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**Class 12**

**Physics**

Chapter 16

# Handwritten Notes

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# STATISTICAL MECHANICS AND THERMODYNAMICS

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## Pressure Exerted by Gas Molecule:

Molecules of gas moves randomly and consistantly inside a container. They colide elastically with each other and with container. When gas molecules collide with the wall, it rebound and its momentum changes. This change in momentum results in a force on wall.

$$F = \Delta P / \Delta t$$

$$P = F / A$$

$$P = \frac{\Delta P}{\Delta t} \times \frac{1}{A}$$

$\therefore \Delta P$  is change in momentum  
 $\therefore P$  is pressuse

## Pressure on a Cubic Container:

Consider a cube of side "l" containing "N" molecules. Mass of single molecule is "m" moving with velocity  $V_{ix}$  along x-axis,  $V_{iy}$  along y-axis and  $V_{iz}$  along z-axis.

$$P_x = F_x / A \text{ --- (i)}$$

$$F_x = \Delta P / \Delta t \text{ --- (ii)}$$

along x-axis, consider one particle,

$$\Delta P = P_f - P_i$$

$$\Delta P = (-mv_{ix}) - (mv_{ix})$$

$$\Delta P = -2mv_{ix}$$

$$\Delta t = 2l / v_{ix}$$

$$\therefore S = vt, t = S/v$$

Put the value of  $\Delta t$  and  $\Delta P$  in equation (ii)

$$F_{ix} = \frac{-2mv_{ix}}{2l/v_{ix}}$$

$$F_{ix} = \frac{-mv_{ix}^2}{l}$$

$\therefore$  this is the force exerted by wall

Force exerted by wall on the molecule is equal and opposite to force exerted by molecule on the wall.

$$F_{ix} = - \left\{ \frac{-mv_{ix}^2}{l} \right\}$$

$$F_{ix} = \frac{mv_{ix}^2}{l}$$

$\therefore$  this is the force exerted by molecule

$$F_{2x} = mv_{2x}^2 / l$$

$$F_{3x} = mv_{3x}^2 / l$$

$$F_{Nx} = mv_{Nx}^2 / l$$

For total force,

$$F_x = F_{1x} + F_{2x} + F_{3x} + \dots + F_{Nx}$$

$$F_x = mv_{1x}^2 / l + mv_{2x}^2 / l + mv_{3x}^2 / l + \dots + mv_{Nx}^2 / l$$

$$F_x = m/l \{ v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \dots + v_{Nx}^2 \}$$

Multiply and divide "N" (Total number of molecules)

$$F_x = \frac{mN}{l} \left\{ \frac{v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \dots + v_{Nx}^2}{N} \right\}$$

$$F_x = \frac{mN}{l} \langle V_x^2 \rangle \text{ --- (iii)}$$

Due to random motion  $\langle V_x^2 \rangle = \langle V_y^2 \rangle = \langle V_z^2 \rangle$

For total velocity,

$$\langle V^2 \rangle = \langle V_x^2 \rangle + \langle V_y^2 \rangle + \langle V_z^2 \rangle$$

$$\langle V^2 \rangle = \langle V_x^2 \rangle + \langle V_x^2 \rangle + \langle V_x^2 \rangle$$

$$\langle V^2 \rangle = 3 \langle V_x^2 \rangle$$

$$1/3 \langle V^2 \rangle = \langle V_x^2 \rangle$$

Put the vale of  $\langle V_x^2 \rangle$  in (iii) equation.

$$F_x = \frac{mN}{l} \frac{1}{3} \langle V^2 \rangle$$

$$F = \frac{mN}{l} \frac{1}{3} \langle V^2 \rangle$$

$$\frac{F}{A} = \frac{mN}{A \cdot l} \frac{1}{3} \langle V^2 \rangle$$

$$P = \frac{mN}{V} \frac{1}{3} \langle V^2 \rangle \quad \therefore V = \text{volume}$$

$$P = \frac{1}{3} \frac{mN}{V} \langle V^2 \rangle$$

## Pressure in terms of average translational Kinetic Energy:

$$P = \frac{1}{3} \frac{Nm}{V} \langle V^2 \rangle$$

$$\therefore K.E = \frac{1}{2} mv^2$$

$$\therefore \times \text{ and } \div \text{ by } 2$$

$$P = \frac{2}{3} \frac{N}{V} \left\langle \frac{1}{2} mV^2 \right\rangle$$

$$P = \frac{2}{3} N_0 \langle K.E \rangle$$

$$\therefore N/V = N_0$$

$$\therefore N_0 \text{ is constant}$$

$$\therefore 2/3 \text{ is constant}$$

$$P = \text{Constant} \langle K.E \rangle$$

$$P \propto \langle K.E \rangle$$

## Pressure in terms of Density:

$$P = \frac{1}{3} \frac{Nm}{V} \langle V^2 \rangle$$

$$\therefore \text{total mass} = Nm$$

$$\therefore \text{total volume} = V$$

$$\therefore \rho = \text{mass/volume}$$

$$\therefore \rho = Nm/V$$

$$P = \frac{1}{3} \rho \langle V^2 \rangle$$

## Interpretation of Temperature:

$$P = \frac{1}{3} \frac{Nm}{V} \langle V^2 \rangle$$

$$PV = \frac{1}{3} Nm \langle V^2 \rangle$$

$$PV = \frac{2}{3} N \left( \frac{1}{2} m \overline{V^2} \right)$$

$$PV = \frac{2}{3} N \langle K.E \rangle \quad \text{--- (iv)}$$

General gas equation

$$PV = nRT$$

$$PV = \frac{NRT}{N_A}$$

$$PV = NKT \quad \text{--- (v)}$$

Comparing equation (iv) and (v)

$$NKT = \frac{2}{3} N \langle K.E \rangle$$

$$T = \frac{2}{3K} \langle K.E \rangle$$

$$T = \text{Constant} \langle K.E \rangle$$

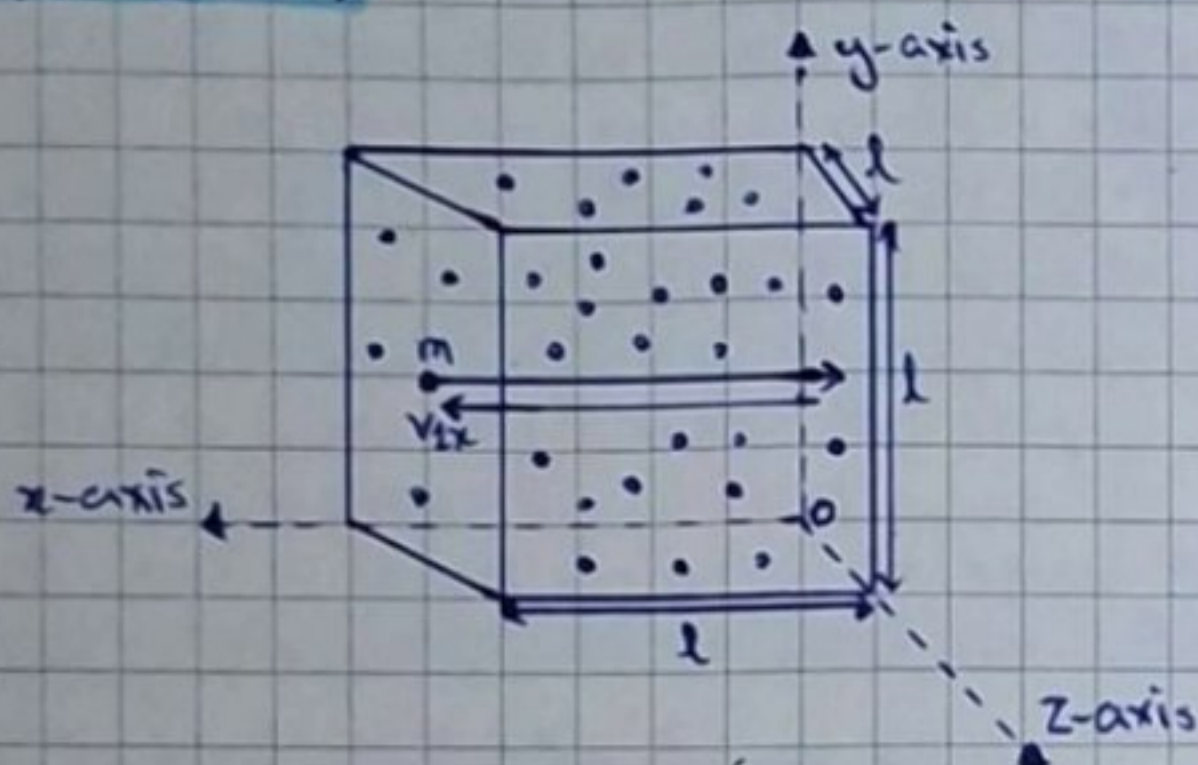
$$T \propto \langle K.E \rangle$$

$$\therefore n = N/N_A$$

$$\therefore K = R/N_A$$

$\therefore 2/3$  is constant

$\therefore K$  is constant



## Root Mean Square Velocity of Gas Molecules

Root Mean Square (R.M.S) velocity of gas molecule is the square root of average of square of velocities of all gas molecules.

$$\text{Pressure of gas: } P = \frac{1}{3} Nm/V \langle V^2 \rangle \quad \text{--- (i)}$$

$$\text{In terms of density: } P = \frac{1}{3} \rho \langle V^2 \rangle \quad \text{--- (ii)}$$

$$\text{General Gas equation: } PV = nRT \quad \text{--- (iii)}$$

Above relations will be used to derive R.M.S velocity.

**Derivation:** Molecules of gas moves with different velocities,  $V_1, V_2, V_3, V_4, \dots, V_n$ .  $N$  is the number of molecules.

$$\text{Velocities} = V_1, V_2, V_3, \dots, V_n$$

$$\text{Square of velocities} = V_1^2, V_2^2, V_3^2, \dots, V_n^2$$

$$\text{Mean Square velocity} = \langle V^2 \rangle$$

$$\langle V^2 \rangle = \frac{V_1^2 + V_2^2 + V_3^2 + \dots + V_n^2}{N}$$

As we know, pressure of gas

$$P = \frac{1}{3} Nm/V \langle V^2 \rangle$$

$$PV = \frac{1}{3} Nm \langle V^2 \rangle \quad \text{--- (iv)}$$

General gas equation,

$$PV = nRT$$

$$PV = \frac{NRT}{N_A}$$

$$\therefore K = R/N_A$$

$$PV = NKT \quad \text{--- (v)}$$

Comparing equation (iv) and equation (v)

$$\frac{1}{3} Nm \langle V^2 \rangle = NKT$$

$$\langle V^2 \rangle = 3KT/m$$

Taking square root on both sides.

$$\sqrt{\langle V^2 \rangle} = \sqrt{3KT/m}$$

$$V_{rms} = \sqrt{\frac{3KT}{m}} \quad \text{--- (vi)}$$

$$V_{rms} = \text{Constant} \sqrt{T}$$

$$V_{rms} \propto \sqrt{T}$$

Put  $K = R/N_A$  in equation (vi)

$$V_{rms} = \sqrt{\frac{3RT}{mN_A}}$$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{rms} = \text{Constant} \frac{1}{\sqrt{M}}$$

$$V_{rms} \propto \frac{1}{\sqrt{M}}$$

Pressure in terms of density,

$$P = \frac{1}{3} \rho \langle V^2 \rangle$$

$$3P = \rho \langle V^2 \rangle$$

Taking square root on both sides

$$\sqrt{\frac{3P}{\rho}} = \sqrt{\langle V^2 \rangle}$$

$$\sqrt{\frac{3P}{\rho}} = V_{rms}$$

$$V_{rms} = \sqrt{\frac{3KT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}$$

$\therefore 3, K, m$  are constants

$$\therefore M = mN_A$$

$\therefore$  If  $T$  is constant

## Modification of Ideal Gas to

## Study Behaviour of Non-

## Ideal Gas

**Ideal Gas:** It is theoretical model of gas that helps to understand behaviour of real gas. It obeys all gas laws perfectly.

$$PV = nRT \quad \text{--- (ii)}$$

**Assumptions of Ideal Gas:** Gas molecules are very small and volume is negligible. No force of attraction between molecule of the gas. Collision of molecules is perfectly elastic. Molecules are in constant random motion.

**Real Gas:** Real gas is the actual gas found in nature.

**Derivation from Ideal Gas:** Molecule of the real gas are like rigid spheres and they have volume. Intermolecular forces are present between molecules. Collision is

not perfectly elastic.

## Properties of Gas:

• **Microscopic Properties:** These are properties related to individual particle of gas. Velocity of single molecule. Position of molecule. Energy of molecule. These properties can't be measured directly. Not observable.

• **Macroscopic Properties:** These are properties of substance that you can see, measure or feel without knowing about individual particles. Temperature, Pressure, Volume, Density etc.

**Comparison:** • Real gas can behave as ideal gas at low pressure and high temperature because intermolecular forces are weak in this case. • No ideal gas exist in real life. • Temperature of ideal gas is given by  $T = \frac{m}{3k} \langle v^2 \rangle$ . • "T" is macroscopic property that is defined by motion of molecules which is microscopic property.

• There is a relation between macroscopic and microscopic properties which give rise to a branch of physics called statistical mechanics. • Ideal gas governs macroscopic properties.  $PV = nRT$ .

**Statistical Mechanics:** It is branch of physics that connects the microscopic details of system such as motion, energy and interaction of particles with the macroscopic observable we measure such as temperature, pressure, volume and entropy.

**Van Der Wal Equation:** Van der wall equation is the modified version of ideal gas equation. Because ideal gas equation  $PV = nRT$  is not valid at high pressure and low temperature due to faulty assumptions.

**Faulty Assumptions:** • Volume of molecule is negligible. • No intermolecular force is present between molecules. • Pressure and volume are affected by these assumptions.

**Volume Correction:** Van Der Wall states that gas molecules are like hard spheres with definite volume. At high pressure when gas is compressed the actual volume of gas can't be neglected. So space available for motion of molecules of gas will be slightly lesser.

$$V_{\text{container}} = V_{\text{free}} + V_{\text{molecules}}$$

$$V_{\text{free}} = V_{\text{container}} - V_{\text{molecules}}$$

$$\text{Volume of 1 mole molecules} = b$$

$$\text{Volume of } n \text{ moles molecules} = nb$$

$$V_{\text{free}} = V - nb \quad \text{--- (i)}$$

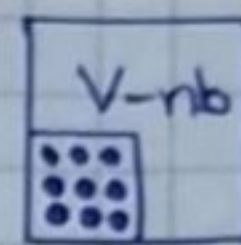
$$V_{\text{free}} = (V - b) \quad \text{--- (ii)} \quad \because n=1 \text{ for 1 mole}$$



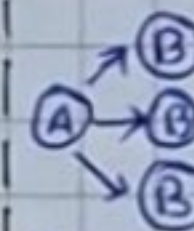
Ideal gas



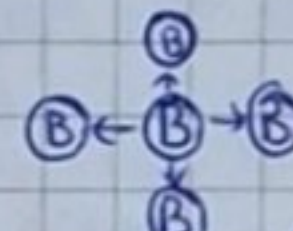
Real gas



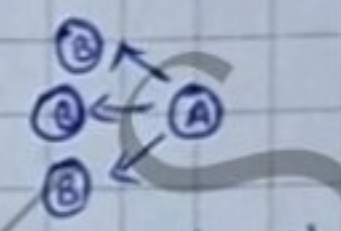
**Pressure Correction:** In real gas molecule of gas attracts each other at low temperature. Inside container, intermolecular forces between gas molecules cancel each other. Particles near the wall of container have net inward force.



Inward pull



Balanced



Inward pull

A = Molecules near wall of container.

B = Molecules in the center

Because a molecule going towards wall is attracted by other molecules

So,  $P_r < P_i$

$$P_r = P_i - P' \quad \text{--- (iii)}$$

$P' \propto$  Concentration of A-type molecules and B-type molecules

$$P' \propto C_A \times C_B$$

$$P' \propto \left(\frac{n}{V}\right) \times \left(\frac{n}{V}\right)$$

$$P' \propto \frac{n^2}{V^2}$$

$$P' = a \frac{n^2}{V^2} \quad \because a \text{ is constant}$$

Equation (iii) becomes

$$P_r = P_i - a \frac{n^2}{V^2}$$

$$P + a \frac{n^2}{V^2} = P_i \quad \text{--- (iv)}$$

Put values in equation  $PV = nRT$  from equation (iv) and equation (iv).

$$(P + a \frac{n^2}{V^2})(V - nb) = nRT$$

For 1 mole,  $n=1$ , so

$$(P + a/V^2)(V - b) = RT$$

This equation is designed for real gas but can be used for ideal gas.

## Gravitational Effects on Ideal Gas Model

In the ideal gas model, at lab scale, gravity is negligible compared to factors like gas pressure and molecular kinetic energy, so it's ignored. However, on large scales such as stars, gravity is crucial for stability and requires more complex models. In stars, outward pressure from fusion, radiation, and quantum effects balance gravity in a state called hydrostatic equilibrium, maintaining their stability.

# Behaviour of Matter under Extreme Physical Condition

In the universe, matter can evolve into extreme physical conditions that alter its behaviour, leading to unique states like degenerate matter, Bose-Einstein condensation, superconductivity and superfluidity, and other quantum phenomena. These conditions are observable at extremely high pressure or at extremely low pressure, by materials of high density or by materials experiencing high gravitational force.

## Degenerate Matter

Degenerate matter happens when gravity squeezes atoms so tightly under extremely high pressure in a star that they can change from their normal state. Extremely high pressure can cause particles to interact closely where quantum mechanical effects, particularly the Pauli Exclusion Principle, become significant. It states that two electrons cannot occupy the same quantum state. Under extreme pressure, electrons are forced tightly against the nucleus and can no longer move freely. This creates a resistance called degeneracy pressure, which counteracts gravity and prevents further compression. Degenerate matter forms in such conditions, typically found in the cores of white dwarfs and neutron stars, where matter is compressed to incredibly high densities.

**White Dwarfs:** White dwarfs are formed when stars with low or medium mass run out of fuel. The star sheds its outer layers, and the core contracts into a small, dense object. In a white dwarf, gravity tries to squeeze the matter inward, but electron degeneracy pressure (a quantum effect that stops electrons from being in the same state) pushes back. The balance keeps the white dwarf stable.

**Neutron Stars:** Neutron stars form from the remains of massive stars after they explode. Gravity compresses the matter so much that electrons and protons combine to form neutrons. The star is held up by neutron degeneracy pressure, which stops it from collapsing further. Even though a neutron star is only about 20 km wide, it can be 1.5 times heavier than the Sun and has an incredibly

high density (about  $10^{17} \text{ kg m}^{-3}$ ). New neutron stars, called hot neutron stars, can have surface temperature of millions of degrees Celsius.

## Bose-Einstein Condensation

Bose-Einstein Condensation (BEC) is a state of matter that forms at extremely low temperatures, such as close to absolute zero, causing particles' kinetic energy to decrease significantly. At this low temperature, bosons (e.g., photons, W bosons, Z bosons, etc.) can condense in the same state known as Bose-Einstein condensation. This is different from degenerate matter, where two fermions, e.g., electrons, cannot exist in the same state. The Bose-Einstein condensation state of matter exhibits unique properties like superfluidity and superconductivity.

## Super Fluidity

Superfluidity is a state in which a liquid experiences no viscosity, no friction and does not lose energy when it flows. It was first time observed by "Pyotr Kapitsa", John R. Allin and Don Misner in 1937. Superfluidity can be observed in some fluids when they are cooled few degrees below 0K. Superfluidity is due to quantum mechanical effect. Example: He-4 and He-3 shows superfluidity when cooled at "2.17K" and 0.0025K respectively. Superfluids are used in high-precision devices like gyroscope. It is also used to trap and slow down speed of light.

## Super Conductivity

In certain conditions, Bose-Einstein condensation leads to superconductivity, where electrical resistance drops to zero, allowing current to flow without resistance. This occurs when two electrons form boson-like entities called Cooper pairs, which can then occupy the same state and move through the material without scattering. Superconductivity is the property of certain materials to conduct current without energy loss when they are cooled below a critical temperature or transition temperature. The low temperature at which and below which the resistivity  $\rho$  of substance becomes zero is called the critical temperature or superconducting transition temperature. Superconductivity was first discovered by Heike Kamerlingh Onnes in 1911, when he observed the sudden disappearance of electrical resistance in mercury at 4.2K. Superconductors are used in powerful electromagnets (32 Tesla), particle accelerators, magnetic levitation trains, fast computer chips etc.