c) 50° (d) 550°

Cosine law.

(30) $\sin\dfrac{\beta}{2} = ?$

(a) $\sqrt{\dfrac{(s-c)(s-a)}{ac}}$ (b) $\sqrt{\dfrac{s(s-b)}{ac}}$

(c) $\sqrt{\dfrac{(s-c)(s-a)}{s(s-b)}}$ (d) $\sqrt{\dfrac{(s-b)(s-c)}{bc}}$

(31) If a = 9350, b=7145, c=8563 then $\dfrac{\alpha}{2}$ =?

(a) 36° app (b) 41° app

(c) 39° app (d) 47° app

(32) Δ =?

(a) $\dfrac{b^2\sin\gamma\sin\alpha}{2\sin\beta}$ (b) $\dfrac{1}{2}bc\sin\alpha$

(c) $\sqrt{s(s-a)(s-b)(s-c)}$ (d) all

(33) $R = ?$

(a) $\dfrac{ab}{2\sin\gamma}$ (b) $\dfrac{b}{2\sin\beta}$

(c) $\dfrac{bc}{2\sin\alpha}$ (d) $\dfrac{abc}{\sin\alpha}$

(34) $r = ?$

(a) $\frac{abc}{4\Delta}$ (b) $\frac{\Delta}{s}$

(c) $\frac{s}{\Delta}$ (d) $\frac{\Delta}{abc}$

(35) $\frac{2\Delta}{bc} = ?$

(a) $cos\alpha$ (b) $cos\beta$

(c) $sin\alpha$ (d) $sin\beta$

(36) $s\tan\frac{\alpha}{2} = ?$

(a) R (b) r

(c) r_1 (d) r_2

(37) $r_1+r_2+r_3-r = ?$

(a) R (b) $4R$

(c) r (d) $4r$

(38) $\sqrt{rr_1r_2r_3} = ?$

(a) $sin\alpha$ (b) $\frac{1}{2}bc\sin\beta$

(c) $\frac{\Delta}{2abc}$ (d) Δ

(39) In a triangle ABC, a =15, b=25, c=20, then the ratio of cosβ to cosγ is

(a) 0 (b) 1

(c) 0.093 (d) 1.342

(40) For a triangle, Δ=888 and s=24, then radius of in circle is;

(a) 4300.85 (b) 37

(c) 190 (d) 4860.8

(41) If for a triangle ABC, a = 3, b = 4 and m < C = 30° then area of triangle = ?

(a) 2 (b) $\frac{1}{2}$

(c) 3 (d) $\frac{1}{3}$

(42) If the angles of a triangle are 30° and 45° and the included side is $\left(\sqrt{3}+1\right)$ then the area of the triangle is _____

(a) $\left(\sqrt{3}+1\right)$ (b) $\frac{\sqrt{3}+1}{3}$

(c) $\frac{\sqrt{3}+1}{2}$ (d) $3+\sqrt{3}$

(43) If an isosceles triangle has a vertical angle of 120° and a base of 100cm long, then altitude will be

(a) 50 (b) $50\sqrt{2}$

(c) $50\sqrt{3}$ (d) $\frac{50}{\sqrt{3}}$

(44) The angle of elevation of the top of a tower from a point is 30°. If advancing 150 m towards the foot of the tower, the angle increases to 60°. Height of the tower = ?

(a) 75 m (b) $75\sqrt{2}m$

(c) $75\sqrt{3}\,m$ (d) 150 m

(45) A chimney 20m high standing on top of a building subtends an angle whose tangent is 6 at a distance 70m from the foot of the building then height of the building = ?

(a) 400 m (b) 100 m

(c) 150 m (d) 200 m

(46) If the angles of a triangle are in ratio 1:2:3 then the sides are in ratio _____

(a) 2:3:1 (b) $\sqrt{3}:2:1$

(c) $2:\sqrt{3}:1$ (d) $1:\sqrt{3}:2$

(47) Rigid rectilinear figure is _____

(a) Circle (b) Triangle

(c) Spare (d) All of these

(48) For an equilateral triangle which of the following is NOT True?

(a) $r:R:r_1 = 1:2:3$ (b) $r: R: r_2 = 1:2:3$

(c) $r:R:r_1:r_2 = 1:2:3:3$ (d) $r:R:r_3 = 1:2:3$

(49) The sides of a triangle are 13, 14 and 15 then r = ?

(a) $\frac{67}{8}$ (b) $\frac{65}{4}$

(c) 4 (d) 24

(50) For an equilateral triangle which of the following must be true?

(a) $r = 4R$ (b) $r = \frac{R}{2}$

(c) $r = \frac{R}{3}$ (d) None of these

INVERSE TRIGONOMETRIC FUNCTIONS

In general, the inverse of a trigonometric function is not a function but restricting their domains to principal functions, they can be made as functions.

Domains and Ranges of Principal Trigonometric Functions

Trigonometric Function	Domain	Range
$Y = \sin x$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$-1 \le Y \le 1$
$Y = \cos x$	$0 \le x \le \pi$	$-1 \le Y \le 1$
$Y = \tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	$Y \in \mathbb{R}$
$Y = \cot x$	$0 < x < \pi$	$Y \in \mathbb{R}$
$Y = \sec x$	$[0, \pi], x \ne \frac{\pi}{2}$	$Y \ge 1$ OR $Y \le -1$
$Y = \operatorname{cosec} x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right], x \ne 0$	$Y \ge 1$ OR $Y \le -1$

- Remember that,

(i). $\sin^{-1}(x) \ne (\sin x)^{-1}$

(ii). $\cos^{-1}(x) \ne (\cos x)^{-1}$

(iii). $\tan^{-1}(x) \ne (\tan x)^{-1}$

Domains and Ranges Of Inverse Trigonometric Functions

Inverse Trigonometric Function	Domain	Range
$Y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le Y \le \frac{\pi}{2}$
$Y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le Y \le \pi$
$Y = \tan^{-1} x$	$x \in \mathbb{R}$	$-\frac{\pi}{2} < Y < \frac{\pi}{2}$
$Y = \cot^{-1} x$	$x \in \mathbb{R}$	$0 < Y < \pi$
$Y = \sec^{-1} x$	$x \le -1$ OR $x \ge 1$	$[0, \pi], Y \ne \frac{\pi}{2}$
$Y = \operatorname{cosec}^{-1} x$	$x \le -1$ OR $x \ge 1$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right], Y \ne 0$

Relations between Inverse Trigonometric Functions

(i). $\operatorname{cosec}^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$

(ii). $\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$

(iii). $\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right), (x > 0)$

(iv). $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$

(v). $\operatorname{cosec}^{-1}(x) + \sec^{-1}(x) = \frac{\pi}{2}$

(vi). $\tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}$

(vii). $\sin^{-1}(x) = \cos^{-1}\left(\sqrt{1-x^2}\right)$

(viii). $2\cos^{-1}(x) = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

(ix). $2\sin^{-1}(x) = \cos^{-1}\left(1-2x^2\right)$

(x). $\sin^{-1}(x) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

Some Important Results

(i). $\sin^{-1}(-x) = -\sin^{-1} x$ $\quad \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$

(ii). $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ $\quad \cot^{-1}(-x) = \pi - \cot^{-1} x$, $\sec^{-1}(-x) = \pi - \sec^{-1} x$

(iii). $\tan^{-1}(-x) = -\tan^{-1}(x)$

Addition Formulas

(i). $\sin^{-1}(A) + \sin^{-1}(B) = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$

(ii). $\sin^{-1}(A) - \sin^{-1}(B) = \sin^{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)$

(iii). $\cos^{-1}(A) + \cos^{-1}(B) = \cos^{-1}\left(AB - \sqrt{\left(1-A^2\right)\left(1-B^2\right)}\right)$

(iv). $\cos^{-1}(A) - \cos^{-1}(B) = \cos^{-1}\left(AB + \sqrt{\left(1-A^2\right)\left(1-B^2\right)}\right)$

(v). $\tan^{-1}(A) + \tan^{-1}(B) = \tan^{-1}\left(\dfrac{A+B}{1-AB}\right)$

(vi). $\tan^{-1}(A) - \tan^{-1}(B) = \tan^{-1}\left(\dfrac{A-B}{1+AB}\right)$

(vii). $2\tan^{-1} A = \tan^{-1}\left(\dfrac{2A}{1-A^2}\right)$

For Practice Exercise Go To Chapter 14

SOLUTIONS OF TRIGONOMETRIC EQUATIONS

Name	Definition
Trigonometric Equation	An equation involving at least one trigonometric function is called trigonometric equation.
Solution of a Trigonometric Equation	The values of the variables which satisfy the given trigonometric equation are called the solution of trigonometric equation.

Solution of a trigonometric equation is of two types;

1. **Principal value**
2. **General values**

Trigonometric Functions	Principal Value	General Values
$\sin\theta$	$\theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$\{\theta+2n\pi\}\cup\{(\pi-\theta)+2n\pi\}$
$\cos\theta$	$\theta\in[0,\pi]$	$\{\theta+2n\pi\}\cup\{(2\pi-\theta)+2n\pi\}$
$\tan\theta$	$\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	$\{\theta+n\pi\}\cup\{(\pi+\theta)+n\pi\}$

Some Important Results

Trigonometric equation	Principal solution	General solutions
$\sin x=0$	$x=0$	$x=n\pi\ ,n\in\mathbb{Z}$
$\cos x=0$	$x=\frac{\pi}{2}$	$x=(2n+1)\frac{\pi}{2},n\in\mathbb{Z}$
$\tan x=0$	$x=0$	$x=n\pi,n\in\mathbb{Z}$

PRACTICE EXERCISE CHAPTER 13+14

(1) Which of the following is a solution of $\cos 3x = \frac{1}{2}$?

(a) $60°$

(b) $\frac{5\pi}{3}$

(c) $\arccos\frac{1}{6}$

(d) $\frac{1}{3}\arccos\frac{1}{2}$

(2) $\sin^{-1}x + \cos^{-1}x = ?$

(a) 0

(b) π

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{3}$

(3) Range of $\text{Tan}^{-1}x$ is

(a) R

(b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(d) $[0, \pi]$

(4) Range of cosec^{-1} x is

(a) $(0, \pi)$

(b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) $[0, \pi]$

(d) none

(5) $\cos ec^{-1}\left(\frac{-2}{\sqrt{2}}\right) = ?$

(a) $-\frac{\pi}{4}$

(b) $-\frac{\pi}{3}$

(c) $-\frac{\pi}{2}$

(d) $\frac{\pi}{4}$

(6) Which of the following can be the solution of $\sin 2x = \cos x$?

(a) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

(b) $\left\{\frac{\pi}{3}, \frac{\pi}{4}\right\}$

(c) $\left\{\frac{2\pi}{3}, \frac{3\pi}{4}\right\}$

(d) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

(7) Solve for x, $\cos x = -\frac{\sqrt{3}}{2}$.

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

(8) $\sin^{-1}(-x) = ?$

(a) $\text{Cos}^{-1}x$ (b) $\cos^{-1}\frac{1}{x}$

(c) $-\text{Sin}^{-1}x$ (d) π-$\text{Cos}^{-1}x$

(9) $\text{Cot}^{-1}x = ?$

(a) $\frac{1}{\tan^{-1}x}$ (b) $\tan^{-1}\left(\frac{1}{x}\right)$

(c) $-Tan^{-1}x$ (d) $Cos^{-1}x$

(10) $\text{Cosec}^{-1}x = ?$

(a) $\sec^{-1}x$ (b) $\text{Sin}^{-1}x$

(c) $\frac{1}{\sin^{-1}(x)}$ (d) $\sin^{-1}\left(\frac{1}{x}\right)$

(11) The value of $\sin\left[\cos^{-1}\left(\frac{3}{4}\right)\right] =$



$16 = a^2 + 9$
$16 - 9 = a^2 \Rightarrow a^2 = 7$
$a = \sqrt{7}$

(a) $\frac{7}{4}$ (b) $\frac{\sqrt{7}}{4}$ $\frac{\sqrt{7}}{4}$ for sin

(c) $\frac{4}{7}$ (d) $\frac{5}{4}$

(12) Domain of $\text{Sec}^{-1}x$=?

(a) $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) $[0, \pi]$ (d) $x \geq 1,\ x \leq -1$

(13) Domain of $\text{Cot}^{-1}x$ is

(a) R (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) (-1, 1) (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(14) Range of $Cos^{-1}x$ is

(a) $[-1,1]$ (b) $[0,\pi]$

(c) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (d) none

(15) $\tan\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)=?$

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$

(c) $-\frac{1}{\sqrt{3}}$ (d) $-\sqrt{3}$

(16) $Sin^{-1}(1)+Cos^{-1}(1)=?$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) -1 (d) 1

(17) The principal value of $\cot^{-1}(-1)$ is

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $-\frac{\pi}{4}$ (d) $\frac{5\pi}{4}$

(18) Trigonometric equation contains _____ trigonometric functions.

(a) at least one (b) at the most one

(c) exactly four (d) exactly one

(19) Trigonometric functions are _____.

(a) 1 – 1 (b) continuous

(c) periodic (d) non-periodic

(20) A trigonometric equation has _____ solution(s).

(a) Only one (b) two

(c) many (d) infinite

(21) To solve a trigonometric equation ,firstly we find the solution over the interval which is ;

(a) domain (b) range

(c) period (d) none of these

(22) The solution of $\sin^2 x=\frac{3}{4}$ is;

(a) $\frac{\pi}{3},\frac{2\pi}{3}$ (b) $\frac{4\pi}{3},\frac{\pi}{4}$

(c) $-\frac{\pi}{4},\frac{2\pi}{3}$ (d) $-\frac{\pi}{4},\frac{2\pi}{5}$

(23) If $x = \tan^{-1}\left(\frac{1}{2}\right), y = \tan^{-1}\left(\frac{1}{3}\right)$, then $x + y = ?$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

(24) $\tan\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = ?$

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$

(c) $\sqrt{3}$ (d) 0

(25) $\cos\left(\tan^{-1}\infty\right) = ?$

(a) 0 (b) ∞

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(26) If $x + y = \frac{5}{6}$, $xy = \frac{1}{6}$ then $\tan^{-1}(x) + \tan^{-1}(y) = ?$

(a) 1 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

(27) $\frac{1}{x}\sin\left(\csc^{-1}\left(\frac{1}{x}\right)\right) = ?$

(a) x (b) $\frac{1}{x}$

(c) 1 (d) -1

(28) Solution of cos 4x = cos 2x is _____

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{n\pi}{12}$ (d) $\frac{n\pi}{3}$

(29) If $4\sin^{-1}(x) + \cos^{-1}(x) = \pi$ then x = ?

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

(30) $\cos^{-1}\left(\cos\frac{6\pi}{7}\right)=?$

(a) $\frac{\pi}{7}$ (b) $\frac{6\pi}{7}$

(c) $\frac{8\pi}{7}$ (d) $\frac{11\pi}{7}$

(31) Number of solutions of $\tan x+\sec x=2\cos x,\ x\in[0,2\pi]$ are _____

(a) 0 (b) 1

(c) 2 (d) 3

$\frac{\sin x}{\cos x}+\frac{1}{\cos x}=2\cos x$

$\frac{1+\sin x}{\cos x}=2\cos x$

$1+\sin x=2(1-\sin^2 x)$

$0=2(1+\sin x)-2+\sin x$

$-2=-2\sin x$

$\sin x=1$

(32) Domain of the function $\sin^{-1}(x)$ is _____

(a) $(-\pi,\pi)$ (b) $[-1,1]$

(c) $[0,2\pi]$ (d) $(-\infty,\infty)$

(33) $\tan\left(\cos^{-1}(x)\right)=?$

(a) $\frac{\sqrt{1-x^2}}{x}$ (b) $\frac{x}{\sqrt{1+x^2}}$

(c) $\frac{\sqrt{1+x^2}}{x}$ (d) $\frac{x}{\sqrt{1-x^2}}$

(34) $\sin\left[2\sin^{-1}(0.8)\right]=?$ $2\sin y\cos y$ $\sin^{-1}(0.8)=y$ $\sin y=0.8$

(a) 1.2 (b) 1.6

(c) 0.48 (d) 0.96

$\cos y=\sqrt{1-0.64}=\sqrt{\ }=0.6$

$=2\times0.8\times0.6$

$=0.96$

(35) $2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)=?$

(a) $\frac{49}{29}$ (b) $\frac{\pi}{2}$

(c) 0 (d) $\frac{\pi}{4}$

$\tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}}$ $\tan^{-1}\frac{\frac{2}{3}}{\frac{8}{9}}$

$\frac{2}{3}\times\frac{9}{8}\frac{3}{4}$

$\tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{28}}$

$\tan^{-1}\frac{\frac{21+4}{28}}{\frac{25}{28}}=$

ANSWER KEY

DIAGNOSTIC TEST

1	c	11	c	21	a	31	a
2	b	12	d	22	d	32	c
3	c	13	b	23	b	33	d
4	d	14	b	24	c	34	d
5	b	15	d	25	b	35	c
6	c	16	b	26	b	36	c
7	a	17	b	27	d	37	b
8	a	18	a	28	c	38	c
9	b	19	a	29	c	39	b
10	b	20	b	30	c	40	c

Evaluate yourself:

Number of Right Answers	Rating
33-40	Excellent
27-32	Very good
20-26	Above average
12-19	Average
below 12	Below average

CHAPTER-1

1	a	12	a	23	c	34	c	45	a
2	c	13	d	24	a	35	a	46	b
3	a	14	b	25	d	36	c	47	c
4	a	15	a	26	b	37	d	48	a
5	c	16	c	27	c	38	c	49	b
6	a	17	a	28	c	39	c	50	c
7	a	18	d	29	a	40	c	51	c
8	c	19	b	30	b	41	b	52	a
9	d	20	d	31	d	42	a	53	d
10	d	21	a	32	b	43	a	54	b
11	b	22	c	33	d	44	a	55	c

CHAPTER-2

1	c	12	b	23	d	34	a	45	c
2	c	13	a	24	b	35	c	46	b
3	a	14	d	25	a	36	b	47	c
4	c	15	b	26	c	37	c	48	a
5	c	16	c	27	d	38	b	49	c
6	a	17	a	28	a	39	c	50	b
7	b	18	d	29	c	40	a	51	c
8	a	19	b	30	d	41	b	52	d
9	d	20	a	31	c	42	d	53	c
10	d	21	d	32	b	43	b	54	c
11	c	22	c	33	c	44	c	55	b

CHAPTER-3

1	c	11	b	21	c	31	a	41	d	51	b
2	e	12	d	22	b	32	d	42	a	52	b
3	e	13	a	23	a	33	a	43	c	53	c
4	a	14	c	24	b	34	a	44	a	54	d
5	b	15	d	25	c	35	b	45	a	55	c
6	d	16	c	26	c	36	b	46	b	56	c
7	d	17	a	27	a	37	c	47	c	57	a
8	d	18	d	28	b	38	a	48	b	58	a
9	a	19	c	29	b	39	c	49	d	59	b
10	c	20	a	30	b	40	c	50	b	60	c

CHAPTER-4

1	c	11	d	21	a	31	c	41	b	51	a
2	d	12	b	22	d	32	d	42	a'	52	b
3	d	13	d	23	c	33	c	43	b	53	a
4	c	14	c	24	c	34	d	44	d	54	a
5	a	15	c	25	a	35	a	45	d	55	d
6	a	16	d	26	c	36	d	46	b	56	b
7	d	17	c	27	c	37	a	47	d	57	c
8	c	18	a	28	d	38	c	48	c	58	b
9	b	19	d	29	a	39	b	49	c	59	a
10	a	20	b	30	b	40	a	50	d	60	c

CHAPTER-5

1	a	6	a	11	a
2	b	7	a	12	c
3	c	8	a	13	d
4	b	9	c	14	a
5	c	10	d	15	b

CHAPTER-6

1	c	12	b	23	d	34	b	45	b	56	c
2	d	13	d	24	b	35	d	46	d	57	b
3	a	14	b	25	c	36	b	47	d	58	c
4	d	15	d	26	d	37	c	48	b	59	c
5	c	16	a	27	a	38	b	49	b	60	b
6	d	17	b	28	d	39	a	50	a	61	c
7	a	18	d	29	b	40	c	51	a	62	b
8	b	19	c	30	a	41	d	52	a	63	
9	b	20	c	31	c	42	c	53	b	64	d
10	a	21	b	32	d	43	b	54	d	65	
11	c	22	d	33	d	44	c	55	a		

CHAPTER-7

1	b	11	c	21	c	31	c	41	c
2	c	12	d	22	d	32	c	42	c
3	b	13	b	23	c	33	b	43	b
4	d	14	d	24	c	34	d	44	b
5	c	15	b	25	b	35	a	45	d
6	c	16	d	26	a	36	c	46	c
7	b	17	b	27	c	37	b	47	b
8	a	18	a	28	c	38	c	48	c
9	d	19	d	29	d	39	c	49	a
10	c	20	b	30	c	40	a	50	c

CHAPTER-8

1	c	11	c	21	d	31	a
2	d	12	a	22	d	32	b
3	a	13	b	23	d	33	d
4	d	14	c	24	b	34	a
5	c	15	a	25	b	35	b
6	b	16	d	26	b	36	a
7	a	17	b	27	b	37	a
8	d	18	c	28	a	38	c
9	c	19	a	29	d	39	a
10	d	20	d	30	c	40	d

CHAPTER-9

1	c	10	b	19	d	28	b	37	b
2	b	11	c	20	b	29	c	38	a
3	d	12	b	21	c	30	d	39	a
4	a	13	b	22	b	31	b	40	b
5	b	14	b	23	a	32	a	41	c
6	c	15	c	24	d	33	b	42	c
7	c	16	c	25	d	34	c	43	b
8	a	17	d	26	b	35	a	44	a
9	a	18	a	27	b	36	d	45	c

CHAPTER-10

1	a	10	c	19	b	28	d	37	c
2	c	11	b	20	b	29	b	38	b
3	c	12	d	21	c	30	c	39	a
4	b	13	b	22	d	31	b	40	c
5	c	14	a	23	a	32	c	41	a
6	c	15	b	24	b	33	c	42	c
7	d	16	c	25	c	34	a	43	a
8	b	17	c	26	b	35	d	44	c
9	a	18	d	27	a	36	b	45	d

CHAPTER-11

1	a	6	b	11	d	16	c	21	a
2	a	7	a	12	b	17	d	22	c
3	b	8	b	13	d	18	b	23	d
4	a	9	c	14	c	19	c	24	c
5	b	10	d	15	a	20	b	25	c

CHAPTER-12

1	b	11	d	21	b	31	a	41	c
2	b	12	a	22	b	32	d	42	c
3	a	13	d	23	d	33	b	43	d
4	b	14	b	24	c	34	b	44	c
5	c	15	a	25	b	35	c	45	a
6	d	16	d	26	d	36	c	46	d
7	d	17	c	27	b	37	b	47	b
8	c	18	d	28	c	38	d	48	c
9	b	19	b	29	c	39	a	49	c
10	a	20	c	30	a	40	b	50	b

CHAPTER-13 & 14

1	d	8	c	15	d	22	a	29	b
2	c	9	b	16	b	23	d	30	b
3	c	10	d	17	a	24	a	31	c
4	d	11	b	18	a	25	a	32	b
5	a	12	d	19	c	26	b	33	a
6	d	13	a	20	d	27	c	34	d
7	c	14	b	21	c	28	d	35	d

KIPS ENTRY TEST SERIES

Part - II

KIPS PUBLICATION

FUNCTIONS AND LIMITS

Introduction

Function /Mapping*	(i). A function is a rule or correspondence, relating two sets in such a way that each element in the first set corresponds to one and only one element in the second set. (ii). A function f from a set X to Y is a rule or a correspondence that assigns to each element x in X a unique element in Y.
Notation for a function	(a) $y = f(x)$ (b) $f : X \to Y$ (c) f is a mapping from X to Y
Domain	From the above notation X is called domain or independent variable of $f(x)$. The domain of a function always belongs to the set of real numbers unless stated.
Range	Y is called range or dependent variable of $f(x)$.
Graph of a function	If a vertical line intersects a graph in the most one point, it is the graph of a function.
Piece-wise functions	Such functions which are defined by two or more rules are called piece-wise defined functions.

* The term function was recognized by a German Mathematician Leibniz (1646-1716).

Types of functions

Some important types of functions are discussed here.

1. Algebraic Functions	*Definition/Domain/Range*
(a) Polynomial Function	The function of the form $P(x)=a_n x^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\cdots a_2x^2+ax+a_0$ is called polynomial function of degree n. $a_n \neq 0$, and n is any non negative integer. Domain and range is real numbers.
(b) Linear Function	If the degree of a polynomial function is 1, then it is called linear function. The general form a linear function is $f(x)=ax+b$, $a\neq 0$ a & b are real numbers. Domain and range of linear function is real numbers. The graph of a linear function is a straight line.
(c) Identity Function	For any set X, a function $I:X\to X$ of the form $I(x)=x$, $\forall x\in X$ is called an identity function. Domain and range of I is X.
(d) Constant Function	Let X and Y be sets of real numbers. A function $C:X\to Y$ defined by $C(x)=a, \forall x\in X, a\in Y$ is called a constant function. Domain is all real numbers. Range is a.(any constant)
(e) Rational Function	A function $R(x)$ of the form $\frac{P(x)}{Q(x)}$, where both $P(x)$ and $Q(x)$ are polynomial functions and $Q(x)\neq 0$,is called a rational function. The domain of a rational function is set of all real numbers for which $Q(x)\neq 0$. The range set of real numbers.

2.Trigonometric Functions	*Definition/Domain/Range*
(a) The sine function	$y=\sin x$,Domain = R ,Range= $-1\le y\le 1$
(b) The cosine function	$y=\cos x$,Domain = R ,Range= $-1\le y\le 1$
(c) The tangent function	$y=\tan x$, $Domain=\left\{x: x\in R\wedge x\neq (2n+1)\frac{\pi}{2}\right\}, n\in Z$ $Range = R$
(d) The cotangent function	$y=\cot x, Domain=\{x: x\in R\wedge x\neq n\pi\}$, $n\in Z$, $Range = R$
(e) The secant function	$y=\sec x$, $Domain=\left\{x: x\in R\wedge x\neq (2n+1)\frac{\pi}{2}\right\}$ $n\in Z$, $Range = y\ge 1, y\le -1$
(f) The cosecant function	$y=\cos ex$, $Domain=\{x: x\in R\wedge x\neq n\pi\}$, $n\in Z$, $Range =$ $y\ge 1, y\le -1$

3.Inverse Trigonometric Functions	Definition/Domain/Range
(a) Inverse sine function	$y = \sin^{-1} x \Leftrightarrow x = \sin y$,Domain = $-1 \le x \le 1$ Range = $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
(b) Inverse cosine function	$y = \cos^{-1} x \Leftrightarrow x = \cos y$, Domain = $-1 \le x \le 1$ Range = $0 \le y \le \pi$
(c) Inverse tangent function	$y = \tan^{-1} x \Leftrightarrow x = \tan y$,Domain= $-\frac{\pi}{2} < y < \frac{\pi}{2}$ Range = $-\infty < x < \infty$

4.Exponential Functions	Definition/Domain/Range
A function, in which the variable appears as exponent, is called an exponential function. $y = e^x$, $y = e^{ax}$, $y = a^x$ are exponential functions. Domain = R . Range = R Range of e^x is always non-negative real numbers.	

5.Logarithmic Function	Definition/Domain/Range
The functions involving *logx* or *lnx* are called logarithmic functions. Domain = Positive Real Numbers Range = All Real Numbers	

6.Hyperbolic Function	Definition/Domain/Range
(a) Hyperbolic sine function	$\sinh x = \frac{e^x - e^{-x}}{2}$,its domain and range is set of all real numbers.
(b) Hyperbolic cosine function	$\cosh x = \frac{e^x + e^{-x}}{2}$,Domain = R Range = $[1, +\infty)$
(c) Hyperbolic tangent function	$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
(d) Hyperbolic cotangent function	$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$
(e) Hyperbolic secant function	$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$
(f) Hyperbolic cosecant function	$co\operatorname{sech} x = \frac{2}{e^x - e^{-x}}$

7.Inverse Hyperbolic Functions	
(a)	$\sinh^{-1} x = \ln\left(x+\sqrt{x^2+1}\right), \forall x$
(b)	$\cosh^{-1} x = \ln\left(x+\sqrt{x^2-1}\right), x \geq 1$
(c)	$\tanh^{-1} x = \ln\left(\sqrt{\frac{1+x}{1-x}}\right), \lvert x\rvert < 1$
(d)	$co\tanh^{-1} x = \ln\left(\sqrt{\frac{x+1}{x-1}}\right), \lvert x\rvert < 1$
(e)	$\operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right), 0 < x \leq 1$
(f)	$co\operatorname{sech}^{-1} x = \ln\left(\frac{1}{x}+\frac{\sqrt{1+x^2}}{\lvert x\rvert}\right), x \neq 0$

Some Other Important Functions

Name of The Function	Definition	Example
Explicit function	If y is easily expressed in terms of independent variable x, then y is called an explicit function.	$y=\sqrt{2x+1}$ $y=x^2+4x+1$
Implicit function	If y cannot expressed in terms of independent variable x, then y is called an implicit function.	$xy^2+2x^2y=0$
Even function [1]	A function $f(x)$ is said to be even if $f(-x)=f(x)$.	$f(x)=\cos x$ $f(x)=x^2+1$
Odd function [2]	A function $f(x)$ is said to be odd if $f(-x)=-f(x)$.	$f(x)=\sin x$ $f(x)=x^3+x$
Greatest integer function	The greatest integer function is denoted by $[x]$,pairs with each real number x the greatest integer not exceeding x.	$[3.2]=3$ $[-0.6543]=-1$
Absolute value function	$\lvert x\rvert=\begin{cases}x, x\geq 0\\ -x, x<0\end{cases}$	$\lvert 3\rvert=3$ $\lvert -4.12\rvert=4.12$
Parametric functions	Some times the relation between x and y can be expressed in terms of a third variable, which is called parameter, such types of functions are parametric functions.	$\left.\begin{matrix}x=at^2\\ y=at\end{matrix}\right\}$ Parabola. $\left.\begin{matrix}x=a\cos\theta\\ y=a\sin\theta\end{matrix}\right\}$ circle $\left.\begin{matrix}x=a\cos\theta\\ y=b\sin\theta\end{matrix}\right\}$ ellipse $\left.\begin{matrix}x=a\sec\theta\\ y=b\tan\theta\end{matrix}\right\}$ hyperbola

[1] Sum, difference and quotient of even (odd) function are even (odd).
[2] Product of an even and odd function is odd.

Some important identities

(i). $\cosh^2 x - \sinh^2 x = 1$

(ii). $\cosh^2 x + \sinh^2 x = \cosh 2x$

(iii). $\sinh 2x = 2\sinh x \cosh x$

(iv). $\coth^2 x - 1 = \operatorname{csc} h^2 x$

(v). $1 - \tanh^2 x = \sec h^2 x$

Algebraic Operations on Functions

Name of operation	Definition
Composition of Functions	For any two defined functions $f(x)$ and $g(x)$ $(f \circ g)(x) = f(x) \circ g(x) = f(g(x))$ is called composition of $f(x)$ with respect to $g(x)$. Remember that, (i). $f(x) \circ g(x) \neq g(x) \circ f(x)$ in general. (ii). $f^2(x) = f(f(x))$ (iii). $f^3(x) = f(f(f(x)))$ and so on .
Inverse of a function	The inverse of a function f,is a relation denoted by f^{-1} ,that has a property that $f(x) \circ f^{-1}(x) = f^{-1}(x) \circ f(x) = x$ Remember that inverse of a function may or may nor be a function.

Limits

Limit of a function	A function $f(x)$ has the limit "L" as $x \to a$ (x approaches to a) if for every positive number ε ,there is a positive number δ such that , $\lvert f(x) - L \rvert < \varepsilon$ if $0 < \lvert x - a \rvert < \delta$. Symbolically $\lim_{x \to a} f(x) = L$ The $\lim_{x \to a} f(x)$ may exist even if it is not defined at $x=a$.

Some fundamental theorems on limits of different functions

(i). $\lim_{x \to a}\left[f(x) \pm g(x)\right] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

(ii). $\lim_{x \to a}\left[f(x).g(x)\right] = \lim_{x \to a} f(x).\lim_{x \to a} g(x)$

(iii). $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \lim_{x \to a} g(x) \neq 0$

(iv). $\lim_{x \to a}\left(Kf(x)\right) = K \lim_{x \to a} f(x)$, K is any real number.

(v).If $f(x) \le g(x)$ for all x then $\lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$.

(vi). $\lim_{x\to a} \ln f(x) = \ln \lim_{x\to a} f(x)$

(vii). $\lim_{x\to a}\left[f(x)\right]^n = \left[\lim_{x\to a} f(x)\right]^n$, where n is an integer.

(viii).Sandwich Theorem (Squeeze Principle):

Let f, g and h be functions such that $f(x) \le g(x) \le h(x)$ for all values of x. if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} h(x) = L$ then $\lim_{x\to a} g(x) = L$.

Limit of a function at infinity:

(a) The limit of a function at infinity means the limit as $x \to +\infty$ (x increases beyond all bounds). If the limit is K we write as follows,

$\lim_{x\to+\infty} f(x) = K$.

(b) If $x \to -\infty$ (x decreases beyond all bounds) and the limit of the function is L we write as follows,

$\lim_{x\to-\infty} f(x) = L$.

Indeterminate forms:

An indeterminate form is an expression of the type:

$\infty - \infty$	$\frac{0}{0}$	$\frac{\infty}{\infty}$
$(\infty)^0$	$(0)^\infty$	$(0)^0$
$(1)^\infty$	$0\times\infty$	$\infty\times\infty$

One sided limits	
The left hand limit	$\lim_{x\to a} f(x) = L$ is read as the limit of $f(x)$ is equal to L as x approaches a from the left i.e. for all x sufficiently close to a, but less than a.
The right hand limit	$\lim_{x\to a^+} f(x) = M$ is read as the limit of $f(x)$ is equal to M as x approaches a from the right *i.e.* for all x sufficiently close to a, but greater than a.
Existence of Limit of a Function	*The limit of a function exists if right hand limit =left hand limit* *i.e. if* $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$
Continuity of a Function at a Number	A function f is said to be continuous at a number "c" iff the following three conditions are satisfied. (i). $f(c)$ is defined. (ii). $\lim_{x\to c} f(x)$ exists. (iii). $\lim_{x\to c} f(x) = f(c)$ If one or more of these conditions fail to hold at "c", then the function is discontinuous.

Methods for Evaluating the Limit of a Function

Rationalization Method	In this method, we rationalize the factor containing the square root. After simplification ,we put the value of x to get the limit of $f(x)$
Factorization Method	Let $\frac{f(x)}{g(x)}, g(x) \neq 0$, we factorize *f(x)* and *g(x)* if possible. Cancel the common factors to get the simplified form of *f(x)*.this form of function in general, is a determinate form. Now we put the value of *x* to get the limit of *f(x)*.
Substitution Method	In this method we try out different forms of substitutions, depending upon the form of the given function and removing, in turn the indeterminate form. After simplification, we put the value of *x* to get the limit of *f(x)*.
Rule of De l'Hospital*	*If* $\lim_{x\to a} f(x) = o$ *and* $\lim_{x\to a} g(x) = o$ *then* $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

Some Standard Limits

(i). $\lim_{x\to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

(ii). $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = e$

(iii). $\lim_{x\to\infty} \left(1 + \frac{p}{x}\right)^x = e^p, p \in \mathbb{R}$

(iv). $\lim_{x\to 0} (1 + x)^{\frac{1}{x}} = e$

(v). $\lim_{x\to 0} (1 + px)^{\frac{1}{x}} = e^p, p \in \mathbb{R}$

(vi). $\lim_{x\to 0} \left(1 + \frac{x}{a}\right)^{\frac{a}{x}} = e$

(vii). $\lim_{x\to 0} e^x = 1$

(viii). $\lim_{x\to\infty} e^x = \infty$

* A French mathematician de l' Hospital (1661-1704)

(ix). $\lim_{x\to-\infty} e^x = 0$

(x). $\lim_{x\to0} \frac{e^x - 1}{x} = 1$

(xi). $\lim_{x\to0} \frac{\ln(1+x)}{x} = 1$

(xii). $\lim_{x\to0} \frac{a^x - 1}{x} = \ln a$

Limits Involving Trigonometric Functions

(i). $\lim_{x\to0} \frac{\sin x}{x} = 1$

(ii). $\lim_{x\to0} \frac{\sin px}{qx} = \frac{p}{q}$,p & q are any real numbers.

(iii). $\lim_{x\to\infty} \frac{\sin x}{x} = 0$

(iv). $\lim_{x\to0} \frac{\tan x}{x} = 1$

(v). $\lim_{x\to0} \cos x = 1$

(vi). $\lim_{x\to0} \sin x = 0$

- In all above cases x is always measured in radians.

PRACTICE EXERCISE

(1) $\lim_{x\to 0}\frac{\sqrt{49+x}-7}{x}=?$

(a) 0
(b) $\frac{1}{14}$
(c) –6
(d) ∞

(2) The domain of the function $h(x)=\sqrt{x^2-4}$ is
(a) Real no's
(b) Real no's – {2}
(b) (c){x/x∈R∧-2<x <2}
(d) $R-(-2,2)$

(3) $\lim_{x\to 0} e^{\frac{1}{x}}=?$
(a) ∞
(b) 0
(c) 1
(d) 1

(4) The domain of the inverse of the function $f(x)=2+\sqrt{x-1}$ is _____?
(a) [1. ∞)
(b) [2, ∞)
(c) (1. ∞)
(d) (2 ,∞)

(5) $\lim_{x\to c}\frac{x^p-c^p}{x-c}=?$
(a) 0
(b) 1
(c) px^{p-1}
(d) pc^{p-1}

(6) $\lim_{x\to 3}\frac{4x-8}{x+2}=?$
(a) –6
(b) 20
(c) –20
(d) $\frac{4}{5}$

(7) Given $f(x)=x^2+3$ then $f(a+b)=?$
(a) $a^2+b^2+2ab+3$
(b) $a^2+b^2-2ab-3$
(c) a^2+b^2-3ab
(d) x^2+3

(8) $\lim_{x\to 0}\frac{5x^4+3x^3+9}{7x^4+5x^2+17}=?$
(a) $\frac{5}{7}$
(b) $\frac{3}{5}$
(c) $\frac{9}{17}$
(d) 0

(9) Let the variable Z take in succession the values $6, 6\frac{1}{2}, 6\frac{2}{3}, 6\frac{4}{5}, \cdots$ then $Z\to ?$
(a) 0
(b) 7
(c) 5
(d) 6

(10) Give $f(x) = 4x+5$, $g(x) = 3x+7$ then $f\left(g\left(x\right)\right) = ?$

(a) $12x+28$ (b) $12x+33$

(c) $4x-31$ (d) $25x+34$

(11) $\lim_{x\to 0} \frac{x^5+3x^3+9x^2+7x+8}{2x^6+17x^4+3x^2+5} = ?$

(a) 0 (b) $\frac{8}{5}$

(c) $\frac{1}{2}$ (d) ∞

(12) $\lim_{z\to 0} \frac{\sin pz}{mz} = ?$

(a) 0 (b) ∞

(c) $\frac{1}{m}$ (d) $\frac{p}{m}$

(13) Which one is true?

(I) $\lim_{x\to\infty} e^x = \infty$ (II) $\lim_{x\to\infty} e^x = 0$ (III) $\lim_{x\to\infty} a^x = a$

(a) I & II only (b) II and III only

(c) I only (d) all

(14) Let $|X|$ denotes the number of elements in X, then $|X \times Y| = ?$

(a) $|X| + |Y|$ (b) $|X|-|Y|$

(c) $|X|\times|Y|$ (d)none of these

(15) Let A = {3.5.7.-9}. B={0.1.-3} and R={(3.0). (5.1).(7.-3). (-9.0)} then domain of R=?

(a) {0.1.-3.0} (b) {0.1.-3}

(c) {3.5.7-9} (d) {3.5} (e) none

(16) $\lim_{x\to 0} \frac{1-\cos 2x}{x^2} = ?$

(a) 0 (b) 1

(c) –1 (d) 2

(17) $\lim_{x\to 3} \frac{x^3-27}{x^2-9} = ?$

(a) 0 (b)

(c) 1 (d) $\frac{9}{2}$

(18) $\lim_{\theta\to 0} \frac{1-\cos p\theta}{1-\cos q\theta} = ?$

(a) $\frac{p^2}{q^2}$ (b) $\frac{q^2}{p^2}$

(c) 0 (d)

(19) $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=?$

(a) $\lim_{h\to 0}\frac{f(a+h)+f(a)}{h}$

(b) $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

(c) lna

(d) e

(20) Inverse function of $y=\frac{x}{x+5}$ is

(a) $\frac{x}{x+5}$

(b) $\frac{x}{x-5}$

(c) $\frac{x}{1-x}$

(d) $\frac{5x}{1-x}$

(21) $\lim_{h\to 0}\cos ec(\pi+h)=?$

(a) 0

(b) 1

(c) -1

(d) ∞

(22) The graph of the function $y = logx$ has an asymptote at _____.

(a) $y=0$

(b) $x=0$

(c) $x=10$

(d) $y=10$

(23) $\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^{x}=?$

(a) e

(b) e^{-1}

(c) e^2

(d) $\frac{1}{e^2}$

(24) The graph of the parametric equations $x=t^2$ and $y=t$
When $-2\le t\le 2$ represents _____.

(a) Circle

(b) parabola

(c) ellipse

(d) straight line

(25) If A= {1,2,..... n}, how many distinct relations can be defined on A?

(a) n^2

(b) 2^n

(c) 2^{n^2}

(d) 0

(26) $\lim_{x\to 0}\frac{e^{\frac{-1}{x^2}}}{1+e^{\frac{-1}{x^2}}}=?$

(a) 0

(b) 1

(c) -1

(d) ∞

(27) $\lim_{x\to 0}(1-4x)^{\frac{1}{x}}=?$

(a) e^4

(b) $\frac{1}{e^4}$

(c) e

(d) e^{-4x}

(28) $\lim_{\theta\to\frac{\pi}{2}} \tan\left(\frac{\theta}{2}\right) = ?$

(a) 1
(b) 0
(c) ∞
(d) -1

(29) The values of b and c for which $f(x+1) - f(x) = 8x + 3$ holds true, where $f(x) = bx^2 + cx + d$, are

(a) $b = 2, c = -1$
(b) $b = 4, c = -1$
(c) $b = -1, c = 4$
(d) none

(30) If $f(x) = x^2 - \frac{1}{x^2}$ then $f(x) = ?$

(a) $-f\left(\frac{1}{x}\right)$
(b) $f\left(\frac{1}{x}\right)$
(c) $f(x)$
(d) $f(x^2)$

(31) The range of the function f(x) = [x] is

(a) N
(b) Z
(c) R
(d) none

(32) $\lim_{x\to 0} \frac{(1-\cos 2x)\sin 5x}{x^2 \sin 3x} = ?$

(a) $\frac{10}{3}$
(b) $\frac{3}{10}$
(c) $\frac{6}{5}$
(d) $\frac{5}{6}$

(33) $\lim_{x\to\infty} \frac{x^2 - 3x + 2}{2x^3 + x - 3} = ?$

(a) 2
(b) $\frac{1}{2}$
(c) 0
(d) does not exist

(34) $\lim_{x\to\infty} \frac{\sin x}{x} = ?$

(a) 0
(b) undefined
(c) 1
(d) ∞

(35) $\lim_{x\to\infty} \left(\frac{x}{x+2}\right)^x = ?$

(a) e
(b) e^{-1}
(c) $\frac{1}{e^2}$
(d) e^5

(36) $\frac{e^x + e^{-x}}{e^x - e^{-x}} = ?$

(a) $tanhx$
(b) $cothx$
(c) $\coth^{-1} x$
(d) $cosechx$

(37) $Cosh^{-1}x =$ _____

(a) $\frac{1}{2}\ln(1+x)$ (b) $\frac{1}{2}\ln(1-x)$

(c) $\frac{1}{2}\ln\left(x+\sqrt{x^2-1}\right)$ (d) $\ln\left(x+\sqrt{x^2-1}\right)$

(38) Which of the following is an implicit function?

(a) $y=x-1$ (b) $y-1=x^2$

(c) $x^2+y+2=0$ (d) $x^2+xy+y^2=9$

(39) If $f(x) = 4x^2+xy+y^2-7$ then $f(x)$ is (an) _____ function

(a) Even (b) odd

(c) both even and odd (d) Neither even nor odd

(40) $f(x)=\frac{3x+1}{2x-1}$ then $f\left(f^{-1}(2)\right)=?$

(a) x (b) ½

(c) 2 (d) 7

(41) Let p be a +ve rational number if x^p is defined then

$\lim_{x\to\infty}\frac{a}{x^p}=?$

(a) 1 (b) p

(c) 0 (d) x

(42) If $f(x)\le g(x)\le h(x)$ and $\lim_{x\to c} f(x)=\lim_{x\to c} h(x)=2$,then $\lim_{x\to c} g(x)=?$

(a) <2 (b) >2

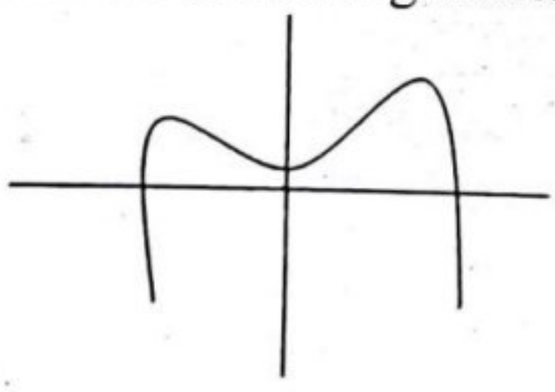
(c) =2 (d) nothing can be said

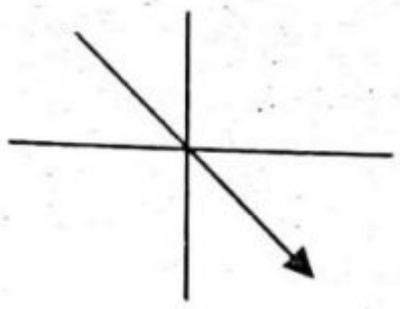
(43) For a function if right hand limit = left hand limit then which of the following must be true

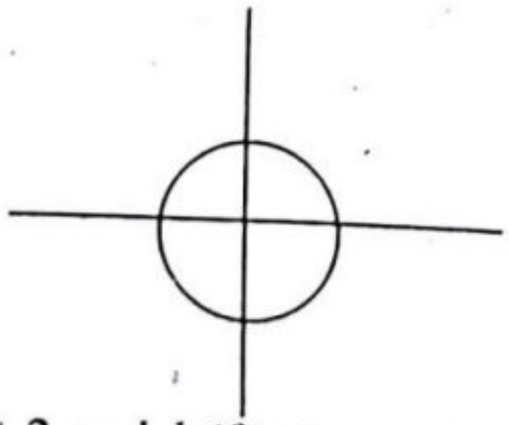
(a) Function is continuous (b) Function has same domain and range

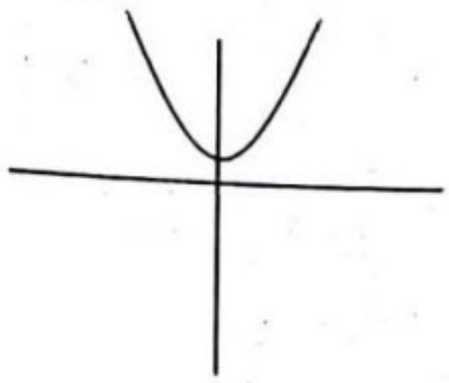
(c) Function is defined (d) its limit exists

(44) Which of the following cannot be the graph of a function?

(a) 

(b) 

(c) 

(d) 

(45) If $h(x)=3x+2$ and $h(f(x))=x$, then $f(2)=?$

(a) 0 (b) 2

(c) -2 (d) 1

(46) The domain of the function $f(x)=\frac{|x+2|}{x+2}$ is;

(a) $\mathbb{R}-\{2\}$ (b) $\mathbb{R}$

(c) $\mathbb{R}-\{\pm 2\}$ (d) $\mathbb{R}-\{-2\}$

(47) $\log_2\left[\log_3\left[\log_2 x\right]\right]=1$ then $x=?$

(a) 4^3 (b) 3^4

(c) 2^9 (d) 2^2

(48) The domain of the function $g(x)=e^{\sqrt{x^2-1}}.\ln(x-1)$ is;

(a) $(1,\infty)$ (b) $[1,\infty)$

(c) $\mathbb{R}-\{1\}$ (d) $[1,\infty]$

(49) The graph of $x^2+y^2=9$ is symmetrical about

(a) x-axis (b) y-axis

(c) origin (d) All of these

(50) The domain of $y=\sqrt{-x}$ is;

(a) $(0,\infty)$ (b)

(c) $(-\infty,0)$ (d) $(-\infty,0]$

(51) The range of the function $f=\{(1,x),(2,y),(3,z)\}$ is _____

(a) $\{1,x,z\}$ (b) $\{1,y,z\}$

(c) $\{1,2,3\}$ (d) $\{x,y,z\}$

(52) The domain of definition of the function $y=\frac{1}{\sqrt{16-x^2}}$ is _____

(a) (-4, 4) (b) [-4, 4]

(c) R – (-4, 4) (d) (4, ∞)

(53) If $f(x)=\frac{x-1}{x+1}, x\neq -1$ then $f^{-1}(x)=?$

(a) $\frac{1+x}{x-1}$ (b) $\frac{1+x}{1-x}$

(c) $\frac{2}{1+x}$ (d) $\frac{1}{x-1}$

(54) If $f(x)=4x-x^2$, then $f(a+1)-f(a-1)$ is equal to

(a) 4(2 – a) (b) 2 (4 – a)

(c) 4 (2 + a) (d) 2 (4 + a)

(55) The range of the function $f(x)=\frac{1+x^2}{x^2}$ is _____

(a) [0, 1] (b) (0, 1)

(c) (1, ∞) (d) [1, ∞]

(56) A function f:R $\rightarrow$ R is defined by $f(x)=\begin{cases}1, & if\ x\in Q\\ -1, & if\ x\in Q'\end{cases}$ then $f(\pi)-f\left(\frac{22}{7}\right)=?$

(a) 0
(b) 2
(c) -2
(d) None of these

(57) $\lim\limits_{x\to 2}[x]=?$

(a) 1
(b) 2
(c) 0
(d) Does not exist

(58) $\lim\limits_{x\to 0}\frac{e^x-e^{-x}}{\sin x}=?$

(a) 1
(b) 2
(c) 0
(d) -1

(59) $\lim\limits_{x\to 0}\frac{4^x-3^x}{3^x-2^x}=?$

(a) $\ell n\left(\frac{4}{3}\right)$
(b) $\ell n\left(\frac{3}{2}\right)$
(c) $\frac{\ell n\left(\frac{4}{3}\right)}{\ell n\left(\frac{3}{2}\right)}$
(d) $\ell n\left(\frac{4}{3}\right)-\ell n\left(\frac{3}{2}\right)$

(60) $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{x-\frac{\pi}{4}}=?$

(a) 2
(b) $\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$
(d) $2\sqrt{2}$

DIFFERENTIATION

Name of Mathematician	Notation used for Derivative
Leibniz	$\frac{dy}{dx}$ or $\frac{df}{dx}$
Newton	$\dot{f}(x)$
Lagrange	$f'(x)$
Cauchy	$Df(x)$

If $y=f(x)$,the derivative of "y" with respect to x is defined as;

(i). $\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

(ii). $\frac{d}{dx}f(x)=\lim_{\delta x\to 0}\frac{f(x+\delta x)-f(x)}{\delta x}$

(iii). $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

The process of taking derivative is called differentiation.

General Rules of Differentiation

(i). $\frac{d}{dx}(c)=0$, where c is a constant

(ii). $\frac{d}{dx}(x)=1$

(iii). $\frac{d}{dx}(cx)=c$

(iv). $\frac{d}{dx}(x^n)=nx^{n-1}$ $n\in\mathbb{Q}$ → power rule

(v). $\frac{d}{dx}(cx^n)=cnx^{n-1}$

(vi). $\frac{d}{dx}(u\pm v\pm w\pm\cdots)=\frac{du}{dx}\pm\frac{dv}{dx}\pm\frac{dw}{dx}\pm\cdots$

(vii). $\frac{d}{dx}(f(x).g(x))=f'(x).g(x)+f(x).g'(x)$ (The product rule).

(viii). $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)g(x)-f(x).g'(x)}{(g(x))^2}$ (the quotient rule)

(ix). $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$ (the chain rule)

(x) $\frac{d}{dx}\left(\frac{1}{f(x)}\right)=-\frac{f'(x)}{[f(x)]^2}$ (reciprocal rule)

Derivatives of Trigonometric Functions

(i). $\frac{d}{dx}\sin x = \cos x$

(ii). $\frac{d}{dx}\cos x = -\sin x$

(iii). $\frac{d}{dx}\tan x = \sec^2 x$

(iv). $\frac{d}{dx}\cot x = -\cos ec^2 x$

(v). $\frac{d}{dx}\cos ecx = -\cos ecx\cot x$

(vi). $\frac{d}{dx}\sec x = \sec x\tan x$

Derivatives of Inverse Trigonometric Functions

(i). $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}, \left[x \in (-1,1)\right]$

(ii). $\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}, \left[x \in (-1,1)\right]$

(iii). $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}, \left[x \in \mathbb{R}\right]$

(iv). $\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}, \left[x \in \mathbb{R}\right]$

(v). $\frac{d}{dx}\cos ec^{-1} x = \frac{-1}{x\sqrt{x^2-1}}, \left[x \in [-1,1]'\right]$

(vi). $\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}, \left[x \in [-1,1]'\right]$

Derivatives of Hyperbolic Functions

(i). $\frac{d}{dx}\sinh x = \cosh x$

(ii). $\frac{d}{dx}\cosh x = \sinh x$

(iii). $\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$

(iv). $\frac{d}{dx}\coth x = -\cos ech^2 x$

(v). $\frac{d}{dx}\cos ec\,h\,x = -\coth x\cos echx$

(vi). $\frac{d}{dx}sec\,h\,x = -\tanh x\,sechx$

Derivatives of Inverse Hyperbolic Functions

(i). $\frac{d}{dx}\sinh^{-1}x=\frac{1}{\sqrt{1+x^2}},x\in\mathbb{R}$

(ii). $\frac{d}{dx}\cosh^{-1}x=\frac{1}{\sqrt{x^2-1}},x>1$

(iii). $\frac{d}{dx}\tanh^{-1}x=\frac{1}{1-x^2},|x|<1$

(iv). $\frac{d}{dx}\coth^{-1}x=\frac{1}{1-x^2},|x|>1$

(v). $\frac{d}{dx}\operatorname{sech}^{-1}x=\frac{-1}{x\sqrt{1-x^2}},0<x<1$

(vi). $\frac{d}{dx}co\operatorname{sech}^{-1}x=\frac{-1}{|x|\sqrt{1+x^2}},x\in\mathbb{R}-\{0\}$

Derivatives of Exponential and Logarithmic Functions

(i). $\frac{d}{dx}e^x=e^x$

(ii). $\frac{d}{dx}e^{f(x)}=e^{f(x)}.f'(x)$

(iii). $\frac{d}{dx}a^x=a^x\ln a$

(iv). $\frac{d}{dx}a^{f(x)}=a^{f(x)}\ln a.f'(x)$

(v). $\frac{d}{dx}\ln x=\frac{1}{x}$

(vi) $\frac{d}{dx}\log_a x=\frac{1}{x\ln a}$

(vii). $\frac{d}{dx}\ln f(x)=\frac{1}{f(x)}.f'(x)$

Higher Derivatives

The second, third and higher derivatives are defined as follows;

(i). Second derivative$=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}=f''(x)=y''=D^2y$

(ii). Third derivative $=\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)=\frac{d^3y}{dx^3}=f'''(x)=y'''=D^3y$

(iii). n^{th} derivative $=\frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right)=\frac{d^ny}{dx^n}=f^{(n)}(x)=y^{(n)}=D^ny$

Series Expansions of Functions

1. **If f is defined interval containing 'a' and its derivatives of all orders exists at x = a, then, Taylor series expansion of a function $f(x)$ at $x=a$ is**

$$f(x)=f(a)+(x-a)f'(a)+\frac{(x-a)^2 f''(a)}{2!}$$

$$+\frac{(x-a)^3 f'''(a)}{3!}+\frac{(x-a)^4 f^{(4)}(a)}{4!}+\cdots+\frac{(x-a)^n f^{(n)}(a)}{n!}+\cdots$$

Or

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)(x-a)^n}{n!}$$

2. **If f is defined in the interval containing 0 and its derivatives exists at x = 0 then, Maclaurin Series expansion of a function is**

$$f(x)=f(0)+xf'(0)+x^2\frac{f''(0)}{2!}+x^3\frac{f'''(0)}{3!}+x^4\frac{f^{(4)}(0)}{4!}+\cdots+\frac{f^{(n)}(0)}{n!}x^n+\cdots \text{ Or}$$

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$$

Some Important Maclaurin Series Expansions:

Function	Maclaurin series
sinx	$x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$
Cosx	$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$
Tanx	$x+\frac{x^3}{3}+\frac{2x^5}{15}+\frac{17x^7}{315}+\cdots$
e^x	$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots$
a^x	$1+x\ln a+\frac{x^2(\ln a)^2}{2!}+\frac{x^3(\ln a)^3}{3!}+\cdots$

Applications of Derivatives

1. Geometrical Meaning of $\frac{dy}{dx}$	If $y=f(x)$ then $\frac{dy}{dx}$ denotes the slope of the tangent to the given curve at any point (x,y).
2. Equation of Tangent to $y=f(x)$ at (x_1,y_1)	$y-y_1=\left(\frac{dy}{dx}\right)_{(x_1,y_1)}(x-x_1)$
3. **Equation of Normal to** $y=f(x)$ **at** (x_1,y_1)	$y-y_1=\frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}(x-x_1)$
4. Increasing Function	(a) Let a function $f(x)$ is defined in the interval (a,b).then $f(x)$ is said to be increasing if $\forall x_1,x_2\in(a,b), x_1>x_2\Rightarrow f(x_1)>f(x_2)$ (b) If $f'(x)>0\forall x\in(a,b)$
5. Decreasing Function	(a) Let a function $f(x)$ is defined in the interval (a,b),then $f(x)$ is said to be decreasing if $\forall x_1,x_2\in(a,b), x_1>x_2\Rightarrow f(x_1)<f(x_2)$ (b) $f'(x)<0\forall x\in(a,b)$
6. Extreme Values Or Maximum Values and Minimum Values Or Maxima and Minima	Let f be a differentiable function in a neighborhood of c where $f'(c)=0$,then 1. f has relative maxima at c if $f''(c)<0$. 2. f has relative minima at c if $f''(c)>0$. Note: Evaluate f at all critical points and endpoints.
7. *Critical Point	The point c where $f'(c)=0$ or undefined is called critical point.
8. Point of Inflexion	The point c where $f''(c)=0$ is called point of inflexion.
9. Differentials	Let $y=f(x)$ is a differentiable function then $dy=f'(x)dx$ is called differential of the function.
10. Stationary point	Any point where f is neither increasing nor decreasing but $f'(c)=0$ is called stationary point

Every critical point is stationary point while every stationary point is not a critical point.

* The only domain points where extreme values exist are critical or end points.

PRACTICE EXERCISE

(1) $\frac{d}{dx}\cos x^{\circ} = ?$

(a) -sinx°
(b) x° sinx°
(c) $\frac{\pi}{180}\sin x$
(d) $-\frac{\pi}{180}\sin x$

(2) If $\frac{d}{dx}\left(3x^3+x\right)=\frac{d}{dx}\left(-3x^2-5\right)$,then x=?

(a) 1 or 2
(b) $\frac{-1}{3}$
(c) 1 or $\frac{-1}{3}$
(d) 1 or 0

(3) If $f(y) = by^3$ then which of the following is true

(I) $\frac{d}{dx}f(y)=3by$ (II) $\frac{d}{dx}f(y)=3by^2$

(III) $\frac{d}{dx}f(y)=2y^2$

(a) I only
(b) II only
(c) I & II only
(d) none

(4) The point at which the curve $y = x^2-4x+3$ has gradient –2 is?

(a) (0,1)
(b) (1,0)
(c) (2,1)
(d) (-3, 2)

(5) The deceleration of a point, when it is momentarily at rest, which is moving along a straight line with a velocity $16-t^2$ is____.

(a) 4m/sec^2
(b) 6m/s^2
(c) 2m/sec^2
(d) 8m/sec^2

(6) If $y=\frac{2}{x}$ then $y+\frac{1}{2}\frac{d^2y}{dx^2}=?$

(a) $\frac{2+x}{x^2}$
(b) $\frac{2+2x}{x^2}$
(c) $\frac{2\left(1+x^2\right)}{x^3}$
(d) $\frac{x^3-1}{2}$

(7) Differentiating x^9 w.r.t, x^2, we get

(a) $5x^9$
(b) $\frac{5}{2}x^6$
(c) $\frac{9}{2}x^7$
(d) $3x^6$

(8) $\frac{d}{dx}e^{e^x} = ?$

(a) e^{2x}
(b) $e^x e^{e^x}$
(c) e^e
(d) $\frac{1}{e^x}$

(9) Derivative of $\ln e^{x^3}$ w.r.t x is

(a) $3x^2$ (b) $3e^{x^2}$

(c) $\frac{1}{3e^x}$ (d) $\frac{1}{e^x}$

(10) $\frac{d}{dx}(c) = ?$ where 'c' is a constant

(a) $2e$ (b) c

(c) 0 (d) ∞

(11) $\frac{dy}{dx} = ?$ If $x = \frac{a(1-t^2)}{2at}, y = \frac{2bt}{1+t^2}$

(a) $\frac{a(t^2-1)}{2bt}$ (b) $\frac{b(1-t^2)}{2at}$

(c) $\frac{2at}{t^2(t-1)}$ (d) none

(12) If $x^{\frac{3}{4}} + y^{\frac{3}{4}} = a^{\frac{3}{4}}$ then $\frac{dy}{dx} = ?$

(a) $\left(\frac{y}{x}\right)^{\frac{3}{4}}$ (b) $-\left(\frac{y}{x}\right)^{\frac{1}{4}}$

(c) $-\left(\frac{x}{y}\right)^{\frac{1}{4}}$ (d) $\left(-\frac{y}{x}\right)^{\frac{1}{4}}$

(13) Given $S = 980t - 490t^2$,the velocity at the instant $t = ½$ is

(a) $490cm$ (b) $490cm/sec$

(c) $-490cm/sec$ (d) $49cm/sec$

(14) If $x = a\cos^4\theta, y = b\sin^4\theta$ then

(a) $a\frac{dy}{dx} = -b\tan^2\theta$ (b) $a\frac{dy}{dx} - b\tan^2\theta = 0$

(c) $\frac{dy}{dx} = \frac{a}{b\tan^2\theta}$ (d) none

(15) $\frac{d}{dx}\ln(\ln x) = ?$

(a) $\frac{1}{x}$ (b) $\frac{1}{x\ln a}$

(c) $\frac{1}{x\ln x}$ (d) $\frac{x}{\ln x}$

(16) $\delta y = ?$ If $y = 3x^2 + 5x - 2, x = 1, dx = -0.5$

(a) 13.92 (b) -4.75

(c) 13.21 (d) 11.76

(17) If $y = 7x^6+3x^4-5x^3+2$, then which one is true
(a) $y_5 = 0$ (b) $y_6 = 0$
(c) $y_7 = 0$ (d) $y_4 = 0$

(18) If $y = vw$ then which is false
(a) $y_1 = vw_1+v_1w$ (b) $y_2 = vw_2+2w_1v_1+v_2w$
(c) $y_3 = vw_3+3v_2w_1+v_3w$ (d) $y_4 = vw_4+4v_1w_3+6v_2w_2+4v_3w_1+v_4w$

(19) If $y = e^{20x}$ then y_4?
(a) $16000\,e^{20x}$ (b) $160000e^{20x}$
(c) $1600000e^{20x}$ (d) $16000000e^{20x}$

(20) If $y = \cos ax$ then $y_8 = ?$
(a) $a^8\cos(ax+4\pi)$ (b) $a^8\cos(ax+2\pi)$
(c) $a^8\cos(ax+5\pi)$ (d) $a^8\cos(ax+3\pi)$

(21) The minimum value of the function $y = x^3-3x^2$ is
(a) $y(1) = -13$ (b) $y(2) = -4$
(c) $y(2) = 4$ (d) $y(3) = 0$

(22) $\frac{d}{dx}a^x = ?$
(a) $\ln a$ (b) $x\ln a$
(c) $a^x \ln a$ (d) $a\ln x$

(23) $\frac{d}{dx}7^{4x-3} = ?$
(a) $\frac{4x-3}{\ln 7}$ (b) $4x\ln 7$
(c) $(4x-3)\ln 7$ (d) $4.7^{4x-3}\ln 7$

(24) $\frac{d}{dx}\sec 2x = ?$
(a) $2.sec2x.tan2x$ (b) $2.sec2x.tan^2x$
(c) $2.secx.tanx$ (d) $sec2x.tan2x$

(25) $\frac{d}{dx}\cos(\cos(\cos x)) = ?$
(a) $-sin[cos(cosx)].sin(cosx)$ (b) $-cos[sin(sinx)].sin(sinx)$
(c) $-sin[cos(cosx)].sin(cosx).sinx$ (d) $sin[cos(cosx)].cos(cosx)$

(26) If $\frac{y}{x} = \tan^{-1}\frac{x}{y}$, then $\frac{dy}{dx} = ?$
(a) $\frac{x}{y}$ (b) $\frac{y}{x}$
(c) xy (d) $\frac{1}{xy}$

(27) $\frac{d}{dx}\cos^{-1}3x = ?$

(a) $\frac{-1}{\sqrt{1-9x^2}}$ (b) $\frac{1}{\sqrt{1-9x^2}}$

(c) $\frac{3}{\sqrt{1-9x^2}}$ (d) $\frac{-3}{\sqrt{1-9x^2}}$

(28) $\frac{d}{dx}\frac{1}{\cos^{-1}x} = ?$

(a) $\frac{-1}{\left(\cos^{-1}x\right)^2}$ (b) $\frac{1}{\left(\cos^{-1}x\right)^2}$

(c) $\frac{1}{\left(\cos^{-1}x\right)^2\sqrt{1-x^2}}$ (d) $\frac{-1}{\left(\cos^{-1}x\right)^2\sqrt{1-x^2}}$

(29) In case of strictly decreasing functions, the derivative is
(a) Positive (b) negative
(c) zero (d) either negative or zero

(30) The derivative of an even function is always
(a) Odd function (b) even function
(c) does not exist (d) both a & b

(31) If $y = lnx$, then $y_4 = ?$

(a) $\frac{-3}{x^4}$ (b) $\frac{-6}{x^4}$

(c) $\frac{-5}{x^4}$ (d) $\frac{4}{x^4}$

(32) If $y = tan\,(p\tan^{-1}x)$ then $(1+x^2)y_1 = ?$

(a) $(1+y^2)$ (b) $(1-y^2)$

(c) $\frac{1}{p\left(1+y^2\right)}$ (d) $p(1+y^2)$

(33) $\frac{d^2y}{dx^2}$ for $x^2+y^2 = a^2$ is

(a) $\frac{-a^2}{y}$ (b) $\frac{-a^2}{y^3}$

(c) $\frac{a^2}{y^2}$ (d) $\frac{y^3}{a^2}$

(34) $\frac{d}{dx}\ln x^x = ?$

(a) $lnx + 1$ (b) $(lnx)^x$

(c) $x^x.lnx$ (d) $x^x.ln(ex)$

(35) $\frac{d}{dx}(\ln x)^{\ln x} = ?$

(a) $(\ln x)^{\ln x}\left(\frac{1+\ln\ln x}{x}\right)$

(c) $(\ln x)^{\ln x}\left(\frac{\ln\ln x}{x}\right)$

(d) none

(36) $\frac{d}{dx}\cosh x = ?$

(a) $\sinh x$

(b) $-\sinh x$

(c) $\operatorname{sech} x$

(d) none of these

(37) $\frac{d}{dx}\operatorname{cosech}^{-1} x = ?$

(a) $\frac{1}{x\sqrt{1+x^2}}$

(b) $\frac{1}{1-x^2}$

(c) $\frac{-1}{\sqrt{1+x^2}}$

(d) $\frac{-1}{x\sqrt{1+x^2}}$

(38) If $y = e^x \sin x$, then $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = ?$

(a) $1-x^2$

(b) 0

(c) $2x$

(d) $x+2$

(39) The Maclaurin series expansion of e^x is

(a) $1+x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots$

(b) $1-x-\frac{x^2}{2}-\frac{x^3}{3}-\cdots$

(c) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$

(d) none of these

(40) The maximum value of the function $f(x) = \sin x \cos x$ is

(a) 1

(b) 2

(c) $\frac{1}{2}$

(d) 0

(41) If $\sqrt{x}+\sqrt{y}=1$ then slope of tangent to the curve at $\left(\frac{1}{4},\frac{1}{4}\right)$ is;

(a) 1

(b) 2

(c) -1

(d) $\frac{1}{2}$

(42) Equation of normal to the curve $y = \sin x$ at $(\pi, 0)$ is;

(a) $x+y+\pi=0$

(b) $x+y-\pi=0$

(c) $x-y=\pi$

(d) $x-y+\pi=0$

(43) A rectangle is of perimeter 176m. What will be its maximum area?

(a) $1936m^2$

(b) $1854m^2$

(c) $2110m^2$

(d) $1836m^2$

(44) A cylinder is contained in a sphere of radius "r". What will be its height if its volume is maximum?

(a) $\frac{r}{\sqrt{3}}$ (b) $r\sqrt{3}$

(c) $\frac{2r}{\sqrt{3}}$ (d) $2\sqrt{3}r$

(45) $\frac{d^{777}y}{dx^{777}}(\cos x) = ?$

$y_1 = -\sin x \quad y_2 = -\cos x \quad y_3 = \sin x \quad y_4 = \cos x$

$4\overline{)777}$ 194, 4, 37, 32, 17, 16, 1

(a) $\cos x$ (b) $\sin x$

(c) $-\sin x$ (d) $\cos x$

(46) If $f(2) = 4$ and $f'(2) = 4$ then $\lim_{x\to 2} \frac{x.f(2) - 2f(x)}{x-2} = ?$

(a) 2 (b) -2

(c) -4 (d) 3

(47) If $y = Sec^{-1}\left(\frac{x+1}{x-1}\right) + Sin^{-1}\left(\frac{x-1}{x+1}\right)$ then $\frac{dy}{dx} = ?$

(a) 1 (b) 0

(c) $\frac{x+1}{x-1}$ (d) $\frac{x-1}{x+1}$

(48) Let $f(x) = \begin{cases} 0, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ then for all x,

(a) $f'(x)$ is differentiable (b) f is differentiable

(c) f' is contineous (d) f is continuous

(49) The point on the curve $y^2=x$, the tangent at which makes an angle of $\frac{\pi}{4}$ with x-axis is _____

(a) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (b) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(c) $(2,4)$ (d) $\left(\frac{1}{4}, \frac{1}{2}\right)$

$2yy_1 = 1$

$y_1 = \frac{1}{2y}$

(50) $f(x)= x^3-27x+5$ is increasing when

(a) $x<-3$ (b) $|x| > 3$

(c) $|x| < 3$ (d) $x \geq 3$

(51) If $f(x) = \sqrt{x^2 - 2x + 1}$, on the interval [0,2], ... then $f'(x) = ?$

(a) 1 (b) -1

(c) 0 (d) Does not exist

(52) Derivative of $Sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is _____

(a) 4 (b) -4

(c) 0 (d) $\frac{1}{2}$

(53) y = |Cosx| is not differentiable for all x _____

(a) $x = \frac{n\pi}{2}$ (b) $x = (2n+1)\frac{\pi}{2}$

(c) $x = n\pi$ (d) none of these

(54) Minimum value of $\sqrt{e^{x^2}-1}$ is .....

(a) 1 (b) -1

(c) 0 (d) 2

(55) If g(x) is inverse of f(x) and $f'(x) = \frac{1}{1+x^2}$ then $g'(x) = ?$

(a) $1 + x^2$ (b) $1 + g(x)$

(c) $1 + [g(x)]^2$ (d) none of these

(56) If U is a variable V is a constant then $\frac{d}{du}(U^v) = ?$

(a) $U.V^{U-1}$ (b) $V.U^{V-1}$

(c) U^{V-1} (d) $V.U^V$

(57) If $y = \sqrt{Sinx + \sqrt{Sinx + \sqrt{Sinx + ... + \infty}}}$ then $\frac{dy}{dx}(2y-1) = ?$

(a) Sinx (b) Cosx

(c) Secx (d) Cosecx

(58) $\frac{d}{dx} Sin^{-1}\left(Cos\left(\frac{\pi}{2} - x\right)\right) = ?$

(a) $\frac{\pi}{2}$ (b) Cosx

(c) 1 (d) –Sinx

(59) If $y = e^{\ln \sin x^2}$ then $\frac{dy}{dx} = ?$

(a) $Cosx^2$ (b) x^2Cosx^2

(c) $2xCosx^2$ (d) $Sinx^2$

(60) The point of inflexion on the curve $a^2y=(x-b)^3$ is _____

(a) (a,0) (b) (b,0)

(c) (a,b) (d) (1,a)

INTEGRATION

- If $\frac{dy}{dx} = f(x)$,then y is the function whose derivative is *f(x)* and is called the **anti-derivative** of *f(x)* or the **indefinite integral** of *f(x)*,denoted by $\int f(x)dx$.
- Similarly if $y = \int f(x)dx$, then $\frac{dy}{dx} = f(x)$.
- Since the derivative of a constant is zero, all indefinite integrals differ by an arbitrary constant.
- Remember the process of finding an integral is called **integration.**

Some General Formulae of Integration

1) $\int x^n dx = \frac{x^{n+1}}{n+1} + c, (n \neq -1) \quad n \in \mathbb{Q}$

2) $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, (n \neq -1)$

3) $\int cf(x)dx = c\int f(x)dx$ (c is a constant)

4) $\int \frac{dx}{x} = \ln x + c$

5) $\int e^x dx = e^x + c$

6) $\int e^{mx} dx = \frac{e^{mx}}{m} + c$

7) $\int a^x dx = \frac{a^x}{\ln a} + c$, a > 0

8) $\int (uv)\,dx = u\int v dx - \int\left(\frac{du}{dx}\int v dx\right)dx$, this is called integration by parts.

Trigonometric Functions

1) $\int \sin x dx = -\cos x + c$

2) $\int \sin mx dx = -\frac{\cos mx}{m} + c$

3) $\int \cos x dx = \sin x + c$

4) $\int \cos mx dx = \frac{\sin mx}{m} + c$

5) $\int \sec^2 x dx = \tan x + c$

6) $\int \cos ec^2 x dx = -\cot x + c$

7) $\int \sec x \tan x dx = \sec x + c$

8) $\int \tan x dx = \ln \sec x + c$

9) $\int \cos ecx \cot x dx = -\cos ecx + c$

10) $\int \sec x dx = \ln|\sec x + \tan x| = \ln \tan\left(x + \frac{\pi}{2}\right) + c$

11) $\int \text{cosec}\, x dx = \ln|\text{cosec}\, x - \cot x| = \ln \tan\left(\frac{x}{2}\right) + c$

12) $\int \tan^2 x dx = \tan x - x + c$

13) $\int \cot^2 x dx = -\cot x - x + c$

14) $\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4} = \frac{1}{2}(x - \sin x \cos x) + c$

15) $\int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2}(x + \sin x \cos x) + c$

Inverse Trigonometric Functions

1) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$ or $-\cos^{-1} x + c$

2) $\int \frac{dx}{\sqrt{x^2-1}} = \sec^{-1} x + c$ OR $-\text{cosec}^{-1} x + c$

3) $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$ OR $-\cot^{-1} x + c$

4) $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

5) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

6) $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

Some Integrals Including Logarithms

1) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c = -\frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + c, x > a$

2) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + c, x < a$

3) $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x + \sqrt{x^2+a^2}\right| + c = \sinh^{-1} x + c$

4) $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| + c$

5) $\int \sqrt{x^2+a^2}\, dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln\left|x + \sqrt{x^2+a^2}\right| + c$

Some Useful Results

1) $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$

2) $\int e^x \left(f(x) + f'(x)\right)dx = e^x f(x) + c$

3) $\int e^{ax} \left(af(x) + f'(x)\right)dx = e^{ax} f(x) + c$

4) $\int e^{ax} \cos bx dx = \frac{e^{ax}\left(a\cos bx + b\sin bx\right)}{a^2 + b^2} + c$

5) $\int e^{ax} \sin bx dx = \frac{e^{ax}\left(a\sin bx - b\cos bx\right)}{a^2 + b^2} + c$

Definite Integrals

1) $\int_a^b f(x)dx = \left|F(x)\right|_a^b = F(b) - F(a)$

2) $\int_a^a f(x)dx = 0$

3) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

4) $\int_a^b cf(x)dx = c\int_a^b f(x)dx$, (c is a constant)

5) $\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

6) $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$,where a<b<c

7) $\int_a^b f(x)dx = \int_a^b f(t)dt$ (independent of variable)

Some Very Important Results

1) $\int_{-a}^a f(x)dx = 0$, if f(x) is an odd function.

2) $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, if f(x) is an even function.

3) $\int_0^{\frac{\pi}{2}} \ln\sin x dx = \int_0^{\frac{\pi}{2}} \ln\cos x dx = -\frac{\pi}{2}\ln 2$

4) $\int_0^{\frac{\pi}{2}} \ln \tan x dx = 0$

5) $\frac{d}{dx}\int_a^x f(t)dt = f(x)$

6) $\int_a^b f(x)dx = \int_{a-c}^{b-c} f(x+c)dx$,[shift property]

7) $f(x) \geq g(x)$ in $[a,b]$ then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$ [domination rule]

Applications of Integration

1) Area Under Curves and The Axes:

I. If 'A_1' is the area bounded by curve $y = f(x)$, ***x-axis*** and between the coordinates $x = a$, $x = b$ then $A_1 = \int_a^b y dx$

II. If 'A_2' is the area bounded by the curve $y = f(x)$, ***y-axis*** and between the co-ordinates $y = a$, $y = b$, then $A_2 = \int_a^b x dy$

III. In choosing the limits of integration, the smaller values of x at ordinate is drawn will be taken as lower limit and the greater as upper limit. (i.e. you are to move from left to right on x-axis)

IV. Only the numerical value of the area will be considered (i.e. if there comes –ve sign with the area after calculating neglect it.)

V. If the curve is symmetrical, then find the area of one symmetrical portion, and then multiply it by the number of symmetrical portions.

To Check Symmetry

a) The given curve is symmetrical w. r. t x-axis if equation $y = f(x)$ involves even powers of y and the curve is symmetrical about y-axis having even powers of x.

b) If the equation involves the even powers of both x & y curve is symmetrical about both axis.

2) Area Under Two Curves:

Let $f(x)$ and $g(x)$ be two curves such that $f(x) > g(x)$ for every $x \in (a,b)$ .then the area of the region bounded by the graphs of $f(x)$ & $g(x)$ and the ordinates $x=a$ & $x=b$ for $a<x<b$ is given by $A = \int_a^b (f(x) - g(x))dx$.

3) Differential Equations:

- An equation involving one dependent variable and its derivatives with respect to one or more independent variables is called a **differential equation**.
- The **order** of the differential equation is the order of the highest derivative that occurs in the equation.
- The **degree** of the differential equation is the degree of the highest order derivative that appears in the equation. (The dependent variable and its derivatives should express in a form free from radicals and fractions.)

4) Solution of Differential equation

- A relation between the dependent and the independent variables of the differential equation i.e. a function of the form $y = f(x)+c$ which satisfies a given differential equation is called its solution.

5) General Solution

- If the solution of differential equation of nth order contains n arbitrary constants. then it is called the general solution.

6) Particular Solution

- A solution obtained by giving particular values to arbitrary constants in the general solution is called a particular solution.

7) Initial Conditions

- The arbitrary constants involving in the solution of differential equations can be determined by the given conditions, such conditions are called initial value conditions.

$\ln e = 1$

$\ln(1) = 0$

PRACTICE EXERCISE

(1) $\int \frac{a}{x^n} dx = ?$

(a) $\frac{ax^{n+1}}{n+1}$

(b) $ax^{n-1}(n-1)$

(c) $\frac{ax^{1-n}}{1-n}$

(d) $(ax)^{n+1}$

(2) $\int \cos^2 ax dx = ?$

(a) $\cos^3\left(\frac{ax}{3}\right)$

(b) $\cos^3\left(\frac{a}{x}\right)$

(c) $\frac{x}{2} + \frac{\sin 2ax}{4a}$

(d) $\frac{\cos ax}{2a}$

(3) $\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2}\right) dx = ?$

(a) $e^x \sec^{-1} x + \frac{e^x}{1+x^2}$

(b) $e^x \tan^{-1} x$

(c) $e^x \sec^{-1} x$

(d) $\frac{e^x}{1+x^2}$

(4) $\int_{\frac{\pi}{6}}^{6} \sec^2 x dx = ?$

(a) 0

(b) 1

(c) ½

(d) -1

(5) $\int_{-1}^{1} (x+1)^2 dx = ?$

(a) 8

(b) 3

(c) $\frac{8}{3}$

(d) $\frac{3}{8}$

(6) $\int_{1}^{6} g(x) dx = -3, \int_{6}^{8} g(x) dx = 10$, then $\int_{8}^{1} -3g(x) dx = ?$

(a) 21

(b) –21

(c) 7

(d) -7

(7) $\int (\tan^4 x + \tan^2 x) dx = ?$

(a) $\frac{\tan^3 x}{3}$

(b) $\frac{\tan^4 x}{4}$

(c) $\frac{\tan^5 x}{5}$

(d) none

(8) Solution of $\frac{dy}{dx} = x^4 + 12$ is

(a) $x^4 + 12 + c$

(b) $\frac{x^5}{5} + 12x + c$

(c) $\frac{x^5 + 12x + c}{5}$

(d) $\frac{x^2 + 12x}{4} + c$

(9) Which one is false

(a) If a is a constant, then $\int af(x)\,dx = a\int f(x)\,dx$

(b) $\int[f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

(c) $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|$

(d) $\int e^x\left(f(x) + f'(x)\right)dx = e^x f'(x)$

(10) The general solution of a differential equation of order 50 is that solution which contains _____ essential arbitrary constants.

(a) 49

(b) 50

(c) 51

(d) 50^2

(11) $\int \sec^2 nx\,dx = ?$

(a) $\frac{n}{3}\sec 3nx$

(b) $n\tan(nx)$

(c) $\tan(nx)$

(d) $\frac{1}{n}\tan(nx)$

(12) $\int \frac{x}{x^2 + 8}\,dx = ?$

(a) $\ln(x^2 + 8)$

(b) $-\ln(x^2 + 8)$

(c) $\ln\left(\sqrt{x^2 + 8}\right)$

(d) $2\ln\left(\sqrt{x^2 + 8}\right)$

(13) $\int 4^{3x}\,dx = ?$

(a) 3.4^{3x}

(b) $\frac{1}{3}4^{3x}$

(c) $\frac{4^{3x}}{\ln 3}$

(d) $\frac{4^{3x}}{3\ln 4}$

(14) For $f(x) \geq 0$ in the interval $a \leq x \leq b$, the definite integral $\int_a^b f(x)\,dx$,represents ___ of the region enclosed by the graph y=f (x), the -axis and the two lines $x = a$ and $x = b$.

(a) Area

(b) volume

(c) graph

(d) integral

(15) If $\Phi_1(x)$ and $\Phi_2(x)$ are any two anti-derivatives of the function $f(x)$ then $\Phi_1(x)-\Phi_2(x)$ =?

(a) Constant
(b) e
(c) lne
(d) 0

(16) $\int \frac{1}{x^2} dx = ?$

(a) $\ln|x|$
(b) $\ln|x^2|$
(c) $-\frac{1}{x}$
(d) $-\frac{2}{x^3}$

(17) $\frac{d}{dx}\int g(x)dx = ?$

(a) $g(x)$
(b) $\frac{1}{g(x)}$
(c) $g'(x)$
(d) $g''(x)$

(18) $\int \frac{x+4}{x+3} dx = ?$

(a) $\ln|x+3|$
(b) $4x+\ln|x+3|$
(c) $x+\ln|x+3|$
(d) $x^2+\ln|x+3|$

(19) $\int \frac{e^{-\frac{1}{x^3}}}{x^4} dx = ?$

(a) $-e^{-\frac{1}{x^3}}$
(b) $\frac{1}{3}e^{-\frac{1}{x^3}}$
(c) $e^{-\frac{1}{x^3}}$
(d) $\frac{1}{12}e^{-\frac{1}{x^3}}$

(20) $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx = ?$

(a) $e^{\tan^{-1}x}$
(b) $\frac{1}{e^{\tan^{-1}x}}$
(c) $\ln|1+x^2|$
(d) $\ln|e^{\tan^{-1}x}|$

(21) $\int \frac{\sin 2x}{a^2+b^2\sin^2 x} dx = ?$

(a) $ln(a^2+b^2.sin^2 x)$
(b) $\frac{1}{b^2}\ln\left(a^2+b^2\sin^2 x\right)$
(c) $\frac{1}{a^2}\ln\left(a^2+b^2\sin^2 x\right)$
(d) $\frac{1}{a^2+b^2}\ln\left(a^2+b^2\sin^2 x\right)$

(22) $\int x\cos x dx = ?$

(a) $xsinx$
(b) $xsinx-cosx$
(c) $xsinx+cosx$
(d) $-xsinx+cosx$

(23) $\int e^x\left(\cos ec^{-1}x - \frac{1}{x\sqrt{x^2-1}}\right)dx = ?$

(a) $e^x \cos ec^{-1}x$
(b) $\cos ec^{-1}x$
(c) $\frac{\cos ec^{-1}x}{e^x}$
(d) $-\frac{\cos ec^{-1}x}{e^x}$

(24) Which one is false?

(a) $\int_a^a f(x)dx = 0$
(b) $\int_a^b f(x)dx = -\int_a^b f(x)dx$
(c) $\int_a^b (f(x)\pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
(d) $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

(25) $\int_0^{\frac{\pi}{3}} \frac{dx}{1-\sin x} = ?$

(a) $\sqrt{3}$
(b) $\sqrt{3}-1$
(c) $\sqrt{3}+1$
(d) 1

(26) $\int \sin^{-1}(\cos x)dx = ?$

(a) $\frac{\pi x}{2} - \frac{x^2}{2}$
(b) $\frac{(\pi x + x^2)}{2}$
(c) $\frac{\pi x}{2}$
(d) $\frac{x^2}{2}$

(27) $\int e^x\left[\frac{1+\sin x}{1+\cos x}\right]dx = ?$

(a) $\ln(\tan x)$
(b) $e^x \tan\left(\frac{x}{2}\right)$
(c) $\sin(\ln x)$
(d) $e^x \cot x$

(28) $\int \tan^{-1}\left(\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}\right)dx = ?$

(a) $2x^2 + c$
(b) $x^2 + c$
(c) $\frac{x^2}{2} + c$
(d) $2x + c$

(29) $\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx = ?$

(a) $\frac{2}{3}\left(\frac{x}{a}\right)^{\frac{3}{2}}$

(b) $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{\frac{3}{2}}$

(c) $\frac{2}{3}\cos^{-1}\left(\frac{x}{a}\right)^{\frac{3}{2}}$

(d) none

(30) $\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx = ?$

(a) $\sin^{-1}(\sin x + \cos x)$

(b) $\sin^{-1}(\sin x - \cos x)$

(c) $\sin^{-1}(\cos x - \sin x)$

(d) $\sin^{-1}(\cos x + \sin x)$

(31) $\int \frac{dx}{x \sin^2(\ln x)} = ?$

(a) $\cot(\ln x)$

(b) $\tan(\ln x)$

(c) $-\cot(\ln x)$

(d) $-\tan(\ln x)$

(32) $\int \frac{\cot(\ln x)}{x} dx = ?$

(a) $\ln(\cos(\ln x))$

(b) $\sin(\ln x)$

(c) $\ln \sin(\ln x)$

(d) none

(33) $\int e^{ax} \cos(bx) dx = ?$

(a) $\frac{e^{ax}(a\cos bx + b\sin bx)}{a^2 + b^2}$

(b) $\frac{e^{ax}(a\cos bx + b\sin bx)}{\sqrt{a^2 + b^2}}$

(c) $\frac{e^{ax}(a\cos bx + b\sin bx)}{\sqrt{a^2 - b^2}}$

(d) none

(34) $\int \frac{x}{\sqrt{4 - x^4}} dx = ?$

(a) $-\cos^{-1}\left(\frac{x^2}{2}\right)$

(b) $\frac{1}{2}\cos^{-1}\left(\frac{x^2}{2}\right)$

(c) $\cos^{-1}\left(\frac{x^2}{2}\right)$

(d) $\frac{1}{2}\sin^{-1}\left(\frac{x^2}{2}\right)$

(35) The area bounded by the curve $y = lnx$, *x-axis* and line $x = e$ is

(a) e

(b) 1

(c) ∞

(d) none

(36) The degree of the differential equation $\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)+y\frac{d^2y}{dx^2}=0$ is

(a) 1
(b) 2
(c) 0
(d) none

(37) Solution of the differential equation $\frac{dy}{dx}=\frac{\sin x}{\sin y}$ is

(a) $cosx=cosy$
(b) $sinx = siny$
(c) $sinx+siny=c$
(d) $cosy-cosx=c$

(38) $\int_{-\pi}^{\pi}\left(\sin^{75}x+x^{125}\right)dx=?$

(a) -75
(b) 125
(c) 0
(d) -125

(39) The slope of tangent to the curve $y = x^2-2x$ at $P(1,1)$ is

(a) 2
(b) 0
(c) -2
(d) none

(40) The area bounded by the x-axis and the curve $y=4x-x^2$ is

(a) $\frac{32}{3}$
(b) 32
(c) $\frac{64}{3}$
(d) 64

(41) $\int e^{e^{e^x}}.e^{e^x}.e^x dx=?$ is

(a) $\frac{1}{2}e^{2e^{e^x}}+C$
(b) $e^{2e^{e^x}}+C$
(c) $e^{e^{e^x}}+C$
(d) $\frac{1}{2}e^{e^{e^x}}+C$

(42) $2\int_{3}^{6}[x]dx=?$ Where [ ] is greatest integer function.

(a) 12
(b) 30
(c) 18
(d) 24

(43) $\int_{-3}^{3}\left(\sin^{93}x+x^{295}\right)dx=?$

(a) 0
(b) any real number
(c) $2\left(8^{290}-1\right)$
(d) $2\left(3^{295}-1\right)$

(44) If $F(x)=\int_{0}^{x}x^2dx$ then $F(3)=?$

(a) 9
(b) 27
(c) 3
(d) 81

(45) In $\int_0^1 \sin^{-1} x dx$ if we substitute $x = \sin t$ then the new integral is;

(a) $\int_0^{\frac{\pi}{2}} t \cos t dt$

(b) $\int_0^{\frac{\pi}{2}} t dt$

(c) $-\int_0^{\frac{\pi}{2}} t \cos t dt$

(d) $-\int_0^{\frac{\pi}{2}} t dt$

(46) $\int_0^2 |x-1| dx = ?$

(a) -1

(b) 2

(c) 3

(d) 1

(47) $\frac{d}{dt}\int_0^t \cos ec\theta d\theta = ?$

(a) $\cos ec\theta + C$

(b) $\cos ect + C$

(c) 0

(d) $\cos ect$

(48) Area between the x-axis and the curve $y = \sqrt{16x - x^2}$ is;

(a) 8π

(b) 16π

(c) 32π

(d) 64π

(49) Area of ellipse $x^2 + 4y^2 = 16$ is;

(a) 8π

(b) 16π

(c) 32π

(d) 64π

(50) Order and degree of the differential equation $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is respectively;

(a) 2,2

(b) 2,3

(c) 3,2

(d) 1,3

(51) $\int_1^e \frac{1 + \ell nx}{2x} dx = ?$

(a) $\frac{1}{4}$

(c) $\frac{3}{4}$

(52) $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = ?$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

(53) $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx = ?$

(a) $\tan(xe^x)$ (b) $\sqrt{\tan(xe^x)} + c$

(c) $\tan^{-1}(xe^x) + c$ (d) none of these

(54) $\int x^2 e^{x^3} \cos(e^{x^3}) dx = ?$

(a) $Sin(e^{x^3}) + c$ (b) $3\sin(e^{x^3}) + c$

(c) $\frac{1}{3}\sin(e^{x^3}) + c$ (d) $e^x Sin(e^{x^3}) + c$

(55) $\int 3e^{3x^2} . x dx = K . e^{3x^2}$ then K = ?

(a) $\frac{1}{6}$ (b) 1

(c) $\frac{1}{2}$ (d) -3

(56) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} Cosx dx = ?$

(a) 0 (b) 1

(c) -1 (d) 2

(57) $\int_{0}^{2\pi} \sqrt{1+\sin\frac{x}{2}}\, dx = ?$

(a) 0 (b) 2

(c) 8 (d) 4

(58) $\int_{a}^{b} \frac{|x|}{x} dx =$ _____ $(a < b)$

(a) b – a (b) a – b

(c) b + a (d) |b| - |a|

(59) $\int_{-3}^{3} |x|\,dx = ?$

(a) 3
(b) 9
(c) 18
(d) 0

(60) If the graph of integral of $y = 3\cos x - 2\sin x$ passes through $\left(\frac{\pi}{2}, 1\right)$ then $\int y dx = ?$

(a) $3Sinx + 2Cosx + c$
(b) $3Sinx + 2Cosx + 1$
(c) $3\sin x + 2\cos x - 2$
(d) $3Sinx + 2Cosx$

(61) $\int \frac{\cos x dx}{\sin^2 x} = ?$

(a) Cosec x + c
(b) c – Cosec x
(c) c + Sin x
(d) $Cosec^2$ x + c

(62) $\int (5 - 2^x) dx = ?$

(a) $5x - 2^x \ln a + c$
(b) $5x - \frac{2^x}{\ln 2} + c$
(c) $5x - \frac{2^x}{\ln \alpha} + c$
(d) $5x + 2^x + c$

(63) $\int \frac{1}{(t-2)^2 + 25} dt = ?$

(a) $\tan^{-1}\left(\frac{t-2}{5}\right) + c$
(b) $\tan^{-1}(t-2) + c$
(c) $\frac{1}{5}\tan^{-1}\left(\frac{t-2}{5}\right) + c$
(d) $5\tan^{-1}\left(\frac{t-2}{5}\right) + c$

(64) $\int e^x (f(x) + f'(x)) dx = e^x Sinx$, then $f(x) = ?$

(a) Sin x
(b) Cos x
(c) – Sin x
(d) – Cos X

(65) $\int_{-\pi}^{\pi} \frac{xCosx}{1 + Sin^2 x} dx = ?$

(a) Zero
(b) 1
(c) 2
(d) -2

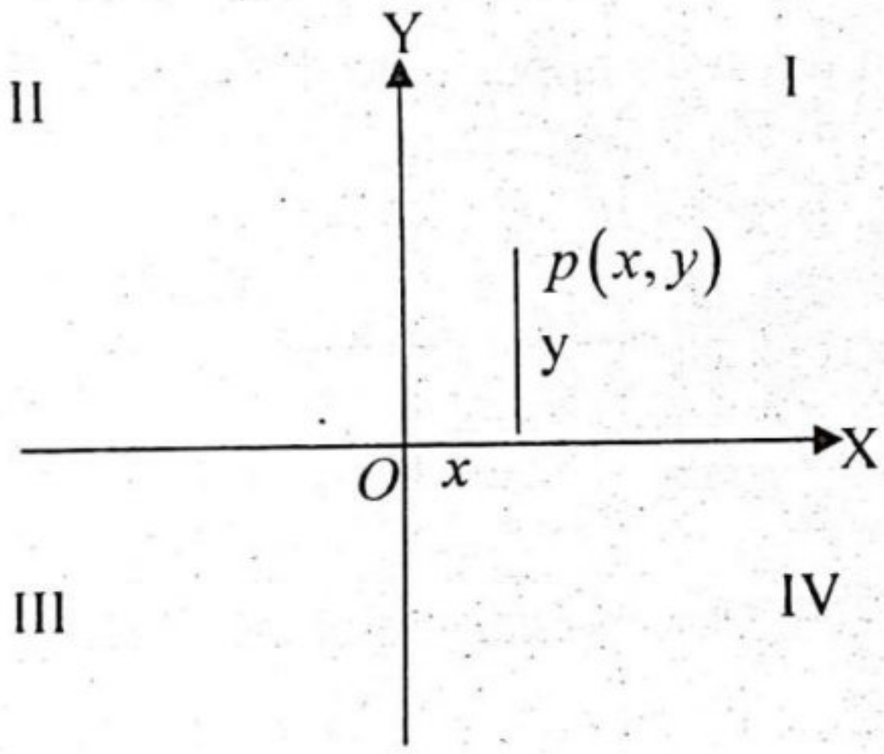
INTRODUCTION TO ANALYTIC GEOMETRY

Coordinate System

The system of coordinates use in analytic geometry is defined by the familiar Cartesian plane used in algebra and trigonometry. As shown in the figure the plane is divided into four quadrants by the two perpendicular lines ($X-axis$ & $Y-axis$) intersecting at the origin O.

They are defined as;

Quadrant I	All points (x, y) with $x>0, y>0$
Quadrant II	All points (x, y) with $x<0, y>0$
Quadrant III	All points (x, y) with $x<0, y<0$
Quadrant IV	All points (x, y) with $x>0, y<0$



Some Terms Related To Cartesian plane

Abscissa	The distance of a point from $Y-axis$ is called *x-coordinate* or *abscissa*.
Ordinate	The distance of a point from $X-axis$ is called *y-coordinate* or *ordinate*.
Cartesian/Rectangular Coordinates	The two distances taken together and enclosed in parentheses (x, y) are called *Cartesian* or *rectangular coordinates*.

Definitions and Formulae:

The Distance Formula	The length or the distance of the line segment joining the points $A(x_1, y_1)$ & $B(x_2, y_2)$ is given by; $d = \|AB\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Distance of a Point from Origin	Let $A(x_1, y_1)$ be any point then its distance from the origin $O(0,0)$ is given by; $\|OA\| = \sqrt{x_1^2 + y_1^2}$
Centroid of a triangle	The point of intersection of medians of a triangle is called centroid. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle then , centroid of triangle $= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$
In centre of a triangle	The point of intersection of angle bisectors of a triangle is called in centre. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle and ***a,b*** and ***c*** are the lengths of the sides opposite to the vertices ***A*** ,***B*** and ***C*** respectively ,then in centre of the triangle $= \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$

The division formula

Internal division	Let $A(x_1, y_1)$ & $B(x_2, y_2)$ be the two points in a plane. The coordinates of the point dividing the line segment AB in the ratio $k_1 : k_2$ internally are $\left(\frac{k_1 x_2 + k_2 x_1}{k_1 + k_2}, \frac{k_1 y_2 + k_2 y_1}{k_1 + k_2} \right)$
External division	Let $A(x_1, y_1)$ & (x_2, y_2) be the two points in a plane. The coordinates of the point dividing the line segment AB in the ratio $k_1 : k_2$ externally are $\left(\frac{k_1 x_2 - k_2 x_1}{k_1 - k_2}, \frac{k_1 y_2 - k_2 y_1}{k_1 - k_2} \right)$.
The midpoint	If $k_1 : k_2 = 1:1$,then the point of division becomes the mid point of $A(x_1, y_1)$ & $B(x_2, y_2)$,so the coordinates of the point are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

Theorems*

(i).The centroid of a triangle is a point that divides each median in the ratio 2:1.
(ii).The medians of a triangle are concurrent.
(iii).The angle bisectors of a triangle are concurrent.
(iv).The division formula is valid in which ever quadrants the points lie.
(v).The angle bisector divides the opposite side in the same ratio as the other two sides have.

Some Terms Related To a Line

Inclination	Inclination is the angle (α), less than $0^o < \alpha < 180^o$ measured anti clockwise, that a line makes with the positive $X-axis$.
Inclinations of some standard straight lines	(a) Inclination of $X-axis$ and any line parallel to $X-axis$ is 0. (b) Inclination of $Y-axis$ or any line parallel to $Y-axis$ is 90.
Slope/Gradient	The tangent of inclination of a line is called slope, denoted by m. *i.e.* $slope = m = \tan\alpha$.
Slope of a line segment	If $A(x_1, y_1)$ & $B(x_2, y_2)$ are any two points on a line ,then the slope of the line segment is defined as; $m = \frac{y_2 - y_1}{x_2 - x_1}$
Slopes of some standard straight lines	(a) The slope of $X-axis$ & any line parallel to $X-axis$ is 0. (b) The slope of $Y-axis$ & any line parallel to $Y-axis$ is undefined.
Theorems related to slope	Let m_1 & m_2 be the slopes of two straight lines l_1 & l_2 respectively, then; (i). $l_1 \parallel l_2$ iff $m_1 = m_2$. (ii). $l_1 \perp l_2$ iff $m_1.m_2 = -1$. (iii). If slope of AB = slope of BC then the points A,B and C are collinear.

* The proofs of all the theorems are beyond the scope of this book.

Equations of Straight Lines under Different Conditions

Equation of $X-axis$	$y=0$
Equation of $Y-axis$	$x=0$
Equation of a line parallel to $X-axis$	$y=k$, k is any constant not equal to zero.
Equation of line parallel to $Y-axis$	$x=h$, h is any constant not equal to zero.
Slope –Intercept form	Let m be the slope and c be the y-intercept of a non-vertical line then the equation of the line is $y=mx+c$
Equation of Line having slope m and passing through Origin	$y=mx$
Point-Slope Form	Equation of a non-vertical line having slope m and passing through a point $p(x_1,y_1)$ is $y-y_1=m(x-x_1)$.
Two Point Form	Equation of a line passing through two points $A(x_1,y_1)$ & $B(x_2,y_2)$ is. (i) $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$ (ii) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0$
Intercepts Form	Equation of line making intercepts a,b on X-and $Y-axes$ respectively is $\frac{x}{a}+\frac{y}{b}=1$
Normal/Perpendicular Form	Equation of line with perpendicular length p from origin and perpendicular making an angle α with positive X-$axis$ is $x\cos\alpha+y\sin\alpha=p$.

General Form of a Straight Line

$ax+by+c=0$ is the general form of a straight line, where a,b,c are any real numbers and a,b cannot zero simultaneously.

Different parameters of different forms of the straight line can be expressed in terms of a, b, c of the general form of a line as follow;

Slope , $m=-\frac{a}{b}$
x-intercept $=-\frac{c}{a}$
y-intercept $=-\frac{c}{b}$
Distance from origin $=p=\frac{\lvert c\rvert}{\sqrt{a^2+b^2}}$

Position of a Point with respect to a Line

(a) The point $p(x_1, y_1)$ lies above, on or below the line $ax + by + c = 0$, according as $ax_1 + by_1 + c \gtreqless 0\ (b > 0)$.

(b) The point $p(x_1, y_1)$ is above or below the line $ax + by + c = 0$ if $ax_1 + by_1 + c$ and b have the same or opposite signs.

(c) The $p(x_1, y_1)$ and the origin are

(i). On the same side of the line according as $ax_1 + by_1 + c$ and c have same signs.

(ii). On the opposite side of the line according as $ax_1 + by_1 + c$ and c have opposite signs.

Two and Three Straight Lines:

Two lines $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ are said to be;

(i). Parallel iff $\frac{a_1}{a_2} = \frac{b_1}{b_2}$. $a_1b_2 = a_2b_1 \Rightarrow a_1b_2 - a_2b_1 = 0$

(ii). Perpendicular iff $a_1a_2 + b_1b_2 = 0$.

(iii). Identical iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

(iv). Intersects each other iff $a_1b_2 - a_2b_1 \neq 0$

The Point of Intersection of Two Straight Non-Parallel Lines:

Let $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ be two non-parallel and coplanar lines then their point of intersection is;

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right), a_1b_2 - a_2b_1 \neq 0$$

Distance of A Point $p(x_1, y_1)$ From A Line:

Theorem: the distance d from the point $p(x_1, y_1)$ to the line $l : ax + by + c = 0$ is given by; $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Distance between two parallel lines:

The distance between two parallel lines $l_1 : ax + by + c = 0$, $l_1 : ax + by + c' = 0$ is given as; $d = \frac{|c - c'|}{\sqrt{a^2 + b^2}}$

Area of a triangular region:

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangular region ABC then the

area of the triangular region is given as; $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

Trapezium:

A quadrilateral having two parallel sides and two non parallel sides is called Trapezium.

Area of Trapezium = $\frac{1}{2}$ (sum of parallel sides) (distance between parallel sides)

Angle between two lines:

Theorem: let l_1 and l_2 be two non-vertical lines such that they are not perpendicular to each axes. If m_1 and m_2 are the slopes of l_1 and l_2 respectively, then the angle from l_1 to l_2 is given by; $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$ or $\theta = \tan^{-1}\frac{m_2 - m_1}{1 + m_1 m_2}$,

Acute angle between two straight lines:

The acute angle between two lines $\begin{matrix} l_1 : a_1x + b_1y + c_1 = 0 \\ l_1 : a_2x + b_2y + c_2 = 0 \end{matrix}$ is given

$$\tan\theta = \left|\frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2}\right|$$

Ortho center	The point of concurrency of altitudes of a triangle is called ortho centre.
Circum center	The point of concurrency of perpendicular bisectors of a triangle is called circum centre.
Theorem: Altitudes of a triangle are concurrent.	
Theorem: Right bisectors of a triangle are concurrent.	
Theorem: If the points A, B, C are collinear then $\Delta = 0$.	

Family of Lines: Let $\ell_1 : a_1x + b_1y + c_1 = 0$ *and* $\ell_2 : a_2x + b_2y + c_2 = 0$ for non zero real h, the equation $(a_1x + b_1y + c_1) + h(a_2x + b_2y + c_2) = 0$ represents family of lines.

Pair of Straight Lines

Homogeneous equation	An equation of the type $f(x, y) = 0$ is called homogeneous equation of degree n (n is positive integer)if. $f(kx, ky) = k^n f(x, y)$
General homogeneous equation of second degree in two variables	$ax^2 + 2hxy + by^2 = 0$, where a, h and b are not simultaneously zero.

Theorem: every homogeneous second degree equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines through the origin. The lines are ;

(i). Real and distinct, if $h^2 > ab$.

(ii). Real and coincident, if $h^2 = ab$.

(iii). Imaginary, if $h^2 < ab$.

Angle between the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ is given as; $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a+b}$.

(i). $\theta = 0$ if $h^2 - ab = 0$. (coincident)

(ii). $\theta = \dfrac{\pi}{2}$ if $a+b=0$. (perpendicular) $\Rightarrow \tan\theta = \infty$

Condition of Concurrency of Three Lines:

Three non-parallel lines

$l_1 : a_1x + b_1y + c_1 = 0$

$l_2 : a_2x + b_2y + c_2 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

$l_3 : a_3x + b_3y + c_3 = 0$

This is necessary and sufficient condition of concurrency.

Transformations:

Changing the position of the reference axis in a coordinate system by **translation**, **rotation** or both, usually to simplify the equation of a curve, is called transformation.

Translation of Axis	Rotation of Axis
Let $p(x,y)$ be a point in xy-coordinate system. If the origin $O(0,0)$ is translated to a new point $O'(h,k)$, then point $p(x,y)$ are translated to $p'(X,Y)$, where $X = x - h$ $Y = y - k$ Remember that in translation of axis, origin is shifted to another point but the axes remain parallel to the old axes.	The xy-coordinate axes are rotated about the origin through an angle $\theta\left(0 < \theta < \dfrac{\pi}{2}\right)$. Then point $p(x,y)$ are transformed to $p'(X,Y)$, where $X = x\cos\theta + y\sin\theta$ $Y = y\cos\theta - x\sin\theta$ In rotation origin remains at its position, while the axes are shifted to new axes.

PRACTICE EXERCISE

(1) Centroid of a triangle with vertices A(2,3) B(-1,6) and C(7,5) is

(a) $\left(\frac{1}{3},\frac{2}{7}\right)$ (b) $\left(\frac{2}{5},\frac{7}{3}\right)$

(c) $\left(\frac{8}{3},\frac{14}{3}\right)$ (d) $\left(\frac{7}{3},\frac{11}{3}\right)$

(2) The points (7,9), (3, -7) and (-3, 3) are the vertices of

(a) A right angled Δ (b) an isosceles Δ

(c) an equilateral Δ (d) a right angled isosceles Δ

(3) The points (4, -2), (-2,4) and (5,5) are the vertices of a triangle. Then the coordinates of the in centre of the triangle are_____?

(a) $\left(\frac{7}{2},\frac{2}{7}\right)$ (b) $\left(\frac{3}{2},\frac{3}{2}\right)$

(c) $\left(\frac{5}{2},\frac{5}{2}\right)$ (d) $(3,-3)$

(4) If the gradient of line passing through A(3, 2) is $\frac{3}{4}$, the points on the line at a distance 5 units from A are;

(a) (-7, 5), (-1, -1) (b) (7, 5), (-1, -1)

(c) (1, 1), (5, -5) (d) (-7, -5),(1, 1)

(5) Slope of a line parallel to y-axis is;

(a) 0 (b) undefined

(c) 1 (d) –1

(6) The equation of line through (-3,5) and $\perp$ to line $x+2y+3=0$ is;

(a) $y+2x+7=0$ (b) $2x-y+11=0$

(c) $3x+7y+20=0$ (d) $4y+13x=19$

(7) Which pair of lines are $\perp$ to each other?

(a) $2x+2y=1$, $3x+3y=1$ (b) $4x+3y-7=0$, $2x+7y+4=0$

(c) $7x-3y=17$, $5x+2y=9$ (d) $x-2y-6=0$, $2x+y-5=0$

(8) Point of intersection of pair of lines $7x+2y=6$ and $3x+y=0$ is;

(a) (6, -18) (b) (17, 9)

(c) (–6, -1) (d) $\left(3,-\frac{1}{2}\right)$

(9) Which point is above the line $\frac{4}{3}x+y=1$?

(a) (-3, -1) (b) (-3, 1)

(c) (3, 1) (d) (-4, -1)

(10) The area of the triangle formed by the lines $7x - y - 10 = 0$, $10x + y - 41 = 0$ and $3x+2y+3=0$ is;

(a) 68 square units
(b) 34 square units
(c) 17 square units
(d) 7.5 square units

(11) Distance of the point (-7, 13) from the line $2x+y+13=0$ is;

(a) $\frac{6}{\sqrt{109}}$
(b) 56
(c) 67
(d) $\frac{12}{\sqrt{5}}$

(12) Which is the intercept form of the equation $px + qy + r = 0$?

(a) $y = mx + r$
(b) $\frac{x}{p} + \frac{y}{q} = 1$
(c) $x\cos\alpha + y\sin\alpha = 1$
(d) none

(13) The slope formula is given by;

(a) $m = \frac{y_2 + y_1}{x_2 - x_1}$
(b) $m = \frac{x_2 - x_1}{y_2 + y_1}$
(c) $m = \frac{y_2 - y_1}{x_2 - x_1}$
(d) $m = \frac{y_2 + y_1}{x_2 + x_1}$

(14) The inclination of any line parallel to y-axis is

(a) $\frac{\pi}{2}$
(b) 0
(c) $\frac{\pi}{4}$
(d) $\frac{3\pi}{2}$

(15) Two lines are parallel if and only if they have same;

(a) Distance
(b) quadrants
(c) slope
(d) vertices

(16) The inclination of the x-axis is;

(a) 180°
(b) 0
(c) 90°
(d) 150°

(17) Any equation of first degree in x and y represents a;

(a) Triangle
(b) rectangle
(c) line
(d) quadratic equation

(18) Find 'b' if the line through (3,4) and (-1, b) is parallel to the line through (2,3) and (-5,1).

(a) 20
(b) 17
(c) $\frac{20}{7}$
(d) $\frac{-11}{15}$

(19) The measure of the angle from the first line to the second line whose slopes are 0 and 3, is;

(a) $\tan^{-1}\left(\frac{1}{3}\right)$ (b) $\tan^{-1} 3$

(c) $\frac{1}{2}\tan^{-1} 7$ (d) $2\tan^{-1} 3$

(20) A (0, b), B (0, 0) and C (a, 0) are vertices of a triangle. Its median AD and BE are mutually perpendicular if;

(a) $a = \sqrt{2}b$ (b) $b = -\sqrt{2}a$

(c) $a = \pm\sqrt{2}b$ (d) $a = \frac{b}{\sqrt{2}}$

(21) The equation of line through the points (2,4) and (7,1) is;

(a) 5y+3x=0 (b) 5y+3x-26=0

(c) $2\sqrt{5}y + 16x - 13 = 0$ (d) 9x+2y+17=0

(22) The equation of the line through (-1, 3) perpendicular to a line having slope $\frac{3}{2}$ is;

(a) 2y-3x+7 = 0 (b) 15y+2x-1 = 0

(c) 13y+2x-3 = 0 (d) 3y+2x-7 = 0

(23) The equation of the line through the point (-2,1) and the point of intersection of the lines 4x+3y-1 = 0 and 2x-6y+5 = 0 is;

(a) x-13y+19 = 0 (b) 8x+51y-35 = 0

(c) 23x-43y+27 = 0 (d) 18x+29y-21 = 0

(24) The equation of the line through (x_2, y_2) parallel to $px+qy+r=0$ is

(a) $px+qy+r+px_2=0$ (b) $px+qy+px_2+qy_2=0$

(c) $px+qy-px_2+r=0$ (d) $px+qy-px_2-qy_2=0$

(25) The equation of line through (x_2, y_2) perpendicular to the line $px+qy+r=0$ is;

(a) $qx_2-py_2-qx+py=0$ (b) $px+qy+px_2+qy_2=0$

(c) $px_2+qy_2+3px-3qy=0$ (d) $px_2+qy_2=0$

(26) The measure of the angle θ from the line $p_1x+q_1y+r_1=0$ to the line $p_2x+q_2y+r_2=0$, is given by tan θ = _____;

(a) $\frac{p_1q_1 + p_2q_2}{p_1q_2 - p_2q_1}$ (b) $\frac{p_1q_2 + q_2q_1}{p_1q_2 - p_2q_1}$

(c) $\frac{p_1q_2 - p_2q_1}{p_1p_2 + q_1q_2}$ (d) none of these

(27) The distance between the parallel lines 4x+3y-2 = 0 and 4x+3y+8 = 0 is;

(a) $\frac{13}{2}$ (b) $\frac{59}{3}$

(c) 2 (d) 12

(28) Slope of line parallel to x-axis is;
(a) 0
(b) 1
(c) ∞
(d) 2

(29) The matrix form of the system of linear equations; $\left.\begin{matrix} 3x+4y-7=0 \\ 2x-5y+8=0 \\ x+y-3=0 \end{matrix}\right\}$

(a) $\begin{bmatrix} 3x+4y-7=0 \\ 2x-5y+8=0 \\ x+y-3=0 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 4 & -7 \\ 2 & -5 & 8 \\ 1 & 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 4 & -7 \\ 2 & -5 & 8 \\ 1 & 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(d) all of these

(30) The equation of the straight line through the point of intersection of $16x-10y-33=0, 12x+14y+29=0$ and the intersection of $x-y+4=0, x-7y+2=0$ is;
(a) 13x+29y – 66 =0
(b) 13x-29y-66=0
(c) 13x+29y+66=0
(d) 13x-29y+66=0

(31) The points (5, -2), (1,2),(-2,5) are;
(a) Collinear
(b) vertices of Δ
(c) vertices of rectangle
(d) all

(32) $y=-\frac{17}{9}x+\frac{37}{9}$ is _____ form of equation 9y+17x-37=0.
(a) Normal
(b) slope – intercept
(c) intercepts
(d) point – slope

(33) The average entry test score of engineering candidates was 592 in the year 1998, while the score was 564 in 2002. assuming that the relationship between time and score is linear, then the average score in the year 2006 will be;
(a) 450
(b) 350
(c) 536
(d) 650

(34) The equation of line passing through point (4, -6) and making an angle of 45° with positive x-axis is;
(a) x+y-10 = 0
(b) 2x-y= 10
(c) x-y+10 = 0
(d) y-x+10 = 0

(35) If a line passes through points (4, 3) and (2, λ) and perpendicular to $y = 2x+3$, then λ =?
(a) –1
(b) 1
(c) –4
(d) 4

(36) If the origin is shifted to (-4,-6) then the point P(-6,-8) will be shifted into;
(a) (-2,1) (b) (-2,-2)
(c) (0,-2) (d) (2,2)

(37) The xy-coordinates are rotated about origin at an angle 180°, then the coordinates of the point P(4,-1) are;
(a) (1,4) (b) (-1,4)
(c) (-4,1) (d) (-4,-1)

(38) After the rotation of axis at 90° the coordinates of the point P(x,y) are (2,-9) then the original point is;
(a) (-9,2) (b) (9, -2)
(c) (9,2) (d) (-2,9)

(39) The angle between the lines represented by $9x^2+24xy+16y^2=0$ is
(a) 90° (b) 0°
(c) 180° (d) none of these

(40) Which of the following is not homogenous?
(a) $3x^2+2y^2+3xy=0$ (b) $x^2+y^2+2gxy=0$
(c) $4x^2+5y^2+3xy=0$ (d) $x^2+4y^2+2gx+2fy=0$

(41) Two points (a, 0) and (0, b) are on a straight line. Which of the following points is on that straight line?
(a) (3a, -2b) (b) (a^2, ab)
(c) (-3a, 2b) (d) (a, b)

(42) The join of (-3, 2) and (4, 6) is cut by x-axis in ratio _____ externally.
(a) 2:3 (b) 1:2
(c) 1:3 (d) 3:2

(43) If A and B are the points (-3, 4) and (2, 1) then the coordinates of the point C on AB such that AC = 2BC are _____
(a) (2, 4) (b) (3, 7)
(c) (7, -2) (d) $\left(\frac{-1}{2}, \frac{5}{2}\right)$

(44) Equation of the straight line making equal intercepts on the axes passing through (3, 2) is _____
(a) $x + y = 2$ (b) $x + y = 3$
(c) $x + y = 5$ (d) $x + y = 15$

(45) If the points (a, 0), (0, b) any (x, y) are collinear then _____
(a) $ax + by = 1$ (b) $bx + ay = 1$
(c) $\frac{x}{a}+\frac{y}{b}=1$ (d) $ax + by = 0$

(46) The line segment joining by the points (-3, -4) and (1, -2) is divide by y – axis in the ratio
(a) 1:3 (b) 2:3
(c) 3:1 (d) 3:2

(47) Area of the triangle whose vertices are (-4, -1), (1, 2) and (4, -3) is _____
(a) 17
(b) 16
(c) 15
(d) 0

(48) The vertices of a ΔABC are (2, 2), (-4, -4) and (5, -8) respectively, the length of the median through C is _____
(a) $\sqrt{65}$
(b) $\sqrt{117}$
(c) $\sqrt{85}$
(d) $\sqrt{113}$

(49) If the sum of the distances of a point from two $\perp$ lines is 1 then the locus is _____
(a) Square
(b) Circle
(c) Straight line
(d) Two intersecting straight lines

(50) Distance between the lines $4x + 3y = 11$ and $8x + 6y = 15$ is _____
(a) $\frac{7}{2}$
(b) 4
(c) $\frac{7}{10}$
(d) $\frac{15}{10}$

(51) Directed distance of a point (-5, -6) from y-axis is
(a) 6
(b) -6
(c) 5
(d) -5

(52) If $\overline{AB} : \overline{BC} = 2:3$ then point C divides the line segment AB externally in ratio
(a) 3:2
(b) 2:3
(c) 3:5
(d) 1:3

(53) Equation of line having slope 3 making x-intercept 5 is
(a) $y = 3x + 5$
(b) $-3x + y + 15 = 0$
(c) $3x - y + 5 = 0$
(d) $x + \frac{1}{3}y - 5 = 0$

(54) Equation of hypotenuse $\overline{AB}$ of length $\sqrt{2}$ in isosceles ΔOAB is
(a) $x + y = 1$
(b) $x + \sqrt{2y} = 1$
(c) $\sqrt{2}x + y = 1$
(d) $x + y = \sqrt{2}$

(55) Joint equation of lines through the origin and perpendicular to the lines $x^2 + xy - 6y^2 = 0$
(a) $6x^2 + xy - y^2 = 0$
(b) $-6x^2 + xy + y^2 = 0$
(c) $-6x^2 - xy - y^2 = 0$
(d) None of these

LINEAR INEQUALITIES AND LINEAR PROGRAMMING

Inequalities

Inequalities are of four types;

Inequality	Symbol
Greater than	$>$
Less than	$<$
Greater than or equal to	$\geq$
Less than or equal to	$\leq$

Linear Inequalities

Generally a linear inequality in two variables x & y can be one of the following four forms;

(i). $ax+by<c$

(ii). $ax+by\leq c$

(iii). $ax+by>c$

(iv). $ax+by\geq c$, where a, b & c are constants also a, b can not zero simultaneously.

Some Important Results Related To Linear Inequality In Two Variables:

(a) $ax+by=c$ is called **associated** or **corresponding equation** of any of the above four linear inequalities.
(b) Every inequality divides a plane into two **half planes**.
(c) A **vertical line** divides the plane into **left and right half planes**.
(d) A non-vertical line divides the plane in to **upper and lower half planes.**
(e) If an inequality involves $<or>$,the half plane is **open**.
(f) If an in equality involves $\leq or\geq$,the half plane is **closed.**
(g) A point of a solution region where two of its boundary lines intersect is called a **corner point** or **vertex.**

Procedure for Graphing a Linear Inequality in Two Variables:

(i).Graph the corresponding equation $ax+by=c$ .used a **dashed line** for this line in the $<or>$ cases, and a **solid line** in the $\leq or\geq$ cases.

(ii).The solution set is the half plane on one side of the boundary line. To determine which side, choose a **test point** (any point in the plane but not on the line) and check to see if the coordinates of the point satisfy the given inequality.

Linear Programming*

Linear programming is a branch of mathematics that deals with an important class of constrained optimization problems.

Some terms related to linear programming;

Problem constraints	The system of linear inequalities involved in the problem concerned is called problem constraints.
Decision variables	Non-negative constraints are called decision variables.
Feasible region	A restricted region under certain constraints is called the feasible region.
Feasible solution	Each point of the feasible region is called feasible solution.
Feasible solution set	The set of all feasible solutions of a system of linear inequalities is called feasible solution set.
Convex region	If the line joining any two points of a region lies entirely with in the region, then the region is called convex.(see figures at the end)
Objective function	A function which is to be maximized or minimized but not both is called objective function.
Optimal solution	The feasible solution which maximizes or minimizes the objective function is called optimal solution.

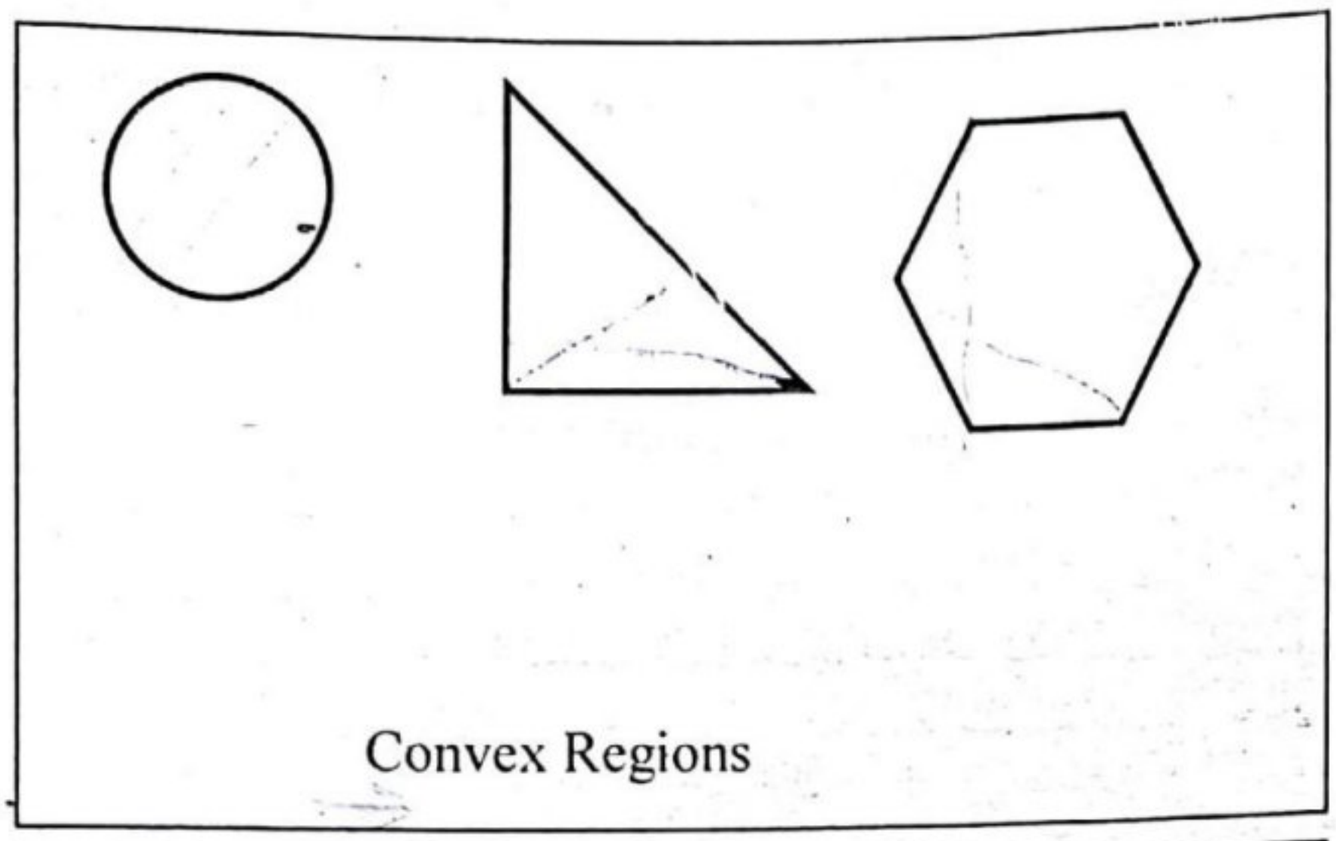
Procedure for Determining Optimal Solution:

(i).Graph the solution set of linear inequality constraints to determine the feasible region.
(ii).Find the corner points of the region.
(iii).Evaluate the objective function at each corner point to find the optimal solution.

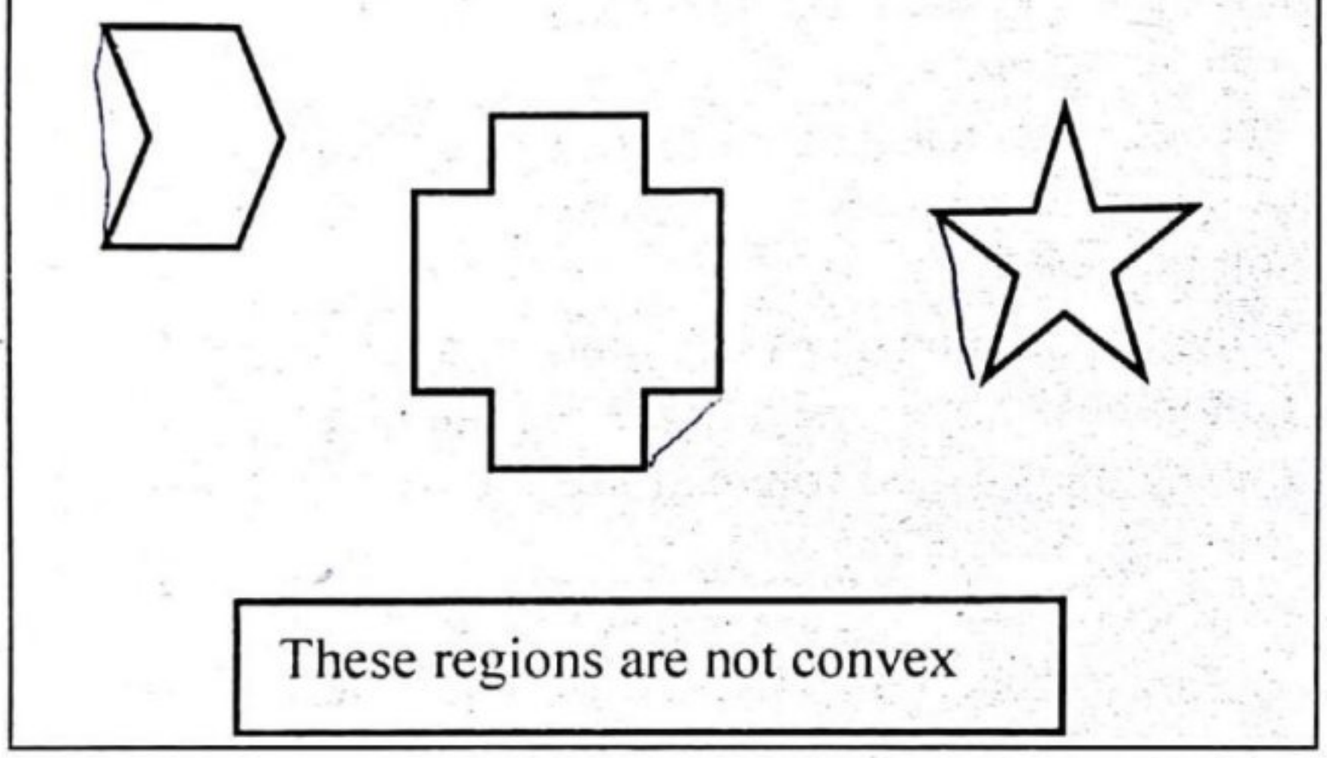
Points to Be Remember

- There are infinite many feasible solutions in the feasible region.
- The intersection point of two boundary lines is a corner point only if it is feasible.
- If a linear program has an optimal solution, it must occur at a corner point of the feasibility region.

* Introduced by a Soviet Mathematician Leonid Kantrovich in 1930's.



Convex Regions



These regions are not convex

PRACTICE EXERCISE

(1) A function which is to be maximized or minimized is called _____ function
(a) Optimal (b) feasible
(c) objective (d) corner

(2) The feasible solution which maximizes or minimizes the objective function is called
(a) Feasible solution (b) optimal solution
(c) either feasible or optimal (d) none of these

(3) Non-negative constraints help in taking_____
(a) Solutions (b) maximization
(c) minimization (d) decision

(4) Optimize means_____ a quantity under certain constraints
(a) Maximize (b) minimize
(c) either maximize or minimize (d) both a and b

(5) Inequalities have_____ symbols
(a) 2 (b) 3
(c) 4 (d) 6

(6) The solution set of $x<4$ is
(a) $-\infty<x<4$ (b) $4<x<\infty$
(c) $0<x<4$ (d) $-\infty<x<\infty$

(7) Which one is an associated equation of $ax+by\leq c$?
(a) $ax+by<c$ (b) $ax+by\leq c$
(c) $ax+by-c<0$ (d) $ax+by-c=o$

(8) Corner point is also called
(a) Origin (b) focus
(c) vertex (d) point of contact

(9) The variables used in the system of linear inequalities are_____
(a) Integers (b) real numbers
(c) non-ve integers (d) none of these

(10) Non-ve constraints are called_____ variables.
(a) Discrete (b) continuous
(c) dependent (d) decision

(11) There are _____ feasible solutions in the feasible region.
(a) 2 (b) 4
(c) 6 (d) infinite.

(12) When $f(x,y)=x+3y$ is maximized, subject to $2x+5y\leq30$, $5x+4y\leq20$, $x\geq0$, $y\geq0$ the maximum value of $f(x,y)$ is_____
(a) (0,0) (b) (0,5)
(c) (5,0) (d) (5,5)

(13) If $H=2x+y$ is minimized, subject to $x+y\geq3$, $7x+5y\leq35$, $x\geq0$, $y\geq0$ then H is minimum at
(a) (0,0) (b) (3,0)
(c) (0,3) (d) (3,3)

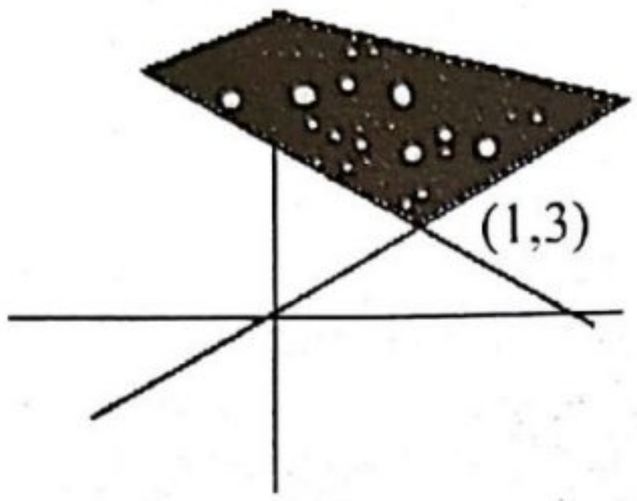
(14) The points above the line $x+2y=4$ satisfy the inequality,

(a) $x+2y<4$ (b) $x+2y\leq 4$

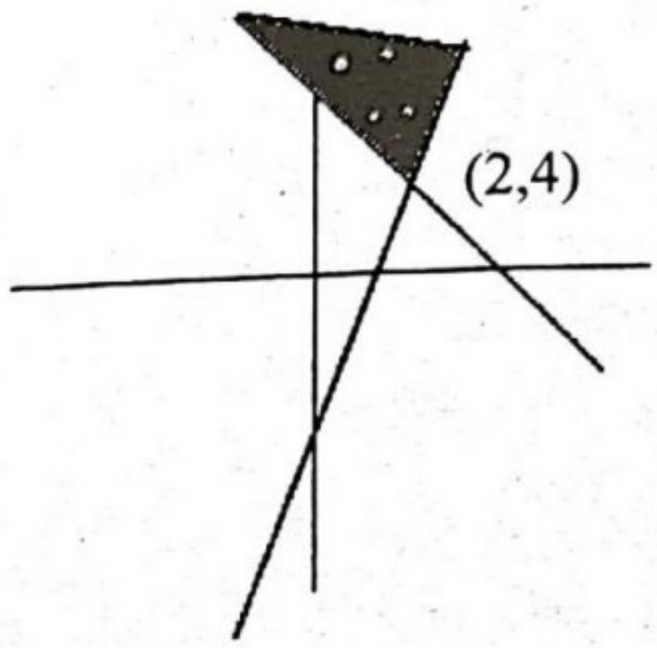
(c) $x+2y>4$ (d) $x+2y\geq 4$

(15) The graph of solutions region of $3x-y\leq 2$ and $x+y\geq 6$ is

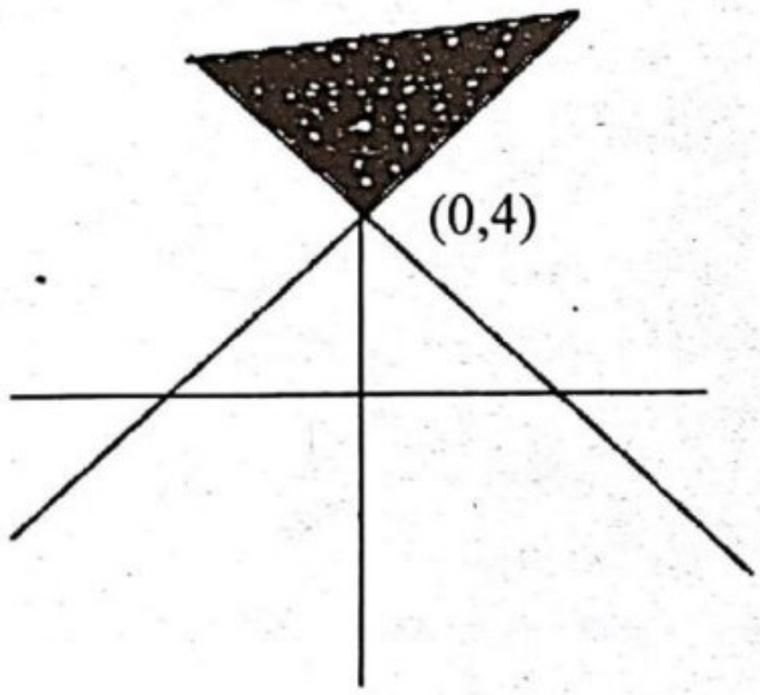
(a)



(b)

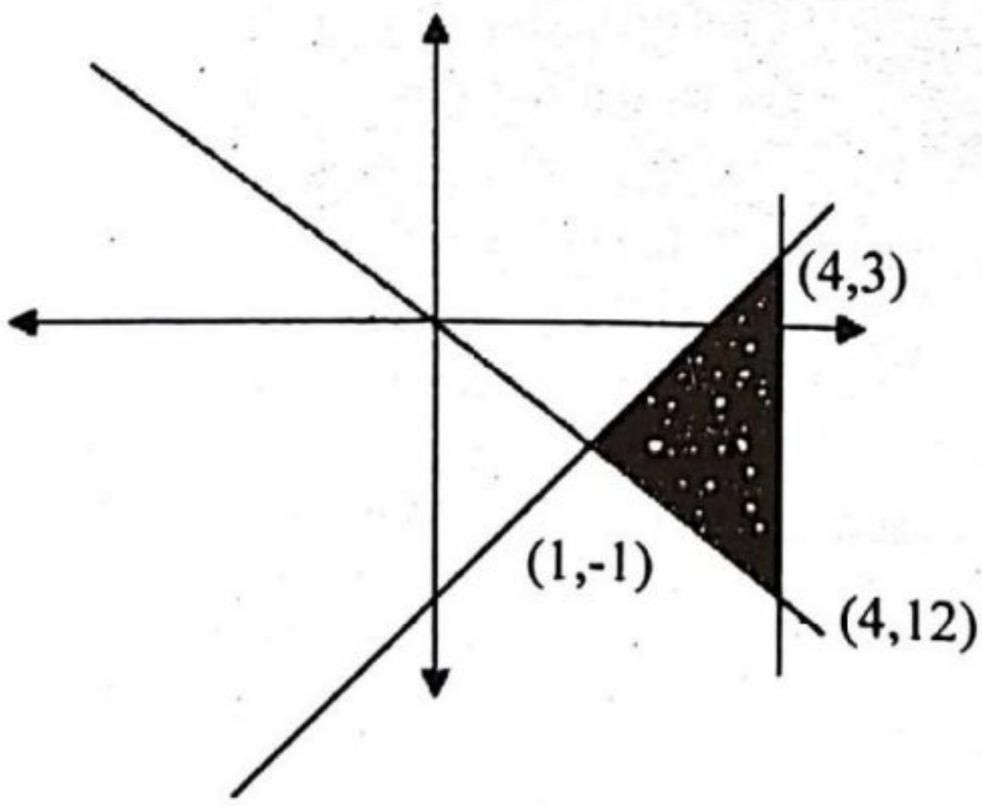


(c)



(d) None

(16)



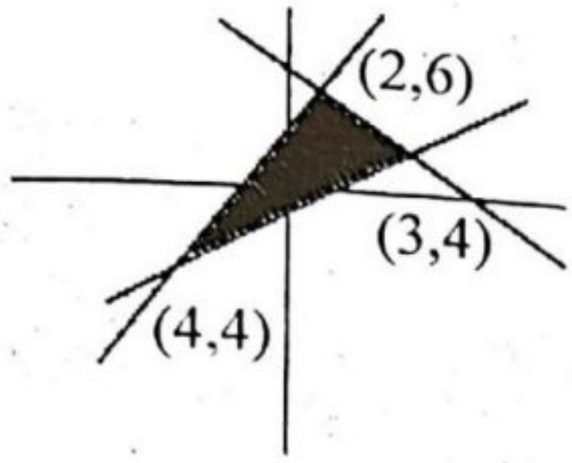
Is the solution region of

(a) $2x-y>5$, $3x+y>0$, $x\leq 4$ (b) $2x-y\geq 5$, $3x+y>0$, $x<4$

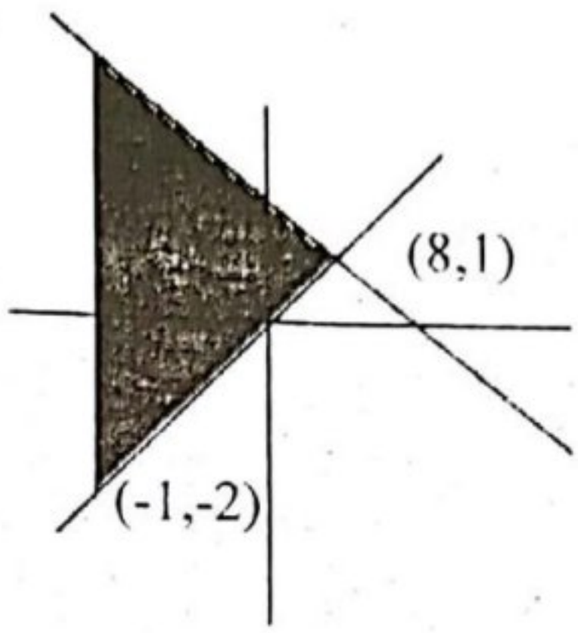
(c) $2x-y\geq 5$, $3x+y\geq 0$, $x\leq 4$ (d) $2x-y\geq 5$, $3x+y>0$, $x<4$

(17) The graph of $x-2y\leq 6$, $2x+y\geq -2$ and $x+2y\leq 10$ is

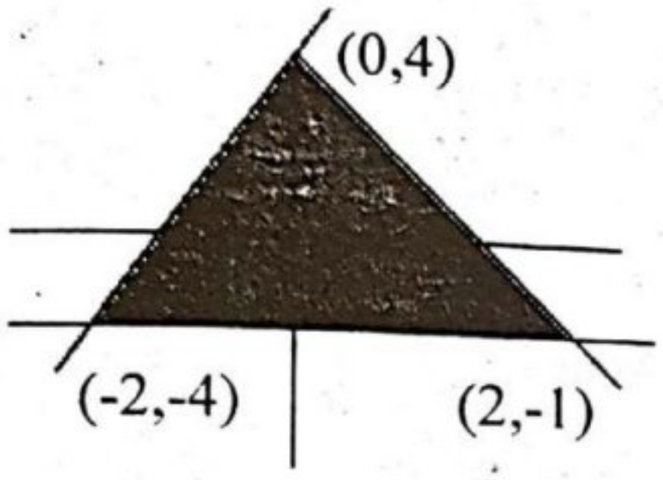
(a)



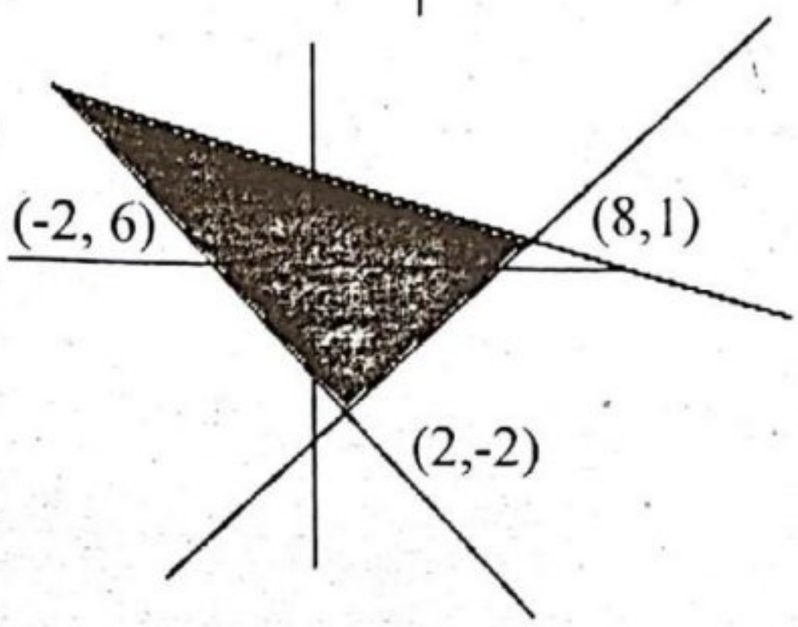
(b)



(c)



(d)

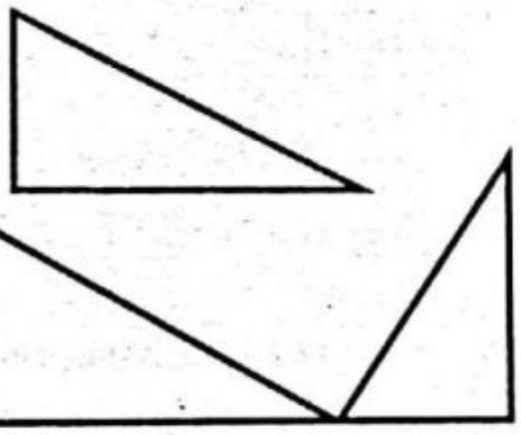


(18) Which of the following is not convex?

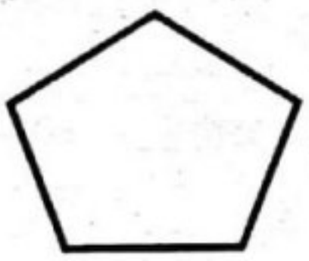
(a)



(b)



(c)



(d)

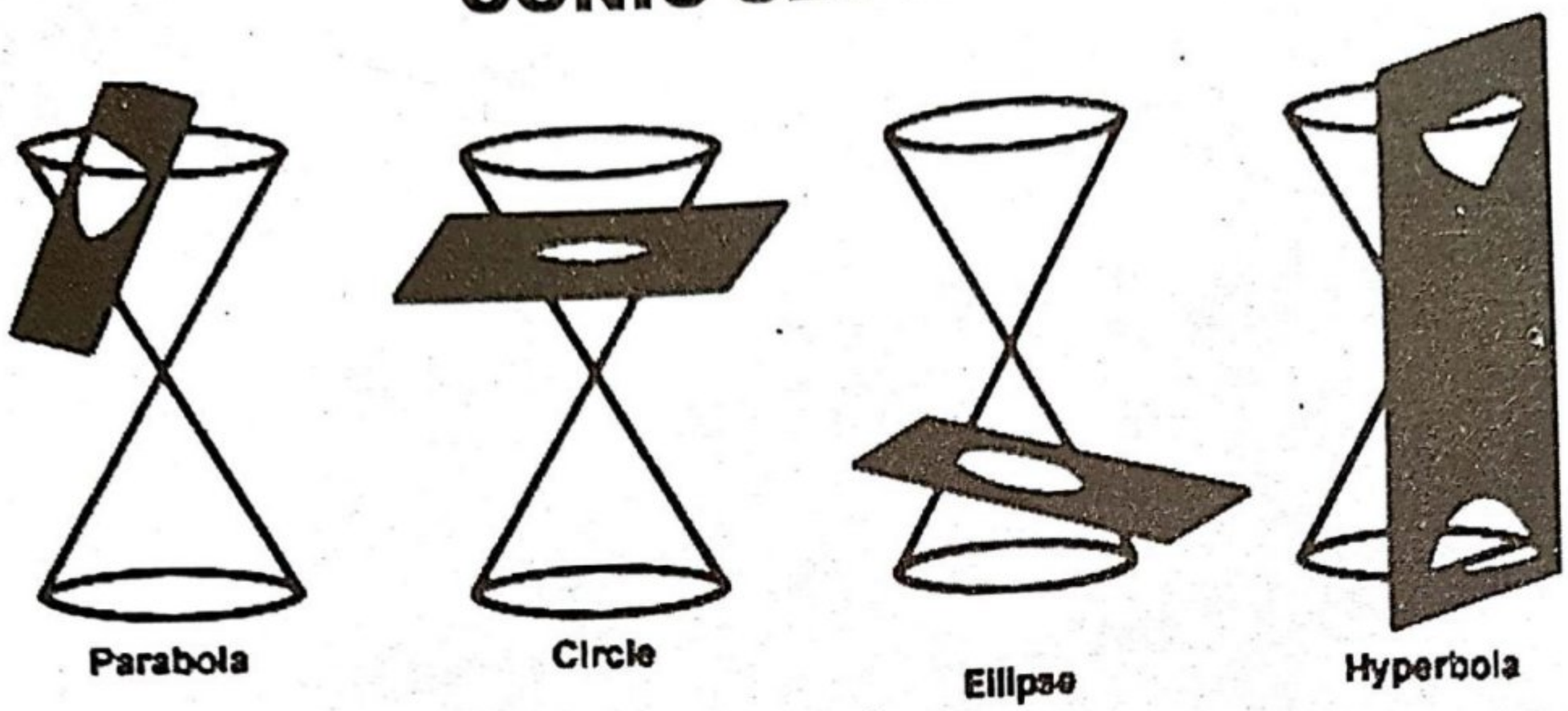
(19) A manufacturer makes two grades of concrete. Each bag of the high-grade concrete contains 10Kg of gravel and 5Kg of cement, while each bag of low-grade concrete contains 12Kg of gravel and 3Kg of cement. There are 1,920 Kg of gravel and 780Kg of cement currently available. The manufacturer can make a profit of \$1.20 of each bag of the high grade and \$ 1.00 on each bag of low grade concrete. Then the number of bags of low grade, high grade and the maximum profit are respectively?

(a) 60,120,240
(b) 120,60,240
(c) 60,120,204
(d) 120,60,204

(20) The largest and smallest value of $F=3x+4y$ subject to $x+3y\leq 15$, $4x+3y\leq 24$, $x,y\geq 0$ are_____ respectively

(a) 25,5
(b) 5,0
(c) 25,10
(d) 25,0

CONIC SECTION



The Cone and Its Sections

Right Circular Cone	The surface generated when one straight line that intersects another fixed straight line is rotated at an oblique angle. The lines are called generators of the cone.
Axis of The Cone	The fixed line of the cone is called its axis.
Elements of The Cone	The possible positions of the generating line in its rotation about the axis are its elements.
Vertex	The common intersection point of all the cone's elements is the cone's vertex.
Nappes	The two symmetrical parts of the generated surface on each side of the vertex are the cone's nappes.
Conic Sections or Conics	The curves that can be obtained by cutting a cone with a plane are called conic sections or simply conics.
Circle	When the intersecting plane cuts completely across one nappe at a right angle to the axis of the cone, the curve is a circle.
Point Circle	If the intersecting plane cuts across the vertex of a cone, the resulting curve is a point circle.
Parabola	When the intersecting plane is parallel to an element, and passing through only one nappe, the resulting curve is parabola.
Ellipse	When the intersecting plane cuts completely across one nappe at an oblique angle to the axis, the curve is an ellipse.
Hyperbola	When the intersecting plane cuts through both nappes parallel to the axis of the cone the resulting curve is hyperbola.

Circle

A set of points such that distance of each point from a fixed point (**centre**) remains constant. The constant distance is called **radius** of the circle.

Equations of Circle:

centre (h,k) & radius $= r$. *(the standard form)*	$(x-h)^2+(y-k)^2=r^2$
Centre $(0,0)$, radius $= r$	$x^2+y^2=r^2$
Parametric equations	$x=r\cos\theta$ $y=r\sin\theta$
If end points of the diameter of the circle are $A(x_1,y_1)$ & $B(x_2,y_2)$	$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

General Form of a Circle:

The general equation of the circle is $x^2+y^2+2gx+2fy+c=0$.

Properties & Results from General Equation

(i).The general equation of the circle involves three constants ,*i.e.* ***g,f & c.***	
(ii). The general equation of a circle is a second degree equation in which coefficients of x^2 & y^2 are 1.	
(iii).The general equation does not contain the term involving the product xy.	
Center of the circle	$(-g,-f)$ *i.e.* $\left(-\frac{\text{coefficient of } x}{2},-\frac{\text{coefficient of } y}{2}\right)$
Radius of the circle	(i). $r=\sqrt{g^2+f^2-c}>0$ Real circle. (ii). $r=\sqrt{g^2+f^2-c}=0$, point circle. (iii).If $g^2+f^2-c<0$,imaginary circle.

Tangents and Normals

Tangent	A straight line that touches the curve at a point without cutting the curve.
Normal	A straight line perpendicular to the curve at the point of tangency.
Equation of tangent to the circle $x^2+y^2=a^2$ at $P(x_1,y_1)$.	$xx_1+yy_1=a^2$
Equation of normal to the circle $x^2+y^2=a^2$ at $P(x_1,y_1)$.	$xy_1-yx_1=0$
Equation of tangent to the circle $x^2+y^2+2gx+2fy+c=0$ at $P(x_1,y_1)$.	$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$
Equation of normal to the circle $x^2+y^2+2gx+2fy+c=0$ at $P(x_1,y_1)$.	$(x-x_1)(y_1+f)-(y-y_1)(x_1+g)=0$

Theorems

(i).The point $p(x_1, y_1)$ lies outside, on or inside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ according as $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c \gtreqless 0$.

(ii).Two tangents can be drawn to a circle from any point out side the circle.

(iii).Let $P(x_1, y_1)$ be a point out side the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, then length of either tangent drawn from $p(x_1, y_1)$ to the circle = $\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

(iv).The line $y = mx + c$ intersects the circle $x^2 + y^2 = a^2$ in at the most two points, the points are

(a) Real and distinct if, $a^2(1 + m^2) - c^2 > 0$

(b) Real and coincident if, $a^2(1 + m^2) - c^2 = 0$

(c) Imaginary if, $a^2(1 + m^2) - c^2 < 0$

Condition of tangency;

(v).The line $y = mx + c$ touches the circle $x^2 + y^2 = a^2$ if, $c = \pm a\sqrt{1 + m^2}$

Important Properties of a Circle*

(i).Perpendicular dropped from the centre of a circle on a chord bisects the chord.

(ii).The perpendicular bisector of any chord of a circle passes through the centre of the circle.

(iii).The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.

(iv).Congruent chords of a circle are equidistant from its centre.

(v).Measure of the central angle of a minor arc is double the measure of the angle subtended in the corresponding major arc.

(vi).An angle in a semi-circle is a right angle.

(vii).The tangent to a circle at any point of the circle is perpendicular to the radial segment at that point.

(viii).The perpendicular at the outer end of a radial segment is tangent to the circle.

(ix).Normal lines of a circle pass through the centre of the circle.

(x).Mid point of the hypotenuse of a right triangle is the circumcentre of the triangle.

(xi).Perpendicular dropped from a point of a circle on a diameter is a mean proportional between the segments into which it divides the diameter.

* Analytic proofs of these properties are beyond the scope of this book.

Contact of Two Circles

Let r_1, r_2 be the radii of two circles and C_1, C_2 be their centers, then

Circles touch externally	$\|C_1C_2\| = r_1 + r_2$
Circles touch internally	$\|C_1C_2\| = \|r_1 - r_2\|$
Circles intersect in two real distinct points	$\|r_1 - r_2\| < \|C_1C_2\| < r_1 + r_2$
Circles do not touch	(i) $\|C_1C_2\| > r_1 + r_2$ (ii) $\|C_1C_2\| < \|r_1 - r_2\|$

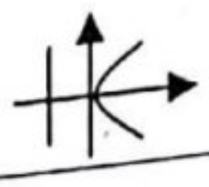
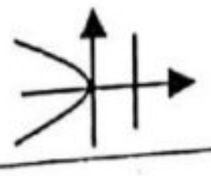
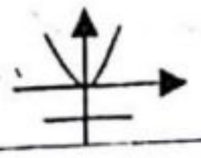
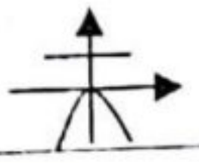
Parabola

A set of points in a plane such that the distance of each point form a fixed point (focus, F) is equal to its distance from a fixed straight line (directrix, L).

Terms related to Parabola:

Axis of parabola	The line through focus and perpendicular to directrix is called axis of parabola.
Vertex	The mid point of the perpendicular line joining focus and directrix is called vertex (or turning point) of parabola.
Focal chord	A chord of the parabola through focus is called focal chord.
Latus rectum	The focal chord perpendicular to the axis of the parabola is called latus rectum of the parabola.
Eccentricity	The ratio of the distance of any point on the parabola to its distance from directrix.

Main Facts about Standard Forms of Parabola:

Equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Focus	$(a,0)$	$(-a,0)$	$(0,a)$	$(0,-a)$
Vertex	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
Length of latus rectum	$4a$	$4a$	$4a$	$4a$
Equation of latus rectum	$x = a$	$x = -a$	$y = a$	$y = -a$
Equation of directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Eccentricity	**1**	**1**	**1**	**1**
Graph				

Theorems on parabola:

(i).The point on the parabola closest to the focus is the vertex.
(ii).The ordinate at any point P of the parabola is a mean proportional between the length of the latus rectum and the abscissa of P.
(iii).The tangent at any point P of a parabola makes equal angles with the line PF and the line through P parallel to the axis of parabola, F being focus.

Ellipse

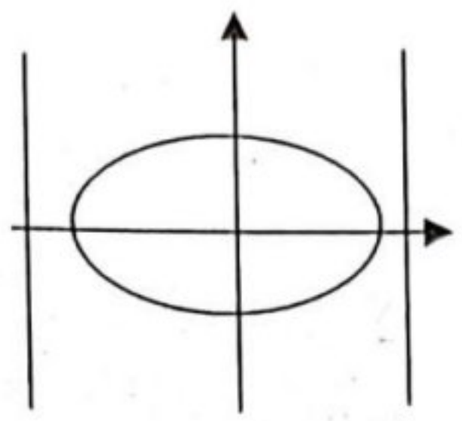
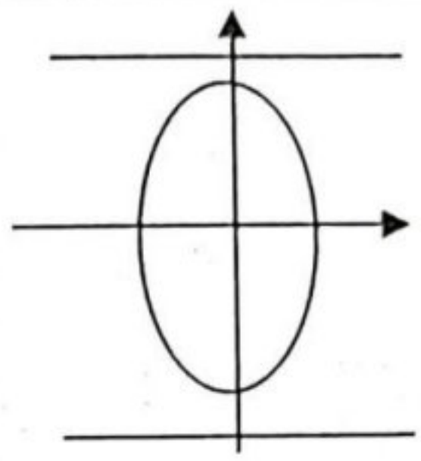
A set of points such that the distance of each point from a fixed point (focus, F) bears a constant ratio (Eccentricity, $0 < e < 1$) to its perpendicular distance from a fixed straight line (directrix, L).

Terms Related To Standard Ellipse:

Vertices	The points on the standard ellipse where it crosses the *x-axis*.
Co vertices	The points on the standard ellipse where it crosses the *y-axis*.
Center	The midpoint of the line joining vertices (or co vertices or Foci).
Major axis	The line joining vertices is called major axis.
Minor axis	The line joining co vertices is called minor axis.
Latera recta	The chords perpendicular to major axis passes through foci are called laters recta.
Eccentricity	The ratio of the distance of any point on the ellipse from the focus to its distance from the directrix.

Main Facts about Standard Forms of Ellipse:

Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$	$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1, a>b$
Foci	$(\pm c,0), c^2=a^2-b^2$	$(0,\pm c), c^2=a^2-b^2$
Eccentricity	$0<e=\frac{c}{a}<1$	$0<e=\frac{c}{a}<1$
Vertices	$(\pm a,0)$	$(0,\pm a)$
Co vertices	$(0,\pm b)$	$(\pm b,0)$
Center	$(0,0)$	$(0,0)$
Length of Major Axis	$2a$	$2a$
Equation of Major Axis	$y=0$	$x=0$
Length of Minor Axis	$2b$	$2b$
Equation of Minor Axis	$x=0$	$y=0$
Length of Latus Rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

Equation of Directrices	$x = \pm\frac{c}{e^2}$	$y = \pm\frac{c}{e^2}$
Graph		

Theorems on Ellipse:

(i).The sum of the focal distances of any point on an ellipse is equal to length of major axis $(2a)$.
(ii).The distance between centre and any focus of the ellipse is denoted by c, and is given as; $c = \sqrt{a^2 - b^2}$.
(iii).The distance between foci $=2c$.

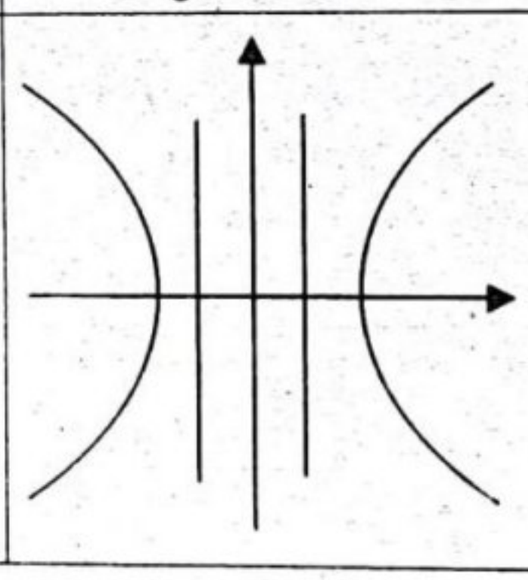
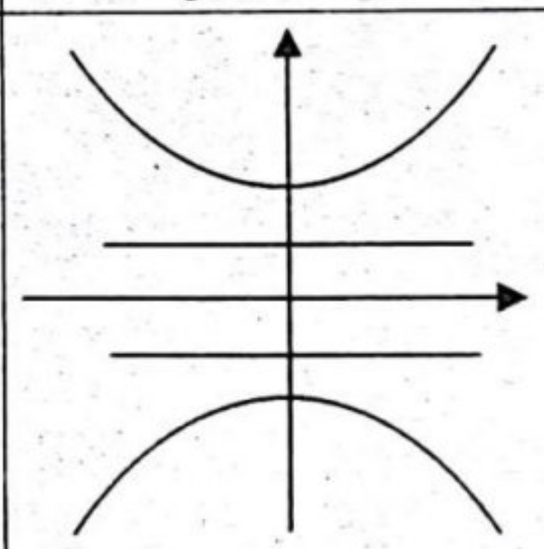
Hyperbola

A set of points such that the distance of each point from a fixed point (focus, F) bears a constant ratio (Eccentricity, $e > 1$) to its perpendicular distance from a fixed straight line (directrix, L).

Terms Related To Standard Hyperbola:

Vertices	The points on the standard hyperbola where it crosses the *x-axis*.
Center	The midpoint of the line joining vertices (or co vertices).
Transverse axis	The line joining vertices is called transverse axis.
Conjugate axis	The line joining co vertices are called conjugate axis.
Latera recta	The chords perpendicular to major axis passes through foci are called latera recta.
Eccentricity	The ratio of the distance of any point on the ellipse from the focus to its distance from the directrix.
Asymptotes	In general, an asymptote is a line that approaches a curve but never touches. Every hyperbola has associated with two lines called asymptotes. Their point of intersection is centre of the hyperbola. (a) Slope of asymptotes $= \pm\frac{b}{a}$,hyperbola opens side. (b) Slope of asymptotes $= \pm\frac{a}{b}$,hyperbola opens up & down

Main Facts about Standard Forms of hyperbola:

Equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
Foci	$(\pm c,0), c^2=a^2+b^2$	$(0,\pm c), c^2=a^2+b^2$
Eccentricity	$e=\frac{c}{a}>1$	$e=\frac{c}{a}>1$
Vertices	$(\pm a,0)$	$(0,\pm a)$
Center	$(0,0)$	$(0,0)$
Length of transverse Axis	$2a$	$2a$
Equation of transverse Axis	$y=0$	$x=0$
Length of conjugate Axis	$2b$	$2b$
Equation of conjugate Axis	$x=0$	$y=0$
Length of Latus Rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Equation of Directrices	$x=\pm\frac{c}{e^2}$	$y=\pm\frac{c}{e^2}$
Graph		

Theorems on Hyperbola:

(i). The difference of the focal distances of any point on the hyperbola is equal to the length of transverse axis ($2a$).
(ii). The distance between centre and any focus of the hyperbola is denoted by c and is given as; $c=\sqrt{a^2+b^2}$.
(iii). The distance between the foci = $2c$
(iv). The product of distances from the foci two any tangent of hyperbola is b^2.

Parametric Equations

Circle $x^2+y^2=a^2$	$x=a\cos\theta$ $y=a\sin\theta$
Parabola $y^2=4ax$	$x=at^2$ $y=2at$
Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$x=a\cos\theta$ $y=b\sin\theta$
Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$x=a\sec\theta$ $y=b\tan\theta$

Tangents and Normals:

Equation of tangent at point $p(x_1,y_1)$.	(i).To parabola $y^2=4ax$ is $yy_1=2a(x+x_1)$ (ii).To ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ (iii).To hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$.
Equation of normal at point $p(x_1,y_1)$.	(i).To parabola $y^2=4ax$ is $y-y_1=\frac{-y_1}{2a}(x-x_1)$. (ii).To ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$. iii).To hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$.
Condition that a line $y=mx+c$ is tangent to a cone	(i).Parabola $y^2=4ax$ if $c=\frac{a}{m}$. (ii).Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $c=\pm\sqrt{a^2m^2+b^2}$. Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if $c=\pm\sqrt{a^2m^2-b^2}$
Generally to find an equation of tangent at a point $p(x_1,y_1)$,make the replacements in the given equation of the curve.	Replace x^2 by xx_1 y^2 by yy_1 xy by $\frac{xy_1+yx_1}{2}$ x by $\frac{x+x_1}{2}$ y by $\frac{y+y_1}{2}$

Identification of a Conic from the General Equation

The general equation of a conic $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents;

(i). A pair of straight lines, if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$.

(ii). A circle if $h^2-ab<0$ & $a=b$.

(iii). An ellipse if $h^2-ab<0$ & $a\neq b$.

(iv). A parabola if $h^2-ab=0$.

(v). A hyperbola if $h^2-ab>0$.

Note:-

Centre of general conic is $\left(\frac{hf-bg}{ab-h^2},\frac{gh-af}{ab-h^2}\right)$

LOCUS

A locus is a set of points satisfying given condition(s).

2-Dimensional	3-Dimensional
1) The set of all points in a plane at a constant distance from a fixed point is a **circle**.	1) The set of all points in space at a constant distance from a fixed point is a **sphere**.
2) The set of all points in a plane at a constant distance from a given straight line is a pair of straight lines parallel to the given line.	2) The set of all points in space at a constant distance from a given straight line is a **cylinder**.
3) The set of all points in a plane such that the sum of their distances from two fixed points is a constant is an **ellipse**.	3) In space an ellipse is changed into an **ellipsoid.**
4) The set of all points in a plane such that the difference of their distances from two fixed points is a constant is a **hyperbola**.	4) In space hyperbola is shifted to a **hyperboloid**.
5) The set of all points such that their distances from a fixed point are equal to their distances from a fixed straight line is a **parabola**.	5) In space parabola is called **paraboloid.**

6) The locus of a point in a plane that is equidistant from two fixed points is the perpendicular bisector (mediator) of the straight line joining those two points. 7) The locus of a point in a plane equidistant from two intersecting straight lines is pair of straight lines bisects the angles b/w two given straight lines.	

Transformations

Translation of Axis	To transform the given equation referred to the new origin $O'(h,k)$ put $\begin{matrix} x = X + h \\ y = Y + k \end{matrix}$ in the given equation of the curve.
Rotation of Axis	To find the new equation when axis are rotated through an angle θ ,put $\begin{matrix} x = X\cos\theta - Y\sin\theta \\ y = X\sin\theta + Y\cos\theta \end{matrix}$ in the given equation.
Elimination of the xy-term	To remove the xy-term from a second degree equation ,determine the value of θ,such that $\tan 2\theta = \frac{2h}{a-b}$

if $a=b$ rotated
then $\theta = 45^\circ$ ↙

PRACTICE EXERCISE

Exercise A (Circle)

(1) Find the equation of tangent to the circle $x^2+y^2+4x+2y=0$ from the point P(-1,2).

(a) $x+y=0$ (b) $x+2y=0$

(c) $2x+y=0$ (d) $2y+x=0$

(2) The equation of the circle passing through the points (5,10),(6,9) and (-2,3) is ;

(a) $x^2+y^2+3x+2y=0$ (b) $x^2+y^2-4x-12y+15=0$.

(c) $x^2+y^2+4x+12y=0$ (d) $x^2+y^2-4x-3y-15=0$

(3) Which one is not the equation of circle

(a) $35x^2+35y^2-30x+17y-2=0$ (b) $17x^2+17y^2+2gx-2fy+17=0$

(c) $x^2+y^2=49$ (d) $x^2+2y^2-2xy+2fy=0$

(4) The circum center of right angled triangle is

(a) Any vertex (b) the vertex where two legs meet

(c) mid point of hypotenuse (d) at which angle bisectors meet

(5) Equation of a circle with center at origin and radius 3 is

(a) $x^2+y^2=9$ (b) $x^2+y^2=3$

(c) $x^2=y^2+3$ (d) $x^2-y^2=9$

(6) Coordinate of center of the circle of equation $x^2+y^2+2x+y+17=0$ is

(a) (1,2) (b) (3,4)

(c) $\left(-1,-\frac{1}{2}\right)$ (d) $\left(1,\frac{1}{2}\right)$

(7) Which is not the equation of circle which touches both the axes and is of radius 13;

(a) $(x+13)^2+(y+13)^2=169$ (b) $(x-13)^2+(y+13)^2=169$

(c) $(x+13)^2+(y-13)^2=169$ (d) None of these

(8) The mid point of the chord cut off from the line $x+y=3$ by the circle $x^2+y^2=9$ is;

(a) $\left(-\frac{3}{2},-\frac{3}{2}\right)$ (b) $\left(\frac{3}{2},-\frac{3}{2}\right)$

(c) $\left(-\frac{3}{2},\frac{3}{2}\right)$ (d) $\left(\frac{3}{2},\frac{3}{2}\right)$

(9) Which point lies inside the circle $2x^2+2y^2+3x+4y-1=0$?

(a) (-1, -2) (b) (2,1)

(c) (-3,1) (d) (1,-3)

(10) The length of the tangent segment from (2,-1) to the circle $x^2+y^2-5x-3y+27=0$ is

(a) 3 (b) 4

(c) 5 (d) 8

(11) Number of tangents drawn to a circle from a point outside the circle is;

(a) 1 (b) 2

(c) 3 (d) 4

(12) The equation of tangent to the circle $x^2+y^2-10x-24y = 0$ at the point (5,-1) is;

(a) $x+y = 0$ (b) $3x-5y+3 = 0$

(c) $9x-7y = -11$ (d) $y+1 = 0$

(13) Equation of tangent to the circle $x^2+y^2=a^2$ for all values of 'm' at point (x_1, y_1) is;

(a) $y = mx+c$ (b) $y^2 = a^2-x^2$

(c) $x^2+y^2-2ax-2by+a^2b^2-r^2 = 0$ (d) $y_1 = mx_1 + a\sqrt{1+m^2}$

(14) Angle between lines $x^2-y^2=0$ is;

(a) 0° (b) 180°

(c) 50° (d) 90°

(15) $3x^3-7xy^2+19y^3=0$ is an equation of;

(a) Degree 1 (b) degree 2

(c) degree 3 (d) degree 4

(16) If g^2+f^2-c is zero, the circle is;

(a) Infinite (b) imaginary

(c) single point (d) unit circle

(17) Radius of the circle $13x^2+13y^2-52x+39y=0$ is

(a) $\frac{125}{\sqrt{9}}$ (b) $\frac{15}{2}$

(c) $\frac{31}{3}$ (d) $\frac{5}{2}$

(18) The equation of circles with (2,3) and (3,4) as end points of diameter is;

(a) $x^2+y^2-9x+12x+12=0$ (b) $x^2+13x+y^2-7y+13=0$

(c) $x^2+y^2-5x-7y+18=0$ (d) $x^2+y^2=36$

(19) The equation of normal to $x^2+y^2=a^2$ at (x_1, y_1) is

(a) $y+y_1=\frac{x_1}{y_1}(x+x_1)$ (b) $y-y_1=\frac{y_1}{x_1}(x+x_1)$

(c) $y-y_1=\frac{y_1}{x_1}(x-x_1)$ (d) none of these

(20) The circle $x^2+y^2-2x-4y+1 = 0$ and $x^2+y^2+4x+4y-1 = 0$;.

(a) Touch internally (b) touch externally

(c) do not touch (d) none

(21) The coordinates of points of intersection of $x^2+y^2=25$ and $3x+y=5$ are;

(a) (5,0) (3,-4) (b) (1,2)(3,4)

(c) (-5,0),(-3,4) (d) (0,5),(3,-4)

(22) The line $3x+2y+K=0$ may touch the circle $x^2+y^2-2x+3y=0$ if K is;

(a) $\frac{13}{2}$ (b) $\frac{5\sqrt{13}}{2}$

(c) $\frac{2\sqrt{3}}{5}$ (d) $\frac{6\sqrt{19}}{3}$

(23) The value of eccentricity of the circle is;

(a) 0 (b) 1

(c) -1 (d) $\sqrt{2}$

(24) The number of constants in equation. $x^2+y^2+2gx+2cy+b=0$ is;

(a) 1 (b) 2

(c) 3 (d) 4

(25) The coordinates of the center and the radius of the circle whose equation is $x^2+y^2+2qcos\theta x+2qsin\theta y+q^2=0$ are respectively_____.

(a) $(qcos\theta, qsin\theta), q$ (b) $(-qcos\theta, -qsin\theta), 0$

(c) $(qcos^2\theta, qsin^2q), \sqrt{q}$ (d) $(-qcos^2\theta, -qsin^2\theta), 0$

(26) Coordinates of the center of circle for equation $2x^2+2y^2+2x-4y+7=0$ are;

(a) $(-2,4)$ (b) $(2,-4)$

(c) $(1,-2)$ (d) $\left(-\frac{1}{2},1\right)$

(27) The equation of circle whose center is at (3,-2) and which touches the line $3x+4y+14=0$ is;

(a) $x^2+y^2-6x+4y+4=0$ (b) $x^2+y^2-9x-7y+13=0$

(c) $x^2+y^2+17x-12y+3=0$ (d) $x^2+y^2+2x+17y+21=0$

(28) The equation of circle with (-3, 4) and (7,1) as end points of diameter is;

(a) $x^2+y^2-4x-13y=0$ (b) $x^2+y^2-2x+7y-3=0$

(c) $x^2+y^2-4x-5y-17=0$ (d) $x^2+y^2+3x+4y+13=0$

(29) The equation of the circle whose radius is $\sqrt{a^2+b^2}$ and center (a,b) is

(a) $x^2+y^2-2ax+2by=0$ (b) $x^2+y^2-2ax-2by=0$

(c) $x^2+y^2+2ax-2by=0$ (d) $x^2+y^2+2ax+2by-c=0$

(30) Length of tangent drawn from (5, 1) to the circle $x^2+y^2+6x-4y-3=0$ is

(a) 81 (b) 29

(c) 7 (d) 49

(31) If the line $x+2by+7=0$ is a diameter of the circle $x^2+y^2-6x+2y=0$, then b =?

(a) 3 (b) –5

(c) –4 (d) 5

(32) If S_1 and S_2 are the equations of circles $x^2+y^2+4x+1=0$ and $x^2+y^2+6x+2y+3=0$, then equation of common chord is;

(a) $x+y+1=0$ (b) $x+y-1=0$

(c) $2x+2y+5=0$ (d) $x+y-3=0$

(33) The length of the chord intercepted by the circle $x^2+y^2+2x-4y-1=0$ on the line $x-y+1=0$ is;

(a) 4 (b) 8

(c) 2 (d) 1

(34) If (x, 3) and (3, 5) are ends of a diameter of a circle and center is (2, y), then the value of (x, y) is;

(a) (1, 4) (b) (4, 1)

(c) (8, 2) (d) (2, 8)

(35) The equation of circle which touches y-axis at (0, 3) and cuts 8 intercept on x-axis, is;

(a) $x^2+y^2+10x+6y+9=0$ (b) $x^2+y^2+6x-10y+9=0$

(c) $x^2+y^2+8x+4y+2=0$ (d) $x^2+y^2-10x-6y+9=0$

(36) If one end of the diameter of the circle $2x^2+2y^2-8x-4y=2=0$ is (2, 3) then other end is _____

(a) (-2, -3) (b) (4, -2)

(c) (2, -1) (d) (3, 2)

(37) Which of the following lines is a normal to the circle $(x-1)^2+(y-2)^2=10$?

(a) $x+y=3$ (b) $(x-1)+(y-2)=10$

(c) $x+2y=0$ (d) $2x+y=3$

(38) If the line $\frac{x}{a}+\frac{y}{b}=1$ touches the circle $x^2+y^2=r^2$ then _____

(a) $a^2+b^2=r^2$ (b) $a^2-b^2=r^2$

(c) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{r^2}$ (d) $\frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{r^2}$

(39) Circle passes through (a, 0), (0, b) and (0, 0) then its centre is

(a) $\left(\frac{a}{2},\frac{b}{2}\right)$ (b) $\left(\frac{b}{2},\frac{a}{2}\right)$

(c) (a. b) (d) (b, a)

(40) The equation of the chord of the circle whose mid-point is (1, -2) is _____

(a) $2x+y=5$ (b) $2x-y=5$

(c) $x+2y=5$ (d) $x-2y=5$

(41) Centre of the circle $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$ is _____

(a) (x_1, y_1) (b) (x_2, y_2)

(c) $\left(\frac{x_1+y_1}{2}\ \frac{x_2+y_2}{2}\right)$ (d) $\left(\frac{x_1+x_2}{2}\ \frac{y_1+y_2}{2}\right)$

(42) $x^2+y^2=r^2$ and $x^2+y^2-10x+16=0$ intersect in two points then

(a) $2<r<8$ (b) $r=2$ or $r=8$

(c) $r<2$ or $r>8$ (d) $r>2$ and $r>8$

(43) The circles $x^2+y^2-2x-4y=0$ and $x^2+y^2-8y-4=0$

(a) Touch internally (b) Touch externally

(c) Intersect (d) Do not intersect

(44) Equation of normal to the circle $x^2 + y^2 = a^2$ at point (x_1, y_1) is _____

(a) $xx_1 + yy_1 = a^2$ (b) $xy_1 - yx_1 = a^2$

(c) $xy_1 - yx_1 = 0$ (d) $xx_1 + yy_1 = 0$

(45) Area of a circle whose centre is $(\cos\theta, \sin\theta)$ and passes through the origin is _____

(a) π (b) 1

(c) πr^2 (d) 2π

(46) The equation of chord of contact of circle $x^2 + y^2 = 21$ when tangent are drawn from (-3, 4), is

(a) $-3x + 4y = 21$ (b) $x + 6y = 21$

(c) $3x - 4y = 21$ (d) $x - 6y = 21$

(47) Distance of chord cut by line $2x + 3y = 13$ from center of circle $x^2 + y^2 = 26$ is

(a) $\sqrt{26}$ (b) 13

(c) $\sqrt{13}$ (d) Can not be determined

(48) Angle subtended in the major arc of circle $x^2 + y^2 = 18$, whose end points are (3, 3), (-3, 3)

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

(49) For what value of m the circle $3x^2 + 3y^2 + 60x + 180y + 75m = 0$ become a point circle

(a) 20 (b) 75

(c) 40 (d) 3

(50) The equation of circle of circumference πa and tangent to both axes in 3rd quadrant is

(a) $(x+a)^2 + (y+a)^2 = a^2$ (b) $(x-a)^2 + (y-a)^2 = a^2$

(c) $(2x+a)^2 + (2y+a)^2 = a^2$ (d) $(2x+a)^2 + (2y+a)^2 = \frac{a^2}{4}$

Exercise B (Conics)

(1) The focus of the parabola $x^2 = 6ay$ is

(a) (0,a) (b) (0,6a)

(c) $\left(0, \frac{2}{3}a\right)$ (d) none

(2) The standard equation of an ellipse with major-axis along x-axis and center at (h,k) is

(a) $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ (b) $\frac{(x-k)^2}{b^2} + \frac{(y-h)^2}{a^2} = 1$

(c) $\frac{(x-k)^2}{b^2} + \frac{(y-h)^2}{a^2} = 0$ (d) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

(3) Foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are

(a) $(\pm c,0)$ (b) (a,b)
(c) $(0,\pm c)$ (d) none

(4) Equations of directrices of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ are;

(a) $x=\pm\frac{a}{e}$ (b) $x=\pm a$
(c) $y=\pm\frac{a}{e}$ (d) $x=\pm\frac{25}{3}$

(5) Equations of directrices of ellipse $\frac{x^2}{3}+\frac{y^2}{2}=1$ are;

(a) $x=\pm\frac{3}{e}$ (b) $x=\pm\frac{e}{3}$
(c) $x=\pm\frac{3}{2}$ (d) none

(6) The coordinates of foci of $a^2x^2+b^2y^2=a^2b^2$ are;

(a) (0,c) (b) $(0,\pm c)$
(c) $\left(0,\pm\frac{a}{e}\right)$ (d) none

(7) The eccentricity of $2x^2+5y^2=1$ is

(a) $\frac{1}{\sqrt{16}}$ (b) $\sqrt{\frac{3}{5}}$
(c) $\frac{2}{5}$ (d) $\frac{3}{5}$

(8) The equation of ellipse with centre at (2,3), length of major axis = 3 and length of minor axis = 2 is;

(a) $\frac{x^2}{9}+\frac{y^2}{4}=1$ (b) $\frac{x^2}{4}+\frac{y^2}{9}=1$
(c) $\frac{x^2}{4}-\frac{y^2}{9}=1$ (d) none

(9) The coordinates of foci of $-\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ are;

(a) $(0,\pm c)$ (b) $(\pm c,0)$
(c) $(\pm b,0)$ (d) none

(10) The eccentricity of $\frac{x^2}{25}-\frac{y^2}{16}=1$ is;

(a) $\frac{\sqrt{41}}{5}$ (b) $\frac{\sqrt{13}}{2}$
(c) $\frac{2}{\sqrt{13}}$ (d) none

(11) The equations of directrices of $-\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ are;

(a) $y=\pm\frac{a}{e}$ (b) $y=\pm\frac{e}{a}$

(c) $x=\pm\frac{a}{e}$ (d) $x=\pm\frac{e}{a}$

(12) The parametric equations of circle $x^2+y^2=a^2$ are;

(a) $x=a\cos\theta, y=b\sin\theta$ (b) $x=at^2, y=2at$

(c) $x=a\sec t, y=b\tan t$ (d) none

(13) The parametric equations of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are;

(a) $x=a\sec t, y=b\tan t$ (b) $x=at^2, y=2at$

(c) $x=a\cos\theta, y=b\sin\theta$ (d) none

(14) The equation of tangent at (x_1,y_1) to the parabola $y^2=4ax$ is;

(a) $yy_1=2a(x+x_1)$ (b) $y_1=4ax_1$

(c) $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ (d) none

(15) The equation of normal to $y^2=4ax$ at (x_1,y_1) is;

(a) $y-y_1=\frac{a^2y_1}{b^2x_1}(x-x_1)$ (b) $y-y_1=-\frac{y_1}{2a}(x-x_1)$

(c) $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ (d) none

(16) In ellipse the relation between a^2, b^2, c^2 is;

(a) $c^2=a^2-b^2$ (b) $a^2=b^2-c^2$

(c) $c^2=a^2+b^2$ (d) none

(17) In hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ The ends of latera recta are;

(a) $\left(\pm ae,\pm\frac{a}{e}\right)$ (b) $\left(\pm ae,\pm\frac{b^2}{a}\right)$

(c) $\left(\pm\frac{2b^2}{a},0\right)$ (d) $\left(\pm\frac{2b^2}{a},\pm c\right)$

(18) In hyperbola the relation between a^2,b^2,c^2 is

(a) $c^2=a^2-b^2$ (b) $c^2+a^2=b^2$

(c) $c^2=a^2+b^2$ (d) none

(19) Length of latus rectum of parabola $y^2-8y-4x=0$ is

(a) 2 (b) 4

(c) 8 (d) 16

(20) If (0,0) is vertex and (0,-5) is focus, then equation of parabola is

(a) $y^2=-5x$ (b) $y^2=5x$

(c) $x^2=-20y$ (d) none

(21) For parabola $y^2=4y-4x$, the vertex is

(a) (1,2) (b) (2,4)

(c) 4,0) (d) (2,0)

(22) Equation of ellipse with length of major axis = 4, length of minor axis = 3 is

(a) $\frac{x^2}{4}+\frac{y^2}{3}=1$ (b) $\frac{x^2}{3}+\frac{y^2}{4}=1$

(c) $\frac{x^2}{\sqrt{3}}+\frac{y^2}{\sqrt{4}}=1$ (d) none

(23) Equation of hyperbola with centre at (h,k) is;

(a) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (b) $\frac{(x-k)^2}{a^2}-\frac{(y-h)^2}{b^2}=1$

(c) $\frac{(x-k)^2}{a^2}+\frac{(y-h)^2}{b^2}=1$ (d) none

(24) The coordinates of the point on parabola $y^2=8x$ whose distance from focus is 4, are;

(a) (2,4) (b) (4,2)

(c) (–2,-4) (d) (4,-2)

(25) The length of latus rectum is one third of length of major axis of an ellipse then its eccentricity is;

(a) $\frac{2}{3}$ (b) $\sqrt{\frac{2}{3}}$

(c) $\frac{1}{3}$ (d) $\frac{3}{4}$

(26) The graph of parabola $x=4y^2$;

(a) opens left (b) opens right

(c) opens up (d) opens down

(27) The gradient of the normal of the parabola $y^2=4ax$ at $(at_1^2, 2at_1)$ is

(a) $\frac{1}{t^1}$ (b) t_1

(c) $-t_1$ (d) $-\frac{1}{t^1}$

(28) If the line $y=4x+c$ is tangent to the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$, then c=?

(a) ± 4 (b) ± 6

(c) ± 1 (d) $\pm\sqrt{132}$

(29) The equation of the chord of hyperbola $25x^2-16y^2=400$ which is bisected at the point (6,2) is;

(a) $16x-75y=418$ (b) $75x-16y=418$

(c) $25x-4y=400$ (d) none

(30) Line $y = mx+1$ is tangent to the parabola $y^2=4x$ if;

(a) $m=1$ (b) $m=2$

(c) $m=4$ (d) $m=3$

(31) $x=3(cost+sint)$, $y=4(cost-sint)$ represents the curve.
(a) an ellipse (b) a parabola
(c) a hyperbola (d) a circle

(32) If $x+2y=1$, (2,1) and 2 are the directrix, focus and eccentricity of the hyperbola, then its equation is
(a) $x^2-16xy-11y^2-12x+6y+21=0$ (b) $3x^2+16xy+15y^2+4x-14y-1=0$
(c) $x^2+16xy+11y^2+12x+6y+21=0$ (d) none

(33) The foci of the ellipse $25(x+1)^2+9(y+2)^2=225$ are;
(a) (-1,2)&(-1,-6) (b) (-2,1)&(-2,-6)
(c) (-1,-2)&(-2,-1) (d) (-1,-2)&(-1,-16)

(34) If $2x+y+\lambda=0$ is normal to the parabola $y^2=-8x$, λ =?
(a) 12 (b) –12
(c) 24 (d) -24

(35) If e is the eccentricity of $\frac{x^2}{8}+\frac{y^2}{4}=1$,then ;
(a) $0<e<1$ (b) $e>1$
(c) $e=1$ (d) none

(36) Which of the following is true for the equation $x^2-4x-3y+13=0$
(a) centre (2,3) (b) focus (2,3)
(c) vertex (2,3) (d) focus (0,0)

(37) The point which is closest to the focus on the parabola is
(a) centre (b) vertex
(c) end point (d) any point

(38) The point of intersection of $3x^2+5y^2=60$ and $9x^2+y^2=124$ is
(a) (2,-2) (b) (40, 2)
(c) (2, -2) (d) $\left(\sqrt{\frac{40}{3}},2\right)$

(39) The distance between the foci of the conic $9x^2-18x+4y^2+8y-23=0$ is;
(a) $\sqrt{\frac{5}{3}}$ (b) $\sqrt{5}$
(c) $2\sqrt{5}$ (d) none of these

(40) If the equation $x^2+6x-8y+17=0$ is transformed, with the reference to (-3,1) as origin and axes remaining parallel to the old axes then the new equation is ;
(a) $x^2-8y=0$ (b) $y^2-8x=0$
(c) $8x^2-y=0$ (d) $8x^2-3y=0$

(41) When the xy-axes are rotated at 45° then the equation $y=\frac{1}{x}$ becomes;
(a) $y=\frac{1}{x}$ (b) $x^2=y$
(c) $x^2=y^2$ (d) $x^2-y^2=2$

(42) The equation of tangent to $x^2+5xy-4y^2+4=0$ *at* $y=-1$ is;
(a) $5x+8y+8=0$ (b) $5x-8y+8=0$
(c) $5x-8y-8=0$ (d) none of these

(43) The equation $2x^2-xy+5x-2y+2=0$ represents _____
(a) circle (b) parabola
(c) ellipse (d) intersecting lines

(44) If $(x-4)^2 + 4(y-3)^2 = 16$ is graphed, the sum of the distance from any point on the curve to the two foci is;
(a) 4 (b) 8
(c) 12 (d) 16

(45) The equation $x^2-y^2+4x+8y-11=0$ with respect to new parallel axes obtained by shifting the origin to (-2, 4) is;
(a) $x^2+y^2=1$ (b) $x^2+y^2+1=0$
(c) $x^2-y^2-1=0$ (d) $x^2-y^2+1=0$

(46) The point on the parabola $y^2 = 8x$ whose distance from the focus is 8, has x – coordinate, is _____
(a) 0 (b) 2
(c) 4 (d) 6

(47) Length of latus rectum of parabola $x^2 - 16y - 64 = 0$ is _____
(a) 4 (b) -16
(c) 16 (d) -4

(48) Length of latus rectum of the parabola $y^2 = ax$ is _____
(a) a (b) 4a
(c) $\frac{a}{4}$ (d) $\frac{a}{2}$

(49) Eccentricity of the conic $x^2 + y^2 - 2xy + 3x + 2y + 2 = 0$ is _____
(a) 1 (b) $\frac{1}{3}$
(c) 3 (d) 0

(50) The locus of the points which are equidistant from (-a, 0) and $x = a$ is
(a) $y^2 = 4ax$ (b) $y^2 = -4ax$
(c) $x^2 + 4ay = 0$ (d) $x^2 - 4ay = 0$

(51) Distance between foci of the ellipse $9x^2 + 16y^2 = 144$ is _____
(a) 10 (b) 16
(c) 9 (d) $2\sqrt{7}$

(52) Eccentricity of the ellipse $3x^2 + 4y^2 = 24$ is _____
(a) $\frac{1}{4}$ (b) $\frac{7}{4}$
(c) $\frac{1}{2}$ (d) $\sqrt{\frac{7}{4}}$

(53) $x^2 - 7xy + 12y^2 = 0$ represents
(a) Circle (b) Pair of parallel straight lines
(c) Pair of $\perp$ st. lines (d) Pair of st. lines

(54) $\sqrt{(x-2)^2+y^2} + \sqrt{(x+2)^2+y^2} = 8$ represents _____
(a) Circle (b) Ellipse
(c) Parabola (d) Pair of lines

$12x^2+24x+4y^2-16y+25=0$

(55) For the ellipse $12x^2 + 4y^2 + 24x - 16y + 25 = 0$

(a) Centre (-1, 2)

(b) Length of major axis = $\sqrt{3}$

(c) Eccentricity $\sqrt{\frac{2}{3}}$

(d) All of these

$12(x^2+2x+1-1)+4(y^2-4y+2-2)+25=0$

$12(x+1)^2-12+4(y-2)^2-8+25=0$

$12(x+1)^2+4(y-2)^2+5=0$

$12(x+1)^2+4(y-2)^2$

(56) The eccentricity of the ellipse if the length of minor axis is equal to the distance between the foci is _____

(a) $\frac{\sqrt{3}}{2}$

(b) $\frac{2}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{2}}$ $\frac{c}{a}$ then verify that by $c^2=a^2-b^2$ $(c=b)$

(d) $\sqrt{2}$

(57) $x\cos\alpha + y\sin\alpha = P$ is tangent to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then _____

(a) $a^2\cos^2\alpha + b^2\sin^2\alpha = P^2$

(b) $a^2\cos^2\alpha + b^2\sin^2\alpha = 2P^2$

(c) $a^2\sin^2\alpha + b^2\cos^2\alpha = P^2$

(d) None of these

(58) The line $2x+\sqrt{6}y=2$ is tangent to $x^2 - 2y^2 = 4$ at the point _____

(a) $\left(\sqrt{6},1\right)$

(b) $\left(7,-2\sqrt{6}\right)$

(c) (2, 3)

(d) $(4, -\sqrt{6})$

(59) Eccentricity of the hyperbola whose length of latus rectum is half of its length of transverse axis is _____

$e=?$

$e=\frac{c}{a}$

$=\frac{\sqrt{a^2+b^2}}{a}$

(a) $\frac{1}{\sqrt{2}}$

(b) $\sqrt{\frac{2}{3}}$

$\frac{2b^2}{a}=\frac{2a}{2}$

$2b^2=a^2$

$b^2=\frac{a^2}{2}$

$=\frac{\sqrt{a^2+\frac{a^2}{4}}}{a}$

$=\frac{\sqrt{\frac{5a^2}{4}}}{a}$

$\frac{\sqrt{5a^2}}{2a}$

$e=\frac{\sqrt{3}}{\sqrt{2}}$

(c) $\sqrt{\frac{3}{2}}$

(d) None of these

(60) The angle between the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is _____

$c^2=a^2+b^2$

$b^2=c^2-a^2$

$=3-2$

$=1$

$1=2$

(a) $\cos^{-1}\left(\frac{1}{e}\right)$

(b) $2\cos^{-1}\left(\frac{1}{e}\right)$

(c) $\sin^{-1}\left(\frac{1}{e}\right)$

(d) $\tan^{-1}\left(\frac{1}{e}\right)$

eqv =

$y=\pm\frac{b}{a}$

$\tan 2\theta=\frac{2h}{a-b}$

VECTORS

As we know, in **mathematics, physics** and **engineering** two important quantities are **scalars and vectors.**

Physical Quantity	Description	Examples
Scalar Quantity /Scalar	The physical quantities, which have magnitude, only are called scalars.	Distance, speed, mass, length, time, volume, temperature and energy
Vector Quantity/Vector	The physical quantities that have magnitude and direction are called vectors.	Displacement, velocity, acceleration, weight,force, momentum,electric and magnetic fields

Geometrical Interpretation of Vector

(a) Geometrically, a vector is represented by a directed line segment $\overrightarrow{AB}$ with A as its initial point and B as its terminal point.

(b) Usually a vector is denoted by an arrow written as $\overrightarrow{AB}$.

(c) A vector can also be represented by a bold face symbol like $\boldsymbol{v}$ or in underline form as $\underline{v}$.

Some Important Terms Related To Vectors

Magnitude/Length/Norm	The magnitude/length/norm of a vector $\overrightarrow{AB}$ or $\underline{v}$,is its absolute value and is denoted as $\lvert\overrightarrow{AB}\rvert$ or AB or $\lvert\underline{v}\rvert$.
Support	The line of infinite length of which the vector is a part, is called its support.
Sense	The sense (direction) of a vector is from its initial point to its terminal point.

Some Types of Vectors

Name	Definition	Representation
Zero Vector/Null Vector	A vector whose initial and terminal points coincide.	A zero vector is represented by $\vec{o}$.
Proper Vector	A non zero vector is called proper vector.	$\vec{a}$ is proper vector iff $\lvert\vec{a}\rvert \neq 0$.
Unit Vector	A vector whose magnitude is 1 is called unit vector.	A unit vector in the direction of $\vec{a}$ is denoted by $\hat{a}$ and is given as $\hat{a} = \frac{\vec{a}}{\lvert\vec{a}\rvert}$.
Fundamental unit vectors	The unit vectors along x-, y- and z-axes are called fundamental unit vectors.	$\underline{i} = [1,0,0]$, $\underline{j} = [0,1,0]$ & $\underline{k} = [0,0,1]$ are unit vectors along x-, y- and z-axes respectively.
Co-Initial Vectors	Two (more than two) vectors are called co-initial if they have the same initial point.	$\overrightarrow{AC}$, $\overrightarrow{AB}$, $\overrightarrow{AD}$ are co-initial vectors.
Negative of A Vector	The vector which has the same magnitude as the vector $\vec{a}$ but has the direction opposite to that of $\vec{a}$.	The negative of $\vec{a}$ is denoted by $-\vec{a}$.

Position Vector	The vector $\overrightarrow{OP}$, whose initial point O is the origin and whose terminal point is P, is called position vector of point P.
Co-Planar Vectors	Two or more vectors are called co-planar if either they lie in the same plane or parallel to the same plane.
Equal Vectors	Two vectors are said to be equal if they have, (i).The same length (ii).Same or parallel support (iii).The same sense
Collinear or Parallel Vectors	Two or more vectors are said to be collinear if they have either the same or parallel supports.
Like And Unlike Vectors	Two or more vectors are said to be **like vectors** when they have the same direction irrespective of their magnitudes and vectors having opposite directions are called **unlike vectors**.
Reciprocal Vectors	Two vectors are reciprocal of each other if they have same direction but their magnitudes are reciprocals of each other.
Free Vector	A vector, in which the initial point is not specified *i.e.* there is no restriction to choose its origin, is called a free vector.
Localized Vector	A vector with a fixed initial point is called a localized vector.

Addition of Vectors

Addition of vectors can be explained by the following laws;

Triangle Law of Vectors	If two vectors are represented in magnitude and direction by the two sides of a triangle, taken in order then their sum (resultant) is represented in magnitude and direction by the third side of the triangle taken in opposite order.
Parallelogram* Law of Vectors	If two vectors are represented in magnitude and direction by two adjacent sides of a parallelogram, then their sum (resultant) is represented in magnitude and direction by the diagonal of the parallelogram with initial point of intersection of the two sides.
Polygon Law of Vectors	If a finite number of vectors are represented in magnitude and direction by the sides of an open polygon, taken in order, then their resultant is represented in magnitude and direction by the closing side of polygon, taken in the opposite order.

Multiplication of a Vector by a Scalar

Let $\vec{a}$ be a vector and k a real number.

i). If k is a positive real number, then $k\vec{a}$ is defined to be a vector whose magnitude is k time that of $\vec{a}$ and direction is the same as that of $\vec{a}$.

ii). If k is a negative real number, then $k\vec{a}$ is defined to be a vector whose magnitude is k time the magnitude of $\vec{a}$ and direction opposite to that of $\vec{a}$.

iii). If $k = 0$, then $k\vec{a}$ is zero vector.

Properties of Vector Addition

Commutative	$\vec{a}+\vec{b}=\vec{b}+\vec{a}$
Associative	$\vec{a}+\left(\vec{b}+\vec{c}\right)=\left(\vec{a}+\vec{b}\right)+\vec{c}$
Difference of Two Vectors	$\vec{a}-\vec{b}=\vec{a}+\left(-\vec{b}\right)$
Distributive Laws	(a) If k is any real number and $\vec{a},\vec{b}$ are any two vectors, then $k\left(\vec{a}+\vec{b}\right)=k\vec{a}+k\vec{b}$. (b) If $\vec{a}$ is any vector and m, n are real numbers, then $(m+n)\vec{a}=m\vec{a}+n\vec{a}$ (c) $m\left(n\vec{a}\right)=n\left(m\vec{a}\right)$

Vector in Plane

- The set of all ordered pairs [x, y] of real numbers is called set of vectors in R^2.
- x and y are components of vector.

* Aristotle used this law to describe the combined action of two forces.

- $\underline{U} = [x, y] = x\underline{\hat{i}} + y\underline{i}$
- $|\underline{U}| = \sqrt{x^2 + y^2}$

Vector in Space

- The set of all ordered triples [x, y, z] of real numbers is called set of vectors in R^3.
- u, y and z are component of vector.
- $\underline{V} = [x, y, z] = x\underline{i} + y\underline{i} + z\underline{k}$
- $|\underline{V}| = \sqrt{x^2 + y^2 + z^2}$

Distance between two points in space

Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in space is given by

$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Direction Ratios, Direction Angles and Direction Cosines of a Vector:

Let $\underline{r} = \overrightarrow{OP} = x\underline{i} + y\underline{j} + z\underline{k}$ be a non-zero vector, then any triplet of numbers proportional to x, y and z are called **direction ratios** of $\underline{r}$.

Let α, β, γ are angles formed by $\underline{r} = \overrightarrow{OP} = x\underline{i} + y\underline{j} + z\underline{k}$ with x-,y- and z-axes respectively such that $0 \le \alpha \le \pi, 0 \le \beta \le \pi$ & $0 \le \gamma \le \pi$, are called direction angles of $\underline{r}$.

The numbers $\cos\alpha, \cos\beta$ and $\cos\gamma$ are called direction cosines of the vector $\underline{r}$. $\cos\alpha, \cos\beta$ and $\cos\gamma$ are also denoted by l, m and n respectively.

Note that;

(i). $\cos\alpha = \dfrac{x}{|\underline{r}|}, \cos\beta = \dfrac{y}{|\underline{r}|}$ and $\cos\gamma = \dfrac{z}{|\underline{r}|}$.

(ii). $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ i.e. $l^2 + m^2 + n^2 = 1$

The ratio formula

(i) Let A & B be two points whose position vectors are $\underline{a}$ and $\underline{b}$ respectively. If a point P divides AB in ratio $p:q$, then the position vector $\underline{r}$ of P is given as;

$$\underline{r} = \frac{q\underline{a} + p\underline{b}}{p + q}.$$

(ii) Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be any two points in space.

The coordinate of point C which divides $\overrightarrow{AB}$ in the ration $m_1 : m_2$ are $\left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}, \dfrac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$

(iii) If $\frac{m_1}{m_2}$ is negative then the points C divides $\overrightarrow{AB}$ externally.

Centroid

Point of intersection of all three medians in a triangle is called centroid.

If a triangle having three vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ then centroid is given by

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

Scalar Product of Two Vectors

Scalar Product/Dot Product/Inner Product	*For any two vectors* $\vec{a}$ *and* $\vec{b}$ $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta$, *where* θ *is the angle between the vectors* $\vec{a}$ *and* $\vec{b}$.
Scalar Product in Terms of Components	if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ & $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
Special Cases	(i). $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$ (ii). $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$ iii). If $\vec{a}.\vec{b} = 0$, then either $\lvert\vec{a}\rvert = 0$ or $\lvert\vec{b}\rvert = 0$ or $\theta = \frac{\pi}{2}$

Properties of Scalar Product

Orthogonal Vectors	Two vectors $\vec{a}$ & $\vec{b}$ are called orthogonal iff their dot product is zero.*i.e.* $\vec{a}.\vec{b} = 0$ Iff $\vec{a} \perp \vec{b}$.
Parallel vectors	If two vectors $\vec{a}$ & $\vec{b}$ are parallel then $\theta = 0$, thus $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert$.
Cosine of the Angle Between Vectors	The angle between the vectors $\vec{a}$ & $\vec{b}$ can be determined by using the definition of the scalar product; $\cos\theta = \frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}, 0 \le \theta \le \pi$

Commutative Property	Scalar product is commutative *i.e.* $\vec{a}.\vec{b} = \vec{b}.\vec{a}$
Associative Property	The scalar product is associative with respect to a scalar k. $\left(k\vec{a}\right).\vec{b} = k\left(\vec{a}.\vec{b}\right) = \vec{a}.\left(k\vec{b}\right)$
Distributive Property	Scalar product is distributive over addition, *i.e.* $\vec{a}.\left(\vec{b}+\vec{c}\right) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$

Applications of Scalar Product

Projection of one Vector upon another Vector	Let $\vec{a}$ & $\vec{b}$ are any two vectors then, (i).The projection of $\vec{a}$ along $\vec{b}$ $= \vec{a}.\hat{b} = \dfrac{\vec{a}.\vec{b}}{\left\|\underline{b}\right\|}$ (ii).The projection of $\vec{b} = \dfrac{\vec{b}.\vec{a}}{\left\|\underline{a}\right\|}$ along $\vec{a}$ $= \vec{b}.\hat{a}$
Angle between two vectors	If $\vec{a} = x_1\underline{i} + y_1\underline{j} + z_1\underline{k}$ and $\vec{b} = x_2\underline{i} + y_2\underline{j} + z_2\underline{k}$ then $\cos\theta = \dfrac{x_1x_2 + y_1y_2 + z_1z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2}\sqrt{x_2^2 + y_2^2 + z_2^2}}$
Work done by a force	The work done by a force $\vec{F}$ when its points of application undergoes a displacement $\vec{d}$ is given by; $w = \vec{F}.\vec{d}$

Vector Product of Two Vectors

Vector Product/Cross Product	The vector or cross product of two vectors $\vec{a}$ & $\vec{b}$ is defined as, $\vec{a}\times\vec{b} = \left\|\vec{a}\right\|\left\|\vec{b}\right\|\sin\theta\hat{n}$,where θ is the angle between $\vec{a}$ & $\vec{b}$ and $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ & $\vec{b}$
Vector Product In Terms of Components	if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ & $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
Special Cases	(i).If $\vec{a}\times\vec{b} = \vec{0}$ then either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ or $\vec{a} \parallel \vec{b}$ (ii). $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$ iii). $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$. iv). $\hat{j}\times\hat{i} = -\hat{k}$, $\hat{k}\times\hat{j} = -\hat{i}$, $\hat{i}\times\hat{k} = -\hat{j}$

Properties of Vector Product

Parallel vectors	If two vectors $\vec{a}$ & $\vec{b}$ are parallel then $\theta=\pi$ or $\theta=0$, thus $\vec{a}\times\vec{b}=0$.
Sine of the Angle Between Vectors	The angle between the vectors $\vec{a}$ & $\vec{b}$ can be determined by using the definition of the vector product; $\sin\theta=\frac{\left\|\vec{a}\times\vec{b}\right\|}{\left\|\vec{a}\right\|\left\|\vec{b}\right\|}, 0\le\theta\le\pi$
Commutative Property	Cross product is not commutative *i.e.* $\vec{a}\times\vec{b}\neq\vec{b}\times\vec{a}$ but $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$
Associative Property	The cross product is associative with respect to scalar *m*. $m\left(\vec{a}\times\vec{b}\right)=\left(m\vec{a}\right)\times\vec{b}=\vec{a}\times\left(m\vec{b}\right)=\left(\vec{a}\times\vec{b}\right)m$
Distributive Property	Vector product is distributive with respect to vector addition. (i). $\vec{a}\times\left(\vec{b}+\vec{c}\right)=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$ (ii). $\left(\vec{b}\times\vec{c}\right)+\vec{a}=\vec{b}\times\vec{a}+\vec{c}\times\vec{a}$

Applications of Vector Product

Geometrical Meaning	Geometrically $\left\|\vec{a}\times\vec{b}\right\|$ represents the area of a parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ & $\vec{b}$
Unit vector. $\perp$ to both $\vec{a}$ & $\vec{b}$	$\frac{\vec{a}\times\vec{b}}{\left\|\vec{a}\times\vec{b}\right\|}$ gives a unit vector $\perp$ to both $\vec{a}$ & $\vec{b}$
Area of a Triangle	The area of triangle having two sides $\vec{a}$ & $\vec{b}$ is equal to $\frac{1}{2}\left\|\vec{a}\times\vec{b}\right\|$.
Moment of a Force	Moment of a force $\vec{F}$ acting at a point A about any other point B is given by $\overrightarrow{BA}\times\vec{F}$.

Scalar Triple Product

Scalar triple product/box product	Let $\vec{a},\vec{b},\vec{c}$ be three vectors, then $\vec{a}.\vec{b}\times\vec{c}$ is called scalar triple product. It is denoted by $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$.
Scalar Triple Product in Terms of Components	If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ & $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$, then $\vec{a}.\vec{b}\times\vec{c}=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$.

Applications of scalar triple product

Geometrical meaning	Geometrically $\vec{a}.\vec{b}\times\vec{c}$ represents the volume of parallelopiped having $\vec{a},\vec{b},\vec{c}$ as its adjacent sides.
Volume of a Tetrahedron	If $\vec{a},\vec{b},\vec{c}$ are the coterminous edges of a tetrahedron then , volume of tetrahedron $= \frac{1}{6}\left\|\vec{a}.\vec{b}\times\vec{c}\right\|$
Coplanar vectors	If $\vec{a},\vec{b},\vec{c}$ are coplanar then $\vec{a}.\vec{b}\times\vec{c}=0$.

Properties of scalar triple product

(i).in scalar triple product the dot and cross product are interchange able.*i.e* $\vec{a}.\vec{b}\times\vec{c}=\vec{a}\times\vec{b}.\vec{c}=\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ Also $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\left[\vec{b}\ \vec{c}\ \vec{a}\right]=\left[\vec{c}\ \vec{a}\ \vec{b}\right]$ $\Rightarrow \vec{a}.\left(\vec{b}\times\vec{c}\right)+\vec{b}.\left(\vec{c}\times\vec{a}\right)+\vec{c}.\left(\vec{a}\times\vec{b}\right)=3\vec{a}.\left(\vec{b}\times\vec{c}\right)$
(ii).If any two vectors are same in scalar triple product then the product is zero. $\vec{a}.\left(\vec{b}\times\vec{a}\right)=0$
(iii).For the fundamental unit vectors $\hat{i}.\hat{j}\times\hat{k}=1,\hat{i}.\hat{j}\times\hat{i}=0$.

Vector triple product*

Vector triple product	$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\left(\vec{A}.\vec{C}\right)\vec{B}-\left(\vec{A}.\vec{B}\right)\vec{C}$

* Only definition is included, further discussion is out of syllabus.

PRACTICE EXERCISE

(1) [Secα, cosecα, 0] is a
(a) Null vector (b) Negative vector
(c) Unit vector (d) Position vector

(2) Which is not a vector?
(a) Force (b) displacement
(c) electric field intensity (d) weight
(e) density

(3) Which is not a scalar?
(a) Kinetic energy (b) torque
(c) entropy (d) work
(e) none

(4) If P(0,1,2), Q(-1,2,1) and R(4,1,-2) are three points then, **PQ+QP+PR+RP**=?
(a) 15 (b) 2
(c) 0 (d) 17

(5) **a**. **b** = 0 iff;
(a) **a** and **b** are parallel (b) **a** and **b** are anti parallel
(c) **a** and **b** are perpendicular to each other (d) can't be determined

(6) If $\mathbf{a} = a_1\underline{i}+a_2\underline{j}+a_3\underline{k}$ and $\mathbf{b}=b_1\underline{i}+b_2\underline{j}+b_3\underline{k}$ then **a. b** = ?
(a) $(a_1b_3-a_3b_2)\underline{i}+(a_3b_1-a_1b_3)\underline{j}+(a_1b_2-a_2b_1)\underline{k}$ (b) $a_1a_2+b_1b_2+c_1c_2$
(c) $a_2b_3-a_3b_2+a_3b_1-a_1b_3$ (d) $a_1b_1+a_2b_2+a_3b_3$

(7) $|\mathbf{a} \times \mathbf{b}| = ?$
(a) Area of a triangular region with **a** and **b** as its adjacent sides
(b) area of a parallelogram with **a** and **b** as its adjacent sides
(c) area of a trapezium with **a** and **b** as its adjacent sides
(d) area of tetrahedron with **a** and **b** as its adjacent sides

(8) Which one is false?
(a) $\text{Sin}(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta$
(b) $\frac{\sin A}{a} = \frac{\sin B}{b}$
(c) $\Delta DEF = 4\Delta ABC$ if D, E, F are the mid points of sides of ΔABC
(d) $\underline{a}.\underline{b} \times \underline{c} = \underline{b}.\underline{c} \times \underline{a} = \underline{c}.\underline{a} \times \underline{b}$

(9) $\underline{i}.\underline{j} \times \underline{k} = ?$
(a) $\underline{i}$ (b) $\underline{j}$
(c) $\underline{k}$ (d) 1

(10) If $\underline{a} = 2\underline{i}+4\underline{j}-\underline{k}$, $\underline{b} = \underline{i}-\underline{j}+7\underline{k}$ and $\underline{c} = 5\underline{i}+2\underline{j}+3\underline{k}$ then $(3\underline{c} + \underline{b}).(2\underline{a}-\underline{c}) = ?$
(a) 32 (b) -66
(c) –19 (d) –27

(11) If $\underline{a} = 3\underline{i}-x\underline{j}+2\underline{k}$ and $\underline{b}=4\underline{i}+6\underline{j}+3\underline{k}$ are perpendicular to each other then x=?
(a) $\frac{18}{7}$ (b) $\frac{2}{3}$
(c) 3 (d) 93

(12) The cosine of the angle between the vectors $\underline{a}$ and $\underline{b}$, where $\underline{a} = \underline{i}+2\underline{j}-\underline{k}$ and $\underline{b} = -\underline{i}+\underline{j}-2\underline{k}$, is

(a) $\frac{\sqrt{3}}{2}$ (b) 0

(c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

(13) The angle between the vectors $2\underline{i}-\underline{j}+\underline{k}$ and $-\underline{i}+\underline{j}$ is;

(a) $\frac{\pi}{2}$ (b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

(14) If $\underline{a} = 2\underline{i}+\underline{j}$ and $\underline{b} = 3\underline{i}-\underline{k}$, then $\underline{a} \times \underline{b}$ =?

(a) $\underline{i}+\underline{j}+2\underline{k}$ (b) $-\underline{i}+\underline{j}-3\underline{k}$

(c) $2\underline{i}+3\underline{j}+4\underline{k}$ (d) $-\underline{i}+2\underline{j}-3\underline{k}$

(15) If $\underline{a} = 2\underline{i}+3\underline{j}+\underline{k}$, $\underline{b}=4\underline{i}+\underline{j}-3\underline{k}$ and $\underline{c} = 5\underline{i}-\underline{k}$ then $(\underline{b}-\underline{a}) \times \underline{c}$=?

(a) $\underline{i}-9\underline{j}-5\underline{k}$ (b) $2\underline{i}-18\underline{j}+10\underline{k}$

(c) $3\underline{i}+7\underline{j}-\underline{k}$ (d) $4\underline{j}+5\underline{k}$

(16) A unit vector perpendicular to both $\underline{a} =2\underline{i}+\underline{j}+\underline{k}$ and $\underline{b}=-\underline{i}-\underline{j}-3\underline{k}$ is;

(a) $\frac{\underline{i}+\underline{j}+\underline{k}}{\sqrt{2}}$ (b) $\frac{2\underline{i}-\underline{j}-3\underline{k}}{\sqrt{15}}$

(c) $\frac{-2\underline{i}+5\underline{j}-\underline{k}}{\sqrt{30}}$ (d) none of these

(17) A vector of magnitude 5 and perpendicular to $\underline{a} =\underline{i}+3\underline{j}-\underline{k}$ and $\underline{b}=3\underline{i}-\underline{j}$ is;

(a) $\frac{5}{\sqrt{110}}\left(-\underline{i}-3\underline{j}-10\underline{k}\right)$ (b) $\frac{5}{\sqrt{17}}\left(-\underline{i}-3\underline{j}-10\underline{k}\right)$

(c) $\frac{5}{\sqrt{17}}\left(-\underline{i}+3\underline{j}-10\underline{k}\right)$ (d) $\frac{6}{\sqrt{17}}\left(-\underline{i}+3\underline{j}-10\underline{k}\right)$

(18) The sine of the angle between the vectors $\underline{a}=5\underline{i}-2\underline{j}+\underline{k}$ and $\underline{b}=3\underline{i}-4\underline{j}+\underline{k}$ is;

(a) $\sqrt{\frac{72}{390}}$ (b) $\frac{14}{29}$

(c) $\frac{61}{\sqrt{23}}$ (d) $\sqrt{\frac{51}{195}}$

(19) The area enclosed by the triangle, ABC whose vertices are A(1,2,-3), B(0,0,0) and C(2,4,7), is;

(a) $\sqrt{676}$ (b) $\frac{\sqrt{845}}{2}$

(c) $\sqrt{184}$ (d) 27

(20) If $\underline{a} = 2\underline{i}+4\underline{j}-\underline{k}$, $\underline{b} = 3\underline{i}-\underline{j}+2\underline{k}$ and $\underline{c} = \underline{i}+3\underline{j}-3\underline{k}$ then $\underline{a}.\underline{b}\times\underline{c}$ =?

(a) 20 (b) 13

(c) 28 (d) 34

(21) $\left[\underline{k}-\underline{i}\ \underline{i}-\underline{j}\ \underline{j}-\underline{k}\right]$=?

(a) 1 (b) –1

(c) $\frac{1}{2}$ (d) 0

(22) The volume of parallelepiped, whose edges are $\underline{a}=4\underline{i}+2\underline{j}+\underline{k}$, $\underline{b}=\underline{i}+3\underline{j}+3\underline{k}$ and $\underline{c}=5\underline{i}+2\underline{j}+4\underline{k}$, is;
(a) 97 (b) 123
(c) 75 (d) 67

(23) The value of λ, which makes $\lambda\underline{i}+\underline{j}+2\underline{k}$, $2\underline{i}+2\underline{j}+\underline{k}$ and $\lambda\underline{i}+\underline{j}+\underline{k}$ coplanar is;
(a) 1 (b) 0
(c) $\frac{11}{2}$ (d) -11

(24) The volume of the tetrahedron that has $\underline{a}=\underline{i}+2\underline{j}+3\underline{k}$, $\underline{b}=3\underline{i}+\underline{j}+2\underline{k}$ and $\underline{c}=2\underline{i}+3\underline{j}+\underline{k}$ as its edges, is;
(a) 19 (b) 33
(c) 25 (d) 23

(25) A force $\underline{F}=2\underline{i}+4\underline{j}-2\underline{k}$ is applied at P(2,3,-1). Its moment about A(1,0,3) is;
(a) $2\underline{i}+3\underline{j}-\underline{k}$ (b) $5\underline{i}+3\underline{j}-\underline{k}$
(c) $10\underline{i}+8\underline{j}+6\underline{k}$ (d) $10\underline{i}-6\underline{j}-2\underline{k}$

(26) The volume of the parallelepiped that has the vectors $-7\underline{i}+\underline{k}$, $2\underline{i}+3\underline{k}$, $\underline{i}+2\underline{j}-\underline{k}$ as coterminous edges is;
(a) 46 (b) 24
(c) 39 (d) 57

(27) If the dot product of vector $\underline{i}+\underline{j}+\underline{k}$ and the unit vector along the sum of vectors $2\underline{i}+4\underline{j}-5\underline{k}$ and $\lambda\underline{i}+2\underline{j}+3\underline{k}$ is 1, then λ=?
(a) 5 (b) 2
(c) 3 (d) 1

(28) The modulus of the sum of three mutually perpendicular unit vectors is;
(a) $\sqrt{3}$ (b) 3
(c) 0 (d) 1

(29) For what values of x and y, $x\underline{a}-5\underline{b}=7\underline{a}+y\underline{b}$, where $\underline{a}$ and $\underline{b}$ are two non-collinear vectors?
(a) -5, 7 (b) 7, -5
(c) -7, 5 (d) 5, -7

(30) For what value of α, the magnitude of the vector $\alpha\underline{i}+(\alpha+1)\underline{j}+(\alpha-1)\underline{k}$ is 2?
(a) $\frac{22}{3}$ (b) $\sqrt{\frac{22}{3}}$
(c) $\sqrt{\frac{3}{22}}$ (d) none

(31) If $\underline{a}$ and $\underline{b}$ are two non-zero vectors then component of $\underline{b}$ along $\underline{a}$ is
(a) $\underline{a}.\underline{b}$ (b) $\hat{a}.\hat{b}$
(c) $\underline{a}.\hat{b}$ (d) $\underline{b}.\hat{a}$

(32) The perimeter of a triangle formed by $3\underline{i}+4\underline{j}-5\underline{k}$, $4\underline{i}-3\underline{j}-5\underline{k}$ and $7\underline{i}+\underline{j}$ is;
(a) $\sqrt{4500}$ (b) $\sqrt{5000}$
(c) $\sqrt{5000}$ (d) 2000

SAMPLE PAPER FOR GIKI

(1) What is the relation between a, b & c if one root of the equation $ax^2+bx+c=0$ is three times the other?

(a) $b^2=ac$
(b) $3b^2=16ac$
(c) $16b^2=3ac$
(d) $b^3=ac$

(2) When the polynomial $2x^4+kx^3-11x^2+4x+12$ is divided by $x-3$, then the remainder is 60, find k;

(a) -1
(b) -2
(c) 1
(d) 2

(3) If $2x^2-9x+14=a(x-1)(x-2)+b(x-1)+c$, then the value of c is;

(a) 14
(b) -7
(c) 7
(d) -14

(4) If $\frac{9}{(x-1)(x+2)^2}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$, then $B=?$

(a) -1
(b) -3
(c) 3
(d) 2

(5) If the first term of an A.P. is 25 and third term is 19. if the sum of the progression is 82 then the number of terms are;

(a) $\frac{41}{3}$
(b) $13\frac{1}{3}$
(c) 4
(d) 14

(6) The sum of first 10-terms of $2,-4,8,\cdots$, is;

(a) -683
(b) -681
(c) -680
(d) -682

(7) How many different numbers can be made by using four out of the digits, $1,2,3,\cdots,9$?

(a) 6561
(b) 4032
(c) 4510
(d) 1054

(8) How many different arrangements of the letters can be made by using all the letters of the word **CONTACT** when the vowels are separate?

(a) 1260
(b) 900
(c) 360
(d) 420

(9) $\sin^3\theta-\cos^3\theta=?$

(a) $1+\sin\theta\cos\theta$
(b) $\sin\theta+\cos\theta$
(c) $(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)$
(d) $\cos^2\theta-\sin^2\theta$

(10) $\sin 1220^\circ=?$

(a) $\sin 140^\circ$
(b) $\sin 180^\circ$
(c) $\sin 90^\circ$
(d) $\sin 270^\circ$

(11) $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=?$

(a) $2\sin\theta$
(b) $2\sec\theta$
(c) $2\cos ec\theta$
(d) $2\cot\theta$

(12) $2\sin^{-1}\left(\frac{1}{2}\right)=?$

(a) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(b) $\cos^{-1}\left(\frac{1}{2}\right)$

(c) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(d) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(13) If $\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}$, then $x=?$

(a) -1

(b) 6

(c) $\frac{1}{6}$

(d) 1

(14) If $F(t)=2t^2+3t-2$, what value of t makes $F(t)=0$?

(a) -2

(b) -1

(c) 1

(d) 2

(15) A body moves in a straight line a distance s meters in t seconds and $s=t^3$. What will be the acceleration after 3-seconds?

(a) $10m/\sec^2$

(b) $9m/\sec^2$

(c) $18m/\sec^2$

(d) none

(16) Differentiate $10\sin x\cos x$ w.r.t. x we get;

(a) $-5\sin 2x$

(b) $5\sin 2x$

(c) $5\cos 2x$

(d) $10\cos 2x$

(17) If $y=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{dy}{dx}=?$

(a) $\frac{2}{x^2}$

(b) $\frac{2}{1+x^2}$

(c) $\frac{2}{1-x^2}$

(d) $\frac{2}{x^2-1}$

(18) $\frac{\cot 49}{\tan 41}=?$

(a) 0

(b) ∞

(c) 1

(d) 0.5

(19) If $y=\sin^{-1}(\cos x)$, then $\frac{dy}{dx}=?$

(a) 0

(b) -1

(c) 1

(d) 2

(20) $\lim_{p\to\infty}\frac{3p^3-1}{p^2+2p^3}=?$

(a) $\frac{2}{3}$

(b) 3

(c) 2

(d) $\frac{3}{2}$

(21) Given that $A=\begin{bmatrix}3 & 2 & 1\\ 0 & -1 & 1\\ 3 & -2 & 4\end{bmatrix}$,then find P if $AX=P$ and $X=\begin{bmatrix}1\\2\\3\end{bmatrix}$;

(a) $\begin{bmatrix}10\\1\\11\end{bmatrix}$

(b) $\begin{bmatrix}10 & 1 & 11\end{bmatrix}$

(c) $\begin{bmatrix}5\\4\\9\end{bmatrix}$

(d) $\begin{bmatrix}5 & 4 & 9\end{bmatrix}$

(22) If ω is a cube root of unity then $\left(1-\omega+\omega^2\right)\left(1+\omega-\omega^2\right)=?$

(a) -3

(b) 1

(c) 0

(d) 4

(23) Find an equation whose roots are ω and $\frac{1}{\omega}$.

(a) $x^2+x+1=0$

(b) $x^2-x+1=0$

(c) $x^2-x-1=0$

(d) $x^2+x-1=0$

(24) Find z_3,if $\frac{1}{z_3}=\frac{1}{z_1}+\frac{1}{z_2}$,and $z_1=2+i, z_2=-2+4i$.

(a) $1.2+1.6i$

(b) $2.1+6.1i$

(c) $2.1-6.1i$

(d) $1.2-1.6i$

(25) The inverse of $-i$ is;

(a) 1

(b) -1

(c) i

(d) $-i$

(26) The inverse of the matrix $\begin{pmatrix}3 & 3\\3 & 3\end{pmatrix}$ is;

(a) $\begin{pmatrix}3 & 3\\3 & 3\end{pmatrix}$

(b) $\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$

(c) $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

(d)none of these

(27) $\frac{2+3i}{3+4i}=?$

(a) $\frac{1}{25}\left(18+\sqrt{-1}\right)$

(b) $\frac{1}{25}\left(18-\sqrt{-1}\right)$

(c) $\frac{2}{25}\left(18+\sqrt{-1}\right)$

(d) $\frac{3}{25}\left(18+\sqrt{-1}\right)$

(28) What is the distance between the foci of $3x^2+4y^2=12$?

(a) $\sqrt{7}$
(b) $2\sqrt{2}$
(c) $\sqrt{14}$
(d) 2

(29) Eccentricity of the conic whose parametric equations are $x=\cos\theta$, $y=\sin\theta$ is;

(a) 1
(b) 2
(c) $\frac{1}{2}$
(d) 0

(30) Which of the following is an equation of tangent to the ellipse $5x^2+3y^2=137$ at a point in the first quadrant whose ordinate is 2?

(a) $25x+6y-137=0$
(b) $6x-25y+20=0$
(c) $25x-6y+137=0$
(d) $6x+25y-20=0$

(31) Find all angles less than four right angles whish satisfy the equation $2\cos^2 x=1+\sin x$;

(a) $60^\circ, 120^\circ, 240^\circ$
(b) $30^\circ, 150^\circ, 270^\circ$
(c) $45^\circ, 135^\circ, 225^\circ$
(d) $60^\circ, 120^\circ, 240^\circ, 300^\circ$

(32) Which of the following is a point of inflection on the curve $y=x^5-5x^4+5x^3-1$?

(a) $(2,2)$
(b) $(0,0)$
(c) $(0,-1)$
(d) $(1,-1)$

(33) The line $y=mx$ is tangent to the circle $x^2+y^2+2gx+2fy+c=0$, and then which of the following must be true?

(a) $c^2=a^2(1+m^2)$
(b) $(g+mf)^2=c(1+m^2)$
(c) $(g+mf)^2=c^2(1+m^2)$
(d) none of these

(34) If $\tan\theta+\sin\theta=x$ and $\tan\theta-\sin\theta=y$, then $(x^2-y^2)^2=?$

(a) xy
(b) $16xy$
(c) $4xy$
(d) $2xy$

(35) If $y=\tan x$, then $\frac{d^2y}{dx^2}=?$

(a) $\sec^2 x+\sec x$
(b) $\tan^2 x+\tan^3 x$
(c) $2\tan x+2\tan^3 x$
(d) none of these

(36) Find area between the x-axis and sine curve from zero to π.

(a) 0
(b) 4
(c) 2
(d) –4

(37) If $y=xe^{-x}$ then $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=?$

(a) 0
(b) 1
(c) –1
(d) 2

(38) Integrate w.r.t. x, $\frac{3-2x}{2x-1}$.

(a) $\ln(2x)+x+c$ (b) $x+\ln(2x-1)+c$

(c) $\ln(2x-1)-x+c$ (d) $x-\ln(2x-1)+c$

(39) The area of the quadrilateral whose vertices are $(2,1),(3,5),(-3,4)$ and $(-2,-2)$ is;

(a) 13 (b) 12

(c) 29 (d) 25

(40) Which of the following line passes through the point $(1,2)$ and making an angle 45 with x-axis?

(a) $x+y+1=0$ (b) $y-x+1=0$

(c) $x-y+1=0$ (d) $x-y-1=0$

(41) What is the length of the perpendicular from the origin to the straight line passing through the points $(6,4)$ and $(9,8)$?

(a) $\frac{5}{12}$ (b) $\frac{12}{5}$

(c) $\frac{9}{4}$ (d) $\frac{8}{5}$

(42) $\cos 165 + \sin 165 = ?$

(a) $\cos 90$ (b) $\sin 90$

(c) $\cos 35$ (d) $\cos 135$

(43) $\int \tan^{-1} x dx = ?$

(a) $\sec^2 x$ (b) $\frac{1}{1+x^2}$

(c) $x\tan^{-1} x$ (d)none of these

(44) What is the acute angle between the lines represented by $3x^2-10xy+7y^2=0$?

(a) $\tan^{-1}\left(\frac{2}{5}\right)$ (b) $\tan^{-1}\left(\frac{4}{5}\right)$

(c) $\tan^{-1}\left(\frac{5}{2}\right)$ (d) $\tan^{-1}\left(\frac{5}{4}\right)$

(45) The coefficient of x^4 in the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is;

(a) $\frac{405}{256}$ (b) $\frac{504}{259}$

(c) $\frac{450}{263}$ (d)none of these

(46) In how many ways can 6 be chosen out of 4 officers and 6 soldiers so that at least one officer is included?

(a) 209 (b) 880

(c) 868 (d) 448

(47) Two identical dices are rolled .The probability that the same number will appear on each of them is;

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$

(c) $\frac{1}{18}$ (d) $\frac{1}{9}$

(48) The fifth term of the sequence $(-1)^n + 1$ is;

(a) -1 (b) 0

(c) 81 (d) -81

(49) The domain of the function $f = \{(1,2),(2,3),(3,4)\}$ is;

(a) $\{1,2,3,4\}$ (b) $\{1,2,3\}$

(c) $\{2,3,4\}$ (d) $\{1,2,4\}$

(50) $\tan\left(\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)+\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)=?$

(a) 0 (b) 1

(c) -1 (d) ∞

SAMPLE PAPER FOR FAST

(1) $\left(\frac{1+i}{1-i}\right)^4 = ?$

(a) i (b) $-i$

(c) 1 (d) -1

(2) The cube roots of -27 are;

(a) $-3,-3\omega^2,-3\omega^8$ (b) $-3\omega^3,-3\omega^{16},-3\omega^2$

(c) $-3\omega^8,3(1+\omega),3(1+\omega^2)$ (d) $-3,-3+\omega,-3+\omega^2$

(3) The value of "K" for which $1+\sqrt{2}$ is one root of the equation $x^2-2x+k=0$,is?

(a) 1 (b) $\sqrt{2}$

(c) -1 (d) $-\sqrt{2}$

(4) If $1+2+3+\cdots+n=55$,then $1^3+2^3+3^3+\cdots+n^3=?$

(a) 165 (b) 385

(c) 255 (d) 3025

(5) $\frac{{}^{15}C_{11}}{{}^{15}C_{10}}=?$

(a) $\frac{15}{11}$ (b) $\frac{5}{11}$

(c) $\frac{5}{10}$ (d) $\frac{15}{10}$

(6) Number containing three digits, repetition allowed are.
(a) 720
(b) 999
(c) 900
(d) 1000

(7) Domain of $\sqrt{4x-4x^2}$ is;
(a) $[0,1]$
(b) $(0,1)$
(c) $\mathbb{R}-[0,1]$
(d) $\mathbb{R}-(0,1)$

(8) In a circle of diameter 20cm, the length of a chord is 10cm. The length of the minor arc of the chord is;
(a) 10π
(b) 3π
(c) $\frac{10\pi}{3}$
(d) $\frac{20\pi}{3}$

(9) If $\sin\theta+\cos\theta=0, \theta=?$
(a) –45
(b) 45
(c) 180
(d) 225

(10) If $\theta=60$,then $\frac{1+\tan^2\theta}{2\tan\theta}=?$
(a) $\frac{\sqrt{3}}{2}$
(b) $\frac{2}{\sqrt{3}}$
(c) $-\sqrt{3}$
(d) $-\frac{\sqrt{3}}{2}$

(11) Slope of a line which is perpendicular to the line joining the points $(2,5)$ and $(3,8)$ is;
(a) 1
(b) 3
(c) –3
(d) $-\frac{1}{3}$

(12) The point of intersection of the lines $3x+2y=8$ and $5x-11y+1=0$ is;
(a) $(1,2)$
(b) $(-1,2)$
(c) $(2,1)$
(d) $(2,-1)$

(13) The length of tangent from the point $(2,5)$ to the circle $2x^2+2y^2-4x-6y-2=0$ is;
(a) 5
(b) 4
(c) $\sqrt{5}$
(d) 3

(14) If θ is the angle between the unit vectors $\hat{a}$ and $\hat{b}$,then $\cos\theta=?$
(a) $\hat{a}+\hat{b}$
(b) $\hat{a}-\hat{b}$
(c) $\hat{a}\times\hat{b}$
(d) $\hat{a}\hat{b}$

(15) $\left(\hat{k}\times\hat{i}\right).\hat{j}=?$
(a) 0
(b) 1
(c) –1
(d) none of these

(16) The inverse of I_3 is;

(a) I_3

(b) $-I_3$

(c) does not exist

(d) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(17) $\begin{vmatrix} \sec x & -1 \\ -1 & \sec x \end{vmatrix} = ?$

(a) $2\sec x$

(b) $\tan^2 x$

(c) $-\tan^2 x$

(d) $-2\sec x$

(18) Let $f(x) = \begin{cases} x-4, x>2 \\ k, x=2 \\ 2-x^2, x<2 \end{cases}$,then find the value of "k" such that the function is continuous at $x=2$.

(a) 0

(b) 1

(c) 2

(d) –2

(19) For $f(x)=x^2, f(a+b)=?$

(a) $f(a)+f(b)$

(b) $f(a)-f(b)$

(c) $f(a).f(b)$

(d)none of these

(20) If $f(x)=3x^2-1$,then $\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=?$

(a) $6x$

(b) 6

(c) –1

(d) –6

(21) $\cos\left(\tan^{-1}(-1)\right)=?$

(a) $-\frac{1}{\sqrt{2}}$

(b) $\frac{1}{\sqrt{2}}$

(c) 1

(d) –1

(22) $\frac{d}{dx}(\log_a x)=?$

(a) $a^x \ln a$

(b) $\frac{1}{x\ln a}$

(c) $\frac{1}{x}$

(d) $\frac{1}{x\ln 10}$

(23) Derivative of $\sin^{-1} x$ exists for;

(a) For all real x

(b) for all real $x>0$

(c) $-1\le x\le 1$

(d) $-1<x<1$

(24) For $y=x.e^x$,the point;

(a) $x=-1$ is a minimum

(b) $x=-1$ is a maximum

(c) $x=0$ is a minimum

(d) $x=0$ is a maximum

(25) On substituting $\cos\theta = t$, $\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta}\sin^3\theta d\theta$, becomes;

(a) $\int_1^0 \sqrt{t}\left(1-t^2\right)dt$

(b) $-\int_1^0 \sqrt{t}\left(1-t^2\right)dt$

(c) $\int_0^{\frac{\pi}{2}} \sqrt{t}\left(1-t^2\right)dt$

(d) $-\int_0^{\frac{\pi}{2}} \sqrt{t}\left(1-t^2\right)dt$

(26) Which one of the following gives the direction cosines of a vector?

(a) 1,1,1

(b) $-1,-1,-1$

(c) $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

(d) $\frac{1}{3},\frac{1}{3},\frac{1}{3}$

(27) $\int e^x \sec x\left(1+\tan x\right)dx = ?$

(a) $e^x \sec x + c$

(b) $e^x \tan x + c$

(c) $e^x \sin x + c$

(d) $e^x + \sec x + c$

(28) one root of the equation $2^{2x} - 10.2^x + 16 = 0$ is;

(a) 1

(b) 0

(c) -1

(d) 2

(29) If α, β are the roots of the equation $x^2 - 2x + 2 = 0$, then $\alpha^2 + \beta^2 = ?$

(a) 2

(b) 1

(c) 0

(d) -2

(30) The number of selection of n-different things taken r at time which includes a particular thing is;

(a) ${}^{n-1}C_r$

(b) ${}^{n-1}C_{r-1}$

(c) ${}^{n-1}C_{r-1}.n$

(d) ${}^{n-1}C_r.r$

(31) For $|x| < 1, 1 + 2x + 3x^2 + 4x^3 + \cdots = ?$

(a) $\frac{1}{1+x}$

(b) $\frac{1}{1+x^2}$

(c) $\frac{1}{1-2x}$

(d) $\frac{1}{(1-x)^2}$

(32) The term independent of x in $\left(2x+\frac{3}{x}\right)^8$ is;

(a) 8^{th}

(b) 9^{th}

(c) 5^{th}

(d) 4^{th}

(33) In an acute angle triangle ABC, $\cos A = ?$

(a) $\frac{a^2+b^2+c^2}{2ab}$

(b) $\frac{a^2+b^2-c^2}{2ab}$

(c) $\frac{b^2+c^2-a^2}{2bc}$

(d) $\frac{a^2+c^2-b^2}{2ac}$

(34) $\sin 38 + \sin 22 = ?$

(a) $\sin 8$ (b) $\cos 8$

(c) $\sin 16$ (d) $\cos 16$

(35) $\cos 2\theta = ?$, when $\sin\theta = \frac{2}{3}$.

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$

(c) $\frac{2}{9}$ (d) $\frac{4}{9}$

(36) The line $y = 2x + c$ is tangent to the circle $x^2 + y^2 = 9$, then c=?

(a) $4\sqrt{5}$ (b) $3\sqrt{5}$

(c) Less than $3\sqrt{5}$ (d) greater than $3\sqrt{5}$

(37) Which of the following is not a value of $2\tan^{-1} x = ?$

(a) $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ (b) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

(c) $\tan^{-1}\left(\frac{2x}{1+x^2}\right)$ (d) $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

(38) $\cos^{-1}\left(\frac{2}{7}\right) + \sin^{-1}(x) = \frac{\pi}{2}$, then x=?

(a) $\frac{2}{7}$ (b) $\frac{7}{2}$

(c) $\frac{-7}{2}$ (d) $-\frac{2}{7}$

(39) The H.M. of 2,3,4 is;

(a) 3 (b) 24

(c) $\frac{36}{13}$ (d)none of these

(40) The value of "a" for which $4x^2 + 4(a+1)x + a^2 = 0$, has equal roots, is;

(a) $-\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $-\frac{1}{4}$

(41) If given that $g(0) = 2, g'(0) = 1$ and $f(x) = xg(x)$, then $f'(0) = ?$

(a) -2 (b) 0

(c) 2 (d) $\frac{1}{2}$

(42) Which of the following matrices is not invertible?

(a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & -1 \\ -1 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$

(43) If $f'(x) = g(x)$ then $\int f(x)g(x)dx = ?$

(a) $[f(x)]^2 + c$ (b) $[g(x)]^2 + c$

(c) $\frac{1}{2}[f(x)]^2 + c$ (d) $\frac{1}{2}[g(x)]^2 + c$

(44) The area between $y = 4\sin x$ from 0 to π is;

(a) 1 (b) 2

(c) 4 (d) 8

(45) A coin is tossed 8 times ,the probability of getting a head 5 times is;

(a) $\frac{7}{16}$ (b) $\frac{7}{64}$

(c) $\frac{7}{128}$ (d) $\frac{7}{32}$

(46) If $|\vec{a}| = 2, |\vec{b}| = 5, |\vec{a} \times \vec{b}| = 8$ then $\vec{a}.\vec{b} = ?$

(a) 16 (b) 20

(c) -16 (d) 6

(47) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx = ?$

(a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

(48) Distance between the parallel lines $\begin{matrix} x + y - 13 = 0 \\ 2x + 2y + 26 = 0 \end{matrix}$ is;

(a) 0 (b) 26 (c) $39\sqrt{2}$ (d) $13\sqrt{2}$

(49) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx = ?$

(a) $\ln|e^x + e^{-x}|$ (b) $\ln|e^x - e^{-x}|$ (c) $\ln(2e^{-x})$ (d) $\ln(2e^x)$

(50) Three dice are rolled simultaneously, the probability that the sum is greater than four is

(a) $\frac{3}{216}$ (b) $\frac{52}{53}$

(c) $\frac{53}{54}$ (d) $\frac{13}{216}$

SAMPLE PAPER FOR NUST

(1) The argument and modulus of $\frac{1+i}{1-i}=?$

(a) $-\frac{\pi}{2},1$

(b) $\frac{\pi}{2},\sqrt{2}$

(c) $0,\sqrt{2}$

(d) $\frac{\pi}{2},1$

(2) $\frac{1+2i+3i^2}{1-2i+3i^2}=?$

(a) 1

(b) i

(c) $-i$

(d) 0

(3) If $x,2x+2,3x+3,\cdots$ is a G.P.,then the fourth term is;

(a) 27

(b) –27

(c)13.5

(d) –13.5

(4) Three numbers are in A.P.,whose sum is 33 and product is 792,then the least number is;

(a)4

(b)8

(c)11

(d)14

(5) The sum of numbers divisible by 9 between 100 and 1000 is;

(a)55350

(b)57228

(c)97015

(d)62140

(6) If one root of the equation $ax^2+3x-7=0$ is reciprocal of the other then the value of $a=?$

(a)2

(b) 3

(c) –7

(d)7

(7) If α,β are the roots of the equation $4x^2+3x+7=0$,then the value of $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=?$

(a) $\frac{-27}{64}$

(b) $\frac{63}{16}$

(c) $\frac{225}{343}$

(d) none of these

(8) There are 9 chairs in a room on which 6 men are to be seated. One of them is a guest for whom there is a particular chair. In how many ways can they be seated?

(a)6720

(b) 60480

(c)30

(d)346

(9) If ${}^nP_r=840,{}^nC_r=35$,then $r=?$

(a) 7

(b) 4

(c) 2

(d) 3

(10) In how many ways n books can be arranged in a line so that two particular books are not together.

(a) $n!-(n-2)!$

(b) $(n-1)!(n-2)$

(c) $n!-2(n-1)$

(d) $(n-2)n!$

(11) A question paper consist of two sections A & B each having 5 questions. In how many ways can a candidate answer 6 questions having selected at least two questions from each section?

(a)80 (b)100
(c)200 (d)150

(12) In the expansion of $\left(3x^2-\frac{2}{x^3}\right)^n$ 9^{th} term is independent of x, then $n=?$

(a)18 (b)20
(c)24 (d)32

(13) In $\frac{(1+x)^2}{(1-x)^3}$ the coefficient of x^n is;

(a) $3n^2+2n+1$ (b $2n^2+2n+1$
(c) n^2+n+1 (d) $2n^2-2n+1$

(14) Which of the following matrices is skew symmetric?

(a) $\begin{pmatrix} 0 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 & 1 \\ -2 & 2 & -3 \\ -3 & 0 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{pmatrix}$

(15) The matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is non singular if;

(a) $\lambda \neq -5$ (b) $\lambda \neq -17$
(c) $\lambda \neq -16$ (d) $\lambda \neq -18$

(16) The value of $\frac{A-B}{B-A}$ will be;

(a) >0 (b) <0
(c) $=0$ (d) A

(17) What is the slope of the tangent to the curve $y=e^{2x^2}$ at $x=0$?

(a)0 (b)1
(c) $\frac{1}{2}$ (d)4

(18) The length of perpendicular from the origin to a line is 5 units and the inclination of the perpendicular is 120^o, then the slope of the line is;

(a) 0 (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{2}$ (d) $\sqrt{3}$

(19) Which of the following is not true?'

(a) $\sin x = -\frac{1}{5}$

(b) $\cos x = 1$

(c) $\sec x = \frac{1}{2}$

(d) $\tan x = 20$

(20) If $\sec\theta + \tan\theta = p, \sec\theta = ?$

(a) $\frac{1}{2}\left(p + \frac{1}{p}\right)$

(b) $p - \frac{1}{p}$

(c) $2\left(p + \frac{1}{p}\right)$

(d) $-p + \frac{1}{p}$

(21) $\cot\left(\frac{\pi}{4} + \theta\right)\cot\left(\frac{\pi}{4} - \theta\right) = ?$

(a) -1

(b) 1

(c) 0

(d) ∞

(22) If $\alpha + \beta - \gamma = \pi$ then $\sin^2\alpha + \sin^2\beta - \sin^2\gamma = ?$

(a) $2\sin\alpha\sin\beta\cos\gamma$

(b) $2\cos\alpha\cos\beta\cos\gamma$

(c) $2\sin\alpha\sin\beta\sin\gamma$

(d) $2\cos\alpha\sin\beta\sin\gamma$

(23) $\cos 105 + \sin 105 = ?$

(a) $\frac{1}{2}$

(b) 1

(c) $\sqrt{2}$

(d) $\frac{1}{\sqrt{2}}$

(24) The period of $\tan 3\theta = ?$

(a) 2π

(b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{3}$

(d) π

(25) If $\cos^{-1}\left(\frac{1}{x}\right) = \theta$, then $\tan\theta = ?$

(a) $\frac{1}{\sqrt{x^2 - 1}}$

(b) $\sqrt{x^2 - 1}$

(c) $\sqrt{1 - x^2}$

(d) $\sqrt{x^2 + 1}$

(26) If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1, x = ?$

(a) 1

(b) 0

(c) $\frac{4}{5}$

(d) $\frac{1}{5}$

(27) $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=?$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

(28) The number of triangles formed by 4 points, when no three points are collinear is;

(a) 4 (b) 6

(c) 8 (d) 9

(29) In ΔABC if $a=16, b=24, c=20, \cos\frac{\beta}{2}=?$

(a) $\frac{3}{4}$ (b) $\frac{11}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

(30) Which of the following is a diameter of the circle $x^2+y^2-6x-8y-9=0$?

(a) $3x-4y=0$ (b) $4x-3y=9$

(c) $x+y=7$ (d) $x-y=1$

(31) The number of tangents from the point $(-1,2)$ to the circle $x^2+y^2+2x-4y+4=0$ are;

(a) 2 (b) 1

(c) 3 (d) 0

(32) If the line $y=mx+c$ touches the circle $x^2+y^2-4y=0$ then the value of "c" is;

(a) $1+\sqrt{1+m^2}$ (b) $1-\sqrt{1+m^2}$

(c) $2\left(1+\sqrt{1+m^2}\right)$ (d) $1+\sqrt{1+m^2}$

(33) $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{7x}=?$

(a) e (b) e^7

(c) e^{-1} (d) e^2

(34) $\frac{d}{dx}\left(\sin^{-1}\left(3x-4x^3\right)\right)=?$

(a) $\frac{3}{\sqrt{1-x^2}}$ (b) $\frac{-3}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\frac{-1}{\sqrt{1-x^2}}$

(35) Which of the following is focus of the parabola $y^2-x-2y+2=0$?

(a) $\left(\frac{1}{4},0\right)$ (b) $(1,2)$

(c) $\left(\frac{3}{4},1\right)$ (d) $\left(\frac{5}{4},1\right)$

(36) If the line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$ then;

(a) $m = 1$ (b) $m = 2$

(c) $m = 4$ (d) $m = 3$

(37) The eccentricity of the hyperbola which passes through the points $(3,0), (3\sqrt{2}, 2)$ is;

(a) $\frac{13}{3}$ (b) $\frac{\sqrt{13}}{3}$

(c) $\sqrt{\frac{13}{3}}$ (d) $\frac{13}{\sqrt{3}}$

(38) $(\cos\theta + i\sin\theta)^2 . (\cos 2\theta + i\sin 2\theta)^5 = ?$

(a) $\cos 10\theta + i\sin 10\theta$ (b) $\cos 12\theta + i\sin 12\theta$

(c) $\cos 7\theta + i\sin 7\theta$ (d) none of these

(39) Which of the following does not represent hyperbola?

(a) $xy = 1$ (b) $x^2 - y^2 = 5$

(c) $(x-1)(y-1) = 3$ (d) $x^2 - y^2 = 0$

(40) If the foci of $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ are concentric, then $b = ?$

(a) 1 (b) 5

(c) $\sqrt{7}$ (d) 9

(41) ${}^nC_r \div {}^nP_r$

(a) $n!$ (b) $(n-r)!$

(c) $\frac{1}{r!}$ (d) $r!$

(42) $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = ?$

(a) $2\sqrt{\sec x} + c$ (b) $2\sqrt{\tan x} + c$

(c) $\frac{2}{\sqrt{\tan x}} + c$ (d) $\frac{2}{\sqrt{\sec x}} + c$

(43) The inverse of the function $y = \sin x$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ is correct for;

(a) $y = \sin x$ (b) $x = \sin y$

(c) $y = \sin^{-1} x$ (d) $x = \frac{1}{\sin y}$

(44) At a point on a curve the product of the slope of the curve and the square of the abscissa of the point is 2. If the curve passes through the point $(1,-1)$ then its equation is;

(a) $y = 1 - 2x$ (b) $y = 1 - \frac{2}{x}$

(c) $y = 1 + 2x$ (d) $y = 1 + \frac{2}{x}$

(45) If $y = x^3$ then $\int_0^4 y dx + \int_0^{64} x dy = ?$

(a) 0
(b) 64
(c) 256
(d) 512

(46) Find the equation of the line through the point $(2,3)$ making angle of 45 with the line $x - 2y = 1$.

(a) $x + 3y = 11$
(b) $3x - y = 3$
(c) $x + y = 1$
(d) both a & b

(47) $\int_0^{\frac{\pi}{2}} \sin^2 2x dx = ?$

(a) $\frac{\pi}{4}$
(b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$
(d) $\frac{\pi}{6}$

(48) The equation of latus rectum of the parabola $x^2 + 4x + 2y = 0$ is;

(a) $y = \frac{1}{2}$
(b) $y = \frac{3}{2}$
(c) $x = \frac{1}{2}$
(d) $x = \frac{3}{2}$

(49) A force $F = 3i + 2j - 4k$ is acting at a point $(1,-1,2)$ then its moment through the point $(2,-1,3)$ is;

(a) $2i - 7j - 2k$
(b) $3i + 4j - 5k$
(c) $2j$
(d) $2k$

(50) Seven men and five women sit in a line. The probability that no two women are adjacent to each other is;

(a) $\frac{7}{99}$
(b) $\frac{17}{99}$
(c) $\frac{17}{98}$
(d) $\frac{17}{97}$

(51) There will be a such triangle which satisfies the condition;

(a) $b \sin A = a, A < \frac{\pi}{2}$
(b) $b \sin A > a, A > \frac{\pi}{2}$
(c) $b \sin A > a, A < \frac{\pi}{2}$
(d) $b \sin A < a, A < \frac{\pi}{2}$

(52) $\lim_{x \to 0} \frac{\sin mx}{\tan nx} = ?$

(a) $\frac{n}{m}$
(b) $\frac{m}{n}$
(c) mn
(d) $(m)^n$

(53) The derivative of $6x^3$ w.r.t. $6x^3$ is;

(a) $18x^2$ (b) $6x^2$

(c) 1 (d) 6

(54) If the sum of the roots of the equation $x^2 + px + q = 0$ is three times the difference of the roots then;

(a) $9p^2 = 2q$ (b) $2q^2 = 9p$

(c) $2p^2 = 9q$ (d) $9q^2 = 2p$

(55) $i^{501} + i^{502} + i^{503} + i^{504} = ?$

(a) 1 (b) -1

(c) 0 (d) $-i$

(56) The 9^{th} term of the series $27 + 9 + 5\frac{2}{5} + \cdots$ is;

(a) $1\frac{10}{17}$ (b) $\frac{10}{17}$

(c) $\frac{16}{27}$ (d) $\frac{17}{27}$

(57) If $\frac{1}{(x+1)^2(x^2-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$ then $D = ?$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{2}$

(58) $\sum_{T=1}^{n} T^3 = ?$

(a) $\frac{T(T+1)}{2}$ (b) $\frac{T^2(T+1)^2}{4}$

(c) $\frac{(n(n+1))^2}{4}$ (d) $\frac{n(n+1)}{2}$

(59) $\frac{\cos 12^\circ + \sin 12^\circ}{\cos 12^\circ - \sin 12^\circ} = ?$

(a) $\tan 42^\circ$ (b) $\tan 52^\circ$

(c) $\tan 57^\circ$ (d) $\tan 33^\circ$

(60) If $\begin{vmatrix} 1 & 2 & 1 \\ 2 & x & 2 \\ 3 & 6 & x \end{vmatrix} = 0$ then $x = ?$

(a) 3 (b) 4

(c) None of these (d) both a & b

ANSWER KEY

CHAPTER-1

1	b	11	b	21	d	31	b	41	c	51	d
2	d	12	d	22	b	32	a	42	c	52	a
3	b	13	c	23	b	33	c	43	d	53	b
4	b	14	c	24	b	34	a	44	c	54	a
5	d	15	c	25	c	35	c	45	a	55	c
6	d	16	d	26	a	36	b	46	d	56	c
7	a	17	d	27	b	37	d	47	c	57	d
8	c	18	a	28	a	38	d	48	a	58	b
9	b	19	b	29	b	39	d	49	d	59	c
10	b	20	d	30	a	40	c	50	d	60	b

CHAPTER-2

1	d	13	b	25	c	37	d	49	d
2	b	14	a	26	b	38	b	50	b
3	d	15	c	27	d	39	c	51	d
4	b	16	b	28	c	40	c	52	a
5	d	17	c	29	b	41	c	53	b
6	c	18	c	30	a	42	c	54	c
7	c	19	b	31	b	43	a	55	c
8	b	20	a	32	d	44	c	56	b
9	a	21	b	33	b	45	c	57	b
10	c	22	c	34	a	46	c	58	c
11	d	23	d	35	a	47	b	59	c
12	b	24	a	36	a	48	c	60	b

CHAPTER-3

1	c	14	a	27	b	40	a	53	a
2	c	15	a	28	c	41	c	54	c
3	b	16	c	29	b	42	d	55	c
4	a	17	a	30	b	43	a	56	d
5	c	18	c	31	c	44	a	57	c
6	a	19	b	32	c	45	a	58	d
7	a	20	a	33	a	46	d	59	b
8	b	21	b	34	d	47	d	60	c
9	d	22	c	35	b	48	c	61	b
10	b	23	a	36	a	49	a	62	b
11	d	24	b	37	d	50	a	63	c
12	c	25	c	38	c	51	c	64	a
13	d	26	a	39	b	52	d	65	a

CHAPTER-4

1	c	11	d	21	b	31	a	41	a	51	c
2	d	12	d	22	d	32	b	42	c	52	d
3	c	13	c	23	b	33	c	43	c	53	b
4	b	14	a	24	d	34	d	44	c	54	a
5	b	15	c	25	a	35	d	45	c	55	a
6	b	16	b	26	c	36	b	46	c		
7	d	17	c	27	c	37	c	47	a		
8	a	18	c	28	a	38	c	48	c		
9	c	19	b	29	b	39	b	49	a		
10	b	20	c	30	c	40	d	50	c		

CHAPTER-5

1	c	5	c	9	b	13	c	17	d
2	b	6	a	10	d	14	c	18	d
3	d	7	d	11	d	15	b	19	c
4	c	8	c	12	b	16	c	20	d

CHAPTER-6

EXERCISE A

1	c	10	c	19	c	28	c	37	a	46	a
2	b	11	b	20	b	29	b	38	c	47	c
3	d	12	d	21	d	30	c	39	a	48	d
4	c	13	d	22	a	31	d	40	d	49	c
5	a	14	d	23	a	32	a	41	d	50	c
6	c	15	c	24	c	33	a	42	a		
7	d	16	c	25	b	34	a	43	a		
8	d	17	d	26	d	35	d	44	c		
9	a	18	c	27	a	36	c	45	a		

EXERCISE B

1	d	13	a	25	b	37	b	49	a
2	d	14	a	26	b	38	d	50	b
3	a	15	b	27	c	39	c	51	d
4	d	16	a	28	d	40	a	52	c
5	d	17	b	29	b	41	d	53	d
6	b	18	c	30	a	42	c	54	b
7	b	19	b	31	a	43	d	55	d
8	d	20	c	32	a	44	b	56	c
9	a	21	a	33	a	45	d	57	a
10	a	22	d	34	c	46	d	58	d
11	a	23	d	35	a	47	c	59	c
12	d	24	a	36	c	48	a	60	b

CHAPTER-7

1	d	11	c	21	d	31	d	41	c	51	a
2	c	12	c	22	b	32	a	42	d	52	c
3	b	13	b	23	d	33	c	43	b	53	d
4	c	14	d	24	b	34	a	44	c	54	a
5	c	15	b	25	d	35	a	45	d	55	d
6	d	16	c	26	a	36	a	46	d		
7	b	17	a	27	d	37	d	47	d		
8	c	18	d	28	a	38	c	48	b		
9	d	19	b	29	b	39	b	49	a		
10	b	20	c	30	b	40	d	50	c		

LOGARITHMIC AND EXPONENTIAL EQUATIONS

1	b	11	c	21	b
2	a	12	d	22	a
3	c	13	b	23	c
4	d	14	d	24	b
5	b	15	a	25	d
6	a	16	c	26	c
7	b	17	b	27	c
8	a	18	c	28	d
9	c	19	c	29	d
10	a	20	b	30	d

QUADRATIC INEQUALITIES

1	a	6	b	11	c
2	c	7	a	12	a
3	d	8	c	13	d
4	d	9	a	14	b
5	c	10	b	15	c

SAMPLE PAPER FOR ECAT

1	d	7	b	13	c	19	d	25	a
2	c	8	c	14	c	20	b	26	b
3	c	9	b	15	d	21	b	27	d
4	d	10	d	16	c	22	a	28	b
5	a	11	b	17	a	23	b	29	a
6	b	12	a	18	c	24	d	30	c

SAMPLE PAPER FOR GIKI

1	b	11	c	21	a	31	b	41	b
2	a	12	b	22	d	32	c	42	d
3	c	13	c	23	a	33	b	43	d
4	a	14	a	24	a	34	b	44	a
5	c	15	c	25	c	35	c	45	a
6	d	16	d	26	d	36	c	46	a
7	a	17	b	27	a	37	a	47	a
8	b	18	c	28	d	38	c	48	b
9	c	19	b	29	d	39	d	49	b
10	a	20	d	30	a	40	c	50	d

SAMPLE PAPER FOR FAST

1	c	11	d	21	b	31	d	41	c
2	b	12	c	22	b	32	c	42	c
3	c	13	d	23	d	33	c	43	c
4	d	14	d	24	a	34	b	44	d
5	b	15	b	25	b	35	b	45	d
6	c	16	a	26	c	36	b	46	d
7	a	17	b	27	a	37	c	47	b
8	c	18	d	28	a	38	a	48	d
9	a	19	d	29	c	39	c	49	b
10	b	20	b	30	b	40	a	50	c

SAMPLE PAPER FOR NUST

1	d	13	b	25	b	37	b	49	a
2	c	14	a	26	d	38	b	50	a
3	d	15	b	27	d	39	d	51	a
4	a	16	b	28	a	40	c	52	b
5	a	17	a	29	a	41	c	53	c
6	c	18	b	30	c	42	b	54	c
7	c	19	c	31	d	43	a	55	c
8	a	20	a	32	c	44	b	56	a
9	b	21	b	33	b	45	c	57	b
10	b	22	a	34	a	46	d	58	c
11	c	23	d	35	d	47	a	59	c
12	b	24	c	36	a	48	b	60	d