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Where Minds Pollinate

Class 12
Physics

Chapter 19
Handwritten Notes

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Electric Potential & Capacitor

Electric Potential Energy

The energy possessd by a charge due to its position in an electric field is called electric potential energy. Just like a stone at some height has gravitational potential energy, a charge kept in an electric field has electric potential energy.

Force on a test charge: Suppose a test charge q_0 is placed in a uniform electric field E. The force on it is: $F = q_0 E$

Work done in Moving a Charge:
If this charge is moved from point B to point A:

$W_{BA} = F \cdot d \cos\theta \qquad \because F = q_0 E$

$W_{BA} = q_0 E d \cos\theta$

Relation with Potential Energy:
The work done is equal to the change in potential energy. Electric potential energy is represented by U and change is ΔU.

$\Delta U = U_A - U_B = W_{BA} = q_0 E d \cos\theta$

Unit of electric potential energy is Joule.

Electric Potential (V)

Electric potential at a point is the potential energy per unit charge at that point. Depends only on the position in the field, not on the value of the charge.

$V = U/q_0$

Potential Difference (ΔV)

The work done per unit positive charge in moving it from one point to another in an electric field is called potential difference.

$\Delta V = V_A - V_B = W_{AB}/q_0 \qquad \because W_{AB} = \Delta U$

$\Delta V = \Delta U/q_0$

SI Unit is volt. If 1 joule of work is done in moving a 1 coulomb charge between two points, then the potential difference between those points is 1 volt.

1V = 1 Joule/1 Coulomb

The following multiples and sub-multiples of volt are commonly used:

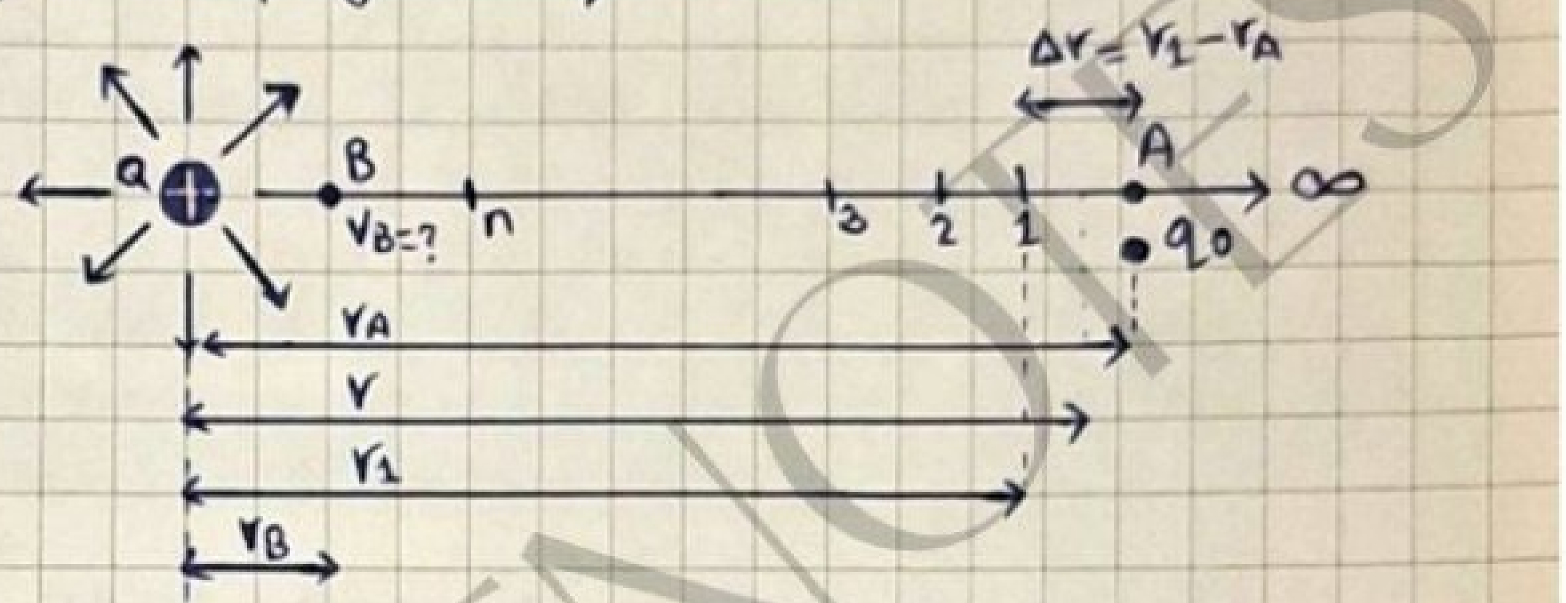
$1mV = 10^{-3} V$ $\qquad 1MV = 10^{6} V$

$1\mu V = 10^{-6} V$ $\qquad 1GV = 10^{9} V$

$1kV = 10^{3} V$

Electric Potential Due to a Point Charge

Electric potential at any point in an electric field is equal to work done in bringing a unit positive charge from infinity (outside of electric field) to that point, keeping it in equilibrium.



Consider a point charge "Q" fixed in space. Move a test charge "q_0" from point "A" to point "B" in equilibrium. Divide distance "A" to "B" in small displacement so the "E" is same in each displacement

$W_{A\to1} = \vec{F} \cdot \vec{d}$

$= Fd \cos 180$

$= Fd \cos(-1)$

$= -Fd = -qE\Delta r \qquad \because E = Q/4\pi\varepsilon_0 r^2$

$W_{A\to1} = \dfrac{-qQ}{4\pi\varepsilon_0 r^2}(r_1 - r_A) \qquad \because \Delta r = (r_1 - r_A)$

To make r^2 into $r_1 r_A$, we do some extra steps

$r = r_A + r_1/2$ — (i)

$\Delta r = r_1 - r_A$ — (ii)

$\Delta r + r_A = r_1$

put in equation (i)

$r = r_A + r_A + \Delta r/2$

$r = 2r_A + \Delta r/2$

Taking square on both sides

$r^2 = (2r_A + \Delta r/2)^2$

$r^2 = [2r_A/2 + \Delta r/2]^2$

$r^2 = [r_A + \Delta r/2]^2$

$r^2 = [r_A^2 + \Delta r^2/4 + 2r_A \Delta r/2] \qquad \because \Delta r^2$ is neglible

$r^2 = [r_A^2 + r_A \Delta r]$

$r^2 = [r_A^2 + r_A(r_1 - r_A)]$

$r^2 = [r_A^2 + r_A r_1 + r_A^2]$

$r^2 = r_A r_1$

Now put the value of r^2 in the equation

$W_{A\to1} = \dfrac{-Qq(r_1 - r_A)}{4\pi\varepsilon_0 r_1 r_A}$

$W_{A\to1} = \dfrac{-Qq}{4\pi\varepsilon_0}\left[\dfrac{r_1}{r_1 r_A} - \dfrac{r_A}{r_1 r_A}\right]$

$W_{A\to1} = \dfrac{-Qq}{4\pi\varepsilon_0}\left\{\dfrac{1}{r_A} - \dfrac{1}{r_1}\right\}$

$$W_{A\to 1} = \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_1} - \frac{1}{r_A}\right\}$$
$$W_{1\to 2} = \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_2} - \frac{1}{r_1}\right\}$$
$$W_{2\to 3} = \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_3} - \frac{1}{r_2}\right\}$$
$$W_{3\to 4} = \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_4} - \frac{1}{r_3}\right\}$$
$$W_{n\to B} = \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_B} - \frac{1}{r_n}\right\}$$
$$W_{A\to B} = \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_1} - \frac{1}{r_A}\right\} + \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_2} - \frac{1}{r_1}\right\} + \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_3} - \frac{1}{r_2}\right\} + \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_4} - \frac{1}{r_3}\right\} \cdots + \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_B} - \frac{1}{r_n}\right\}$$
$$W_{A\to B} = \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_1} - \frac{1}{r_A} + \frac{1}{r_2} - \frac{1}{r_1} + \frac{1}{r_3} - \frac{1}{r_2} + \frac{1}{r_4} - \frac{1}{r_3} \cdots + \frac{1}{r_B} - \frac{1}{r_n}\right\}$$
$$W_{A\to B} = \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_B} - \frac{1}{r_A}\right\}$$
$$W_{\infty\to B} = \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_B} - \frac{1}{\infty}\right\}$$
$$W_{\infty\to B} = \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r_B}\right\}$$
$$V_B = W_{\infty\to B}/q_0$$
$$V_B = \frac{1}{q_0}\frac{Qq_0}{4\pi\varepsilon_0}\left\{\frac{1}{r_B}\right\}$$
$$V_B = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{1}{r_B}\right\}$$

For general equations:

$$U = \frac{Qq}{4\pi\varepsilon_0}\left\{\frac{1}{r}\right\}$$
$$V = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Q}{r}\right\}$$

Capacitor

A device which is used for storing electrical charge is called capicator.

Parallel Plate Capacitor: A parallel plate capacitor consists of two parallel metal plates separated by small distance. A medium between plate is air or some other medium called dielectric.

Working: Capacitor is charged by connecting its plates to opposite terminal of battery. Some electrons are transfered from one plate through battery. Positive and negative charge will appear on plates. Mutual attraction between charges keep them bound on inner surface of two plates and charge stored. "Q" is the charge stored on plate of capacitor and "V" is potential difference between plates of capacitor.

$Q \propto V$

$Q = CV$ — (i)

"C" is constant of proportionality called capacitance of the capacitor.

$C = Q/V$ — (ii)

Capacitance: Ratio of charge stored to the potential difference between plates is called capacitance of capacitor. To produce "1V" potential difference between plates of capacitor, the amount of charge stored will be equal to capacitance. If potential difference is 1 volt.

$Q = CV$

$Q = C \quad \because V = 1$

For example, If $Q = 8C$ and $V = 4$, So $C = 8/4 = 2F$. If $Q = 16C$ and $V = 8$, So $C = 16/8 = 2F$ and If $Q = 24C$ and $12V$, So $C = 24/12 = 2F$.

SI unit of capacitance "Farad". Capacitance depends on area of plate, separation between plates. and medium between plates.

Capacitance of Parrallel Plate Capacitor: As we know,

$E = -\Delta V/\Delta r$

$E = -(V_B - V_A)/d$

$E = V_A - V_B/d$ — (i)

We also know,

$E = \sigma/\varepsilon_0 \quad \because \sigma = Q/A$

$E = Q/A\varepsilon_0$ — (ii)

Comparing equation (i) and equation (ii)

$V/d = Q/A\varepsilon_0$

$A\varepsilon_0/d = Q/V \quad \because Q/V = C$

$C = A\varepsilon_0/d$

$C_{vac} = A\varepsilon_0/d$

For a medium,

$C_m = A\varepsilon_m/d \quad \because \varepsilon_m = \varepsilon_0\varepsilon_r$

$C_m = A\varepsilon_0\varepsilon_r/d$

$C_m = C_v \times \varepsilon_r \quad \because C_m > C_{vac}$

$C_m/C_v = \varepsilon_r \quad \because \varepsilon_r > 1$

Combination of Capacitors

Series Combination: When the capacitors are connected plate to plate, i.e. right plate of one capacitor is connected to left plate of the next capacitor, then it is called series combination.

$Q_1 = Q_2 = Q_3 = Q$ (battery will supply same charge Q to each capacitor)

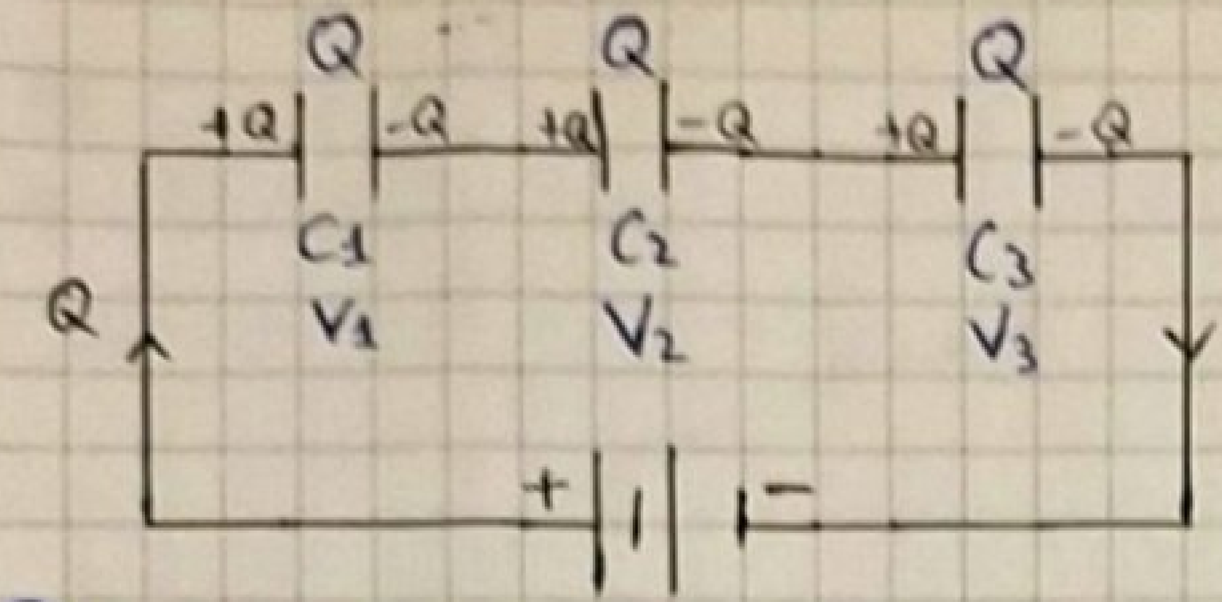
$V = V_1 + V_2 + V_3 \quad \because V = Q/C$

$V = Q/C_1 + Q/C_2 + Q/C_3$

$V = Q\{1/C_1 + 1/C_2 + 1/C_3\}$

$V/Q = 1/C_1 + 1/C_2 + 1/C_3 \quad \because V/Q = 1/C$

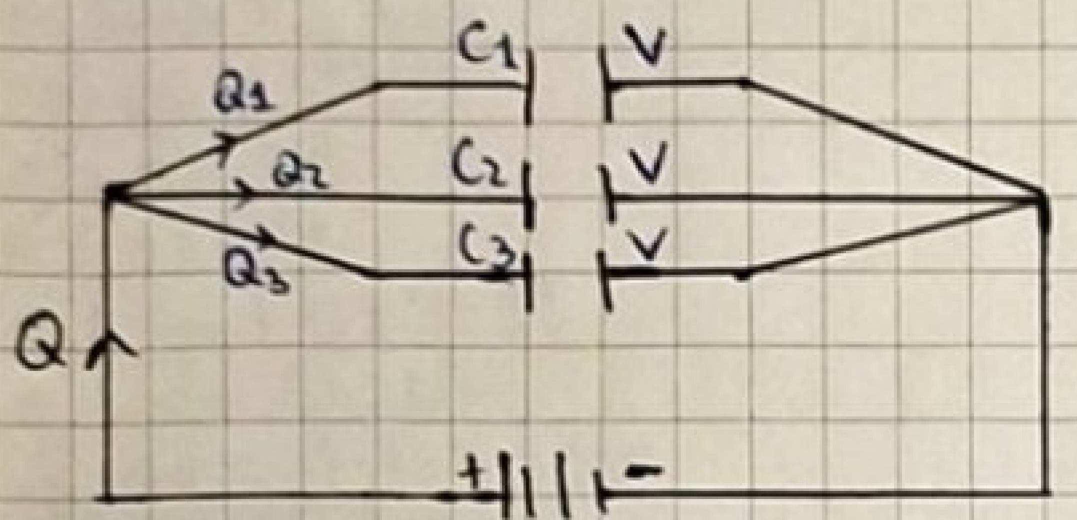
$1/C_e = 1/C_1 + 1/C_2 + 1/C_3 \ldots$



Parallel Combination:

When two or more capacitors are connected between the same two points, then the combination is called parallel combination.

$Q = Q_1 + Q_2 + Q_3$ $\quad \because Q = CV$
$Q = C_1V + C_2V + C_3V$
$Q = V(C_1 + C_2 + C_3)$
$Q/V = C_1 + C_3 + C_3$ $\quad \because Q/V = C$
$C_e = C_1 + C_2 + C_3$
$C_e = C_1 + C_2 + C_3 \ldots$



Energy Stored in a Capacitor

A capcitor stores energy in the form of an electric field between its plates. The energy stored is called electrical potential energy. When a capacitor is uncharged, its potential difference is zero. The initial charge placed on a capacitor experiances a change in voltage $\Delta V_i = 0$. The final charge placed on a capacitor experiances $\Delta V_f = V$, since the capacitor now has its full voltage "V" on it. The charge Q on capacitor is directly propotional to its potential difference V. Therefore graph is straight-line through the origin.

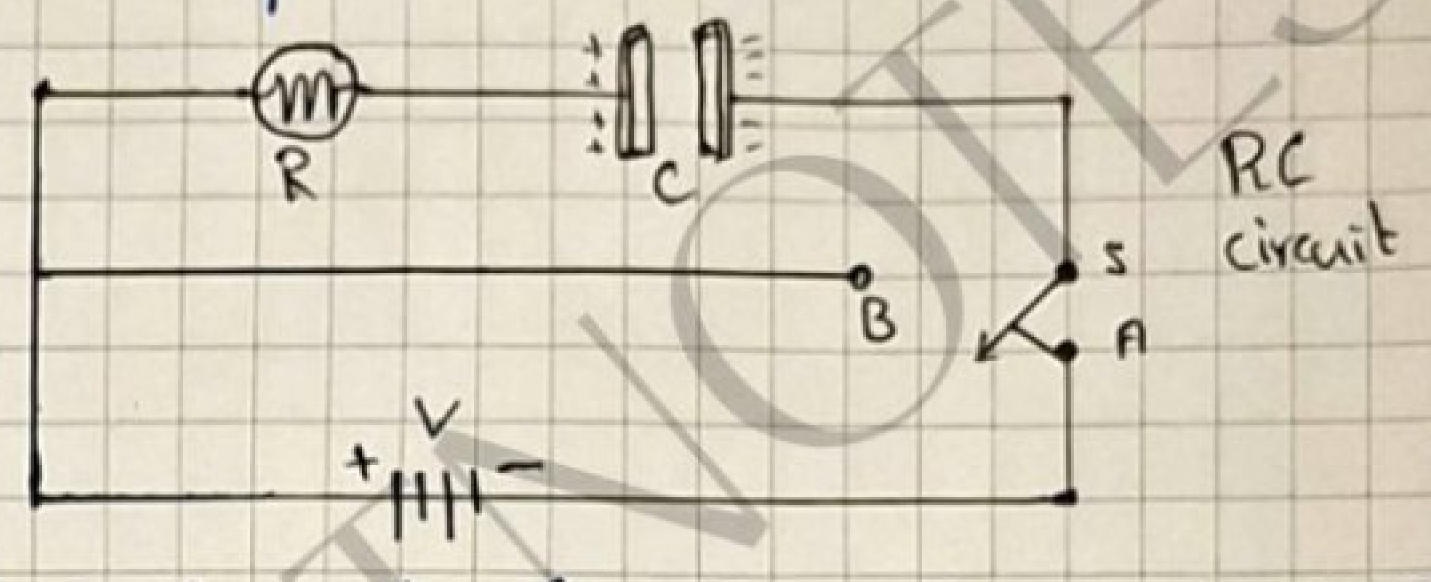
E = area of right-angled ΔOAB
$E = 1/2$ Base $\times$ Height
$E = 1/2\ Q \times V$ $\quad \because$ Base $= Q$
Substitue Q with CV $\quad \because$ Height $= V$
$E = 1/2\ (CV) \times V$
$E = 1/2\ CV^2$
Substitute the $V = Q/C$
$E = Q^2/2C$

These equations are valid for calculating energy stored in capacitor.

Uses of Capacitors

Used for filtering unwanted frequencies in radio and TV set, in camera flashes to store and quickly release a large amount of electrical energy, refrigrator capacitors keep the compressor running, to increase the speed of fan, in rectification circuit to act as a filter to reduce ripple voltage and etc etc.

Charging and Discharging of a Capacitor



A circuit consits of resistor and capacitor is RC circuit.

Charging of Capacitor:

When switch is connected to "A" terminal battery will start charging capacitor through resistor "R". At time $t = 0$, charge $q = 0$. After time "t", charge builds up on plates and repel extra charges that are arriving and the current drops. Charging will stop when potential difference is equal to e.m.f and maximum charge q_0 is stored. Equation for amount of charge stored:

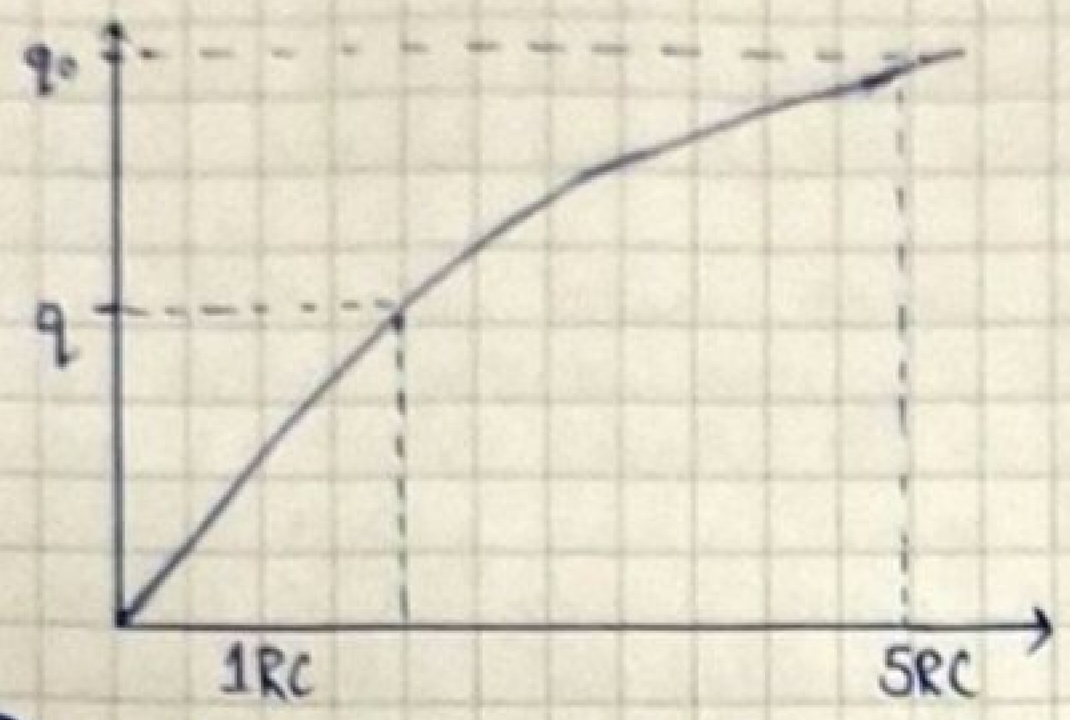
$q = q_0\{1 - e^{-t/RC}\}$

Time Constant:

The time taken to charge a capacitor to 63.2% of its maximum value "q_0" is called time constant. It is equal to product of "R" and "C". $\tau = RC$. It determines the charging time of a capacitor and depends upon ressistance "R" and capacitance "C".

At $t = RC$,
$q = q_0\{1 - e^{-RC/RC}\}$
$q = q_0\{1 - e^{-}\}$
$q = q_0\{1 - 1/e\}$
$q = q_0\{1 - 1/2.718\}$
$q = q_0\{0.632\}$
$q/q_0 = 0.632 \times 100$
$q/q_0 = 63.2\%$
$q_1 = 63.2\%\ q_0$
At $t = 2RC$,
$q_2 = 86\%\ q_0$
At $t = 3RC$
$q_3 = 94\%\ q_0$
At $t = 4RC$
$q_4 = 98\%\ q_0$

At $t = 5RC$
$q = 99\% \, q_0$



Discharging of Capacitor:

When switch "S" is connected to terminal "B", battery is disconnected and capacitor start discharging. The equation:

$q = q_0 e^{-t/RC}$

At time $t = RC$

$q = q_0 e^{-RC/RC}$

$q = q_0 e^{-1}$

$q = q_0 \, 1/e$

$q = q_0 \, [1/2.718]$

$q = q_0 \, (0.368)$

$q/q_0 = 0.368 \times 100$

$q_1 = 36.8\% \, q_0$

At time $t = 2RC$

$q_2 = 14\% \, q_0$

At time $t = 3RC$

$q_3 = 6\% \, q_0$

At time $t = 4RC$

$q_4 = 2\% \, q_0$

At time $t = 5RC$

$q_5 = 1\% \, q_0$