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Where Minds Pollinate

Class 12
Mathematics

Chapter 9
Exercise 9.1
Handwritten Solution

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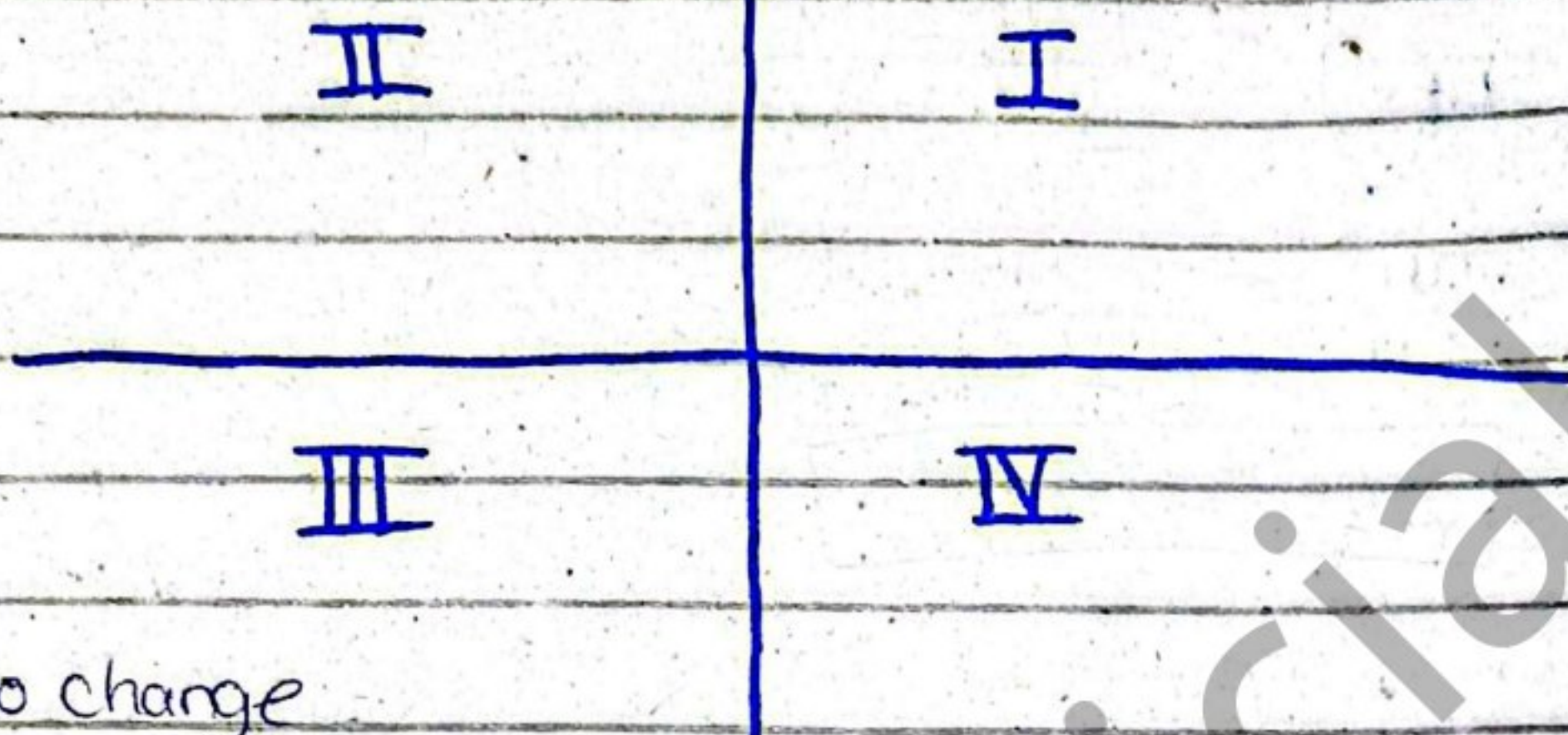


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Exercise 9.1



Q I no change

Q II $\pi - \theta$

Q III $\pi + \theta$

Q IV $2\pi - \theta$

Q4 (i) ~~Q4 (i)~~ $\sqrt{2} \sin x = 1$

Solution:- $\sin x = \frac{1}{\sqrt{2}}$

$$x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$x = 45^\circ \times \frac{\pi}{180}$$

$$\boxed{x = \frac{\pi}{4}} \text{ Reference angle}$$

Since, $\sin x$ ~~is +ve in~~ ~~is +ve in~~ Q I and Q II

Q-I $\Rightarrow \boxed{x = \frac{\pi}{4}}$

Q-II $x = \pi - \frac{\pi}{4} \Rightarrow \boxed{x = \frac{3\pi}{4}}$

S.S. = $\left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\}$ Answer.

ii) $5 \sin x = 2$

Solution:- $\sin x = \frac{2}{5} \Rightarrow x = \sin^{-1}\left(\frac{2}{5}\right) = 23.6^\circ$

Reference angle = $23.6^\circ \times \frac{\pi}{180} \Rightarrow \boxed{\frac{59\pi}{450}}$

Q-I $\Rightarrow x = \frac{59\pi}{450}$

Q-II $\Rightarrow x = \pi - \frac{59\pi}{450}$
 $= \frac{391\pi}{450}$

Sol set = $\left\{ \frac{59\pi}{450}, \frac{391\pi}{450} \right\}$ Answer.

iii) $2 \cos \theta + 1 = 0$

Solution:- $\cos \theta = -\frac{1}{2}$
 $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

neglect -ve. $\theta = 60^\circ \times \frac{\pi}{180}$

$\boxed{\theta = \frac{\pi}{3}}$

$\cos \theta$ -ve in II & III

Q-II $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Q-III $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Sol set = $\left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$ Answer.

iv) $\cot x + \sqrt{3} = 0$

Sol $\cot x = -\sqrt{3}$
 $\tan x = -\frac{1}{\sqrt{3}}$

$x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$x = 30^\circ \times \frac{\pi}{180}$

$\boxed{x = \frac{\pi}{6}}$

Q-II $x = \pi - \frac{\pi}{6} \Rightarrow \boxed{\frac{5\pi}{6}}$

Q-IV $x = 2\pi - \frac{\pi}{6} = \boxed{\frac{11\pi}{6}}$

Sol set = $\left\{ \frac{5\pi}{6}, \frac{11\pi}{6} \right\}$ Answer.

(v) $\cos x = 0.71$

Solution: $\cos x = 0.7071067812$

$\cos x = \frac{1}{\sqrt{2}}$

$x = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$x = 45^\circ \times \frac{\pi}{180} = \boxed{\frac{\pi}{4}}$

Q-I $\Rightarrow \boxed{x = \frac{\pi}{4}}$

Sol set = $\left\{ \frac{\pi}{4}, \frac{7\pi}{4} \right\}$

Answers.

Q-IV $x = 2\pi - \frac{\pi}{4} = \boxed{\frac{7\pi}{4}}$

(vi) $\sqrt{6} \operatorname{cosec} x = \sqrt{8}$

Sol $\operatorname{cosec} x = \frac{\sqrt{8}}{\sqrt{6}} = \frac{2}{\sqrt{3}}$

$\sin x = \frac{\sqrt{3}}{2}$

$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \Rightarrow x = \frac{\pi}{3}$

Q-I $x = \frac{\pi}{3}$

Q-II $x = \pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$

Sol set = $\left\{ \frac{\pi}{3}, \frac{2\pi}{3} \right\}$

(vii) $4 \cos x - 3 = 0$

Sol $\cos x = \frac{3}{4}$

$x = \cos^{-1}\left(\frac{3}{4}\right)$

$x = 41.4^\circ$

Reference angle = $41.4^\circ \times \frac{\pi}{180^\circ}$

$= \frac{23\pi}{100}$

$$\textcircled{Q-I} \Rightarrow x = \frac{23\pi}{100}$$

$$\textcircled{Q-IV} \Rightarrow x = 2\pi - \frac{23\pi}{100} \Rightarrow \frac{177\pi}{100}$$

$$\text{Sol set} = \left\{ \frac{23\pi}{100}, \frac{177\pi}{100} \right\}$$

$$\text{viii) } \sqrt{2} \operatorname{cosec} x + 2 = 0$$

$$\text{Sol: } \operatorname{cosec} x = \frac{-2}{\sqrt{2}} = -\sqrt{2}$$

$$\sin x = -\frac{1}{\sqrt{2}} \Rightarrow x = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \boxed{\frac{7\pi}{4}}$$

$$\textcircled{Q-III} \quad x = \pi + \frac{\pi}{4} \Rightarrow \boxed{\frac{5\pi}{4}}$$

$$\textcircled{Q-IV} \quad x = 2\pi - \frac{\pi}{4} \Rightarrow \boxed{\frac{7\pi}{4}}$$

$$\text{S.S.} = \left\{ \frac{5\pi}{4}, \frac{7\pi}{4} \right\} \text{ Answer}$$

$$\text{ix) } \tan^2 P = 3$$

$$\text{Sol: } \tan P = \pm\sqrt{3}$$

$$\text{Case I } \tan P = +\sqrt{3}$$

$$P = \tan^{-1} \sqrt{3}$$

$$\boxed{P = \frac{\pi}{3}}$$

$$\textcircled{Q-I} \quad \boxed{x = \frac{\pi}{3}}$$

$$\textcircled{Q-III} \quad x = \pi + \frac{\pi}{3}$$

$$\boxed{x = \frac{4\pi}{3}}$$

$$\text{Case II } \tan P = -\sqrt{3}$$

$$\textcircled{Q-II} \quad x = \pi - \frac{\pi}{3}$$

$$\boxed{x = \frac{2\pi}{3}}$$

$$\textcircled{Q-IV} \quad x = 2\pi - \frac{\pi}{3}$$

$$\boxed{x = \frac{5\pi}{3}}$$

$$\text{S.S.} = \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\} \text{ Answer}$$

$$(2) (i) \sin 3\theta + \sin \theta = 0 \quad \therefore [0, 2\pi)$$

$$\text{Sol} \quad 2 \sin\left(\frac{3\theta + \theta}{2}\right) \cos\left(\frac{3\theta - \theta}{2}\right) = 0$$

$$\sin 2\theta \cdot \cos \theta = 0$$

$$\sin 2\theta \cdot \cos \theta = 0$$

$$\sin 2\theta = 0 \quad \cos \theta = 0$$

reference angle:-

$$2\theta = \sin^{-1}(0)$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

$$2\theta = 0$$

sin is +ve in (I) & (II)

cos is +ve in (I) & (IV)

$$\text{if } \theta - I \Rightarrow \theta = 0^\circ \quad \boxed{\theta = 0^\circ}$$

$$\text{if } \theta - I \Rightarrow \theta = \frac{\pi}{2}$$

$$\text{if } \theta - II \Rightarrow 2\theta = \pi - 0$$

$$\text{if } \theta - IV \Rightarrow \theta = 2\pi - \frac{\pi}{2} = \boxed{\frac{3\pi}{2}}$$

$$\boxed{\theta = \frac{\pi}{2}}$$

$$\text{S. Set} = \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 0 \right\}$$

Answer

$$(ii) \sin 2\theta = 2 \cos \theta$$

$$\text{Sol} \quad 2 \sin \theta \cos \theta = 2 \cos \theta = 0$$

$$2 \cos \theta [\sin \theta - 1] = 0$$

$$\cos \theta = 0$$

or

$$\sin \theta - 1 = 0$$

Q-I

$$\sin \theta = 1$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \sin^{-1}(1)$$

$$= \frac{\pi}{2}$$

$$\boxed{\theta = \frac{\pi}{2}}$$

Q-I

$$\boxed{\theta = \frac{\pi}{2}}$$

$$\text{Q-IV} \quad \theta = 2\pi - \frac{\pi}{2}$$

$$= \boxed{\frac{3\pi}{2}}$$

Q-II

$$\theta = \pi - \frac{\pi}{2}$$

$$\boxed{\theta = \frac{\pi}{2}}$$

$$\text{S. Set} = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

(iii) $\sin 2\theta + \sin \theta = 0$

Solution: $2\sin\theta\cos\theta + \sin\theta = 0$
 $\sin\theta [2\cos\theta + 1] = 0$

$\sin\theta = 0$

$\theta = \sin^{-1}(0)$

$\boxed{\theta = 0}$

QI $\theta = 0$

QII $\theta = \pi - 0$

$\boxed{\theta = \pi}$

$\cos\theta = -\frac{1}{2}$

$\theta = \cos^{-1}(-\frac{1}{2})$

$\theta = 60^\circ \text{ or } \frac{2\pi}{3} \Rightarrow \boxed{\theta = \frac{2\pi}{3}}$

QII $\theta = \pi - \frac{2\pi}{3} = \frac{2\pi}{3}$

Q-III $\theta = \pi + \frac{2\pi}{3} = \frac{4\pi}{3}$

Sol: $\theta = \{0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}\}$ Answer

(iv) $\cos 2\theta + \cos \theta = 0$

Solution: $\cos 2\theta = 2\cos^2\theta - 1$
 $2\cos^2\theta - 1 + \cos\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

Let $\cos\theta = x$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0$

$x = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}(\frac{1}{2})$

$\boxed{\theta = \frac{\pi}{3}}$

QI $\Rightarrow \theta = \frac{\pi}{3}$

QIV $\Rightarrow \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$x = -1$

$\cos\theta = -1$

$\theta = \cos^{-1}(-1)$

$\boxed{\theta = \pi}$

QII $\theta = \pi - 0 = \pi$

QIII $\theta = \pi + 0 = \pi$

Sol: $\theta = \{\frac{\pi}{3}, \frac{5\pi}{3}, \pi\}$

$$(v) 2\sin^3\theta = \sin^2\theta$$

sol $2\sin^3\theta - \sin^2\theta$

$$\sin^2\theta[2\sin\theta - 1] = 0$$

$$\sin\theta = 0$$

$$\theta = \sin^{-1}(0)$$

$$\boxed{\theta = 0}$$

$$\theta = I \quad \theta = 0^\circ$$

$$\theta = II \quad \theta = \pi$$

$$\text{sol set} = \left\{0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$$

$$2\sin\theta - 1 = 0$$

$$\boxed{\sin\theta = \frac{1}{2}}$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{6}$$

$$\theta = I \quad \theta = \frac{\pi}{6}$$

$$\theta = II \quad \theta = \pi - \frac{\pi}{6}$$

$$\boxed{\theta = \frac{5\pi}{6}}$$

$$(vi) \cos 2\theta = 3\cos\theta - 2$$

sol $2\cos^2\theta - 1 - 3\cos\theta + 2 = 0$

$$2\cos^2\theta - 3\cos\theta + 1 = 0$$

$$\text{let } \cos\theta = x$$

$$2x^2 - 3x + 1 = 0$$

$$(2x-1)(x-1) = 0$$

$$x = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\theta = I \quad \theta = \frac{\pi}{3}$$

$$\theta = II \quad \theta = \frac{5\pi}{3}$$

$$\text{sol set} = \left\{0, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$$

$$x = 1$$

$$\cos\theta = 1$$

$$\theta = \cos^{-1}(1)$$

$$\boxed{\theta = 0^\circ}$$

$$\theta = I \quad \theta = 0^\circ$$

$$\theta = IV = 2\pi - 0$$

$$= 2\pi \notin [0, 2\pi)$$

vii) $\cos 3\theta = 2 - \cos \theta$ IMP

Solution:-

$\therefore \cos 3\theta = 4\cos^3 \theta - 3\cos \theta$

$4\cos^3 \theta - 3\cos \theta + \cos \theta = 2$

$4\cos^3 \theta - 2\cos \theta = 2$

$2[2\cos^3 \theta - \cos \theta] = 2$

$2\cos^3 \theta - \cos \theta = 1$

Let $x = \cos \theta$

$2x^3 - x - 1 = 0$

	2	0	-1	-1
1		2	2	1
	2	2	1	0

$2x^2 + 2x + 1 = 0$

neglect due to imaginary roots

so

$\cos \theta = 1$

$\theta = \cos^{-1}(1)$

$\boxed{\theta = 0}$

QI $\theta = 0$

QII $\theta = 2\pi - 0$

$= 2\pi \notin [0, 2\pi)$

Sol set = $\{0\}$ ✓

viii) $\cos 3\theta = \cos 5\theta$

Sol:- $\cos 5\theta - \cos 3\theta = 0$

$-2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right) = 0$

$-2\sin\left(\frac{5\theta+3\theta}{2}\right)\sin\left(\frac{5\theta-3\theta}{2}\right) = 0$

$-2\sin 4\theta \sin \theta = 0$

$\sin 4\theta \sin \theta = 0$

$$\sin 4\theta = 0$$

or

$$\sin \theta = 0$$

$$4\theta = \sin^{-1}(0)$$

ref angle = 0

$$Q-I \Rightarrow 4\theta = 0$$

$$\boxed{\theta = 0}$$

$$Q-II \quad 4\theta = \pi - 0$$

$$\boxed{\theta = \frac{\pi}{4}}$$

$$Q-I \quad \theta = 0^\circ$$

$$Q-II \quad \theta = \pi - 0$$

$$\boxed{= \pi}$$

$$S.S = \left\{0, \pi, \frac{\pi}{4}\right\}$$

$$(ix) \cos \theta \sin \theta = \frac{1}{2}$$

Sol

$$2 \cos \theta \sin \theta = 1$$

$$\sin 2\theta = 1$$

$$2\theta = \sin^{-1}(1)$$

Ref angle = $\frac{\pi}{2}$

$$Q-I \Rightarrow 2\theta = \frac{\pi}{2}$$

$$\boxed{\theta = \frac{\pi}{4}}$$

$$Q-II \quad 2\theta = \pi - \frac{\pi}{2}$$

$$\boxed{\theta = \frac{\pi}{4}}$$

$$S.S = \left\{\frac{\pi}{4}\right\} \text{ answer.}$$

$$(x) \cos^2 \theta + 4 \sin \theta = 4$$

Sol

$$1 - \sin^2 \theta + 4 \sin \theta = 4$$

$$-\sin^2 \theta + 4 \sin \theta - 3 = 0$$

$$\sin^2 \theta - 4 \sin \theta + 3 = 0$$

$$\text{Let } \sin \theta = x$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$$\sin \theta = 1$$

$$\theta = \sin^{-1}(1)$$

$$\text{Ref angle} = \frac{\pi}{2}$$

$$\text{Q I} = \frac{\pi}{2}$$

$$\text{Q II} \quad \theta = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\sin \theta = 3 \notin [-1, 1] \quad \text{Range not possible}$$

$$\text{S.S} = \left\{ \frac{\pi}{2} \right\} \text{ answer.}$$

$$\text{xi) } \cos P - \sqrt{3} \sin P = 2$$

$$\text{Sol} \quad \div 2 \quad \frac{1}{2} \cos P - \frac{\sqrt{3}}{2} \sin P = 1$$

$$\therefore \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \quad \therefore \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{3}\right) \cos P - \sin\left(\frac{\pi}{3}\right) \sin P = 1$$

$$\cos\left(\frac{\pi}{3} + P\right) = 1$$

$$\frac{\pi}{3} + P = \cos^{-1}(1)$$

$$\text{S.S} = \left\{ \frac{5\pi}{3} \right\}$$

$$\text{Reference angle} = 0$$

$$\text{Q I} = \frac{\pi}{3} + P = 0 \Rightarrow P = -\frac{\pi}{3} \quad \text{not possible}$$

$$\text{Q IV} \quad P = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$\text{xii) } 3\tan^2 \theta - 1 = 0$$

$$\text{Sol} \quad \tan^2 \theta = \frac{1}{3}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\tan \theta = -\frac{1}{\sqrt{3}}$$

~~Reference~~

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\pi}{6}$$

$$= \frac{\pi}{6}$$

$$\text{Q I} \Rightarrow \theta = \frac{\pi}{6}$$

$$\text{Q II} \quad \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\text{Q-III } \alpha = \pi + \frac{\pi}{6}$$

$$\boxed{\alpha = \frac{7\pi}{6}}$$

$$\text{QII } \alpha = 2\pi - \frac{\pi}{6}$$

$$\boxed{\alpha = \frac{11\pi}{6}}$$

$$\text{S.S} = \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\} \text{ Answer.}$$

IMPORTANT

$$\text{Q3) (i) } 6\sin\phi + 8\cos\phi = 7$$

$$\text{Let } 8\cos\phi = 7 - 6\sin\phi$$

Squaring both sides

$$(8\cos\phi)^2 = (7 - 6\sin\phi)^2$$

$$64\cos^2\phi = 49 + 36\sin^2\phi - 2(7)(6)\sin\phi$$

$$64\cos^2\phi = 49 + 36\sin^2\phi - 84\sin\phi$$

$$64(1 - \sin^2\phi) = 49 + 36\sin^2\phi - 84\sin\phi$$

$$64 - 64\sin^2\phi = 49 + 36\sin^2\phi - 84\sin\phi$$

$$0 = 49 + 36\sin^2\phi - 84\sin\phi - 64 + 64\sin^2\phi$$

$$100\sin^2\phi - 84\sin\phi - 15 = 0$$

$$\text{Let } \sin\phi = x$$

$$100x^2 - 84x - 15 = 0$$

$$x = \frac{84 \pm \sqrt{(-84)^2 - 4(100)(-15)}}{2(100)}$$

$$x = \frac{84 + 114.26}{200}$$

$$, x = \frac{84 - 114.26}{200}$$

$$x = 0.9913$$

$$, x = -0.1513$$

$$\sin\phi = 0.9913$$

$$\sin\phi = -0.1513$$

$$\phi = \sin^{-1}(0.9913)$$

$$\phi = \sin^{-1}(-0.1513)$$

$$\text{Reference angle} = 82.43$$

$$\text{Reference} = 8.7$$

Case # I $\sin$ is +ve in Q I & II

$$\underline{QI} = 82.43^\circ$$

$$\underline{QII} \quad \phi = \pi - 82.43$$

$$\phi = 97.57^\circ$$

put QI in Question

$$6 \sin(82.43) + 8 \cos(82.43) = 7$$

$$7 = 7$$

put QII in Question

$$6 \sin(97.57) + 8 \cos(97.57) = 7$$

$$4.89 \neq 7 \text{ False}$$

Case II $\sin$ is -ve in Q III & IV

$$\underline{Q-III} \quad \phi_3 = \pi + 8.7 = 188.7^\circ$$

$$\underline{QIV} = \phi_4 = 2\pi - 8.7 = 351.3^\circ$$

check ϕ_3 in Question

$$6 \sin(188.7^\circ) + 8 \cos(188.7^\circ) = 7$$

$$-8.8 \neq 7 \text{ False}$$

check ϕ_4 in Question

$$6 \sin(351.3) + 8 \cos(351.3) = 7$$

$$7 = 7 \text{ True}$$

$$\text{Sol set} = \left\{ \frac{82.43\pi}{180}, \frac{351.3\pi}{180} \right\} \checkmark$$

$$\text{ii) } \cos \phi + \cos 3\phi = 0$$

$$\text{Sol } 2 \cos\left(\frac{3\phi + \phi}{2}\right) \cos\left(\frac{3\phi - \phi}{2}\right) = 0$$

$$\cos 2\phi \cos \phi = 0$$

$$\cos 2\phi$$

$$2\phi = \cos^{-1}(0)$$

$$\cos \phi = 0$$

$$\phi = \cos^{-1}(0)$$

i Reference angle = $\frac{\pi}{2}$

Q I $\Rightarrow 2\alpha = \frac{\pi}{2}$

$\alpha = \frac{\pi}{4}$

Q II $2\alpha = 2\pi - \frac{\pi}{2}$

$\alpha = \frac{3\pi}{4}$

; Reference angle = $\frac{\pi}{2}$

Q-I $\Rightarrow \alpha = \frac{\pi}{2}$

Q IV $\alpha = 2\pi - \frac{\pi}{2}$

$\alpha = \frac{3\pi}{2}$

f. set = $\left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2} \right\}$ Answer

(iii) $\sin 4\phi + \sin 3\phi = 0$ $\phi \in [0, 2\pi]$

Sol $2 \sin \left(\frac{4\phi + 3\phi}{2} \right) \cos \left(\frac{4\phi - 3\phi}{2} \right) = 0$

$\sin 7\frac{\phi}{2} \cos \frac{\phi}{2} = 0$

$\sin \left(\frac{7\phi}{2} \right) = 0$

$\frac{7\phi}{2} = 0 \Rightarrow \phi = 0$

$\frac{7\phi}{2} = \pi \Rightarrow \phi = \frac{2\pi}{7}$

$\frac{7\phi}{2} = 2\pi \Rightarrow \phi = \frac{4\pi}{7}$

$\cos \left(\frac{\phi}{2} \right) = 0$

Ref angle: $\frac{\pi}{2}$

if Q I $\frac{\phi}{2} = \frac{\pi}{2}$

$\phi = \pi$

if Q IV $\frac{\phi}{2} = 2\pi - \frac{\pi}{2}$

$\frac{\phi}{2} = \frac{3\pi}{2}$

$\phi = 3\pi$

f. set = $\left\{ 0, \frac{2\pi}{7}, \frac{4\pi}{7}, \pi \right\}$ Answer

(iv) $\sin \alpha + \sin 3\alpha = \sin 2\alpha$

Sol $2 \sin \left(\frac{3\alpha + \alpha}{2} \right) \cos \left(\frac{3\alpha - \alpha}{2} \right) = \sin 2\alpha$

$2 \sin 2\alpha \cos \alpha - \sin 2\alpha = 0$

$\sin 2\alpha [2 \cos \alpha - 1] = 0$

$$\sin 2\theta = 0$$

$$; 2\cos\theta - 1 = 0$$

$$2\theta = \sin^{-1}(0)$$

$$\cos\theta = 1/2$$

$$\text{So } 2\theta = 0 \Rightarrow \boxed{\theta = 0}$$

$$\theta = \cos^{-1}(1/2)$$

$$2\theta = \pi \Rightarrow \boxed{\theta = \pi/2}$$

$$\text{Ref angle} = \pi/3$$

$$2\theta = 2\pi \Rightarrow \boxed{\theta = \pi}$$

$$\text{Q.I} = \theta = \pi/3$$

Q.IV

$$\theta = 2\pi - \pi/3$$

$$\boxed{\theta = 5\pi/3}$$

$$\theta = \{0, \pi/2, \pi/3, 5\pi/3, \pi\}$$

$$\textcircled{v} \tan\theta + \cot\theta = 8\cos 2\theta$$

$$\text{Solution } \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 8\cos 2\theta$$

$$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cos\theta} = 8\cos 2\theta$$

$$1 = 8\cos 2\theta \sin\theta \cos\theta$$

$$1 = 8\cos 2\theta \frac{2\sin\theta \cos\theta}{2}$$

$$1 = 4\cos 2\theta \sin 2\theta$$

$$1 = 2[2\sin 2\theta \cos 2\theta]$$

$$1 = 2\sin 4\theta$$

$$\sin 4\theta = 1/2 \Rightarrow 4\theta = \sin^{-1}(1/2)$$

$$\text{Reference angle} = \pi/6$$

$$\text{Q.I} = 4\theta = \pi/6 \Rightarrow \boxed{\theta = \frac{\pi}{24}}$$

$$\text{Q.II} = 4\theta = \pi - \frac{\pi}{6} \Rightarrow \boxed{\theta = \frac{5\pi}{24}}$$

$$\text{Sol set} = \left\{ \frac{\pi}{24}, \frac{5\pi}{24} \right\} \text{ Answer.}$$

(vi) $\sin 2\phi + \sin 3\phi + \sin 5\phi = 0$

Sol: $2\sin\left(\frac{3\phi+2\phi}{2}\right)\cos\left(\frac{3\phi-2\phi}{2}\right) + \sin 5\phi = 0$

$$2\sin\frac{5\phi}{2}\cos\frac{\phi}{2} + 2\sin\frac{5\phi}{2}\cos\frac{5\phi}{2} = 0$$

$$2\sin\frac{5\phi}{2}\left[\cos\frac{\phi}{2} + \cos\frac{5\phi}{2}\right] = 0$$

$$2\sin\frac{5\phi}{2} = 0$$

$$; \cos\frac{\phi}{2} + \cos\frac{5\phi}{2} = 0$$

$$\sin\left(\frac{5\phi}{2}\right) = 0$$

$$2\cos\left(\frac{5\phi}{2} + \frac{\phi}{2}\right)\cos\left(\frac{5\phi}{2} - \frac{\phi}{2}\right) = 0$$

$$\frac{5\phi}{2} = 0$$

$$\cos 3\phi \cos \phi = 0$$

$$\boxed{\phi = 0}$$

$$\frac{5\phi}{2} = \pi \Rightarrow \boxed{\phi = \frac{2\pi}{5}}$$

$$\cos 3\phi = 0$$

$$; \cos \phi = 0$$

$$\frac{5\phi}{2} = 2\pi \Rightarrow \boxed{\phi = \frac{4\pi}{5}}$$

$$\text{Ref angle} = \pi/2$$

$$; \text{ref angle} = \pi$$

$$\text{QI } \frac{3\phi}{2} = \pi/2$$

$$; \text{I } \boxed{\phi = \pi/2}$$

$$\boxed{\phi = \pi/3}$$

$$; \text{QIV } \boxed{\phi = 3\pi/2}$$

$$S.S = \left\{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{\pi}{3}, \pi, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$$

$$\text{QIV } \frac{3\phi}{2} = 2\pi - \pi/2$$

$$\boxed{\phi = \pi}$$

(vii) $5\cos \phi + 4 = 2\sin^2 \phi$

Sol: $5\cos \phi + 4 = 2\sin^2 \phi$

$$5\cos \phi + 4 = 2[1 - \cos^2 \phi]$$

$$5\cos \phi + 4 - 2 + 2\cos^2 \phi = 0$$

$$2\cos^2 \phi + 5\cos \phi + 2 = 0$$

$$\text{Let } \cos \phi = x$$

$$2x^2 + 5x + 2 = 0$$

$$x = \frac{-5 \pm \sqrt{25 - 4(2)}}{2(2)}$$

$$x = -\frac{5 \pm 3}{4}$$

$$x = -\frac{5+3}{4}$$

$$; x = -\frac{5-3}{4}$$

$$\boxed{\cos \theta = -\frac{1}{2}}$$

$$; \boxed{\cos \theta = -2}$$

Not possible $\notin [-1, 1]$.

$$\text{Ref angle} = \cos^{-1}(\frac{1}{2})$$

$$= \frac{\pi}{3}$$

$\cos(-ve)$ in II & III.

$$\text{if } \theta \text{ in II} \Rightarrow \theta = \pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$$

$$\text{S. Set} = \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

Answer

$$\text{if } \theta \text{ in III} \Rightarrow \theta = \pi + \frac{\pi}{3} = \boxed{\frac{4\pi}{3}}$$

$$\text{viii) } \cos 2\theta + 3 = 5 \cos \theta$$

$$2\cos^2 \theta - 1 + 3 - 5 \cos \theta = 0$$

$$\text{for } 2\cos^2 \theta - 5 \cos \theta + 2 = 0$$

$$\text{Let } \cos \theta = x$$

$$2x^2 - 5x + 2 = 0$$

$$2x^2 - 4x - x + 2 = 0$$

$$2x(x-2) - 1(x-2) = 0$$

$$(2x-1)(x-2) = 0$$

$$\cos \theta = \frac{1}{2} ; \cos \theta = 2 \notin [-1, 1]$$

$$\text{Ref angle} = \cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$$

$$\text{Q-I } \boxed{\theta = \frac{\pi}{3}}$$

$$\text{Q-IV } \theta = 2\pi - \frac{\pi}{3}$$

$$\boxed{\theta = \frac{5\pi}{3}}$$

$$\text{S. Set} = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\} \text{ Answer.}$$

(ix) $\sin^2 \theta = 3 \cos^2 \theta$

for

$$\sin^2 \theta = 3(1 - \sin^2 \theta)$$

$$\sin^2 \theta + 3 \sin^2 \theta - 3 = 0$$

$$4 \sin^2 \theta - 3 = 0$$

$$\sin^2 \theta = \frac{3}{4}$$

$$\sin \theta = \pm \frac{\sqrt{3}}{2}$$

Ref angle = $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{3}}$

For +ve

Q-I

$$\theta = \boxed{\frac{\pi}{3}}$$

Q-II

$$\theta = \pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$$

For -ve

Q-III

$$\theta = \pi + \frac{\pi}{3} = \boxed{\frac{4\pi}{3}}$$

Q-IV

$$\theta = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$$

Set $\left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$

(x) $\cos^2 \theta - 2 \cos \theta + 1 = 0$

for

Let $\cos \theta = x$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x-1 = 0$$

$$\boxed{x = 1}$$

$$\cos \theta = 1$$

$$\theta = \cos^{-1}(1)$$

Reference angle = 0

Q-I

$$\theta = \boxed{0}$$

Q-IV

$$\theta = \boxed{2\pi}$$

Set $\{0, 2\pi\}$

vi) $\tan^2 \theta - \tan \theta = 6$

Let $\tan \theta = x$

$$x^2 - x - 6 = 0$$

$$x^2 - 3x + 2x - 6 = 0$$

$$x(x-3) + 2(x-3) = 0$$

$$(x+2)(x-3) = 0$$

$$\boxed{x = -2} \quad \boxed{x = 3}$$

$$\tan \theta = -2$$

$$\tan \theta = +3$$

Reference angle = $\tan^{-1}(-2)$

$$= 63.43$$

$\tan$ is -ve in QII & IV

$$\theta = \pi - 63.43$$

$$\boxed{\theta = \frac{116.57\pi}{180}}$$

For QIV

$$\theta = 2\pi - 63.43$$

$$\boxed{\theta = \frac{296.57\pi}{180}}$$

Ref angle = $\tan^{-1}(3)$

$$= 71.56$$

QI $\theta = \frac{71.56\pi}{180}$

QIII $\Rightarrow \theta = \pi + 71.56$

$$\boxed{\theta = \frac{251.56\pi}{180}}$$

f. set: $\left\{ \frac{116.57\pi}{180}, \frac{296.57\pi}{180}, \frac{71.56\pi}{180}, \frac{251.56\pi}{180} \right\}$

Q4)(i) $3\cos x + \sin x = 2$ $x \in [-\pi, \pi]$ $\star$

$$3\cos x = 2 - \sin x$$

sq b/s

$$(3\cos x)^2 = (2 - \sin x)^2$$

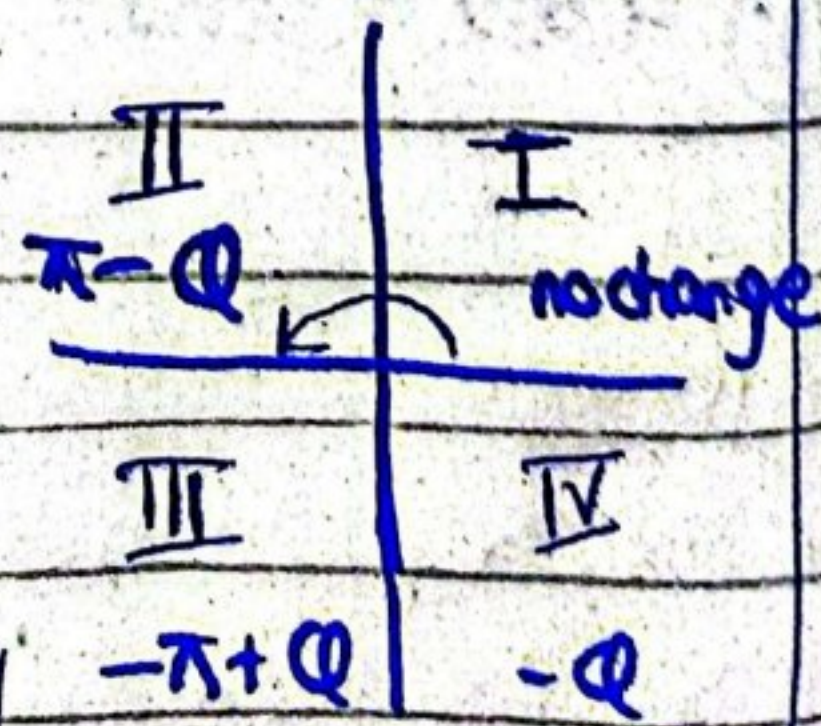
$$9\cos^2 x = 4 + \sin^2 x - 4\sin x$$

$$9(1 - \sin^2 x) = 4 + \sin^2 x - 4\sin x$$

$$9 - 9\sin^2 x = 4 + \sin^2 x - 4\sin x$$

$$0 = 4 + \sin^2 x - 4\sin x + 9\sin^2 x - 9$$

Quadrant change
due to change in interval



$$10 \sin^2 x - 4 \sin x - 5 = 0$$

$$\sin x = \frac{4 \pm \sqrt{16 - 4(10)(-5)}}{2(10)}$$

$$\sin x = \frac{4 \pm 6\sqrt{6}}{20}$$

$$\sin x = \frac{4 + 6\sqrt{6}}{20}$$

$$; \sin x = \frac{4 - 6\sqrt{6}}{20}$$

$$\sin x = 0.9348$$

$$; \sin x = -0.5348$$

Ref angle: $\sin^{-1}(0.9348)$
 $= 69.19^\circ$ ✓

~~Ref angle = $\pi - 32.33$~~
 Ref angle = 32.33

QI $x = 69.19^\circ$ ✓

QIII $x = -\pi + 32.33$

QII $x = \pi - 69.19^\circ$

$x = -147.67^\circ$

$x = 110.81^\circ$ ✗

QIV $x = -32.33^\circ$

Check

$$3 \cos(69.19^\circ) + \sin(69.19^\circ) = 2$$

$$2 = 2 \text{ True}$$

Check $x = -147.67^\circ$

$$3 \cos(-147.67^\circ) + \sin(-147.67^\circ) = 2$$

$$-3.06 \neq 2 \text{ False}$$

Check

$$x = 110.81^\circ$$

$$3 \cos(110.81^\circ) + \sin(110.81^\circ) = 2$$

$$0.13 \neq 2 \text{ False}$$

Check $x = -32.33^\circ$

$$3 \cos(-32.33^\circ) + \sin(-32.33^\circ) = 2$$

$$2 = 2 \text{ True}$$

S.S. = $\left\{ \frac{69.19\pi}{180}, \frac{-32.33\pi}{180} \right\}$ Answers.

(ii) $4 \cos x + 3 \sin x = 2$ $[-\pi, \pi]$

$$(4 \cos x)^2 = (2 - 3 \sin x)^2$$

$$16 \cos^2 x = 4 + 9 \sin^2 x - 12 \sin x$$

$$9 \sin^2 x - 12 \sin x + 4 - 16(1 - \sin^2 x) = 0$$

$$9 \sin^2 x - 12 \sin x + 4 - 16 + 16 \sin^2 x = 0$$

$$25 \sin^2 x - 12 \sin x - 12 = 0$$

$$\sin x = \frac{12 \pm \sqrt{144 - 4(25)(-12)}}{2(25)}$$

$$\sin x = \frac{12 \pm 8\sqrt{21}}{50}$$

$$\sin x = \frac{12 + 8\sqrt{21}}{50}$$

$$\sin x = -0.4932$$

$$\text{now } \sin x = \frac{12 + 8\sqrt{21}}{50} \Rightarrow 0.9732$$

Case I $\sin x = 0.9732$

Ref angle = $\sin^{-1}(0.9732)$
= 76.70

Q-I $\Rightarrow x = 76.70^\circ$

Q-II $x = \pi - 76.70^\circ$
= 103.3

Case II $\sin x = -0.4932$

Ref angle = $\sin^{-1}(0.4932)$
= 29.55

III $x = -\pi + 29.55$

$x = -150.45$

IV

$x = -29.55$

check $x = 76.70$

$$4 \cos(76.70) + 3 \sin(76.70) = 2$$

$$3.82 \neq 2 \text{ False}$$

check $x = 103.3$

$$4 \cos(103.3) + 3 \sin(103.3) = 2$$

$$2 = 2 \text{ True}$$

check $x = -150.45$

$$4 \cos(-150.45) + 3 \sin(-150.45) = 2$$

$$-4.9 \neq 2 \text{ False}$$

check $x = -29.55$

$$4 \cos(-29.55) + 3 \sin(-29.55) = 2$$

$$2 = 2 \text{ True}$$

sol set = $\left\{ \frac{-29.55\pi}{180}, \frac{103.3\pi}{180} \right\}$ Angles.

iii) $3\sin x - 2\cos x = 1$ $[-\pi, \pi]$

Sol: $(3\sin x)^2 = (1 + 2\cos x)^2$

$$9\sin^2 x = 1 + 4\cos^2 x + 4\cos x$$

$$4\cos^2 x + 4\cos x + 1 - 9(1 - \cos^2 x) = 0$$

$$4\cos^2 x + 4\cos x + 1 - 9 + 9\cos^2 x = 0$$

$$13\cos^2 x + 4\cos x - 8 = 0$$

$$\cos x = \frac{-4 \pm \sqrt{16 - 4(13)(-8)}}{2(13)}$$

$$\cos x = \frac{-4 \pm 12\sqrt{3}}{26}$$

$$\cos x = \frac{-4 + 12\sqrt{3}}{26}$$

$$\cos x = +0.6458$$

Ref angle = $\cos^{-1}(0.6458)$
= 49.79

$\cos$ is +ve in I & IV

QI $\Rightarrow x = 49.79$

QIV $\Rightarrow x = -49.79$

check put $x = 49.79$

$$3\sin(49.79) - 2\cos(49.79) = 1$$

$1 = 1$ True

check $x = -49.79$

$$3\sin(-49.79) - 2\cos(-49.79) = 1$$

$-3.58 \neq 1$ False

$$S.F = \left\{ \frac{-162.41\pi}{180}, \frac{49.79\pi}{180} \right\}$$

4

$$\cos x = \frac{-4 - 12\sqrt{3}}{26}$$

$$\cos x = -0.9532$$

Ref angle = $\cos^{-1}(0.9532)$
= 17.59 ~~49.79~~

$\cos(-ve)$ in II & III

QII $\Rightarrow x = 180 - 17.59$

$$x = 162.41$$

QIII $x = -180 + 17.59$

$$x = -162.41$$

check $x = 162.41$

$$3\sin(162.41) - 2\cos(162.41) = 1$$

$2.8 \neq 1$ False

put $x = -162.41$

$$3\sin(-162.41) - 2\cos(-162.41) = 1$$

$1 = 1$ True

(iv) $\sqrt{3} \cos x + \sin x = \sqrt{2}$

Sol Can also do with previous method.

Method #2 $\div$ by 2

$$\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \frac{\sqrt{2}}{2}$$

$$\sin 60^\circ \cos x + \cos 60^\circ \sin x = \frac{1}{\sqrt{2}}$$

$$\therefore 60^\circ \times \frac{\pi}{180} = \frac{\pi}{3}$$

$$\sin\left(\frac{\pi}{3} + x\right) = \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{3} + x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

Reference angle = 45° or $\frac{\pi}{4}$

OI $\frac{\pi}{3} + x = \frac{\pi}{4}$

$$x = \frac{\pi}{4} - \frac{\pi}{3} \Rightarrow \boxed{-\frac{\pi}{12}}$$

OII $\frac{\pi}{3} + x = \pi - \frac{\pi}{4}$

$$\text{Set} = \left\{ \frac{\pi}{12}, \frac{5\pi}{12} \right\}$$

$$x = \frac{3\pi}{4} - \frac{\pi}{3} \Rightarrow \boxed{x = \frac{5\pi}{12}}$$

Answer.

(v) $7 \cos x - 6 \sin x = 4$

Sol $(7 \cos x)^2 = (4 + 6 \sin x)^2$

$$49 \cos^2 x = 16 + 36 \sin^2 x + 48 \sin x$$

$$36 \sin^2 x + 48 \sin x + 16 - 49(1 - \sin^2 x) = 0$$

$$36 \sin^2 x + 48 \sin x + 16 - 49 + 49 \sin^2 x = 0$$

$$85 \sin^2 x + 48 \sin x - 33 = 0$$

$$\sin x = \frac{-48 \pm \sqrt{(48)^2 - 4(85)(-33)}}{2(85)}$$

$$\sin x = \frac{-48 + 14\sqrt{69}}{170}$$

$$, \frac{-48 - 14\sqrt{69}}{170}$$

$$\sin x = 0.4017$$

$$\text{Ref angle} = \sin^{-1}(0.4017) \\ = 23.68$$

$$\text{Q I} = 23.68$$

$$\text{Q II} \Rightarrow \pi - 23.68 \\ = \boxed{156.32}$$

$$\text{Check } x = 23.68$$

$$7 \cos(23.68) - 6 \sin(23.68) = 4 \\ 4 = 4 \text{ true}$$

$$\text{put } x = 156.32$$

$$7 \cos(156.32) - 6 \sin(156.32) = 4 \\ = -8.82 \neq 4 \text{ false}$$

$$\text{g.g.} = \left\{ \frac{-104.95}{180}, \frac{23.68\pi}{180} \right\} \text{ Answer}$$

$$\text{(vi) } 4 \cos x + 2 \sin x = \sqrt{5}$$

$$\text{for } (4 \cos x)^2 = (\sqrt{5} - 2 \sin x)^2$$

$$16 \cos^2 x = 5 + 4 \sin^2 x - 4\sqrt{5} \sin x$$

$$4 \sin^2 x - 4\sqrt{5} \sin x + 5 - 16(1 - \sin^2 x) = 0$$

$$4 \sin^2 x - 4\sqrt{5} \sin x + 5 - 16 + 16 \sin^2 x = 0$$

$$20 \sin^2 x - 4\sqrt{5} \sin x - 11 = 0$$

$$\sin x = \frac{4\sqrt{5} \pm \sqrt{80 - 4(20)(-11)}}{2(20)}$$

$$\sin x = \frac{4\sqrt{5} \pm 8\sqrt{5}}{40}$$

$$\sin x = \frac{4\sqrt{5} + 8\sqrt{5}}{40}$$

$$\sin x = -0.9664$$

$$\text{Ref angle} = \sin^{-1}(0.9664) \\ = 75.10$$

$$\text{Q-III } x = -180 + 75.10$$

$$x = -104.9$$

$$\text{Q IV } \boxed{x = -75.10}$$

$$\text{check } x = -104.9$$

$$7 \cos(-104.9) - 6 \sin(-104.9) = 4 \\ 4 = 4 \text{ True}$$

$$\text{put } x = -75.10$$

$$7 \cos(-75.10) - 6 \sin(-75.10) = 4 \\ 7.59 \neq 4 \text{ false}$$

$$\sin x = 0.9982$$

$$\text{Ref angle} = \sin^{-1}(0.9982) \\ = 86.56^\circ$$

$$\text{Q-I} \Rightarrow x = 86.56^\circ$$

$$\text{Q-II} \Rightarrow x = 180 - 86.56^\circ \\ x = 93.44^\circ$$

$$\text{Check } x = 86.56^\circ$$

$$4\cos(86.56^\circ) + 2\sin(86.56^\circ) = \sqrt{5}$$

$$2.236 = 2.236 \text{ True}$$

$$\text{put } x = 93.44^\circ$$

$$4\cos(93.44^\circ) + 2\sin(93.44^\circ) = \sqrt{5}$$

$$1.75 \neq \sqrt{5} \text{ False}$$

$$\sin x = -0.5509$$

$$\text{Ref angle} = \sin^{-1}(-0.5509) \\ = 33.42^\circ$$

$$\text{Q-III } x = -180 + 33.42$$

$$\boxed{x = -146.58^\circ}$$

$$\text{Q-IV } x = -33.42^\circ$$

$$\text{check } x = -146.58$$

$$4\cos(-146.58) + 2\sin(-146.58) = \sqrt{5}$$

$$-4.44 \neq \sqrt{5} \text{ False}$$

$$\text{Check } x = -33.42$$

$$4\cos(-33.42) + 2\sin(-33.42) = \sqrt{5}$$

$$2.2370 \approx \sqrt{5} \text{ True}$$

$$\text{Set } \left\{ \frac{-33.42\pi}{180}, \frac{86.56\pi}{180} \right\} \text{ d.}$$

$$\text{vii) } \sec x + 5 \tan x + 12 = 0$$

$$\text{Sol } \frac{1}{\cos x} + 5 \frac{\sin x}{\cos x} = -12$$

$$\frac{1 + 5\sin x}{\cos x} = -12$$

$$(1 + 5\sin x)^2 = (-12\cos x)^2$$

$$1 + 10\sin x + 25\sin^2 x = 144\cos^2 x$$

$$1 + 10\sin x + 25\sin^2 x - 144(1 - \sin^2 x) = 0$$

$$169\sin^2 x + 10\sin x - 143 = 0$$

$$\sin x = \frac{-10 \pm \sqrt{100 - 4(169)(-143)}}{2(169)}$$

$$2(169)$$

$$\sin x = \frac{-10 \pm 48\sqrt{42}}{338}$$

$$338$$

$$\sin x = \frac{-10 + 48\sqrt{42}}{338}$$

$$\sin x = 0.8907$$

$$\text{Ref angle} = \sin^{-1}(0.8907) \\ = 62.96^\circ$$

$$\text{Q-I} \Rightarrow x = 62.96^\circ$$

$$\text{Q-II} \Rightarrow x = 180^\circ - 62.96^\circ$$

$$\boxed{x = 117.04^\circ}$$

$$\text{check } x = 62.96^\circ$$

$$\sec x + \tan x + 12 = 0$$

$$23.99 \neq 0 \text{ False}$$

$$\text{check } x = 117.04^\circ$$

$$\sec x + \tan x + 12 = 0$$

$$0 \approx 0 \text{ True}$$

$$\sin x = \frac{-10 - 48\sqrt{42}}{338}$$

$$\sin x = -0.9499$$

$$\text{Ref angle} = \sin^{-1}(0.9499) \\ = 71.78^\circ$$

$$\text{Q-III} \quad x = -180^\circ + 71.78^\circ$$

$$x = -108.22^\circ$$

$$\text{Q-IV} \quad x = -71.78^\circ$$

$$\text{check } x = -108.22^\circ$$

$$\frac{1}{\cos x} + \tan x + 12 = 0$$

$$23.99 \neq 0 \text{ False}$$

$$\text{check } x = -71.78^\circ$$

$$\frac{1}{\cos x} + \tan x + 12 = 0$$

$$0 \approx 0 \text{ True}$$

$$\text{S.S.} = \left\{ \frac{-71.78\pi}{180}, \frac{117.04\pi}{180} \right\} \text{ Answer}$$

$$\text{viii) } \sin 2x - \cos x = 0$$

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x [2 \sin x - 1] = 0$$

$$\cos x = 0$$

$$\text{Ref angle} = \cos^{-1}(0) \\ = \frac{\pi}{2}$$

$$\text{Q-I} \Rightarrow x = \frac{\pi}{2}$$

$$\text{Q-IV} \quad x = -\frac{\pi}{2}$$

$$2 \sin x - 1 = 0$$

$$\sin x = \frac{1}{2}$$

$$\text{Ref angle} = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{\frac{\pi}{6}}$$

$$\text{Q-I} \Rightarrow x = \frac{\pi}{6}$$

$$\text{Q-II} \Rightarrow x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\text{S.S.} = \left\{ \pm \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \right\} \text{ A}$$

(ix) $\cos^2 x - \sin^2 x = 1$

Sol: $1 - \sin^2 x - \sin^2 x = 1$

$$1 - 2\sin^2 x = 1$$

$$-2\sin^2 x = 1 - 1$$

$$\sin^2 x = 0$$

$$\sin x = 0 \Rightarrow x = \sin^{-1}(0) \Rightarrow x = 0$$

$$x = \pi$$

$$x = -\pi \therefore [-\pi, \pi]$$

S. Set $\{0, \pm\pi\}$

(x) $3\sin^2 x - 2\sin x = 0$

Sol $\sin x (3\sin x - 2) = 0$

$$\sin x = 0$$

$$x = \sin^{-1}(0)$$

$$x = 0, \pi, -\pi$$

$$3\sin x - 2 = 0$$

$$\sin x = \frac{2}{3}$$

$$\text{Ref angle} = \sin^{-1}\left(\frac{2}{3}\right)$$

$$= 41.81^\circ$$

sin is +ve in Q I & II

$$\text{Q I} \Rightarrow x = 41.81^\circ$$

$$\text{Q II} \Rightarrow x = 180^\circ - 41.81^\circ$$

$$x = 138.18^\circ$$

S. Set $\{0, \pm\pi, \frac{41.81\pi}{180}, \frac{138.18\pi}{180}\}$

Answer

Q 5) (i) $\cos 50 + \cos 10 = \cos 30$

Sol $2\cos\left(\frac{50+10}{2}\right)\cos\left(\frac{50-10}{2}\right) - \cos 30 = 0$

$$2\cos 30 \cos 20 - \cos 30 = 0$$

$$\cos 30 [2\cos 20 - 1] = 0$$

$$\cos 30 = 0$$

$$\cos 20 = \frac{1}{2}$$

$$\text{Ref angle} = \cos^{-1}(0) = \frac{\pi}{2}$$

$$\text{Q I } 3\theta = \frac{\pi}{2}$$

$$\boxed{\theta = \frac{\pi}{6}}$$

$$\text{Q IV } 3\theta = 2\pi - \frac{\pi}{2}$$

$$\boxed{\theta = \frac{3\pi}{6}}$$

$$\text{Ref angle} = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\text{Q I } 2\theta = \frac{\pi}{3} \Rightarrow \boxed{\theta = \frac{\pi}{6}}$$

$$\text{Q IV } 2\theta = 2\pi - \frac{\pi}{3}$$

$$\boxed{\theta = \frac{5\pi}{6}}$$

$\therefore \text{Set} = \left\{ \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6} \right\}$ Answer.

$$\textcircled{\text{ii}} \sin \theta = 2 \sin \left(\frac{\pi}{3} + \theta \right)$$

$$\sin \theta = 2 \left[\sin \frac{\pi}{3} \cos \theta - \cos \frac{\pi}{3} \sin \theta \right]$$

$$\sin \theta = 2 \left[\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right]$$

$$\sin \theta = \sqrt{3} \cos \theta - \sin \theta$$

$$\frac{2 \sin \theta}{\cos \theta} = \sqrt{3} \Rightarrow \tan \theta = \frac{\sqrt{3}}{2}$$

$$\text{Ref angle} = \tan^{-1} \left(\frac{\sqrt{3}}{2} \right) = 40.89^\circ$$

$\tan$ is +ve in I & III

$$\text{Q I } \Rightarrow 40.89^\circ$$

$$\text{Q III } \Rightarrow \pi + 40.89^\circ$$

$$\boxed{\theta = 220.89^\circ}$$

$$\text{S.S.} = \left\{ \frac{40.89\pi}{180}, \frac{220.89\pi}{180} \right\}$$

Answer.

$$\textcircled{\text{iii}} \cos 3\theta + 2 \cos \theta = 0$$

$$\text{for } (4 \cos^3 \theta - 3 \cos \theta) + 2 \cos \theta = 0$$

$$4 \cos^3 \theta - \cos \theta = 0$$

$$\cos \theta [4 \cos^2 \theta - 1] = 0$$

$$\cos \theta = 0$$

$$\text{Ref angle} = \frac{\pi}{2}$$

$$\text{Q I } \theta = \frac{\pi}{2}$$

$$\text{Q IV } \theta = 2\pi - \frac{\pi}{2}$$

$$\boxed{\theta = \frac{3\pi}{2}}$$

$$\cos \theta = -\frac{1}{2}$$

$$\text{Ref angle} = \frac{\pi}{3}$$

$$\text{Q II } \theta = \pi - \frac{\pi}{3}$$

$$\boxed{\theta = \frac{2\pi}{3}}$$

$$\text{Q III } \theta = \pi + \frac{\pi}{3}$$

$$\boxed{\theta = \frac{4\pi}{3}}$$

$$\cos^2 \theta = \frac{1}{4}$$

$$\cos \theta = \pm \frac{1}{2}$$

$$\cos \theta = +\frac{1}{2}$$

$$\text{Ref angle} = \frac{\pi}{3}$$

$$\text{Q I } \theta = \frac{\pi}{3}$$

$$\text{Q IV } \theta = 2\pi - \frac{\pi}{3}$$

$$\boxed{\theta = \frac{5\pi}{3}}$$

$$\text{S.S} = \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

Answers

$$\text{(iv) } \cos 3\theta = 3\sin 2\theta$$

$$\text{Solve } (3\sin \theta - 4\sin^3 \theta) = 3(2\sin \theta \cos \theta)$$

$$3\sin \theta - 4\sin^3 \theta - 6\sin \theta \cos \theta = 0$$

$$\sin \theta [3 - 4\sin^2 \theta - 6\cos \theta] = 0$$

$$\sin \theta = 0$$

$$\theta = 0, \pi, 2\pi$$

$$3 - 4(1 - \cos^2 \theta) - 6\cos \theta = 0$$

$$3 - 4 + 4\cos^2 \theta - 6\cos \theta = 0$$

$$4\cos^2 \theta - 6\cos \theta - 1 = 0$$

$$\cos \theta = \frac{+6 \pm \sqrt{36 - 4(4)(-1)}}{2(4)}$$

$$\cos \theta = \frac{3 + \sqrt{13}}{4} ; \frac{3 - \sqrt{13}}{4}$$

$$\cos \theta = 1.6513 \notin [-1, 1]$$

$$\boxed{\cos \theta = -0.1513}$$

~~neglect~~
sol not exist

$$\cos \theta = -0.1513$$

$$\text{Ref angle} = \cos^{-1}(0.1513) \\ = \boxed{81.29}$$

cos is -ve in II & III

Q-II $\theta = \pi - 81.29$

$$\boxed{\theta = 98.71^\circ}$$

Q-III $\theta = 180 + 81.29$

$$\boxed{\theta = 261.29^\circ}$$

$$\text{S.S} = \left[0, \pi, 2\pi, \frac{98.71\pi}{180}, \frac{261.98\pi}{180} \right]$$

Answer.

✓ $\cos \theta - \cos 3\theta = \tan^2 \theta$

Sol $-2 \sin\left(\frac{\theta+3\theta}{2}\right) \sin\left(\frac{\theta-3\theta}{2}\right) = \frac{\sin^2 \theta}{\cos^2 \theta}$

$$2 \sin 2\theta \sin \theta = \frac{\sin^2 \theta}{\cos^2 \theta} = 0$$

$$2[2 \sin \theta \cos \theta] \sin \theta = \frac{\sin^2 \theta}{\cos^2 \theta} = 0$$

$$4 \sin^2 \theta \cos \theta = \frac{\sin^2 \theta}{\cos^2 \theta} = 0$$

$$\sin^2 \theta \left[4 \cos \theta - \frac{1}{\cos^2 \theta} \right] = 0$$

$$\sin^2 \theta = 0$$

$$\boxed{\sin \theta = 0, \pi, 2\pi}$$

$$4 \cos \theta - \frac{1}{\cos^2 \theta} = 0$$

$$\frac{4 \cos^3 \theta - 1}{\cos^2 \theta} = 0$$

$$4 \cos^3 \theta - 1 = 0$$

$$\cos \theta = \sqrt[3]{\frac{1}{4}}$$

$$\text{Ref angle} = 50.95$$

$\cos \theta$ is +ve in Q I & IV

if Q-I $\theta = 50.95^\circ$

Q-IV $\theta = 360 - 50.95^\circ$

$\theta = 309.04^\circ$

$\therefore \text{Set } z = \left\{ 0, \pi, 2\pi, \frac{50.95\pi}{180}, \frac{309.04\pi}{180} \right\}$ Answer.

(vi) $\sin 4\theta + \cos 3\theta = 0$

sol

~~sin 4\theta~~

$\therefore \sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$

$\sin 4\theta + \sin\left(\frac{\pi}{2} - 3\theta\right) = 0$

$2\sin\left(\frac{4\theta + \frac{\pi}{2} - 3\theta}{2}\right) \cos\left(\frac{4\theta - \frac{\pi}{2} + 3\theta}{2}\right) = 0$

$\sin\left(\frac{\theta + \frac{\pi}{2}}{2}\right) \cos\left(\frac{7\theta - \frac{\pi}{2}}{2}\right) = 0$

$\sin\left(\frac{\theta + \frac{\pi}{2}}{2}\right) = 0$;

$\cos\left(\frac{7\theta - \frac{\pi}{2}}{2}\right) = 0$

$\cos^{-1}(0) = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{\theta + \frac{\pi}{2}}{2} = \sin^{-1}(0)$

$\frac{7\theta - \frac{\pi}{2}}{2} = \frac{\pi}{2}$

$\frac{\theta + \frac{\pi}{2}}{2} = 0$

~~$\theta = \frac{\pi}{4}$~~

$\theta = -\frac{\pi}{2}$

$\theta = \frac{3\pi}{14}$

$\frac{\theta + \frac{\pi}{2}}{2} = \pi$

$\frac{7\theta - \frac{\pi}{2}}{2} = \frac{3\pi}{2}$

$\theta = \frac{3\pi}{2}$

$\theta = \frac{\pi}{2}$

$\frac{\theta + \frac{\pi}{2}}{2} = 2\pi$

$\theta = \frac{\pi}{2}$

$\therefore \text{Set } z = \left\{ \pm \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{3\pi}{14} \right\}$

Answer.

V.IHP

(vii) $2\sin\theta - \tan\theta = 6\cot\theta$

for $\therefore \cot\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta}$ Formula

$$2\sin\theta - \frac{\sin\theta}{\cos\theta} = 6\left(\frac{1+\cos\theta}{\sin\theta}\right)$$

$$2\sin^2\theta - \frac{\sin^2\theta}{\cos\theta} = 6 + 6\cos\theta$$

$$\frac{2\sin^2\theta\cos\theta - \sin^2\theta}{\cos\theta} = 6 + 6\cos\theta$$

$$2\sin^2\theta\cos\theta - \sin^2\theta = 6\cos\theta + 6\cos^2\theta$$

$$2(1-\cos^2\theta)\cos\theta - (1-\cos^2\theta) = 6\cos\theta + 6\cos^2\theta$$

$$2\cos\theta - 2\cos^3\theta - 1 + \cos^2\theta = 6\cos\theta + 6\cos^2\theta = 0$$

$$-2\cos^3\theta - 5\cos^2\theta - 4\cos\theta - 1 = 0$$

$$2\cos^3\theta + 5\cos^2\theta + 4\cos\theta + 1 = 0$$

Let $\cos\theta = x$

$$2x^3 + 5x^2 + 4x + 1 = 0$$

	2	5	4	1
-1		-2	-3	-1
	2	3	1	0

$$2x^2 + 3x + 1 = 0$$

$$2x^2 + 2x + x + 1 = 0$$

$$2x(x+1) + 1(x+1) = 0$$

$$(2x+1)(x+1) = 0$$

$$2x+1 = 0 \quad ; \quad x+1 = 0$$

$$\cos\theta = -\frac{1}{2}$$

$$\cos\theta = -1$$

$$\theta = \cos^{-1}(-1)$$

$$\theta = \pi$$

$$\text{Ref angle} = \cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$$

$$\text{QII} \Rightarrow \boxed{\theta = \frac{2\pi}{3}}$$

$$\text{QIII} \Rightarrow \boxed{\theta = \frac{4\pi}{3}}$$

$$\text{S.S} = \left\{ \pi, \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

$$\text{(viii)} \cos^2 \theta + \csc^2 \theta = 0$$

$$\text{Soli} \quad \cos^2 \theta - \sin^2 \theta + \frac{1}{\sin^2 \theta} = 0$$

$$1 - \sin^2 \theta - \sin^2 \theta + \frac{1}{\sin^2 \theta} = 0$$

$$1 - 2\sin^2 \theta + \frac{1}{\sin^2 \theta} = 0$$

$$\frac{\sin^2 \theta - 2\sin^4 \theta + 1}{\sin^2 \theta} = 0$$

$$-2\sin^4 \theta + \sin^2 \theta + 1 = 0$$

$$2\sin^4 \theta - \sin^2 \theta - 1 = 0$$

$$\text{Let } \sin x = 0$$

$$2x^4 - x^2 - 1 = 0$$

$$2x^4 - 2x^2 + 1x^2 - 1 = 0$$

$$(2x^2 + 1)(x^2 - 1) = 0$$

$$x^2 = -\frac{1}{2}$$

$$x^2 = 1$$

$$\boxed{x = \pm 1}$$

$$\boxed{x = \frac{1}{\sqrt{2}}i}$$

Not possible

$$\sin \theta = 1$$

$$\text{Ref angle} = \sin^{-1}(1) = \frac{\pi}{2}$$

$$\sin \theta = -1$$

$$\text{Ref angle} = \sin^{-1}(1)$$

$$\text{QIII} \quad \theta = \pi + \frac{\pi}{2}$$

$$\boxed{\theta = \frac{3\pi}{2}}$$

$$\text{QI} \Rightarrow \theta = \frac{\pi}{2}$$

$$\text{QII} \Rightarrow \theta = \pi - \frac{\pi}{2}$$

$$\boxed{\theta = \frac{\pi}{2}}$$

$$\text{QIV}$$

$$\theta = 2\pi - \frac{\pi}{2}$$

$$\boxed{\theta = \frac{3\pi}{2}}$$

$$\text{S.Set} = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\} \text{ Answers.}$$

$$\text{Q1} \sin x + \cos x = 6$$

$$\text{Sol Q6) } \textcircled{1} 10 \cos x - 24 \sin x = 13$$

$$\text{Sol: } (10 \cos x)^2 = (13 + 24 \sin x)^2$$

$$100 \cos^2 x = 169 + 576 \sin^2 x + 624 \sin x$$

$$100(1 - \sin^2 x) = 169 + 576 \sin^2 x + 624 \sin x$$

$$-576 \sin^2 x + 624 \sin x + 69 = 0$$

$$\sin x = \frac{-624 \pm \sqrt{389376 - 4(676)(69)}}{2(676)}$$

$$\sin x = \frac{-624 \pm 450.33}{1352}$$

$$\sin x = \frac{-624 + 450.33}{1352}$$

$$\sin x = \frac{-624 - 450.33}{1352}$$

$$\sin x = -0.1284$$

$$\text{Ref angle} = \sin^{-1}(0.1284) \\ = 7.3771$$

$$\text{I. } \sin x = -0.7946$$

$$\text{Ref angle} = \sin^{-1}(0.7946) \\ = 52.6174$$

$$\text{QIII } x = \pi + 7.3771$$

$$x = 187.3771$$

$$\text{QIII } x = \pi + 52.6174$$

$$x = 232.6174$$

$$\text{QIV } x = 360 - 7.3771$$

$$x = 352.6229$$

$$\text{QIV } x = 360 - 52.6174$$

$$x = 307.3826$$

~~~~~



check  $x = 187.37$   
 $10 \cos x - 24 \sin x = 13$   
 $-6 \neq 13$  False

$x = 352.6229$   
 $10 \cos x - 24 \sin x = 13$   
 $13 = 13$  True

check  $x = 232.61$   
 $10 \cos x - 24 \sin x = 13$   
 $13 = 13$

check  $x = 307.3826$   
 $10 \cos x - 24 \sin x = 13$   
 $25.1 \neq 13$   
 False

Sol:  $\left\{ \frac{232.61\pi}{180} + 2n\pi \right\} \cup \left\{ \frac{352.63\pi}{180} + 2n\pi \right\}$

ii)  $8 \sin x + 15 \cos x = 6$

Sol:  $(15 \cos x)^2 = (6 - 8 \sin x)^2$   
 $225(1 - \sin^2 x) = 36 + 64 \sin^2 x - 96 \sin x$   
 $64 \sin^2 x - 96 \sin x + 36 - 225 + 225 \sin^2 x = 0$   
 $289 \sin^2 x - 96 \sin x - 189 = 0$

$\sin x = \frac{96 \pm \sqrt{96^2 - 4(289)(-189)}}{2(289)}$

$\sin x = \frac{96 \pm 30\sqrt{253}}{578}$

$\sin x = 0.9916$

Ref angle  $= \sin^{-1}(0.9916)$   
 $= 82.56$

Q-I  $x = 82.56$

Q-II  $x = 180 - 82.56$   
 $= 97.44$

$\sin x = -0.6594$

Ref angle  $= \sin^{-1}(0.6594)$   
 $= 41.25$

Q-III  $x = 180 + 41.25$   
 $= 221.25$

Q-IV  $x = 360 - 41.25$   
 $= 318.75$



check  $x = 82.56$

$8 \sin x + 15 \cos x = 6$

$9.87 \neq 6$  False

check  $x = 97.44$

$8 \sin x + 15 \cos x = 6$

$6 = 6$  True

check  $x = 221.25$

$8 \sin x + 15 \cos x = 6$

$-16.5 \neq 6$  False

check  $x = 318.75$

$8 \sin x + 15 \cos x = 6$

$6 = 6$  True

$$S.S. = \left\{ \frac{97.44\pi}{180} + 2n\pi \right\} \cup \left\{ \frac{318.75\pi}{180} + 2n\pi \right\} \quad A$$

iii)  $\cos x - \sin 4x = 0$

Sol:  $\cos x - \sin 2(2x) = 0$

$\cos x - 2 \sin 2x \cos 2x = 0$

$\cos x - 2 [2 \sin x \cos x] [\cos^2 x - \sin^2 x] = 0$

$\cos x - 4 \sin x \cos x [\cos^2 x - [1 - \cos^2 x]] = 0$

$\cos x - 4 \sin x \cos x [\cos^2 x - 1 + \cos^2 x] = 0$

$\cos x - 4 \sin x \cos x [2\cos^2 x - 1] = 0$

$\cos x - 8 \sin x \cos^3 x + 4 \sin x \cos x = 0$

$\cos x [1 - 8 \sin x \cos^2 x + 4 \sin x] = 0$

$\cos x = 0 \quad \text{or} \quad 1 - 8 \sin x \cos^2 x + 4 \sin x = 0$

Ref:  $x = \cos^{-1}(0)$   
 $= \frac{\pi}{2}$

Q-I  $x = \frac{\pi}{2}$

Q-IV  $x = 2\pi - \frac{\pi}{2}$

$x = \frac{3\pi}{2}$

$1 - 8 \sin x (1 - \sin^2 x) + 4 \sin x = 0$

$1 - 8 \sin x + 8 \sin^3 x + 4 \sin x = 0$

$8 \sin^3 x - 4 \sin x + 1 = 0$

Let  $\sin x = y$



$$8y^3 - 4y + 1 = 0$$

$$\begin{array}{c|ccc} \frac{1}{2} & 8 & 0 & -4 & 1 \\ & 4 & 2 & -1 & \\ \hline & 8 & 4 & -2 & 0 \end{array}$$

$$y = \frac{1}{2}$$

or

$$8y^2 + 4y - 2 = 0$$

$$2[4y^2 + 2y - 1] = 0$$

$$a=4 \quad b=2 \quad c=-1$$

$$y = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{2(4)}$$

$$y = \frac{-2 \pm \sqrt{20}}{8}$$

$$y = 0.3090$$

$$y = -0.8090$$

$$\sin x = \frac{1}{2}$$

$$x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$R.f = \frac{\pi}{6}$$

$$\text{Q I } x = \frac{\pi}{6}$$

$$\text{Q II } x = \pi - \frac{\pi}{6}$$

$$x = \frac{5\pi}{6}$$

$$\sin x = 0.3090$$

$$x = \sin^{-1}(0.3090)$$

$$R.f = \frac{18\pi}{180} \Rightarrow \frac{\pi}{10}$$

$$\text{Q I } x = \frac{\pi}{10}$$

$$\text{Q II } x = \pi - \frac{\pi}{10}$$

$$x = \frac{9\pi}{10}$$

$$\sin x = -0.8090$$

$$\text{Q III } x = \pi + \frac{3\pi}{10}$$

$$\text{Ref angle} = \frac{54\pi}{180} = \frac{3\pi}{10}$$

$$\text{Q III } x = \pi + \frac{3\pi}{10}$$

$$x = \frac{13\pi}{10}$$

$$\text{Q IV } x = 2\pi - \frac{3\pi}{10}$$

$$x = \frac{17\pi}{10}$$

$$S.S = \left\{ \frac{\pi}{2} + 2n\pi \right\} \cup \left\{ \frac{3\pi}{2} + 2n\pi \right\} \cup \left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\} \cup$$

$$\left\{ \frac{\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{9\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{13\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{17\pi}{10} + 2n\pi \right\}$$



iv)  $5\sin x + 3\cos x = 4$

sol  $(5\sin x)^2 = (4 - 3\cos x)^2$

$$25\sin^2 x = 16 + 9\cos^2 x - 24\cos x$$

$$9\cos^2 x - 24\cos x + 16 - 25(1 - \cos^2 x) = 0$$

$$9\cos^2 x - 24\cos x + 16 - 25 + 25\cos^2 x = 0$$

$$34\cos^2 x - 24\cos x - 9 = 0$$

$$\cos x = \frac{24 \pm \sqrt{(-24)^2 - 4(34)(-9)}}{2(34)}$$

$$\cos x = \frac{24 \pm 30\sqrt{2}}{68}$$

$$\cos x = 0.9768$$

$$\cos x = -0.2709$$

Ref angle  $= \cos^{-1}(0.9768)$   
 $= 12.36^\circ$

Ref  $= \cos^{-1}(0.2709)$   
 $= 74.28^\circ$

Q I  $\Rightarrow x = 12.36^\circ$

Q II  $x = 180 - 74.28$   
 $x = 105.72^\circ$

Q IV  $\Rightarrow x = 360 - 12.36$

Q III  $x = 180 + 74.28$

$$x = 347.64^\circ$$

$$x = 254.28^\circ$$

check  $x = 12.36^\circ$

$$5\sin x + 3\cos x = 4$$
  
 $4 = 4$  True

check  $x = 105.72^\circ$

$$5\sin x + 3\cos x = 4$$
  
 $4 = 4$  True

check  $x = 347.64^\circ$

$$5\sin x + 3\cos x = 4$$
  
 $1.86 \neq 4$  False

check  $x = 254.28^\circ$

$$5\sin x + 3\cos x = 4$$

$$-5.62 \neq 4$$
 False

$$S.S = \left\{ \frac{12.36\pi}{180} + 2n\pi \right\} \cup \left\{ \frac{105.72\pi}{180} + 2n\pi \right\}$$

Answer.



$$\textcircled{v} \quad 4\cos x + 3\sin x = 2$$

$$\text{sq} \quad (4\cos x)^2 = (2 - 3\sin x)^2$$

$$16\cos^2 x = 4 + 9\sin^2 x - 12\sin x$$

$$9\sin^2 x - 12\sin x + 4 - 16(1 - \sin^2 x) = 0$$

$$9\sin^2 x - 12\sin x + 4 - 16 + 16\sin^2 x = 0$$

$$25\sin^2 x - 12\sin x - 12 = 0$$

$$\sin x = \frac{12 \pm \sqrt{144 - 4(25)(-12)}}{2(25)}$$

$$\sin x = \frac{6 + 4\sqrt{21}}{25}$$

$$; \sin x = \frac{6 - 4\sqrt{21}}{25}$$

$$\boxed{\sin x = 0.9732}$$

$$; \boxed{\sin x = -0.4932}$$

$$\text{Ref angle} = \sin^{-1}(0.9732)$$

$$= 76.70^\circ$$

$$\text{Ref angle} = \sin^{-1}(0.4932)$$

$$= 29.55^\circ$$

$$\textcircled{I} \quad x = 76.70^\circ$$

$$\textcircled{III} \quad x = 180 + 29.55$$

$$x = 209.55$$

$$\textcircled{II} \quad x = 180 - 76.70^\circ$$

$$\textcircled{IV} \quad x = 360 - 29.55$$

$$\boxed{x = 103.3^\circ}$$

$$\boxed{x = 330.45^\circ}$$

$$\text{Check } x = 76.70^\circ$$

$$4\cos x + 3\sin x = 2$$

$$3.8 \neq 2 \text{ False}$$

$$\text{Check } x = 209.55^\circ$$

$$4\cos x + 3\sin x = 2$$

$$-4.95 \neq 2 \text{ False}$$

$$\text{Check } x = 103.3^\circ$$

$$4\cos x + 3\sin x = 2$$

$$2 = 2 \text{ True}$$

$$\text{Check } x = 330.45^\circ$$

$$4\cos x + 3\sin x = 2$$

$$2 = 2$$

$$\text{S.S.} = \left\{ \frac{103.3\pi}{180} + 2n\pi \right\} \cup \left\{ \frac{330.45\pi}{180} + 2n\pi \right\}$$

Answers.



(vii)  $\tan k = 3 \sin 2k$

$\frac{\sin k}{\cos k} = 3[2 \sin k \cos k]$

$\sin k = 6 \sin k \cos^2 k$

$6 \sin k \cos^2 k - \sin k = 0$

$\sin k [6 \cos^2 k - 1] = 0$

$\sin k = 0$

$k = \sin^{-1}(0)$

$k = n\pi$

$6 \cos^2 k - 1 = 0$

$\cos k = \pm \frac{1}{\sqrt{6}}$

$\cos k = +\frac{1}{\sqrt{6}} ; \cos k = -\frac{1}{\sqrt{6}}$

Ref angle =  $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$   
 $= 65.90^\circ$

Ref angle =  $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$   
 $= 65.90^\circ$

Q I  $\Rightarrow k = 65.90^\circ$

Q II  $k = 180 - 65.90^\circ$

Q IV  $k = 360 - 65.90^\circ$

$k = 294.1$

$k = 114.1$

Q III  $k = 180 + 65.90^\circ$

$k = 245.9$

$SS = \{n\pi\} \cup \left\{\frac{65.90\pi}{180} + 2n\pi\right\} \cup \left\{\frac{294.1\pi}{180} + 2n\pi\right\} \cup$

$\left\{\frac{114.1\pi}{180} + 2n\pi\right\} \cup \left\{\frac{245.9\pi}{180} + 2n\pi\right\} A$

(viii)  $5 \sin 3\beta + 6 \cos 3\beta = 1$

$(6 \cos 3\beta)^2 = (1 - 5 \sin 3\beta)^2$

$36 \cos^2 3\beta = 1 + 25 \sin^2 3\beta - 10 \sin 3\beta$

$25 \sin^2 3\beta - 10 \sin 3\beta + 1 - 36(1 - \sin^2 3\beta) = 0$

$25 \sin^2 3\beta - 10 \sin 3\beta + 1 - 36 + 36 \sin^2 3\beta = 0$

$61 \sin^2 3\beta - 10 \sin 3\beta - 35 = 0$



$$\sin 3\beta = \frac{10 \pm \sqrt{100 - 4(61)(-35)}}{2(61)}$$

$$\sin 3\beta = \frac{5 + 12\sqrt{15}}{61}$$

$$\sin 3\beta = \frac{5 - 12\sqrt{15}}{61}$$

$$\sin 3\beta = 0.8438$$

$$\sin 3\beta = -0.6799$$

$$\text{ref angle} = \sin^{-1}(0.8438) = 57.54$$

$$\text{ref angle} = \sin^{-1}(0.6799) = 42.83$$

is in Q-I & II

$$\text{QI} \Rightarrow 3\beta = 57.54$$

$$\beta = 19.18^\circ$$

$$\text{QII} \Rightarrow 3\beta = 180 - 57.54$$

$$\beta = 40.82^\circ$$

check  $\beta = 19.18^\circ$

$$5\sin 3\beta + 6\cos 3\beta = 1$$

$$7.4 \neq 1 \text{ False}$$

check  $\beta = 40.82^\circ$

$$5\sin 3\beta + 6\cos 3\beta = 1$$

$$1 \approx 1 \text{ True}$$

$$\text{QIII} \Rightarrow 3\beta = 180 + 42.83$$

$$\beta = 74.27^\circ$$

$$\text{QIV} \Rightarrow 3\beta = 360 - 42.83$$

$$\beta = 105.72^\circ$$

check  $\beta = 74.27^\circ$

$$5\sin 3\beta + 6\cos 3\beta = 1$$

$$-7.7 \neq 1 \text{ False}$$

check  $\beta = 105.72^\circ$

$$5\sin 3\beta + 6\cos 3\beta = 1$$

$$1 \approx 1 \text{ True}$$

$$\beta = \left\{ \frac{40.82\pi}{180} + 2n\pi \right\} \cup \left\{ \frac{105.72\pi}{180} + 2n\pi \right\}$$

Answer.

$$(ix) 2\cos 4x + 8\cos 2x = 3$$

Let  $2\cos 2(2x) + 8[\cos^2 x - \sin^2 x] = 3$

$$2[\cos^2 2x - \sin^2 2x] + 8[\cos^2 x - \sin^2 x] = 3$$

$$2[1 - \sin^2 2x - \sin^2 2x] + 8[1 - \sin^2 x - \sin^2 x] = 3$$

$$2 - 4\sin^2 2x + 8 - 16\sin^2 x - 3 = 0$$

$$-4\sin^2 2x - 16\sin^2 x + 7 = 0$$

$$-4[\sin 2x]^2 - 16\sin^2 x + 7 = 0$$

$$-4[2\sin x \cos x]^2 - 16\sin^2 x + 7 = 0$$

$$-4[4\sin^2 x \cos^2 x] - 16\sin^2 x + 7 = 0$$



$$-16\sin^2 x (1 - \sin^2 x) - 16\sin^2 x + 7 = 0$$

$$-16\sin^2 x + 16\sin^4 x - 16\sin^2 x + 7 = 0$$

$$16\sin^4 x - 32\sin^2 x + 7 = 0$$

$$\text{Let } \sin^2 x = y$$

$$16y^2 - 32y + 7 = 0$$

$$y = \frac{+32 \pm \sqrt{(32)^2 - 4(16)(7)}}{2(16)}$$

$$y = \frac{32 \pm 24}{32}$$

$$y = \frac{32 + 24}{32}$$

$$y = \frac{7}{4}$$

$$\sin^2 x = \frac{7}{4}$$

$$\sin x = \pm \sqrt{\frac{7}{4}}$$

$$\sin x = +1.3228 \notin [-1, 1]$$

$$\sin x = -1.3228 \notin [-1, 1]$$

$$\text{Now } \sin x = +\frac{1}{2}$$

$$\text{Ref angle} = \sin^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{6}$$

$$\text{Q I } x = \frac{\pi}{6}$$

$$\text{Q II } x = \pi - \frac{\pi}{6}$$

$$\boxed{x = \frac{5\pi}{6}}$$

$$; y = \frac{32 - 24}{32}$$

$$; y = \frac{1}{4}$$

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm \sqrt{\frac{1}{4}}$$

$$\boxed{\sin x = \pm \frac{1}{2}}$$

$$\sin x = -\frac{1}{2}$$

$$\text{Ref angle} = \sin^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{6}$$

$$\text{Q III } x = \pi + \frac{\pi}{6}$$

$$\boxed{x = \frac{7\pi}{6}}$$

$$\text{Q IV } x = 2\pi - \frac{\pi}{6}$$

$$\boxed{x = \frac{11\pi}{6}}$$



$$S.S = \left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{7\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{11\pi}{6} + 2n\pi \right\}$$

Answer.

x)  $\tan 3x = \sqrt{3}$

sol Ref angle =  $\tan^{-1}(\sqrt{3})$

$$= \frac{\pi}{3}$$

Q I  $\Rightarrow 3x = \frac{\pi}{3} \Rightarrow \boxed{x = \frac{\pi}{9}}$

S.S =  $\left\{ \frac{\pi}{9} + n\pi \right\} \cup \left\{ \frac{4\pi}{9} + n\pi \right\}$

A

Q III  $3x = \pi + \frac{\pi}{3} \Rightarrow \boxed{x = \frac{4\pi}{9}}$

(xi)  $\tan 4x = 3$

sol  $\tan 4x = \pm \sqrt{3}$

$\tan 4x = +\sqrt{3}$

Ref angle =  $\tan^{-1}(\sqrt{3})$   
 $= \frac{\pi}{3}$

$\tan 4x = -\sqrt{3}$

R.F =  $\tan^{-1}(\sqrt{3})$   
 $= \frac{\pi}{3}$

Q I  $\Rightarrow 4x = \frac{\pi}{3} \Rightarrow \boxed{x = \frac{\pi}{12}}$

Q II  $4x = \pi - \frac{\pi}{3} \Rightarrow \boxed{x = \frac{2\pi}{3}}$

Q III  $\Rightarrow 4x = \pi + \frac{\pi}{3}$

$$\boxed{x = \frac{\pi}{3}}$$

Q IV  $\Rightarrow 4x = 2\pi - \frac{\pi}{3}$

$$\boxed{x = \frac{5\pi}{12}}$$

S.S =  $\left\{ \frac{\pi}{12} + n\pi \right\} \cup \left\{ \frac{2\pi}{3} + n\pi \right\} \cup \left\{ \frac{\pi}{3} + n\pi \right\} \cup \left\{ \frac{5\pi}{12} + n\pi \right\}$

A

(xii)  $\sec u + \tan u = 3$

sol  $\frac{1}{\cos u} + \frac{\sin u}{\cos u} = 3$

$\frac{1 + \sin u}{\cos u} = 3$



$$(1 + \sin u)^2 = (3 \cos u)^2$$

$$1 + \sin^2 u + 2 \sin u = 9(1 - \sin^2 u)$$

$$\sin^2 u + 2 \sin u + 1 - 9 + 9 \sin^2 u = 0$$

$$10 \sin^2 u + 2 \sin u - 8 = 0$$

$$5 \sin^2 u + \sin u - 4 = 0$$

$$\text{Let } \sin u = x$$

$$5x^2 + x - 4 = 0$$

$$(5x + 4)(x - 1) = 0$$

$$5x + 4 = 0$$

$$x = -\frac{4}{5}$$

$$\sin u = -\frac{4}{5}$$

$$\text{Ref angle} = 53.13$$

$$\text{QI} \Rightarrow u = 53.13$$

$$\text{QII} \Rightarrow u = 180 - 53.13$$

$$u = 126.87$$

$$\text{Check } u = 53.13$$

$$\sec u + \tan u = 3$$

$$3 \approx 3 \text{ True}$$

$$\text{Check } u = 126.87$$

$$\sec u + \tan u = 3$$

$$-3 \neq 3 \text{ False}$$

$$x - 1 = 0$$

$$x = 1$$

$$\sin u = 1$$

$$\text{Ref angle} = \frac{\pi}{2}$$

$$\text{QIII} \Rightarrow u = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$$

$$\text{QIV} \Rightarrow u = 2\pi - \frac{\pi}{2}$$

$$u = \frac{3\pi}{2}$$

$$\text{Check } u = \frac{3\pi}{2} = 270^\circ$$

$$\sec u + \tan u = 3$$

$$\infty \neq 3$$

$$\text{False}$$

$$\text{Sol set} = \left\{ \frac{53.13\pi}{180} + 2n\pi \right\} \text{ Answers}$$



xiii)  $4\sin 2x - 3\cos 2x = 2$

sol  $(4\sin 2x) = (2 + 3\cos 2x)^2$

~~16\sin^2 2x~~  $16\sin^2 2x = 4 + 9\cos^2 2x + 12\cos 2x$

$16[1 - \cos^2 2x] = 4 + 9\cos^2 2x + 12\cos 2x$

$9\cos^2 2x + 12\cos 2x + 4 - 16 + 16\cos^2 2x = 0$

$25\cos^2 2x + 12\cos 2x - 12 = 0$

let  $\cos 2x = y$

$25y^2 + 12y - 12 = 0$

$y = \frac{-12 \pm \sqrt{144 - 4(25)(-12)}}{2(25)}$

$y = \frac{-12 \pm 8\sqrt{21}}{50}$

$\cos 2x = \frac{-12 + 8\sqrt{21}}{50}$

;  $\cos 2x = \frac{-12 - 8\sqrt{21}}{50}$

$\cos 2x = 0.4932$

$\cos 2x = -0.9732$

R.f =  $\cos^{-1}(0.4932)$   
=  $60.44$

Ref angle =  $\cos^{-1}(0.9732)$   
=  $13.29$

QI  $2x = 60.44$

$x = 30.22$

QII  $\Rightarrow 2x = 180 - 13.29$

$x = 83.355$

QIV  $2x = 360 - 60.44$

$x = 149.78$

COIII  $2x = 180 + 13.29$

$x = 96.64^\circ$

check  $x = 30.22^\circ$

$4\sin 2x - 3\cos 2x = 2$   
 $2 = 2$

check  $x = 83.35$

$4\sin 2x - 3\cos 2x = 2$   
 $3.8 \neq 2$  False

check  $x = 149.78$

$-4.9 \neq 2$  False

check  $x = 96.64$

$2 = 2$  True

S.S =  $\left\{ \frac{30.22\pi}{180} + 2n\pi \right\} \cup \left\{ \frac{96.64\pi}{180} + 2n\pi \right\}$

A



(xiv)  $\cos 4x + \cos 6x = \cos 5x$

Sol:-  $2\cos\left(\frac{10x}{2}\right)\cos\left(\frac{2x}{2}\right) - \cos 5x = 0$

$$\cos 5x [2\cos x - 1] = 0$$

$$\cos 5x = 0$$

$$5x = \cos^{-1}(0)$$

$$5x = (2n+1)\frac{\pi}{2}$$

$$x = (2n+1)\frac{\pi}{10}$$

$$2\cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$\text{Ref angle} = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Q.I  $x = \frac{\pi}{3}$

Q.IV  $x = 2\pi - \frac{\pi}{3}$

Sol:-  $\left\{ (2n+1)\frac{\pi}{10} \right\} \cup \left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\}$   $x = \frac{5\pi}{3}$

(xv)

Sol

$$4\cos^2 x + 1 = 4\cos x$$

$$4\cos^2 x - 4\cos x + 1 = 0$$

$$\cos x = \frac{4 \pm \sqrt{16 - 4(4)(1)}}{2(4)}$$

$$\cos x = \frac{4}{8} \Rightarrow \frac{1}{2}$$

$$x = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \text{Ref angle} = \frac{\pi}{3}$$

Sol Q.I  $x = \frac{\pi}{3}$

Q.IV  $x = 2\pi - \frac{\pi}{3} \Rightarrow \frac{5\pi}{3}$

Sol:-  $\left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\}$  Answer.

(xvi)

Sol

$$3\tan^2 x + 5\sec x + 1 = 0$$

$$3(\sec^2 x - 1) + 5\sec x + 1 = 0$$

$$3\sec^2 x - 3 + 5\sec x + 1 = 0$$

$$3\sec^2 x + 5\sec x - 2 = 0$$



$$\begin{aligned} \text{Let } \sec x &= y \\ 3y^2 + 5y - 2 &= 0 \\ 3y(y+2) - 1(y+2) &= 0 \\ (3y-1)(y+2) &= 0 \end{aligned}$$

$$y = \frac{1}{3}$$

$$\sec x = \frac{1}{3}$$

$$\cos x = 3$$

$x = \cos^{-1}(3) \notin [-1, 1]$   
solution doesn't exist

$$\text{S.P.} = \left\{ \frac{2\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{4\pi}{3} + 2n\pi \right\}$$

Answer

$$(xvii) \sqrt{2} \tan x \cos x - \tan x = 0$$

$$\text{for } \tan x [\sqrt{2} \cos x - 1] = 0$$

$$\tan x = 0$$

$$x = \tan^{-1}(0)$$

$$x = n\pi$$

$$\text{S.P.} = \left\{ \frac{\pi}{4} + 2n\pi \right\} \cup \left\{ \frac{7\pi}{4} + 2n\pi \right\} \cup \{n\pi\}$$

Answer

$$y = -2$$

$$\sec x = -2$$

$$\cos x = -\frac{1}{2}$$

$$\text{Ref} = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\text{QII} \Rightarrow x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\text{QIII} \Rightarrow x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$\cos x = \frac{1}{\sqrt{2}}$$

$$\text{Ref angle} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\text{QI} \Rightarrow x = \frac{\pi}{4}$$

$$\text{QIV} \Rightarrow x = 2\pi - \frac{\pi}{4}$$

$$x = \frac{7\pi}{4}$$



Q7) (i) Solution:  $\cos(3\theta) = -1$   $[0, 2\pi)$

$$3\theta = \cos^{-1}(-1)$$

$$3\theta = \pi \Rightarrow \boxed{\theta = \frac{\pi}{3}}$$

(ii) Solution:

$$\sin(2\theta) = -1$$

$$2\theta = \sin^{-1}(-1)$$

$$2\theta = \frac{3\pi}{2} + 2n\pi$$

$$\boxed{\theta = \frac{3\pi}{4} + n\pi}$$

(iii)  $\sin \theta = \sin\left(\frac{2\pi}{3}\right)$   $[0, 2\pi)$

Sol:  $\sin \theta = \frac{\sqrt{3}}{2}$

Ref angle =  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

Q-I  $\boxed{\theta = \frac{\pi}{3}}$

Q-II  $\pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$

S.S =  $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$  Ans

(iv)  $\sin \theta = \sin 3\theta$

Sol:  $\sin \theta - \sin 3\theta = 0$

$$2 \cos\left(\frac{\theta+3\theta}{2}\right) \sin\left(\frac{\theta-3\theta}{2}\right) = 0$$

$$2 \cos 2\theta \sin(-\theta) = 0$$

$$-2 \cos 2\theta \sin \theta = 0$$

$$\cos 2\theta = 0$$

$$2\theta = \cos^{-1}(0)$$

$$\boxed{\theta = \frac{(2n+1)\pi}{4}}$$

$$\sin \theta = 0$$

$$\theta = \sin^{-1}(0)$$

$$\boxed{\theta = n\pi}$$

S.S =  $\left\{n\pi\right\} \cup \left\{\frac{(2n+1)\pi}{4}\right\}$  Answer.



✓) Sol:

$$\sin 3\theta = 0 \Rightarrow 3\theta = \sin^{-1}(0)$$

$$3\theta = n\pi$$

$$\boxed{\theta = \frac{n\pi}{3}} \text{ Answer}$$

✓) ✓)

$$\cos 2\theta = \frac{1}{2}$$

Sol:

$$2\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

Reference angle =  $\frac{\pi}{4}$

QI

$$2\theta = \frac{\pi}{4} + 2n\pi$$

$$\boxed{\theta = \frac{\pi}{8} + n\pi}$$

QIV

$$2\theta = 2\pi - \frac{\pi}{4} \Rightarrow 2\theta = \frac{7\pi}{4} + 2n\pi$$

$$\boxed{\theta = \frac{7\pi}{8} + n\pi}$$

$$S.S = \left\{ \frac{\pi}{8} + n\pi \right\} \cup \left\{ \frac{7\pi}{8} + n\pi \right\}$$

✓) ✓)

$$\tan\left(\theta + \frac{\pi}{4}\right) = 0 \quad [0, 2\pi)$$

Sol

$$\theta + \frac{\pi}{4} = \tan^{-1}(0)$$

$$\theta + \frac{\pi}{4} = 0$$

$$, \quad \theta + \frac{\pi}{4} = \pi$$

$$\theta = -\frac{\pi}{4} \notin [0, 2\pi)$$

$$\theta = \pi - \frac{\pi}{4}$$

$$\boxed{\theta = \frac{3\pi}{4}}$$

$$S.S = \left\{ \frac{3\pi}{4} \right\} \text{ Answer.}$$

✓) ✓)

$$\tan(2x) = \sqrt{3}$$

Sol

$$2x = \tan^{-1}(\sqrt{3})$$

Ref angle =  $\frac{\pi}{3}$

$$QI \Rightarrow 2x = \frac{\pi}{3}$$

$$\boxed{x = \frac{\pi}{6}}$$

$$\boxed{x = \frac{\pi}{6} + n\pi}$$



Q-III

$$2x = \pi + \frac{\pi}{3}$$

$$x = \frac{4\pi}{6} + n\pi$$

$$S.S = \left\{ \frac{\pi}{6} + n\pi \right\} \cup \left\{ \frac{4\pi}{6} + n\pi \right\} \text{ Answer}$$

(ix)

$$\tan 3\alpha = -\frac{1}{\sqrt{3}} \quad [0, 2\pi)$$

for so ~~3\alpha~~

$$3\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Ref angle =  $\frac{\pi}{6}$

Q-II  $3\alpha = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$$\alpha = \frac{5\pi}{18} + n\pi$$

Q-IV

$$3\alpha = 2\pi - \frac{\pi}{6} \Rightarrow \frac{11\pi}{6}$$

$$\alpha = \frac{11\pi}{18} + n\pi$$

$$S.S = \left\{ \frac{5\pi}{18} + n\pi \right\} \cup \left\{ \frac{11\pi}{18} + n\pi \right\}$$

Answer

(x)  $\sec(2x) = -2$

Translate  $\sec \alpha = -2$

Let  $\alpha = 2x$

$$\cos \alpha = -\frac{1}{2}$$

$$\alpha = \cos^{-1}\left(-\frac{1}{2}\right)$$

Ref angle =  $\frac{\pi}{3}$

Q-II  $2x = \pi - \frac{\pi}{3} \Rightarrow 2x = \frac{2\pi}{3} + 2n\pi$

$$x = \frac{\pi}{3} + n\pi$$

Q-III  $2x = \pi + \frac{\pi}{3} \Rightarrow \frac{4\pi}{3} + 2n\pi$

$$2x = \frac{4\pi}{3} + 2n\pi \Rightarrow x = \frac{2\pi}{3} + n\pi$$

$$S.S = \left\{ \frac{\pi}{3} + n\pi \right\} \cup \left\{ \frac{2\pi}{3} + n\pi \right\} \text{ Answer}$$



(xi)

sol:

$$\sin(\theta + \pi) = -\frac{1}{2}$$

$$\text{Let } x = \theta + \pi$$

$$\sin x = -\frac{1}{2}$$

$$x = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \boxed{\frac{\pi}{6}}$$

$$\text{QIII} \Rightarrow x = \pi + \frac{\pi}{6} = \boxed{\frac{7\pi}{6}}$$

$$\theta + \pi = \frac{7\pi}{6}$$

$$\theta = \frac{7\pi}{6} - \pi \Rightarrow \boxed{\frac{\pi}{6}}$$

QIV

$$x = 2\pi - \frac{\pi}{6}$$

$$\theta + \pi = \frac{11\pi}{6}$$

$$\theta = \frac{11\pi}{6} - \pi \Rightarrow \boxed{\frac{5\pi}{6}}$$

∴ Let  $\left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$  Answer.

(xii)

$$\sin\left(\theta - \frac{\pi}{6}\right) = \sin\left(\theta + \frac{\pi}{6}\right)$$

sol:

$$\sin\left(\theta - \frac{\pi}{6}\right) - \sin\left(\theta + \frac{\pi}{6}\right) = 0$$

$$2 \cos\left(\frac{\theta - \frac{\pi}{6} + \theta + \frac{\pi}{6}}{2}\right) \sin\left(\frac{\theta - \frac{\pi}{6} - \theta - \frac{\pi}{6}}{2}\right) = 0$$

$$2 \cos\left(\frac{7\theta}{2}\right) \sin\left(-\frac{\pi}{6}\right) = 0$$

$$2 \cos \theta \sin \frac{\pi}{6} = 0$$

$$2 \cos \theta \left(\frac{1}{2}\right) = 0$$

$$\cos \theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\boxed{\theta = (2n+1)\frac{\pi}{2}} \text{ Answer}$$



(XIII)

$$\cos(\theta - \frac{\pi}{3}) = \frac{1}{2}$$

Sol

$$\text{Let } x = \theta - \frac{\pi}{3}$$

$$\cos x = \frac{1}{2}$$

$$\text{Ref angle} = \cos^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$$

$$\text{QI} = x = \frac{\pi}{3}$$

$$\theta - \frac{\pi}{3} = \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3} + \frac{\pi}{3}$$

$$\boxed{\theta = \frac{2\pi}{3}}$$

QIV

$$x = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$$

$$\theta - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$\theta = 2\pi \notin [0, 2\pi)$$

S. Set  $z \left\{ \frac{2\pi}{3} \right\}$  Answer