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Where Minds Pollinate

Class 12

Physics

Chapter 17

Handwritten Notes

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SIMPLE HARMONIC MOTION

Simple Harmonic Motion

Such oscillatory motion in which acceleration of a particle is directly propotional to its displacement from mean position and is always directed towards mean position.

$a \propto -x$ —(i)

Some examples are, motion of simple pendulum, motion of mass attached to a spring.

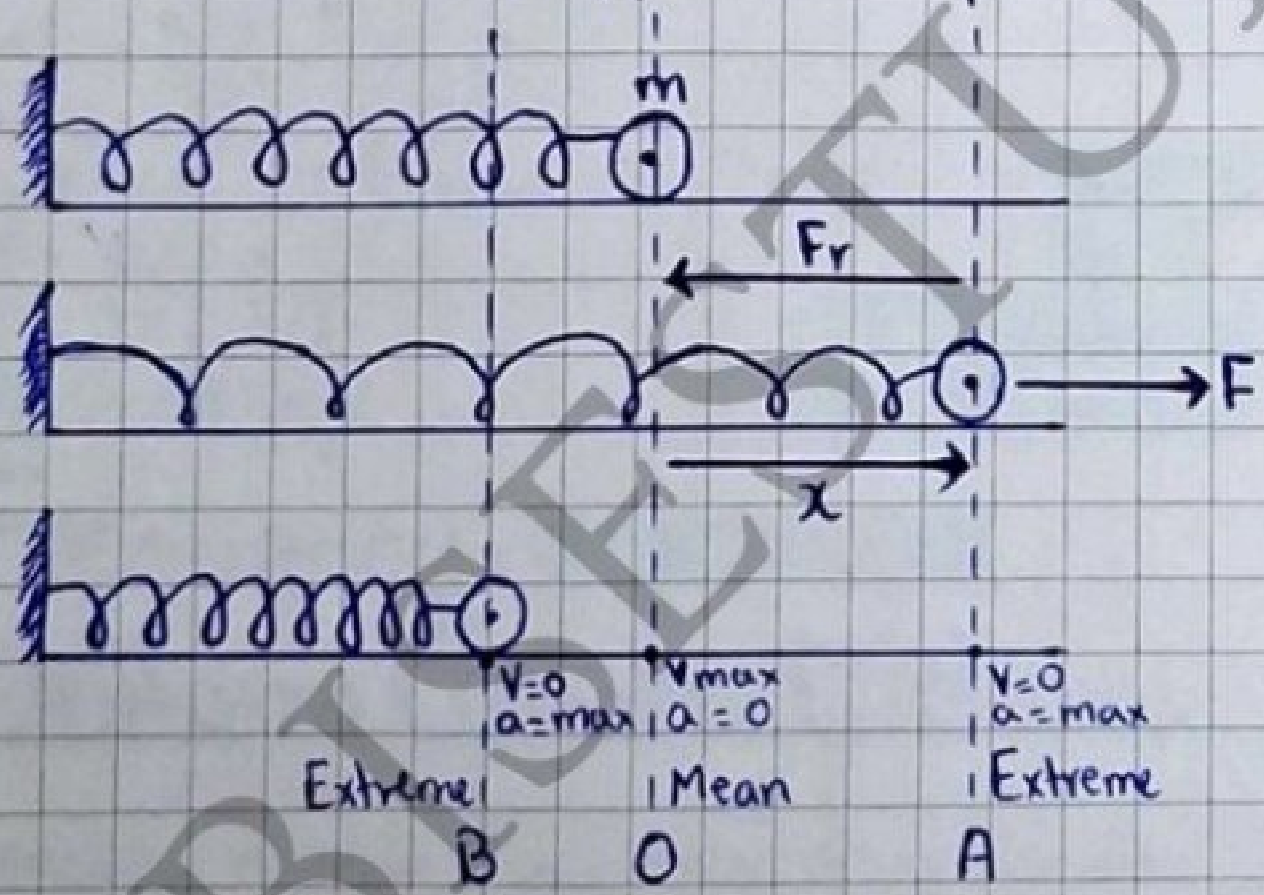
Mass attached to a spring: Let a mass "m" attached to a spring, mass of spring is ignored, mass "m" is free to move on frictionless horizontol surface. When there is no net force mass is at equilibrium position. When we apply a force "F" to displace mass towards right from mean position, it covers some displacement "x" and restoring force is produced in the spring.

Restoring force $\propto$ Displacement

$F \propto -x$

$F = -kx$ —(ii)

"K" is spring constant. Negative sign show that direction of force is opposite to displacement.



When the mass is released, it moves towards mean position. Due to inertia, it can not stop at mean position and start moving towards left side. Now spring begins to compress and mass slows down. At left extreme position mass comes to rest and again a restoring force is produced that will push back the mass towards the mean position. This results a back and forth motion about mean position. Acceleration in the body is given by Newton's 2nd law.

$F = ma$ —(iii)

Comparing equation (ii) and (iii)

$ma = -kx$

$a = -k/m\, x$ —(iv)

Another formula of acceleration,

$a = -\omega^2 x$ —(v)

Comparing equation (v) and (iv)

$-\omega^2 x = -k/m\, x$

$\omega = \sqrt{k/m}$ $\quad\because k/m$ is constant

So, equation (iv) becomes

$a = -\text{Constant}(x)$

$a \propto -x$

Velocity: Velocity is zero at point "A" and "B" and maximum at point "O".

Acceleration: Acceleration is maximum at point "A" and "B" and zero at point "O".

Time Period: Time taken to complete one complete vibration is time period.

As,

$\omega = 2\pi f$ $\quad \because \omega = 2\pi/T \Rightarrow \omega = 2\pi f$

$\omega = 2\pi/T$ $\quad \because T = 1/f \quad \because f = 1/T$

$T = 2\pi/\omega$

$T = 2\pi/\sqrt{k/m}$

$T = 2\pi\sqrt{m/k}$

Frequency: As we know, frequency is indirect proportional to time and vice versa, so,

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Simple Pendulum: Ideal pendulum consists of small but heavy mass "m" which is supported by light and inextensible string of length "l". The other end of string is attached to fixed frictionless support.

Motion: At equilibrium position weight is equal and opposite to tension and net force is zero.

$T = mg$ —(i)

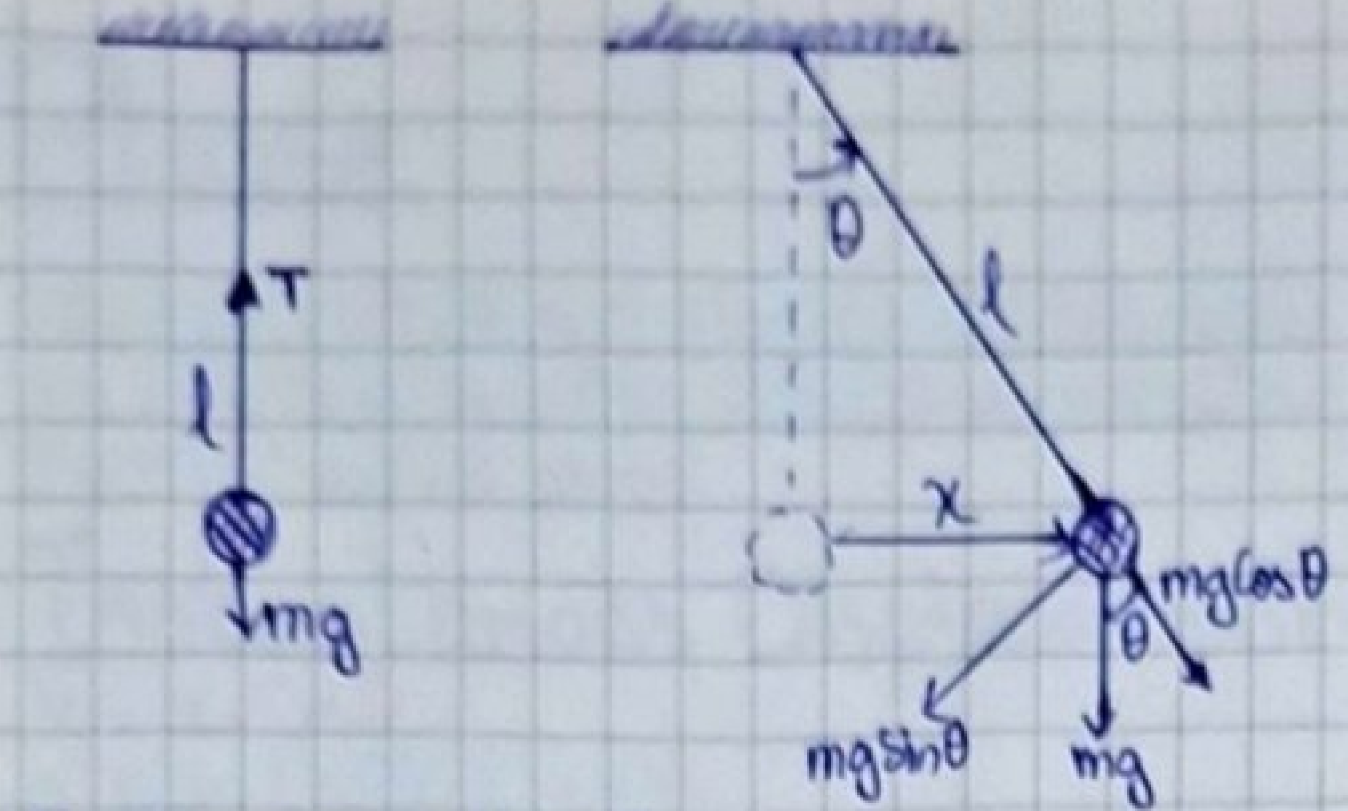
When bob is displaced slightly from mean position, only one component of weight is canceled by tension.

$T = mg\cos\theta$ —(ii)

Remaining component of weight "$mg\sin\theta$" is restoring force that makes it to accelerate towards mean position.

$F = -mg\sin\theta$ —(iii)

Negative sign shows that the force is acting towards the mean position.



For small value of "θ" θ

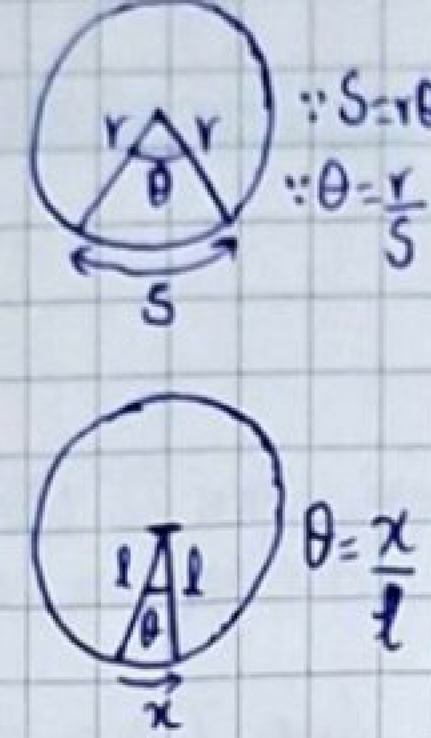
$\sin\theta \approx \theta$

put in equation (iii)

$F = -mg\theta$ — (iv)

put value in equation (iv)

$F = -mg\,x/l$ — (v)



From Newton's 2nd Law,

$F = ma$, put in equation (v)

$ma = -mg\,x/l$

$a = -g\,x/l$ —— (vi)

"g" and "l" is constant, so

$a = \text{constant}(-x)$

$a \propto -x$

Time Period: Time taken to complete one vibration is called time period.

$a = -g\,x/l$ — (vi)

Acceleration of a body executing SHM.

$a = -x\omega^2$ — (vii)

Comparing equation (vi) and (vii)

$-x\omega^2 = -g\,x/l$

$\omega^2 = g/l$

$\omega = \sqrt{g/l}$ — (viii)

but, $\omega = 2\pi f$ and $\omega = 2\pi/T$ ∵ $T \propto 1/\sqrt{g}$

$T = 2\pi/\omega$ ∵ $T \propto \sqrt{l}$

$T = 2\pi/\sqrt{g/l}$ ∵ "T" is independent of mass of the bob

$T = 2\pi\sqrt{l/g}$

Frequency: $f = 1/2\pi\,\sqrt{g/l}$

Uniform Circular Motion:

Consider a point "P" is moving on a circular path of radius "x_o" in xy plane at constant angular velocity "ω". At the same time, "Q" be the projection (shadow) of "P" on x-axis undergoes oscillation along the diameter of circular path between $-x_o$ and $+x_o$. It is seen that the time period of one revolution of point P on the circular path is equal to the time period of one oscillation of point Q on the diameter. Therefore, the angular speed of P is the same as the linear speed of Q. Thus, the expression for displacement velocity and acceleration P also hold for the Q.

Expression for Displacement: As the point P moves on the circle, at some instant t, the angle made by the line OP with x-axis is $\theta = \omega t$, the instantaneous displacement x of Q at that instant can be calculated as:

From ΔOPQ,

$\cos\theta = x/x_o$

$x = x_o\cos\theta$

$x = x_o\cos\omega t$ — (i)

Expression for Velocity: Velocity of the point P at instant t is $v_p = x_o\omega$, which is directed along the tangent. Instantaneous velocity v of the point Q is the projection of v_p on the x-axis, which is the horizontal component of v_p.

$v = v_p\cos(90° - \theta)$

$v = v_p\sin\theta$ ∵ $\sin^2\theta + \cos^2\theta = 1$

$v = v_p\sqrt{1-\cos^2\theta}$ $\quad \sin^2\theta = 1-\cos^2\theta$

$v = x_o\omega\sqrt{1-\cos^2\theta}$

Using equation (i)

$v = \omega\sqrt{x_o^2 - x^2}$

Expression for Acceleration: Since ω is constant, so the acceleration of the point P is centripetal, i.e. $a_p = x_o\omega^2$, which is directed towards O. The acceleration "a" of Q is the horizontal component of a_p.

$a = -a_p\cos\theta$

$a = -x_o\omega^2\cos\theta$

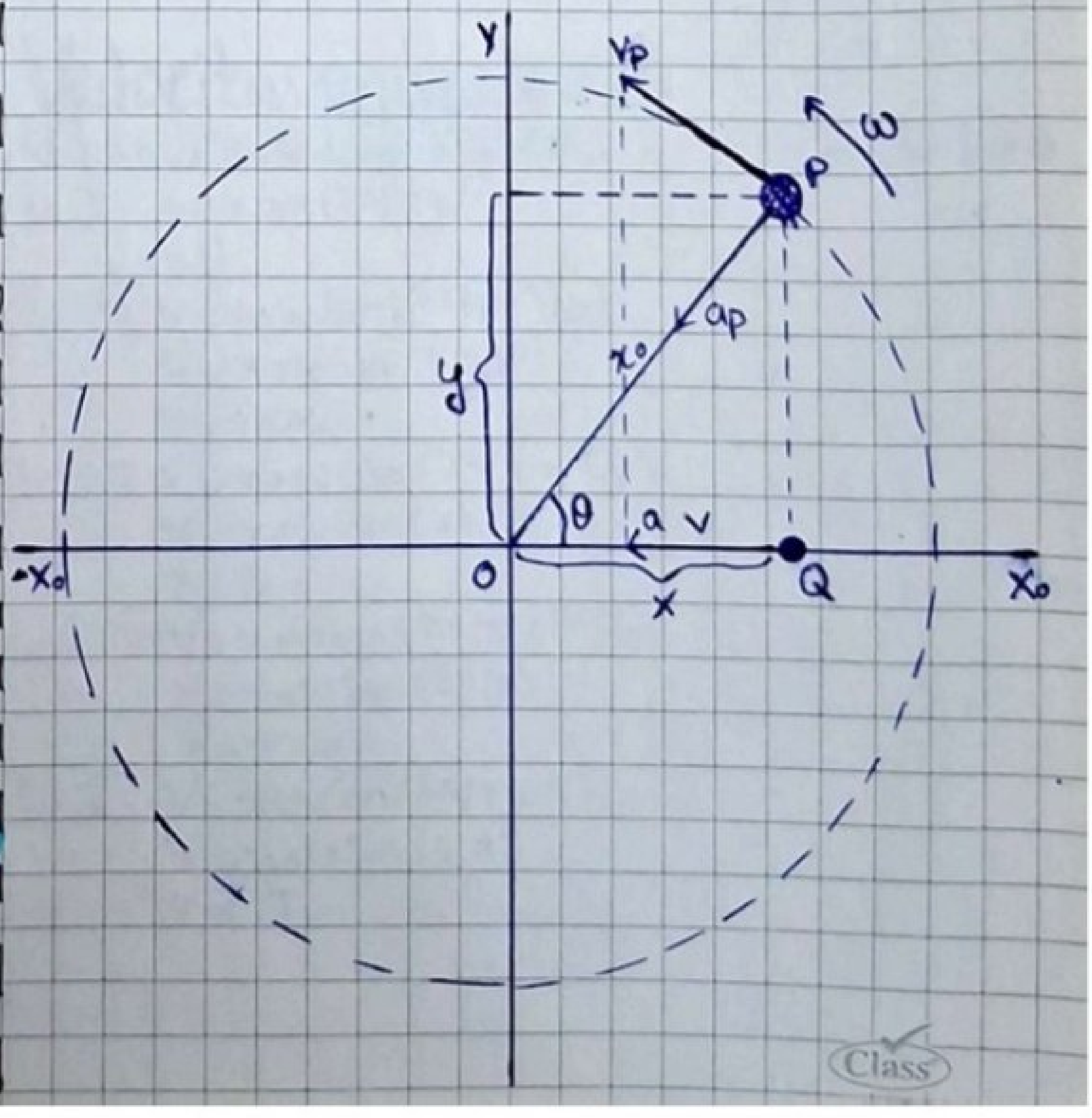
Since the velocity is decreasing, so the negative sign shows the acceleration of Q is directed towards O. Using $\cos\theta = x/x_o$,

$a = -\omega^2 x$

As, ω is constant so,

$a \propto -x$

When a body moves in a circular path, its projection (shadow) undergoes simple harmonic motion on the diameter of the circle.



Phase

The angle that specifies the displacement as well as direction of a point executing SHM is called phase.

Instantanoes Displacement:

$x = x_0 \cos\theta$

$x = x_0 \cos(\omega t)$ — (i)

$\theta = \omega t$ is phase $\quad \because \omega = \theta/t = 2\pi/T$

Initial Phase:

If particle is already at some initial angle ϕ.

$x = x_0 \cos(\omega t + \phi)$ — (ii)

ϕ is initial phase.

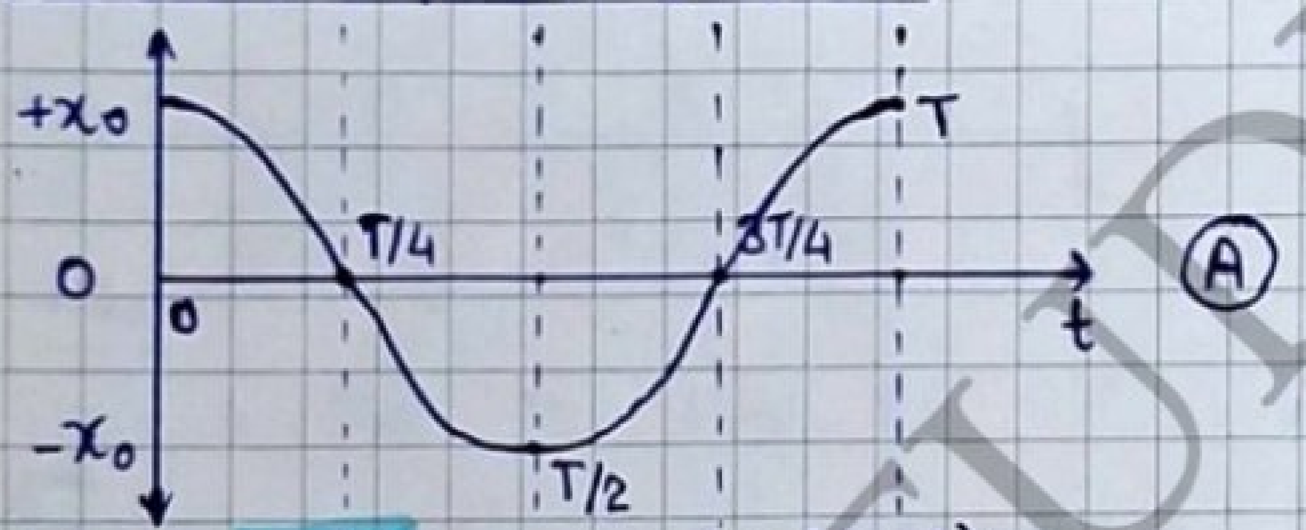
Physical Significance:

It indicates the object's position and direction of motion with in its cycle.

Take value of $t = 0, T/4, T/2, 3T/4$ and T and check displacement from mean position.

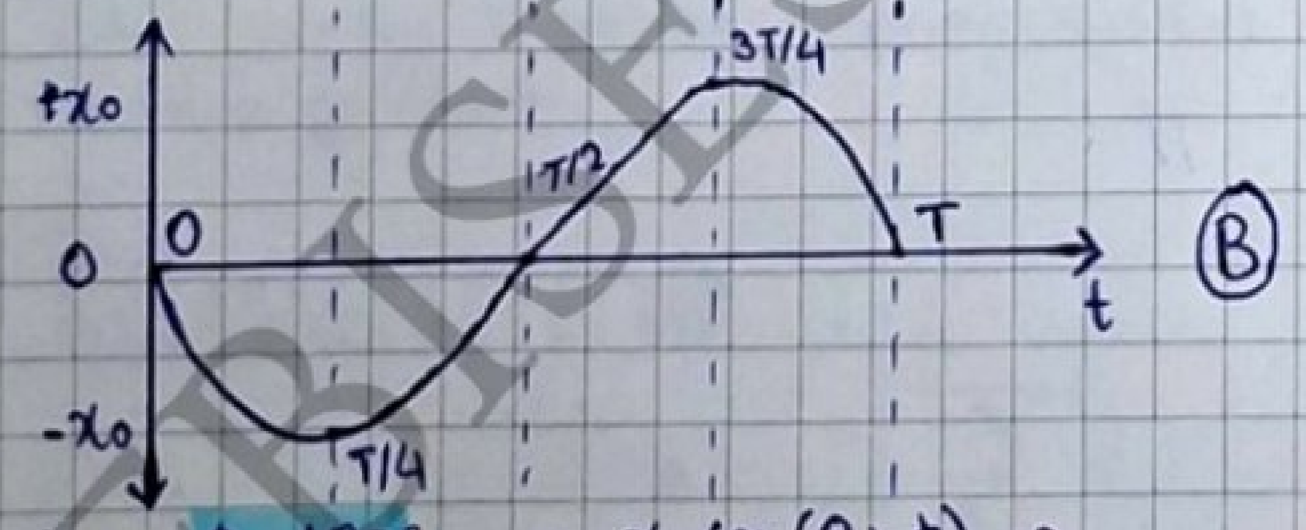
When, $\phi = 0°$ $\quad x = x_0 \cos(\omega t + \phi) = ?$

t	0	T/4	T/2	3T/4	T
θ+ϕ	0°	π/2	π	3π/2	2π
x	$+x_0$	0	$-x_0$	0	$+x_0$



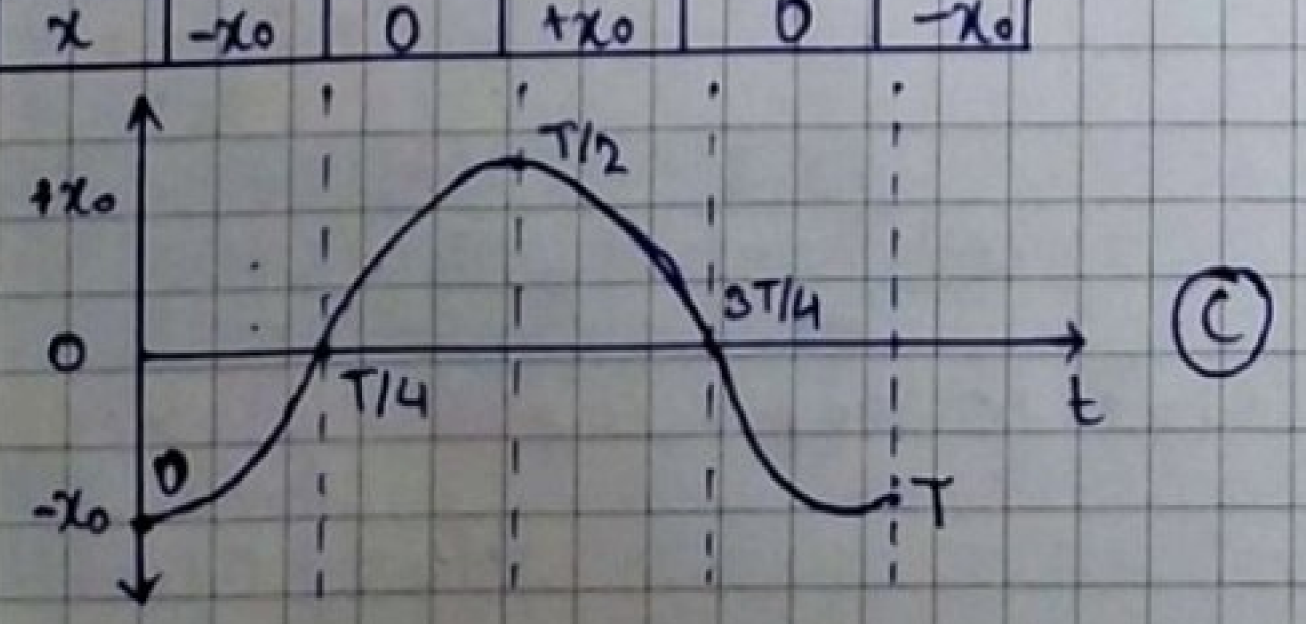
When $\phi = 90°$ $\quad x = x_0 \cos(\theta + \phi) = ?$

t	0	T/4	T/2	3T/4	T
θ+ϕ	π/2	π	3π/2	2π	5π/2
x	0	$-x_0$	0	$+x_0$	0



When $\phi = 180°$ $\quad x = x_0 \cos(\theta + \phi) = ?$

t	0	T/4	T/2	3T/4	T
θ+ϕ	π	3π/2	2π	5π/2	3π
x	$-x_0$	0	$+x_0$	0	$-x_0$



Phase Difference:

In terms of waves phase difference refers to the difference in angular position of two waves at same time. Curve in the figure "B" leads the curve in figure "A" by 90 so phase difference is 90°. Curve in figure "C" leads the curve in figure "A" by 180° so phase difference is 180°.

Graphical Representation for SHM

Displacement: $x = x_0 \cos(\omega t)$

$t = 0$, $x = x_0 \cos(2\pi/T \times 0)$

$x = x_0 \cos(0)$

$x = +x_0$

$t = T/4$, $x = x_0 \cos(2\pi/T \times T/4)$

$x = x_0 \cos(\pi/2)$

$x = 0$

$t = T/2$, $x = x_0 \cos(2\pi/T \times T/2)$

$x = x_0 \cos(\pi)$

$x = -x_0$

$t = 3T/4$, $x = x_0 \cos(2\pi/T \times 3T/4)$

$x = x_0 \cos(3\pi/2)$

$x = 0$

$t = T$, $x = x_0 \cos(2\pi/T \times T)$

$x = x_0 \cos(2\pi)$

$x = +x_0$

Velocity: $V = x_0\omega\sqrt{1-\cos^2\theta}$

$V = x_0\omega \sin\theta = x_0\omega \sin(\omega t)$ $\quad \because 1 - \cos^2\theta = \sin^2\theta$

$t = 0$, $x_0\omega \sin(2\pi/T \times 0)$

$V = 0$

$t = T/4$, $V = x_0\omega \sin(2\pi/T \times T/4)$

$V = x_0\omega \sin(\pi/2)$

$V = +x_0\omega$

$t = T/2$, $V = x_0\omega \sin(2\pi/T \times T/2)$

$V = x_0\omega \sin(\pi)$

$V = 0$

$t = 3T/4$, $V = x_0\omega \sin(2\pi/T \times 3T/4)$

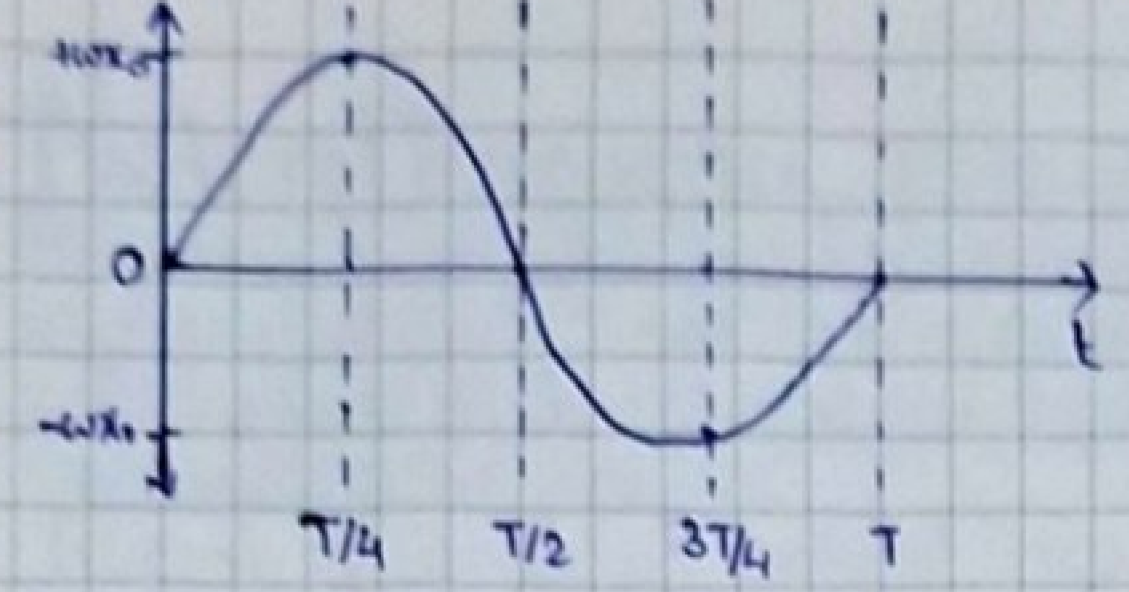
$V = x_0\omega \sin(3\pi/2)$

$V = -x_0\omega$

$t = T$, $V = x_0\omega \sin(2\pi/T \times T)$

$V = x_0\omega \sin(2\pi)$

$V = 0$



Acceleration: $a = -x\omega^2 = -x_0\omega^2\cos(\omega t)$

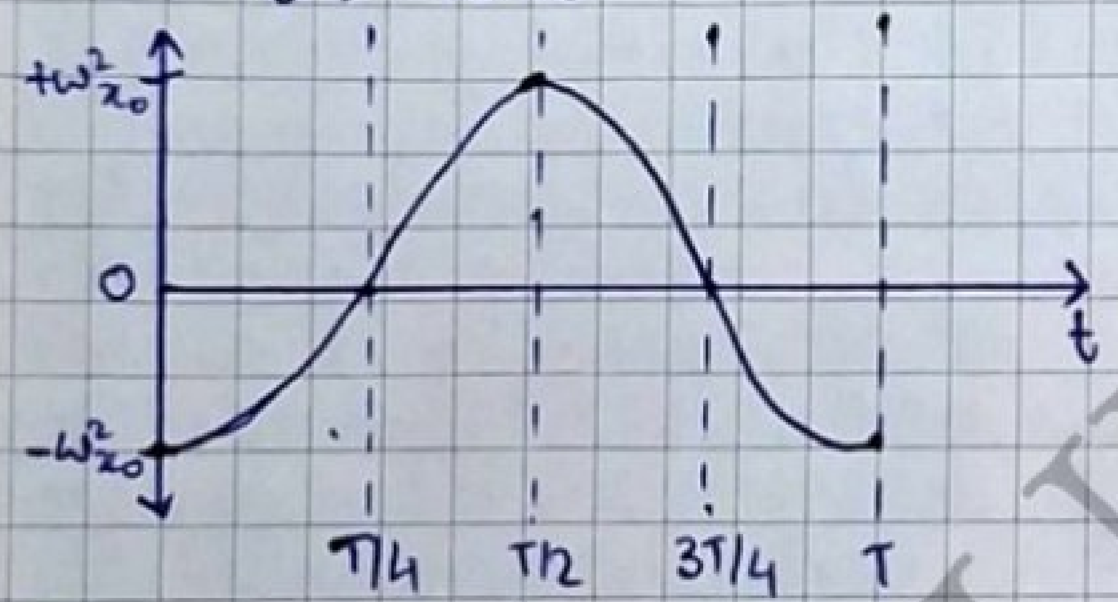
<u>t=0</u>, $a = -x_0\omega^2\cos(2\pi/T \times 0)$
$a = -x_0\omega^2\cos(0)$
$a = -x_0\omega^2$

<u>t=T/4</u>, $a = -x_0\omega^2\cos(2\pi/T \times T/4)$
$a = -x_0\omega^2\cos(\pi/2)$
$a = 0$

<u>t=T/2</u>, $a = -x_0\omega^2\cos(2\pi/T \times T/2)$
$a = -x_0\omega^2\cos(\pi)$
$a = +x_0\omega^2$

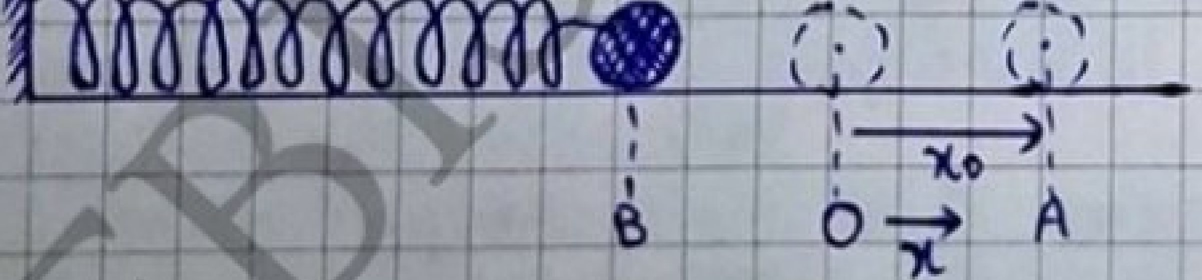
<u>t=3T/4</u>, $a = -x_0\omega^2\cos(2\pi/T \times 3T/4)$
$a = -x_0\omega^2(3\pi/2)$
$a = 0$

<u>t=T</u>, $a = -x_0\omega^2\cos(2\pi/T \times T)$
$a = -x_0\omega^2\cos(2\pi)$
$a = -x_0\omega^2$



Energy Conservation in SHM

Total energy of vibrating spring system remains constant.



Hook's Law, $F_r = -kx$
$F_{op} = -F_r$
$F = -[-kx]$
$F = kx$
At mean position, $x=0$, $F=0$
At extreme position, $x=x_0$, $F=kx_0$
At any position, $x=x$, $F=kx$
Potential energy at any position, PE = W
$P.E = F_{av}d$
$P.E = \left[\frac{0+kx}{2}\right]x$
$P.E = kx/2\ x$
$P.E = kx^2/2$
$P.E = 1/2\ kx^2$

Potential energy at mean position,
$P.E = 1/2\ kx^2$
$P.E = 1/2\ k(0)$ $\quad \because x=0$
$P.E = 0$

Potential energy at extreme position,
$P.E = 1/2\ kx^2$
$P.E = 1/2\ k(x_0)^2$ $\quad \because x = x_0$
$P.E = 1/2\ kx_0^2$

Kinetic Energy at any point, $K.E = 1/2\ mv^2$
$K.E = 1/2\ m(\omega\sqrt{x_0^2-x^2})^2$ $\quad \because v = \omega\sqrt{x_0^2-x^2}$
$K.E = 1/2\ m\omega^2(x_0^2-x^2)$ $\quad \because \omega^2 = k/m$
$K.E = 1/2\ m\, k/m\, (x_0^2-x^2)$
$K.E = 1/2\ k(x_0^2-x^2)$

Kinetic Energy at mean position,
$K.E = 1/2\ k(x_0^2-x^2)$
$K.E = 1/2\ k(x_0^2-0)$ $\quad \because x=0$
$K.E = 1/2\ kx_0^2$

Kinetic energy at extreme position,
$K.E = 1/2\ k(x_0^2-x^2)$
$K.E = 1/2\ k(x_0^2-x_0^2)$ $\quad \because x=x_0$
$K.E = 0$

Total Energy at mean position,
$E_T = K.E_{mean} + P.E_{mean}$
$E_T = 1/2\ kx_0^2 + 0$
$E_T = 1/2\ kx_0^2$

Total Energy at extreme position,
$E_T = K.E_{ex} + P.E_{ex}$
$E_T = 0 + 1/2\ kx_0^2$
$E_T = 1/2\ kx_0^2$

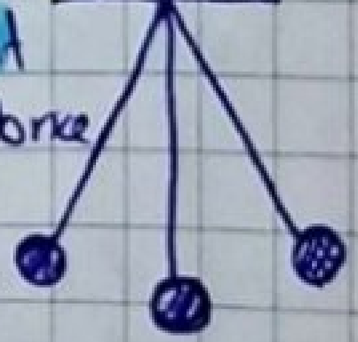
Total Energy at any position,
$E_T = P.E_{any} + K.E_{any}$
$E_T = 1/2\ kx^2 + 1/2\ k(x_0^2+x^2)$
$E_T = 1/2\ kx^2 + 1/2\ kx_0^2 + 1/2\ kx^2$
$E_T = 1/2\ kx_0^2$

It can also be written as:
$T.E = 1/2\ m\omega^2 x_0^2$ $\quad \because \omega = \sqrt{k/m}$, $k = m\omega^2$

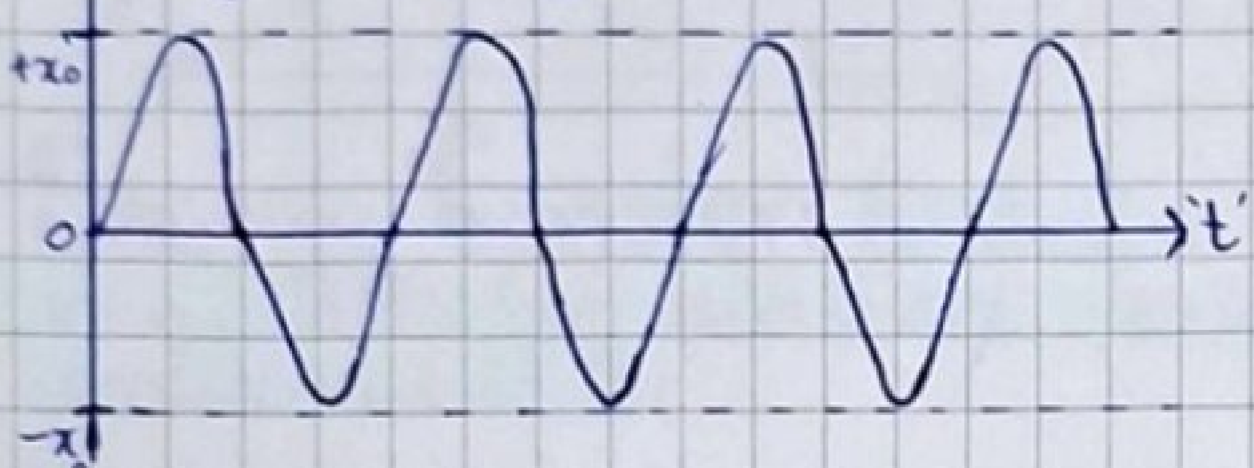
Total energy of the mass-spring system is constant and is propotional to square of the amplitude of oscillation. This statement is ~~constant~~ equally valid for all bodies executing SHM.

Free Oscillations

A body is said to be executing free oscillations when it oscillates under influence of a restoring force without any external force acting on it.
Every oscillator has its natural frequency of vibration with which it vibrates freely after initial disturbnce. For example, an ideal simple pendulum vibrates freely with its natuarl frequency.

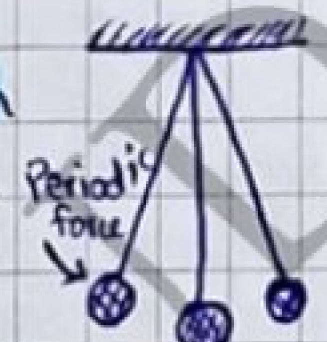


Natural Frequency: A natural frequency is the frequency of oscillating object at which it tends to vibrate without external force or damping when it is disturbed from equilibrium position.
In case of pendulum natural frequency depends upon the length of pendulum.
$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ $\quad \because T = 2\pi\sqrt{\frac{l}{g}}$
$f = \text{Constant } 1/\sqrt{l}$
Free oscillations possess constant ~~damping~~ amplitude and time period. It do not ~~go~~ undergo damping.



Forced Oscillations

If external force makes the object to oscillate at frequency of applied force rather than its natural frequency, then such oscillations are called forced oscillations.
When we apply a periodic force on oscillator, amplitude of oscillations changed and it start oscillations with frequency of applied force.

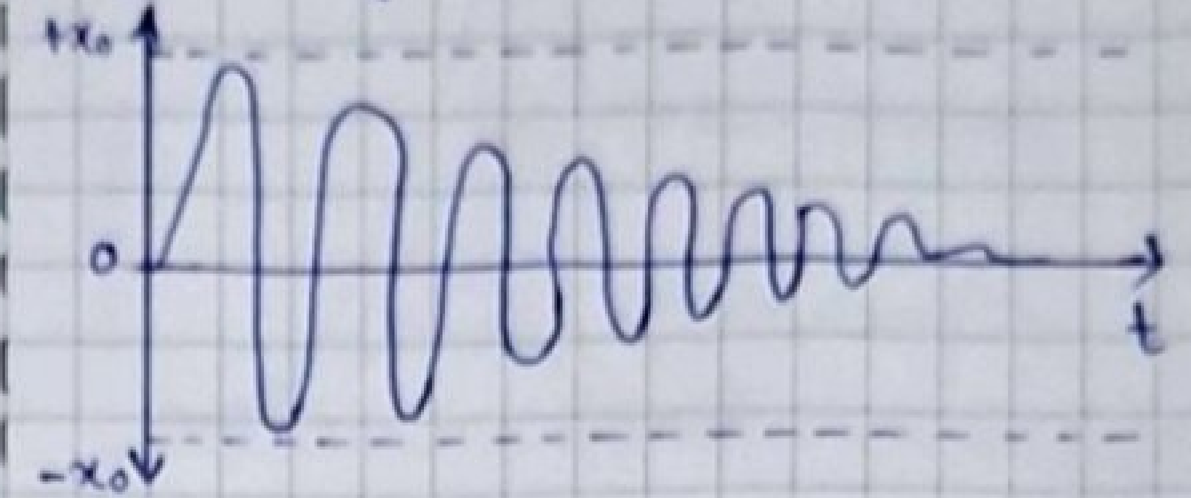


Frequency of oscillations is not necessarily same as natural frequency.
For example, a child pushing a swing periodically.

Driven Frequency: Driven frequency is the frequency of applied force to a system to make it oscillate.
If driven frequency is more than or less than natural frequency, amplitude of vibrations will be smaller.
If driven frequency is equal to natural frequency of vibrations, amplitude of vibrations will be greater.

Damped Oscillations:

If amplitude of oscillations decreases under damping force, then such oscillations are called damped oscillations. For example, when a simple pendulum is being set into oscillations, after some time it stop oscillating due to resistive forces called "damping forces". As oscillator vibrates, it performs work against frictional forces, which results in gradually decrease of oscillator's energy.

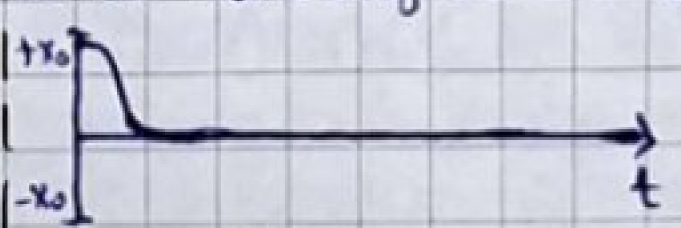


Damping forces: Damping forces includes, air friction, frictional force between moving parts, internal force such as those in spring that tend to dissipate energy as heat.

Light/Under Damping: Damping is light when amplitude of vibrations decreases gradually with time. Damping force is less than the oscillating force. For example, a swing gradually comes to rest from oscillations.

Heavy/Over Damping: In heavy damping oscillator merely moves back to its equilibrium position gradually without completing a single oscillation. Damping force is greater than oscillating force. For example, door damps to prevent from slamming.

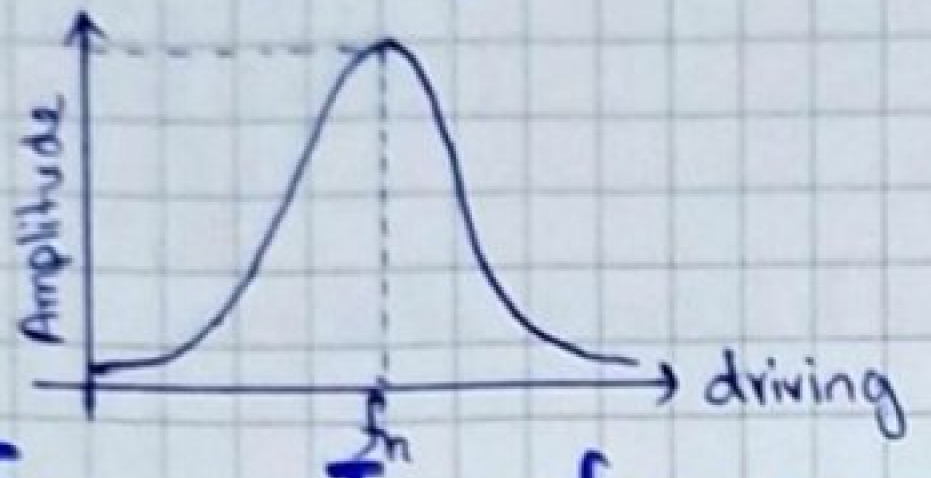
Critical Damping: When an oscillator comes to rest without any oscillations in the shortest time under damping force, then damping is called critical damping. Damping force is equal to oscillating force. For example, car suspension system prevent the car from oscillating after traveling over a bump.



Resonance

Resonance is a phenomenon in which amplitude of vibration of any oscillator atain its maximum value when driving frequency become equal to the natural frequency of oscillator. The natural frequency of oscillator is called resonance frequency (f_r).

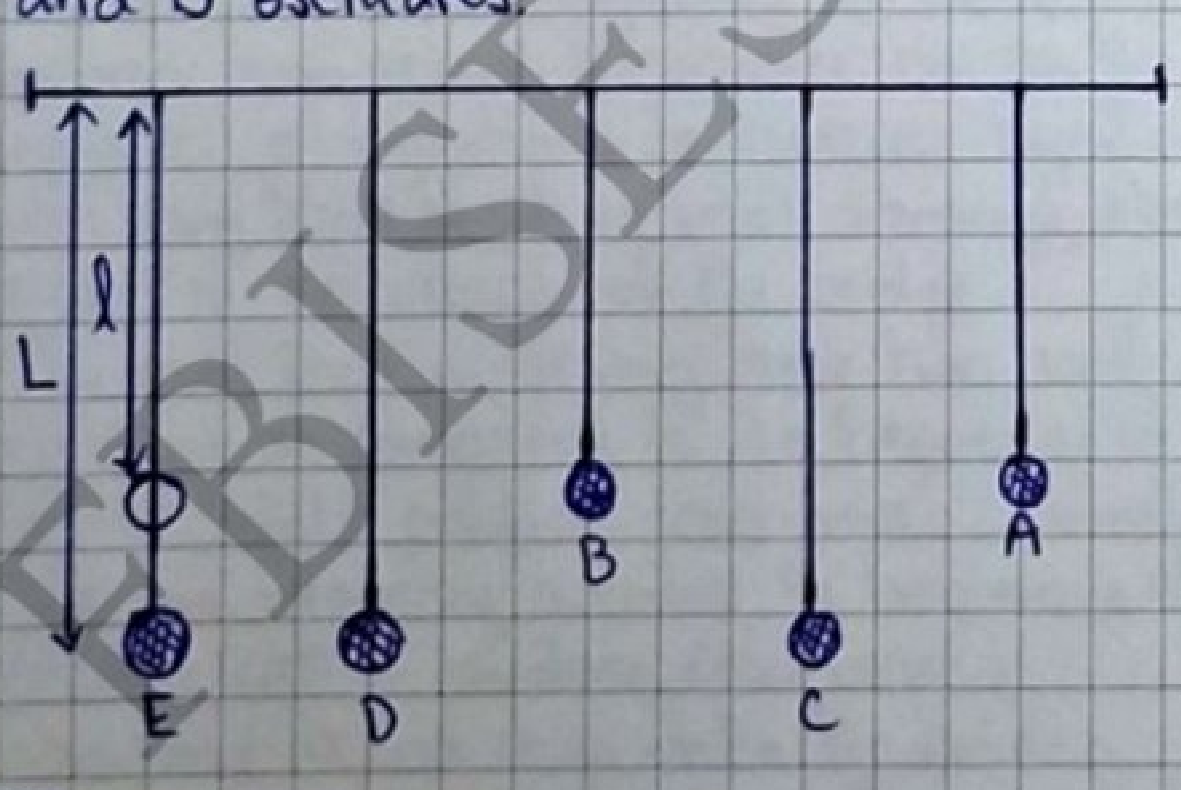
Explanation: Force is required to start oscillations of oscillator. Driving force have its own frequency with which oscillator start vibrations rathar than natural frequency of oscillator. When frequency of driving force gradually increases from zero, oscillator begins to vibrate with small amplitude. As we increase frequency, amplitude increases. Amplitude is greater when driving frequency is closer to natural frequency. When driving frequency become equal to the natural frequency amplitude is maximum called "resonance". If you continue incresing driving frequency, amplitude starts decreasing.



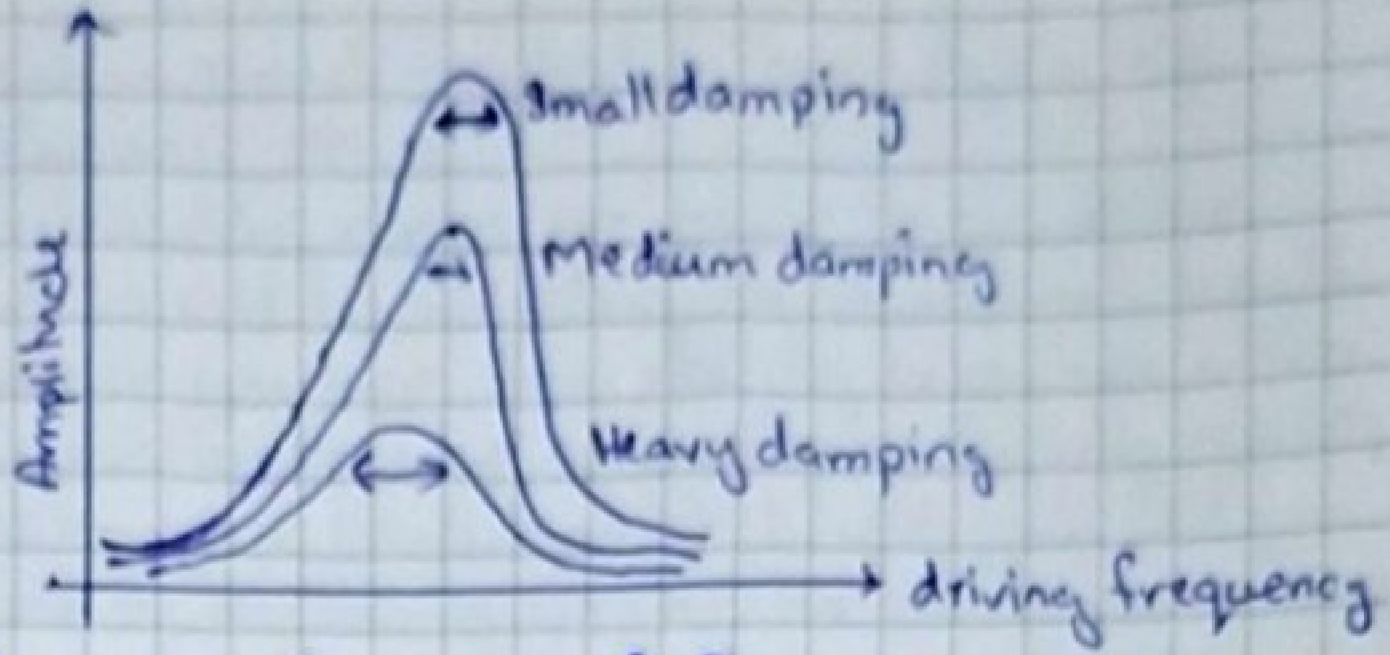
Energy Transfer:
Efficency of energy traster from driving force to oscillator is maximum at "f_r"

Experiment:
Take five pendulum A, B, C, D, E, suspended vertically from the same horizontal string. Length of A and B is equal to "l" and length of "C" and D is equal to "L".

- If length of E is made equal to length of A and B and set into vibration perpendicular to plane of page. Pendulum A and B will start vibration after some time but not C and D. This is due to resonance that A, B and E have same length and same natural frequency.
- Similarly when you mde E equal to C and D. and set vibrations, only C and D vibrates but not A and B. This is because pendulum C and D and E have same length, same natural frequency so, due to resonance pendulm C and D oscillates.



Effect of Damping on resonace
Damping reduces amplitude of an oscillator at resonance frequency and broaden the resonance curve. Graph between driving frequency and amplitude is given. Smaller damping results in a sharper resonance curve and large amplitude, while heavy damping decreases amplitude.



Applications of Resonance

Tuning of Radio:
We turn the knob of radio to tune a station. Acutally we are changing natural frequency of electrical circuit of receiver to make it equal to the transmission frequency of radio station. When both these frequencies match, resonance occurs. Energy absoption is maximum and we hear only this station.

MRI:
Magnetic Resonance and Imaging is a medical diagnostic tool. In this process nuclei of hydrogen are made to oscillate by incoming strong radio waves. When resonance occurs energy absorption is max. The pattern of energy absorped can be used to produce computer generated images.

Food in Microwave Ovens:
In microwave oven a microwave frequency equal to natural frequency of vibration of water and fat molecules used. When food is placed in oven, resonance occur and water molecules absorb max energy from microwaves. It causes food to heat up.

Resonance in Guitar:
When we strike guitar strings, vibrations are produced. Vibration transmit to hollow wooden box. This creates resonance and sound get amplified.

Avoid Resonance

Resonance in Bridge:
Solidiers while marching on the bridge are ordered to break their steps. This is because vibrations created by rhythmic march on the bridge cause the bridge to oscillate with its natural frequency. Thus resonance occurs and amplitude of vibrations increases. It can cause bridge to collapse. A strong and continous wind drove oscillations of "Tacoma Narrow" bridge at great amplitude until break.

Resonance in Airplane Wing:
This wing is very flexible. If the periodic vibrations of wind have a frequency equal to natural structural frequency of wing, resonance occurs. At resonance amplitude of vibration of wing become too large. It eventually leads to its destruction. To avoid this, its designed so that both frequencies do not match.