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Where Minds Pollinate

Class 12
Physics

Chapter 17

Exercise Solution

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Short Questions

1) If we halve the length of a simple pendulum to its original length, what is the alteration in the period of this pendulum? What is the new frequency?

$T = 2\pi\sqrt{l/g}$

The new time period will be:

$T' = 2\pi\sqrt{\frac{l/2}{g}}$

$T' = 2\pi\sqrt{l/2g}$

$T' = 1/\sqrt{2}\,(2\pi\sqrt{l/g})$

$T' = 1/\sqrt{2}\,T$

$T' = T/\sqrt{2}$

As frequency is the reciprocal of time period, so

$f' = 1/T'$

$f' = 1/\frac{T}{\sqrt{2}}$

$f' = \sqrt{2}/T$

2) If the amplitude of vibration of a body executing SHM is doubled, what will happen to the maximum kinetic energy?

The maximum kinetic energy ($K.E_{max}$) in SHM is given by: $K.E_{mx} = 1/2\, m\omega^2 A^2$

If amplitude is doubled, the new kinetic energy $K.E'_{mx}$ will be:

$K.E'_{mx} = 1/2\, m\omega^2 (2A)^2$

$K.E'_{mx} = 4(1/2\, m\omega^2 A^2)$

$K.E'_{mx} = 4K.E_{max}$

The maximum kinetic energy becomes four times greater if the amplitude is doubled.

3) When marching soldiers are about to cross a bridge, they break steps. Why?

When marching solidiers are about to cross a bridge, they are instructed to break steps (walk out of sync) to prevent resonance. Marching in union creates periodic and rhythmic forces on the bridge. If the frequency of their steps matches the natural frequency of the bridge, resonance can occur. Resonance causes the amplitude of vibrations to increase significantly which can lead to structural damage, vibrational instability or collapse of the bridge.

4) Suppose that a driving force has half frequency as compared to the frequency of an oscillator. Will it produce resonance? Similarly, if the driving frequency is twice the frequency of the oscillator, will it produce resonance?

Resonance occurs only when the driving frequency is equal to natural frequency of the oscillator.

$f_{driving} = f_{natural}$

Case 1: Driving frequency = 1/2 × natural frequency. The driving force is too slow and it fails to match the natural timing of the oscillator. Hence no resonance occurs.

Case 2: Driving frequency = 2 × natural frequency. The driving force is to fast so the system cannot respond in sync. Again, no resonance occurs.

5) Pendulum clocks are made to run at the correct rate by adjusting the pendulum's length. Suppose you move from one city to another where the acceleration due to gravity is slightly greater, taking your pendulum clock with you. Will you have to lengthen or shorten the pendulum to keep the correct time, other factors are remaining constant? Explain your answer.

Time period is given by:

$T = 2\pi\sqrt{l/g}$

$T^2 = 4\pi^2(l/g)$

$g = 4\pi^2/T^2\, l$

$g \propto l$

This shows that gravitational acceleration is directly propotional to the length of the pendulum. Hence, if the acceleration due to gravity is slightly greater at any city then we have to lengthen the pendulum to keep the correct time.

6) Two mass-spring systems vibrate with simple harmonic motion. If the spring constants are equal and the mass of one system is twice that of the other, which system has a greater period?

Time period for mass-spring system having mass m_1 of one system for simple harmonic motion is

$T_1 = 2\pi\sqrt{m_1/k}$

Time period for mass-spring system haveing mass m_2 of 2nd system for simple harmonic motion is

$T_2 = 2\pi\sqrt{m_2/k}$

Given that $m_2 = m_1$

$T_2 = 2\pi\sqrt{m_2/k} = 2\pi\sqrt{2m_1/k} = \sqrt{2}\,(2\pi\sqrt{m_1/k})$

$T_2 = \sqrt{2}T_1$

Hence system vith the larger mass has a greater period. Specifically, 22 times longer.

7. Give some applications in which resonance plays an important role.

Resonance plays important role in various fields.

Music & Instruments: Resonance amplifies sound in guitars, voilins and wind instruments.

Radio/TV Recievers: LC circuits use resonance to tune into specific frequencies.

MRI Scans: Magnetic resonance imaging relies on atomic resonance for medical diagnostics.

Structural Engineering: Bridges and buildings avoid resonant frequencies to prevent collapse.

8. A simple pendulum is set into vibrations and left untouched, eventually stops, why?

A simple pendulum is set into vibrations and left untouched, eventually stops. This is because of damping forces like air resistance and friction at the pivot. These forces gradually remove energy from the system, decreasing the amplitude of oscillation until the pendulum comes to rest.

9. Under what conditions motion of simple pendulum is SHM?

The motion of a simple pendulum approx

simple harmonic motion under the following two conditions:

1. Small Angular Displacement ($\theta \leq 10°$): Restoring force must be proportional to displacement ($F \propto -x$), which is true only when $\sin\theta \approx \theta$ (in radians). For larger angles, the motion becomes non-linear and deviates from SHM.

2. Negligible Damping: Air resistance and friction at the pivot must be minimal so that energy loss doesn't distort the sinusoidal motion.

10. At what positions is the velocity of a particle executing simple harmonic motion a) maximum b) minimum?

For a particle executing simple harmonic motion (SHM):

a) Velocity is maximum at the equilibrium position (ie when the displacement is zero). At this point, all the energy is kinetic, and the particle moves fastest.

b) Velocity is minimum at the extreme positions (ie maximum displacement). At these points, the particle momentarily stops before changing direction.

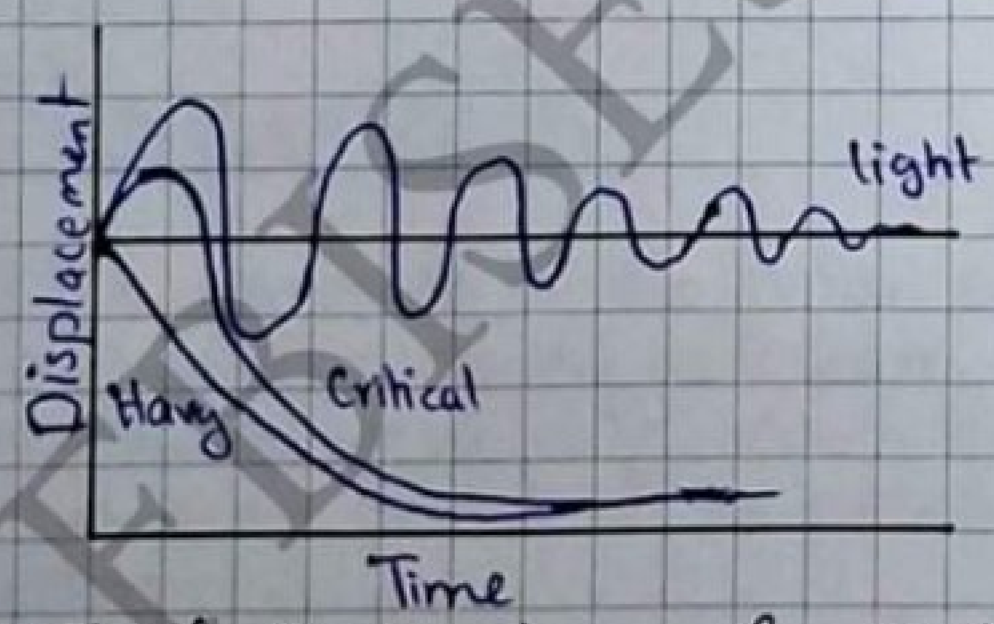
11. Show that the motion of projection of a body revolving in a circle describe SHM.

The projection of a body moving with uniform circular motion onto any diameter of the circle follows SHM because:

1. Displacement follows: $x = x_0 \cos\omega t$
2. Accelleration satisfies the SHM condition: $a = -\omega^2 x_0$

This show that the projection exhibits SHM since its acceleration is proportional and opposite to displacement.

12. Sketch the displacement-time graph to illustrate light, critical and heavy damping.



13. Justify the importance of critical damping in car suspension system.

Critical damping in a car suspension system is important because it allows the suspension to return to its equilibrium position as quickly as possible without oscillating after hitting a bump.

If underdamped: The car would bounce several times, making the ride uncomfortable and reducing tire contact with the road.

If overdamped: The suspension would return slowly, making the ride feel stiff and reducing the vehicle's ability to absorb consecutive bumps.

At critical damping: The system absorbs shocks efficiently, providing comfort, stability, and maximum road grip.

14. Differentiate free and forced oscillations.

	Forced Osci.	Free Osci.
Definition	Occur when a periodic external force drives the system.	Occur when a system oscillates under its own natural frequency without external force.
Energy Souce	Energy is continously supplied by the external force.	Energy comes from initial displacement or disturbence.
Frequency	Equal to the frequency of the external driving force.	Equal to the system's natural frequency.
Example	A pendulum swinging after being pushed once.	A child being pushed repeatedly on a swing.

15. How the period of a simple pendulum changes if mass of its bob is doubled? What change arises in time period of mass attached to a spring if same is done here?

1. Simple Pendulum: Time period of simple pendulum is given by: $T = 2\pi\sqrt{l/g}$ which shows that no change in time period if mass is doubled because time period is independant of mass.

2. Mass-Spring System: Time period of mass spring system is given by: $T = 2\pi\sqrt{m/k}$. Doubling mass increases T by:

$T' = 2\pi\sqrt{2m/k}$

$T' = \sqrt{2}(2\pi\sqrt{m/k})$

$T' = \sqrt{2}T$

Conceptual Questions

1. Why amplitude of vibrations is maximum at resonance?

At resonance, the natural frequency of the system equals the frequency of applied periodic force. This makes the driving force and velocity is the same phase, allowing maximum energy transfer to the systems. As a result, the vibrations become very large and the amplitude reaches its maximum value.

2. What is effect of damping on resonance?

Damping reduces the sharpness and height of the resonance peak. When damping is small, the amplitude at resonance is very large. As damping increases, the maximum amplitude decreases, and the system vibrates over a wider range of frequencies instead of sharply at one frequency.

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3. How does sharpness of resonance curve depends on damping?

The sharpness of the resonance curve is inversely related to damping. When damping is small, the resonance curve is sharp and narrow, showing a clear peak at the natural frequency. As damping increases, the curve becomes flatter and broader, meaning the sharpness of ressonance decreases.

4. Give necessary conditions for resonance to occur?

The necessary conditions for resonance to occur are:

1. The system must have a natural frequency of vibration.
2. An external periodic force must act on the system.
3. The frequency of the applied force must be equal to the natural frequency of the system.

5. What is meant by sharpness of resonance?

Sharpness of resonance means how quickly the amplitude falls on either side of the resonant frequency. A system with high sharpness shows a large amplitude only for a narrow range of frequencies, while a system with low sharpness vibrates for a vibrates for a wider range of frequencies.

6. What is difference between driving frequency and natural frequency?

Natural Frequency: The frequency at which a body vibrates freely without any external force. It depends on the mass and stiffness of the system.

Driving Frequency: The frequency of an external periodic force applied to the system. It is controlled by the source that drives the vibrations.

Numericals

1) The amplitude of the motion of a mass attached to a spring is 2.48m, while maximum speed of its mass is $4.36 ms^{-1}$. What is the period of the motion?

Data:

$x_0 = 2.48m$

$v_{max} = 4.36ms^{-1}$

$T = ?$

Solution:

$T = 2\pi/\omega$

$T = 2\pi/v/x_0$ $\quad (v = x_0\omega)$

$T = 2\pi x_0/v$

$T = \frac{2(3.14)(2.48)}{4.36}$

$T = 3.57s$

2) A particle moves, whose displacement as a function of time is: $x = 3.0\cos(2t)$, where distance is measured in meter and time in second........

Data:

(a) Calculate the amplitude, the frequency, the angular frequency and the period of this particle.

(b) Calculate the time at which the particle reaches the midpoint (i.e. $x=0$) and the turning point.

$x = 3.0\cos(2t)$ —— (i)

$x_0 = ?$

$f = ?$

$\omega = ?$

$T = ?$

$t = ?$ when $x = 0$

Solution:

General equation for displacement is given by,

$x = x_0\cos(\omega t)$ —— (ii)

Comparing equation (i) and (ii)

$x_0 = 3.0m$

$\omega = 2 rads^{-1}$

For frequency,

$\omega = 2\pi f$

$f = \omega/2\pi$

$f = 2/2(3.14)$

$f = 0.32Hz$

For time period,

$T = 1/f$

$T = 1/0.32$

$T = 3.1s$

Time to reach midpoint ($x=0$).

The midpoint occurs when the cosine function is 0.

$\cos 2t = 0$

$2t = \cos^{-1} 0$

$2t = 90 = \pi/2$

$2t = \pi/2 + n\pi \quad (n = 0,1,2,3 \ldots)$

$t = \pi/4 + n\pi/2$

For time $n=0$,

$t = \pi/4 + 0$

$t = 0.785$

Time to reach turning points ($x = \pm A$) $\quad \because A = x_0$

The turning points occur when cosine function is ± 1.

$\cos 2t = \pm 1$

$2t = \cos^{-1}(\pm 1)$

$2t = n\pi \quad (n = 0,1,2,3 \ldots)$

$t = n\pi/2$

For time $n=0$,

$t = 0s$

For time $n=1$,

$t = \pi/2$

$t = 1.57s$

3) A particle executes simple harmonic motion, that moves back and forth along x-axis between points $-0.20m$ and $+0.20m$. The period of the motion is 1.2s. At the time $t=0$, the particle is at $+0.20m$ and its velocity is zero.

(a) What is the frequency of the motion and the angular frequency?

b) What is the amplitude of the motion?

c) At what time will the particle reach the point $x=0$? At what time will the particle reach the point $x=0.10m$?

d) What is the speed of the particle when it is at $x=0$? What is the speed of the particle when it reaches the point $x=0.10m$

Data:

$x_o = \pm 0.20m$

$T = 1.2s$

$x = +0.20m$ at $t=0$

Solution:

a) $f = ?$ $\omega = ?$

$f = 1/T$

$f = 1/1.2$

$f = 0.833Hz$

$\omega = 2\pi f$

$\omega = 2(3.14)(0.833)$

$\omega = 5.23 rads^{-1}$

b) $x_o = ?$

$x_o = 0.20m$

c) $t = ?$ when $x = 0$

$x = x_o \cos(\omega t)$

$x = 0.20\cos(5.23t)$

$0 = 0.20\cos(5.23t)$

$\cos(5.23t) = 0$

$5.23t = \pi/2$

$t = \frac{\pi}{(2)(5.23)}$

$t = 0.30s$

Now, $t = ?$ when $x = 0.10m$

$0.10 = 0.20\cos(5.23t)$

$0.5 = \cos(5.23t)$

$5.23t = \pi/3$

$t = \frac{\pi}{(3)(5.23)}$

$t = 0.20s$

d) $v = ?$ when $x = 0$ $\because t = 0.30s$

$v = x_o \sin\omega(\omega t)$

$v = (0.20)(5.23)\sin(5.23t)$

$v = 1.046\sin(5.23(0.30))$

$v = 1.046\sin 1.57$

$v = 1.05ms^{-1}$

For, $v = ?$ when $x = 0.10m$ $\because t = 0.20s$

$v = 1.046\sin(5.23(0.20))$

$v = 0.91ms^{-1}$

4) A mass of 8.0kg is attached to a spring and oscillate with amplitude of 0.25m and has a frequency of 0.60Hz. What is the energy of the motion?

Data:

$m = 0.8kg$

$x_o = 0.25m$

$f = 0.60Hz$

$E = ?$

Solution:

$E = 1/2 m\omega^2 x_o^2$

$E = 1/2 m(2\pi f)^2 x_o^2$

$E = 1/2 m 4\pi^2 f^2 x_o^2$

$E = 2m\pi^2 f^2 x_o^2$

$E = 2(0.8)(3.14)^2(0.60)^2(0.25)^2$

$E = 0.355J$

5) Calculate the period and frequency of a 3.5m long pendulum at the following locations:

a) at Karachi, where $g = 9.832 ms^{-2}$

b) at K-2, where $g = 9.782 ms^{-2}$

Data:

$T = ?$

$f = ?$

$l = 3.5m$

Solution:

a) $T = 2\pi\sqrt{l/g}$

$T = 2\pi\sqrt{3.5/9.832}$

$T = 3.747s$

$f = 1/T$

$f = 1/3.747$

$f = 0.2669Hz$

b) $T = 2\pi\sqrt{l/g}$

$T = 2\pi\sqrt{3.5/9.782}$

$T = 3.757s$

$f = 1/T$

$f = 1/3.757$

$f = 0.2662Hz$

6) In a car engine, a piston executes SHM with amplitude of 0.41m. The engine is running at angular frequency of 2400rpm ($251 rads^{-1}$). What is the maximum speed of the piston?

Data:

$x_o = 0.41m$

$\omega = 251 rads^{-1}$

$v = ?$

Solution:

$v = x_o\omega$

$v = (0.41)(251)$

$v = 102 ms^{-1}$

7) A ball connected to a spring executes SHM. At $t=0$, its displacement is 0.50m and its acceleration is $-0.72ms^{-2}$. The phase constant for its motion is 0.84 rad. What's the ball's displacement at $t = 3.4s$

Data:

At $t=0$,

$x = 0.50m$

$a = -0.72ms^{-2}$

$\varphi = 0.84 rad$

$x = ?$ when $t = 3.4s$

Solution:

In SHM,

$a = -\omega^2 x$

$\omega^2 = -a/x$

$\omega^2 = -\frac{-0.72}{0.50}$

$\omega^2 = 1.44$

$\omega = 1.2\,rad\,s^{-1}$

$x = x_o \cos(\omega t + \phi)$

$x_o = \dfrac{x}{\cos(\omega t + \phi)}$

$x_o = \dfrac{0.50}{\cos((1.2)(0) + (0.84))}$

$x_o = \dfrac{0.50}{\cos(0.84)} = 0.75$

$x = x_o \cos(\omega t + \phi)$

$x = (0.75)\cos((1.2)(3.4) + 0.84)$

$x = 0.15\,m$

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