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Where Minds Pollinate

Class 12

Mathematics

Chapter 8

Exercise 8.3

Handwritten Solution

Exercise 8.3

Addition & Subtraction Formulas for Inverse Trignometric Functions

There are total 12 formulas, we will do each formula's proof.

(1) $\sin^{-1}(x) + \sin^{-1}(y) = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$

let L.H.S

$= \sin^{-1}(x) + \sin^{-1}(y)$

let $\sin^{-1}(x) = \alpha$ & $\sin^{-1}(y) = \beta$ — eq (1)

$x = \sin\alpha$ $\quad$ $y = \sin\beta$

$\because \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ — eq (i)

We know the values of $\sin\alpha$ & $\sin\beta$, but not the values of $\cos\alpha$ & $\cos\beta$

We will find $\cos\alpha$, but there are two ways to find $\cos\alpha$ way 1 is by the pythagoras theorem & way 2 is by using the identity.

Way 1 (Pythagoras)

$\sin\alpha = x$

$\sin\alpha = \frac{x}{1} = \frac{P}{H}$

$(H)^2 = (P)^2 + (B)^2$

$(1)^2 = (x)^2 + B^2$

$1 - x^2 = B^2$

$B = \sqrt{1-x^2}$

$\cos\alpha = \frac{B}{H}$

$\cos\alpha = \frac{\sqrt{1-x^2}}{1}$

$\boxed{\cos\alpha = \sqrt{1-x^2}}$

$\boxed{\cos\beta = \sqrt{1-y^2}}$

The angle changed, so the value of $\sin\beta$ also changed.

Way 2 (Identity)

$\sin^2\alpha + \cos^2\alpha = 1$

$\cos^2\alpha = 1 - (x)^2$

$\sqrt{\cos^2\alpha} = \sqrt{1-x^2}$

$\boxed{\cos\alpha = \sqrt{1-x^2}}$

This way is shorter but for some identities you won't remember so do the way 1.

$\cos^2\beta = 1 - \sin^2\beta$

$\sqrt{\cos^2\beta} = \sqrt{1-(y)^2}$

$\boxed{\cos\beta = \sqrt{1-y^2}}$

Now putting all the values in the formula.

$\sin(\alpha+\beta) = x\sqrt{1-y^2} + y\sqrt{1-x^2}$

$\alpha+\beta = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

$\boxed{\sin^{-1}x + \sin^{-1}(y) = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)}$ $\because$ as we let in eq (1)

(2) $\sin^{-1}(x) - \sin^{-1}(y) = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$

The method is the same as (1) just the '−' sign is the difference

let L.H.S

$\sin^{-1}(x) - \sin^{-1}(y)$

let $\sin^{-1}(x) = \alpha$ & $\sin^{-1}(y) = \beta$

$x = \sin\alpha$ $y = \sin\beta$

$\because \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Using identity,

$\sin^2\alpha + \cos^2\alpha = 1$

$\cos^2\alpha = 1-x^2$

$\boxed{\cos\alpha = \sqrt{1-x^2}}$

$\sin^2\beta + \cos^2\beta = 1$

$\cos^2\beta = 1-y^2$

$\boxed{\cos\beta = \sqrt{1-y^2}}$

Putting in the values,

$\sin(\alpha-\beta) = \left(x\sqrt{1-y^2} - y\sqrt{1+x^2}\right)$

$\alpha - \beta = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1+x^2}\right)$

$\boxed{\sin^{-1}(x) - \sin^{-1}(y) = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1+x^2}\right)}$

(3) $\cos^{-1}(x) + \cos^{-1}(y) = \cos^{-1}\left[xy - \sqrt{(1-x^2)(1-y^2)}\right]$

let, $\cos^{-1}(x) = \alpha$ & $\cos^{-1}(y) = \beta$

$x = \cos\alpha$ $y = \cos\beta$

$\because \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Using identity,

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha = 1-x^2$

$\boxed{\sin\alpha = \sqrt{1-x^2}}$

$\sin^2\beta + \cos^2\beta = 1$

$\sin^2\beta = 1-y^2$

$\boxed{\sin\beta = \sqrt{1-y^2}}$

$\cos(\alpha+\beta) = xy - \left(\sqrt{1-x^2}\right)\left(\sqrt{1-y^2}\right)$

$\alpha+\beta = \cos^{-1}\left[xy - \sqrt{(1-x^2)(1-y^2)}\right]$

$\boxed{\cos^{-1}(x) + \cos^{-1}(y) = \cos^{-1}\left[xy - \sqrt{(1-x^2)(1-x^2)}\right]}$

(4) $\cos^{-1}(x) - \cos^{-1}(y) = \cos^{-1}\left[xy + \sqrt{(1-x^2)(1-y^2)}\right]$

Almost same as (3) jus the difference of minus

let $\cos^{-1}(x) = \alpha$ & $\cos^{-1}(y) = \beta$

$x = \cos\alpha$ $\quad$ $y = \cos\beta$

$\because \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Using identity,

$\cos^2\alpha + \sin^2\alpha = 1$

$\sin^2\alpha = 1 - x^2$

$$\boxed{\sin\alpha = \sqrt{1-x^2}}$$

$\cos^2\beta + \sin^2\beta = 1$

$\sin^2\beta = 1 - y^2$

$$\boxed{\sin\beta = \sqrt{1-y^2}}$$

Putting in the values,

$\cos(\alpha-\beta) = xy + (\sqrt{1-x^2})(\sqrt{1-y^2})$

$\alpha - \beta = \cos^{-1}\left[xy + \sqrt{(1-x^2)(1-y^2)}\right]$

$$\boxed{\cos^{-1}(x) - \cos^{-1}(y) = \cos^{-1}\left[xy + \sqrt{(1-x^2)(1-y^2)}\right]}$$

(5) $\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

let $\tan^{-1}(x) = \alpha$ & $\tan^{-1}(y) = \beta$

$x = \tan\alpha$ $\quad$ $y = \tan\beta$

$\because \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

Putting in the values,

$\tan(\alpha+\beta) = \dfrac{x+y}{1-xy}$

$\alpha+\beta = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$

$$\boxed{\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)}$$

(6) $\tan^{-1}(x) - \tan^{-1}(y) = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

Almost same just the diff of minus sign.

let $\tan^{-1}(x) = \alpha$ & $\tan^{-1}(y) = \beta$

$x = \tan\alpha$ $\quad$ $y = \tan\beta$

$\because \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$$\tan(\alpha-\beta) = \frac{x-y}{1+xy}$$

$$\alpha-\beta = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

$$\boxed{\tan^{-1}(x) - \tan^{-1}(y) = \tan^{-1}\left(\frac{x-y}{1+xy}\right)}$$

(7) $\cot^{-1}(x) + \cot^{-1}(y) = \cot^{-1}\left(\frac{xy-1}{x+y}\right)$

let $\cot^{-1}(x) = \alpha$ & $\cot^{-1}(y) = \beta$

$x = \cot\alpha \qquad y = \cot\beta$

$\tan\alpha = \frac{1}{x} \qquad \tan\beta = \frac{1}{y}$

$$\because \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha+\beta) = \frac{\frac{1}{x}+\frac{1}{y}}{1-\left(\frac{1}{x}\right)\left(\frac{1}{y}\right)}$$

$$\tan(\alpha+\beta) = \frac{\frac{x+y}{xy}}{1-\frac{1}{xy}}$$

$$\alpha+\beta = \tan^{-1}\left(\frac{\frac{x+y}{\cancel{xy}}}{\frac{xy-1}{\cancel{xy}}}\right)$$

$$\cot^{-1}(x)+\cot^{-1}(y) = \tan^{-1}\left(\frac{x+y}{xy-1}\right)$$

$$\boxed{\cot^{-1}(x)+\cot^{-1}(y) = \cot^{-1}\left(\frac{xy-1}{x+y}\right)}$$

(8) $\cot^{-1}(x) - \cot^{-1}(y) = \cot^{-1}\left(\frac{xy+1}{y-x}\right)$

let $\cot^{-1}(x) = \alpha$ & $\cot^{-1}(y) = \beta$

$x = \cot\alpha \qquad y = \cot\beta$

$\tan\alpha = \frac{1}{x} \qquad \tan\beta = \frac{1}{y}$

$$\because \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\frac{1}{x}-\frac{1}{y}}{1+\left(\frac{1}{x}\right)\left(\frac{1}{y}\right)}$$

$$\tan(\alpha-\beta) = \frac{\frac{y-x}{xy}}{1+\frac{1}{xy}}$$

$$\tan(\alpha-\beta) = \frac{\frac{y-x}{\cancel{xy}}}{\frac{xy+1}{\cancel{xy}}}$$

$$\tan(\alpha-\beta) = \frac{y-x}{xy+1}$$

$$\alpha-\beta = \tan^{-1}\left(\frac{y-x}{xy+1}\right)$$

$$\boxed{\cot^{-1}(x) - \cot^{-1}(y) = \cot^{-1}\left(\frac{xy+1}{y-x}\right)}$$

~~(9) $\sec^{-1}(x) + \sec^{-1}(y) = \cos^{-1}\left[\frac{1-\sqrt{(x^2-1)(y^2-1)}}{xy}\right]$~~

~~let $\sec^{-1}(x) = \alpha$ & $\sec^{-1}(y) = \beta$~~

~~$x = \sec\alpha$ $\quad$ $y = \sec\beta$~~

~~$\cos\alpha = \frac{1}{x}$ $\quad$ $\cos\beta = \frac{1}{y}$~~

~~$\because \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$~~

~~Using identity,~~

~~$\sin^2\alpha + \cos^2\alpha = 1$~~

~~$\sin^2\alpha = 1 - \left(\frac{1}{x}\right)^2$~~

~~$\boxed{\sin\alpha = \sqrt{1-\frac{1}{x^2}}}$~~

~~$\sin^2\beta + \cos^2\beta = 1$~~

~~$\sin^2\beta = 1 - \left(\frac{1}{y}\right)^2$~~

~~$\boxed{\sin\beta = \sqrt{1-\frac{1}{y^2}}}$~~

~~Putting in the values,~~

~~$\cos(\alpha+\beta) = \left(\frac{1}{x}\right)\left(\frac{1}{y}\right) - \left(\sqrt{1-\frac{1}{y^2}}\right)\left(\sqrt{1-\frac{1}{y^2}}\right)$~~

~~$\alpha+\beta = \cos^{-1}\left[\frac{1}{xy} - \sqrt{\left(1-\frac{1}{x^2}\right)\left(1-\frac{1}{y^2}\right)}\right]$~~

~~$\sec^{-1}(x) + \sec^{-1}(y) = \cos^{-1}$~~

* (9) $\sec^{-1}(x) + \sec^{-1}(y) = \cos^{-1}\left[\frac{1-\sqrt{(x^2-1)(y^2-1)}}{xy}\right]$ IMP *

let $\sec^{-1}(x) = \alpha$ & $\sec^{-1}(y) = \beta$

$x = \sec\alpha$ $\quad$ $y = \sec\beta$

$\cos\alpha = \frac{1}{x}$ $\quad$ $\cos\beta = \frac{1}{y}$

$\because \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Using identity,

$\sin^2\alpha + \cos^2\alpha = 1$ $\quad$ $\sin^2\beta + \cos^2\beta = 1$

$\sin^2\alpha = 1 - \left(\frac{1}{x}\right)^2$ $\quad$ $\sin^2\beta = 1 - \left(\frac{1}{y}\right)^2$

$\sin\alpha = \sqrt{1-\frac{1}{x^2}}$ $\quad$ $\sin\beta = \sqrt{1-\frac{1}{y^2}}$

$\sin\alpha = \sqrt{\frac{x^2-1}{x^2}}$ $\quad$ $\sin\beta = \sqrt{\frac{y^2-1}{y^2}}$

$\boxed{\sin\alpha = \frac{\sqrt{x^2-1}}{x}}$ $\quad$ $\boxed{\sin\beta = \frac{\sqrt{y^2-1}}{y}}$

$$\cos(\alpha+\beta) = \left(\frac{1}{x}\right)\left(\frac{1}{y}\right) - \left(\frac{\sqrt{x^2-1}}{x}\right)\left(\frac{\sqrt{y^2-1}}{y}\right)$$

$$\cos(\alpha+\beta) = \frac{1}{xy} - \frac{(\sqrt{x^2-1})(\sqrt{y^2-1})}{xy}$$

$$\cos(\alpha+\beta) = \frac{1-\sqrt{(x^2-1)(y^2-1)}}{xy}$$

$$\alpha+\beta = \cos^{-1}\left[\frac{1-\sqrt{(x^2-1)(y^2-1)}}{xy}\right]$$

$$\boxed{\sec^{-1}(x)+\sec^{-1}(y) = \cos^{-1}\left[\frac{1-\sqrt{(x^2-1)(y^2-1)}}{xy}\right]}$$

(10) $\sec^{-1}(x) - \sec^{-1}(y) = \cos^{-1}\left[\frac{1+\sqrt{(x^2-1)(y^2-1)}}{xy}\right]$

let $\sec^{-1}(x) = \alpha$ & $\sec^{-1}(y) = \beta$

$x = \sec\alpha$ $\quad$ $y = \sec\beta$

$\cos\alpha = \frac{1}{x}$ $\quad$ $\cos\beta = \frac{1}{y}$

$\because \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Using identity,

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin^2\alpha = 1 - \left(\frac{1}{x}\right)^2$$

$$\sin\alpha = \sqrt{1-\frac{1}{x^2}}$$

$$\sin\alpha = \sqrt{\frac{x^2-1}{x^2}}$$

$$\boxed{\sin\alpha = \frac{\sqrt{x^2-1}}{x}}$$

$$\sin^2\beta + \cos^2\beta = 1$$

$$\sqrt{\sin^2\beta} = \sqrt{1-\frac{1}{y^2}}$$

$$\sin\beta = \sqrt{\frac{y^2-1}{y^2}}$$

$$\boxed{\sin\beta = \frac{\sqrt{y^2-1}}{y}}$$

Putting in the values

$$\cos(\alpha-\beta) = \left(\frac{1}{x}\right)\left(\frac{1}{y}\right) + \left(\frac{\sqrt{x^2-1}}{x}\right)\left(\frac{\sqrt{y^2-1}}{y}\right)$$

$$\cos(\alpha-\beta) = \frac{1}{xy} + \frac{\sqrt{(x^2-1)(y^2-1)}}{xy}$$

$$\cos(\alpha-\beta) = \frac{1+\sqrt{(x^2-1)(y^2-1)}}{xy}$$

$$\alpha-\beta = \cos^{-1}\left(\frac{1+\sqrt{(x^2-1)(y^2-1)}}{xy}\right)$$

$$\sec^{-1}(x) - \sec^{-1}(y) = \cos^{-1}\left(\frac{1+\sqrt{(x^2-1)(y^2-1)}}{xy}\right)$$

(11) $\operatorname{cosec}^{-1}(x) + \operatorname{cosec}^{-1}(y) = \sin^{-1}\left[\frac{\sqrt{y^2-1}+\sqrt{x^2-1}}{xy}\right]$

let $\operatorname{cosec}^{-1}(x) = \alpha$ $\operatorname{cosec}^{-1}(y) = \beta$

$x = \operatorname{cosec}\alpha$ $y = \operatorname{cosec}\beta$

$\sin\alpha = \frac{1}{x}$ $\sin\beta = \frac{1}{y}$

$\because \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

using identity,

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\cos^2\alpha = 1 - \frac{1}{x^2}$$

$$\sqrt{\cos^2\alpha} = \sqrt{\frac{x^2-1}{x^2}}$$

$$\boxed{\cos\alpha = \frac{\sqrt{x^2-1}}{x}}$$

$$\sin^2\beta + \cos^2\beta = 1$$

$$\cos^2\beta = 1 - \frac{1}{y^2}$$

$$\sqrt{\cos^2\beta} = \sqrt{\frac{y^2-1}{y^2}}$$

$$\boxed{\cos\beta = \frac{\sqrt{y^2-1}}{y}}$$

Putting in the values,

$$\sin(\alpha+\beta) = \left(\frac{1}{x}\right)\left(\frac{\sqrt{y^2-1}}{y}\right) + \left(\frac{\sqrt{x^2-1}}{x}\right)\left(\frac{1}{y}\right)$$

$$\sin(\alpha+\beta) = \frac{\sqrt{y^2-1}+\sqrt{x^2-1}}{xy}$$

$$\alpha+\beta = \sin^{-1}\left[\frac{\sqrt{(y^2-1)}+\sqrt{(x^2-1)}}{xy}\right]$$

$$\boxed{\operatorname{cosec}^{-1}(x) + \operatorname{cosec}^{-1}(y) = \sin^{-1}\left(\frac{\sqrt{y^2-1}+\sqrt{x^2-1}}{xy}\right)}$$

(12) $\operatorname{cosec}^{-1}(x) - \operatorname{cosec}^{-1}(y) = \sin^{-1}\left[\frac{\sqrt{y^2-1}-\sqrt{x^2-1}}{xy}\right]$

only difference is the minus sign

$$\operatorname{cosec}^{-1}(x) - \operatorname{cosec}^{-1}(y) = \sin^{-1}\left[\frac{\sqrt{y^2-1}-\sqrt{x^2-1}}{xy}\right]$$

1. $\sin^{-1}(x) + \sin^{-1}(y) = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$
2. $\sin^{-1}(x) - \sin^{-1}(y) = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$
3. $\cos^{-1}(x) + \cos^{-1}(y) = \cos^{-1}\left[xy - \sqrt{(1-x^2)(1-y^2)}\right]$
4. $\cos^{-1}(x) - \cos^{-1}(y) = \cos^{-1}\left[xy + \sqrt{(1-x^2)(1-y^2)}\right]$
5. $\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$
6. $\tan^{-1}(x) - \tan^{-1}(y) = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$
7. $\cot^{-1}(x) + \cot^{-1}(y) = \cot^{-1}\left(\frac{xy-1}{x+y}\right)$
8. $\cot^{-1}(x) - \cot^{-1}(y) = \cot^{-1}\left(\frac{xy+1}{y-x}\right)$
9. $\sec^{-1}(x) + \sec^{-1}(y) = \cos^{-1}\left[\frac{1-\sqrt{(x^2-1)(y^2-1)}}{xy}\right]$
10. $\sec^{-1}(x) - \sec^{-1}(y) = \cos^{-1}\left[\frac{1+\sqrt{(x^2-1)(y^2-1)}}{xy}\right]$
11. $\operatorname{cosec}^{-1}(x) + \operatorname{cosec}^{-1}(y) = \sin^{-1}\left[\frac{\sqrt{y^2-1} + \sqrt{x^2-1}}{xy}\right]$
12. $\operatorname{cosec}^{-1}(x) - \operatorname{cosec}^{-1}(y) = \sin^{-1}\left[\frac{\sqrt{y^2-1} - \sqrt{x^2-1}}{xy}\right]$

Q1

(i) $\sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) = \frac{\pi}{2}$

$= \sin^{-1}\left[\frac{1}{3}\sqrt{1-\left(\frac{2\sqrt{2}}{3}\right)^2} + \frac{2\sqrt{2}}{3}\sqrt{1-\left(\frac{1}{3}\right)^2}\right]$

$= \sin^{-1}\left[\frac{1}{3}\sqrt{\frac{1}{9}} + \frac{2\sqrt{2}}{3}\sqrt{\frac{8}{9}}\right]$

$= \sin^{-1}\left[\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) + \frac{2\sqrt{2}}{3}\left(\frac{2\sqrt{2}}{3}\right)\right]$

$= \sin^{-1}\left(\frac{1}{9} + \frac{4(\sqrt{2})^2}{9}\right)$

$= \sin^{-1}\left(\frac{1}{9} + \frac{8}{9}\right)$

$= \sin^{-1}\left(\frac{9}{9}\right)$

$= \sin^{-1}(1)$

$= \frac{\pi}{2}$ Hence true. L.H.S = R.H.S

(ii)
$$= \cos^{-1}\left[\frac{3}{5}\left(\frac{4}{5}\right) - \sqrt{\left(1-\frac{9}{25}\right)\left(1-\frac{16}{25}\right)}\right]$$
$$= \cos^{-1}\left[\frac{12}{25} - \sqrt{\left(\frac{16}{25}\right)\left(\frac{9}{25}\right)}\right]$$
$$= \cos^{-1}\left(\frac{12}{25}-\frac{12}{25}\right)$$
$$= \cos^{-1}(0)$$
$= \frac{\pi}{2}$ Proved

(iii)
$$= \tan^{-1}\left[\frac{\frac{3}{4}+\frac{1}{7}}{1-\left(\frac{3}{4}\right)\left(\frac{1}{7}\right)}\right]$$
$$= \tan^{-1}\left(\frac{25}{25}\right)$$
$$= \tan^{-1}(1)$$
$= \frac{\pi}{4}$ Proved

(iv) $= \cot^{-1}\left[\frac{(2+\sqrt{3})(2-\sqrt{3})-1}{2+\sqrt{3}+2-\sqrt{3}}\right]$
$$= \cot^{-1}\left(\frac{(2)^2-(\sqrt{3})^2-1}{4}\right)$$
$$= \cot^{-1}\left(\frac{4-3-1}{4}\right)$$
$$= \cot^{-1}\left(\frac{0}{4}\right)$$
$= \cot^{-1}(0)$ ∵ $\tan^{-1}\left(\frac{1}{0}\right) = \tan^{-1}(\infty)$

$= \frac{\pi}{2}$ Proved

(v) $: \sin^{-1}\left(\frac{\sqrt{(\sqrt{2})^2-1}+\sqrt{(\sqrt{2})^2-1}}{(\sqrt{2})(\sqrt{2})}\right)$
$$= \sin^{-1}\left(\frac{\sqrt{2-1}+\sqrt{2-1}}{(\sqrt{2})^2}\right)$$
$$= \sin^{-1}\left(\frac{\sqrt{1}+\sqrt{1}}{2}\right)$$
$$= \sin^{-1}\left(\frac{1+1}{2}\right)$$
$$= \sin^{-1}(1)$$
$= \frac{\pi}{2}$ Proved.

(vi) $= \cos^{-1}\left(\frac{1-\sqrt{((2)^2-1)((2)^2-1)}}{(2)(2)}\right)$
$$= \cos^{-1}\left(\frac{1-\sqrt{(3)(3)}}{4}\right)$$
$$= \cos^{-1}\left(\frac{1-\sqrt{9}}{4}\right)$$
$$= \cos^{-1}\left(\frac{1-3}{4}\right)$$
$$= \cos^{-1}\left(-\frac{1}{2}\right)$$
$= \frac{2\pi}{3}$ Proved.

(vii) $\sin^{-1}(x)+\sin^{-1}(-x)=0$
$$= \sin^{-1}\left(x\sqrt{1-(-x)^2}+(-x)\sqrt{1-(x)^2}\right)$$
$$= \sin^{-1}\left(\cancel{x\sqrt{1-x^2}}\cancel{-x\sqrt{1-x^2}}\right)$$
$$= \sin^{-1}(0)$$
$= 0$ Proved.

(viii) $\cos^{-1}(x)+\cos^{-1}(-x)=\pi$
$$= \cos^{-1}\left[x(-x)-\sqrt{(1-(x)^2)(1-(-x)^2)}\right]$$
$$= \cos^{-1}\left[-x^2-\sqrt{(1-x^2)(1-x^2)}\right]$$
$$= \cos^{-1}\left[-x^2-\sqrt{(1-x^2)^2}\right]$$
$$= \cos^{-1}\left(-x^2-(1-x^2)\right)$$
$$= \cos^{-1}\left(\cancel{-x^2}-1\cancel{+x^2}\right)$$
$$= \cos^{-1}(-1)$$
$= \pi$ Proved.

(ix) $\tan^{-1}\left(\frac{\alpha}{\beta}\right)-\tan^{-1}\left(\frac{\alpha-\beta}{\alpha+\beta}\right)=\frac{\pi}{4}$
$$= \tan^{-1}\left(\frac{\frac{\alpha}{\beta}-\left(\frac{\alpha-\beta}{\alpha+\beta}\right)}{1+\left(\frac{\alpha}{\beta}\right)\left(\frac{\alpha-\beta}{\alpha+\beta}\right)}\right)$$
$$= \tan^{-1}\left(\frac{\frac{\alpha(\alpha+\beta)-(\alpha-\beta)\beta}{\beta(\alpha+\beta)}}{1+\frac{\alpha(\alpha-\beta)}{\beta(\alpha+\beta)}}\right)$$
$$= \tan^{-1}\left(\frac{\frac{\alpha^2+\alpha\beta-\alpha\beta+\beta^2}{\alpha\beta+\beta^2}}{1+\frac{\alpha^2-\alpha\beta}{\alpha\beta+\beta^2}}\right)$$
$$= \tan^{-1}\left(\frac{\frac{\alpha^2+\beta^2}{\alpha\beta+\beta^2}}{\frac{\cancel{\alpha\beta}+\beta^2+\alpha^2\cancel{-\alpha\beta}}{\alpha\beta+\beta^2}}\right)$$
$$= \tan^{-1}\left(\frac{\frac{\alpha^2+\beta^2}{\cancel{\alpha\beta+\beta^2}}}{\frac{\alpha^2+\beta^2}{\cancel{\alpha\beta+\beta^2}}}\right)$$
$$= \tan^{-1}(1)$$
$$= \frac{\pi}{4}$$

(x) $\cot^{-1}(1) - \cot^{-1}(\sqrt{3}) = \frac{\pi}{12}$

$= \cot^{-1}\left(\frac{(1)(\sqrt{3})+1}{(\sqrt{3})-(1)}\right)$

$= \cot^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

$\cot^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

$= \cot^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$

$= \cot^{-1}\left(\frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2-(1)^2}\right)$

$= \cot^{-1}\left(\frac{(\sqrt{3})^2+2(\sqrt{3})(1)+(1)^2}{3-1}\right)$

$= \cot^{-1}\left(\frac{3+2\sqrt{3}+1}{2}\right)$

$= \cot^{-1}\left(\frac{4+2\sqrt{3}}{2}\right)$

$= \cot^{-1}\left(\frac{2(2+\sqrt{3})}{2}\right)$

$= \cot^{-1}(2+\sqrt{3})$

$= \tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right)$

$= \frac{\pi}{12}$ Proved

(xi) $\csc^{-1}(\sqrt{2}) - \csc^{-1}(2) = \frac{\pi}{12}$

$= \sin^{-1}\left(\frac{\sqrt{(2)^2-1} - \sqrt{(\sqrt{2})^2-1}}{(\sqrt{2})(2)}\right)$

$= \sin^{-1}\left(\frac{\sqrt{4-1} - \sqrt{2-1}}{2\sqrt{2}}\right)$

$= \sin^{-1}\left(\frac{\sqrt{3}-\sqrt{1}}{2\sqrt{2}}\right)$

$= \frac{\pi}{12}$ Proved

(xii) $\cos^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$= \cos^{-1}\left[\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \sqrt{\left[1-\left(\frac{\sqrt{3}}{2}\right)^2\right]\left[1-\left(\frac{1}{2}\right)^2\right]}\right]$

$= \cos^{-1}\left[\frac{\sqrt{3}}{4} + \sqrt{\left(1-\frac{3}{4}\right)\left(1-\frac{1}{4}\right)}\right]$

$= \cos^{-1}\left[\frac{\sqrt{3}}{4} + \sqrt{\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)}\right]$

$= \cos^{-1}\left[\frac{\sqrt{3}}{4} + \sqrt{\frac{3}{16}}\right]$

$= \cos^{-1}\left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right)$

$= \cos^{-1}\left(\frac{2\sqrt{3}}{4}\right)$

$= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{6}$ Proved

Q2 Evaluate by

(i) Apply inverse secant ---

$\sec^{-1}\left(\frac{x+1}{x}\right) + \sec^{-1}\left(\frac{x-1}{x}\right)$

$= \cos^{-1}\left[\frac{1 - \sqrt{\left[\left(\frac{x-1}{x}\right)^2-1\right]\left[\left(\frac{x+1}{x}\right)^2-1\right]}}{\left(\frac{x+1}{x}\right)\left(\frac{x-1}{x}\right)}\right]$

$= \cos^{-1}\left(\frac{1 - \sqrt{\left(\frac{x^2-2x+1}{x^2}-1\right)\left(\frac{x^2+2x+1}{x^2}-1\right)}}{\frac{(x+1)(x-1)}{x^2}}\right)$

$= \cos^{-1}\left(\frac{1 - \sqrt{\left(\frac{\cancel{x^2}-2x+1-\cancel{x^2}}{x^2}\right)\left(\frac{\cancel{x^2}+2x+1-\cancel{x^2}}{x^2}\right)}}{\frac{x^2-x+x-1}{x^2}}\right)$

$= \cos^{-1}\left[\frac{1 - \sqrt{\left(\frac{-2x+1}{x^2}\right)\left(\frac{2x+1}{x^2}\right)}}{\frac{x^2-1}{x^2}}\right]$

$= \cos^{-1}\left(\frac{1 - \sqrt{\frac{-4x^2-2x+2x+1}{x^4}}}{\frac{x^2-1}{x^2}}\right)$

$= \cos^{-1}\left[\left(1 - \sqrt{\frac{1-4x^2}{x^4}}\right) \div \left(\frac{x^2-1}{x^2}\right)\right]$

$= \cos^{-1}\left[\left(1 - \frac{\sqrt{1-4x^2}}{x^2}\right) \times \frac{x^2}{x^2-1}\right]$

$= \cos^{-1}\left[\frac{x^2-\sqrt{1-4x^2}}{x^2} \times \frac{x^2}{x^2-1}\right]$

$= \cos^{-1}\left[\frac{x^2-\sqrt{1-4x^2}}{x^2-1}\right]$

Putt $x = 0$

$= \cos^{-1}\left(\frac{(0)^2-\sqrt{1-4(0)^2}}{(0)^2-1}\right)$

$= \cos^{-1}\left(\frac{0-\sqrt{1-0}}{0-1}\right)$

$= \cos^{-1}\left(\frac{-\sqrt{1}}{-1}\right)$

$= \cos^{-1}\left(\frac{-1}{-1}\right)$

$= \cos^{-1}(-1)$

$= 0$ Proved.

(ii)

$= \sin^{-1}\left[\frac{\sqrt{\left(\frac{x+1}{x}\right)^2-1} + \sqrt{\left(\frac{2x+1}{x}\right)^2-1}}{\left(\frac{2x+1}{x}\right)\left(\frac{x+1}{x}\right)}\right]$

$= \sin^{-1}\left(\frac{\sqrt{\frac{x^2+2x+1}{x^2}-1} + \sqrt{\frac{4x^2+4x+1}{x^2}-1}}{\frac{2x^2+2x+x+1}{x^2}}\right)$

$$= \sin^{-1}\left(\frac{\sqrt{\frac{x^2+2x+1-x^2}{x^2}} + \sqrt{\frac{4x^2+4x+1-x^2}{x^2}}}{\frac{2x^2+3x+1}{x^2}}\right)$$

$$= \sin^{-1}\left[\left(\sqrt{\frac{2x+1}{x^2}} + \sqrt{\frac{3x^2+4x+1}{x^2}}\right) \div \left(\frac{2x^2+3x+1}{x^2}\right)\right]$$

$$= \sin^{-1}\left[\left(\frac{\sqrt{2x+1}}{x} + \frac{\sqrt{3x^2+4x+1}}{x}\right) \times \left(\frac{x^2}{2x^2+3x+1}\right)\right]$$

$$= \sin^{-1}\left[\left(\frac{\sqrt{2x+1}+\sqrt{3x^2+4x+1}}{x}\right)\left(\frac{x^2}{(2x^2+3x+1)}\right)\right]$$

$$= \sin^{-1}\left[\left(\sqrt{2x+1}+\sqrt{3x^2+4x+1}\right)\left(\frac{x}{2x^2+3x+1}\right)\right]$$

Put $x=0$,

$$= \sin^{-1}\left[\left(\sqrt{2(0)+1} + \sqrt{3(0)^2+4(0)+1}\right)\left(\frac{0}{2(0)^2+3(0)+1}\right)\right]$$

$$= \sin^{-1}\left((\sqrt{1}+\sqrt{1})\ (0)\right)$$

$$= \sin^{-1}(0)$$

$$= 0 \quad \text{Proved}$$

(iii) - - -

$$= \cot^{-1}\left[\frac{(x-1)(2x-1)-1}{(2x-1)+(x-1)}\right]$$

$$= \cot^{-1}\left(\frac{2x^2-x-2x+1-1}{2x-1+x-1}\right)$$

$$= \cot^{-1}\left(\frac{2x^2-3x}{3x-2}\right)$$

Put $x=+1$,

$$= \cot^{-1}\left(\frac{2(1)^2-3(1)}{3(1)-2}\right)$$

$$= \cot^{-1}\left(\frac{2-3}{3-2}\right)$$

$$= \cot^{-1}\left(\frac{-1}{1}\right)$$

$$= \cot^{-1}(-1)$$

$$= \frac{-\pi}{4}$$

Put $x=-1$,

$$= \cot^{-1}\left(\frac{2(-1)^2-3(-1)}{3(-1)-2}\right)$$

$$= \cot^{-1}\left(\frac{2+3}{-3-2}\right)$$

$$= \cot^{-1}\left(\frac{5}{-5}\right)$$

$$= \cot^{-1}\left(\frac{1}{-1}\right)$$

$$= \frac{-\pi}{4} \quad \text{Proved}$$

Q3 Solve the following - - -

(i) $\sin^{-1}(x-1) = \frac{\pi}{2}$

$$x-1 = \sin\left(\frac{\pi}{2}\right)$$

$$x-1 = 1$$

$$x = 1+1$$

$$\boxed{x=2}$$

(ii) $\sin^{-1}(x) = \cos^{-1}\left(\frac{5}{13}\right)$

$$x = \sin\left[\cos^{-1}\left(\frac{5}{13}\right)\right] \quad -\ast$$

let $\theta = \cos^{-1}\left(\frac{5}{13}\right)$ — eq(i)

$$\cos\theta = \frac{5}{13} = \frac{B}{H}$$

$$(H)^2 = (B)^2 + (P)^2$$

$$(13)^2 = (5)^2 + (P)^2$$

$$169 - 25 = P^2$$

$$\sqrt{P^2} = \sqrt{144}$$

$$P = 12$$

$$\sin\theta = \frac{P}{H}$$

$$\sin\theta = \frac{12}{13}$$

$x = \sin\theta$: Putting eq(i) in ∗

$$\boxed{x = \frac{12}{13}}$$

(iii) $\tan^{-1}x = \sin^{-1}\left(\frac{24}{25}\right)$

$$x = \tan\left(\sin^{-1}\left(\frac{24}{25}\right)\right) - \ast$$

let $\theta = \sin^{-1}\left(\frac{24}{25}\right)$ — eq(i)

$$\sin\theta = \frac{24}{25} = \frac{P}{H}$$

$$(H)^2 = (P)^2 + (B)^2$$

$$(25)^2 = (24)^2 = B^2$$

$$\sqrt{49} = \sqrt{B^2}$$

$$B = 7$$

$$\tan\theta = \frac{P}{B}$$

$$\tan\theta = \frac{24}{7}$$

$x = \tan\theta$: Putting eq(i) in ∗

$$\boxed{x = \frac{24}{7}}$$

(iv) $\cos^{-1}\left(x - \frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$$x - \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$$

$$x - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$x = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$$

$$\boxed{x = \sqrt{3}}$$

(v) $\tan^{-1}\left(x + \frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$$x + \frac{\sqrt{2}}{2} = \tan\left(\frac{\pi}{4}\right)$$

$$x + \frac{\sqrt{2}}{2} = 1$$

$$x = 1 - \frac{\sqrt{2}}{2}$$

$$\boxed{x = \frac{2-\sqrt{2}}{2}}$$

(vi) $\cos^{-1}\left(x-\frac{1}{2}\right)=\frac{\pi}{3}$

$x-\frac{1}{2}=\cos\left(\frac{\pi}{3}\right)$

$x-\frac{1}{2}=\frac{1}{2}$

$x=\frac{1}{2}+\frac{1}{2}$

$x=\frac{2}{2}$

$\boxed{x=1}$

(vii) $6\cot^{-1}(2x)-5\pi=0$

$6\cot^{-1}(2x)=5\pi$

$\cot^{-1}(2x)=\frac{5\pi}{6}$

$2x=\cot(150)$

$2x=\frac{1}{\tan(150)}$

$2x=-\sqrt{3}$

$x=\frac{-\sqrt{3}}{2}$

(viii) $9\left(\sin^{-1}x\right)^2-\pi^2=0$

$9(\sin^{-1}x)^2=\pi^2$

$(\sin^{-1}x)^2=\frac{\pi^2}{9}$

$\sqrt{(\sin^{-1}x)^2}=\pm\sqrt{\frac{\pi^2}{9}}$

$\sin^{-1}x=\pm\frac{\pi}{3}$

$x=\sin\left(\pm\frac{\pi}{3}\right)$

$x=\sin(\pm 60)$

$x=\pm\frac{\sqrt{3}}{2}$

(ix) $\pi-2\sin^{-1}x=2\pi$

$-2\sin^{-1}x=2\pi-\pi$

$2\sin^{-1}x=-2\pi+\pi$

$\sin^{-1}x=\frac{\pi-2\pi}{2}$

$\sin^{-1}x=\frac{-\pi}{2}$

$x=\sin\left(\frac{-\pi}{2}\right)$

$x=-1$

(x) $4(\tan^{-1}x)^2-3\pi\tan^{-1}x-\pi^2=0$

let $\tan^{-1}x=y$

$4y^2-3\pi y-\pi^2=0$

$a=4,\ b=-3\pi,\ c=-\pi^2$

$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$y=\frac{-(-3\pi)\pm\sqrt{(-3\pi)^2-4(4)(-\pi^2)}}{2(4)}$

$y=\frac{3\pi\pm\sqrt{9\pi^2+16\pi^2}}{8}$

$y=\frac{3\pi\pm\sqrt{25\pi^2}}{8}$

$y=\frac{3\pi\pm 5\pi}{8}$

$y=\frac{3\pi+5\pi}{8}$	$y=\frac{3\pi-5\pi}{8}$
$y=\frac{8\pi}{8}$	$y=\frac{-2\pi}{8}$
$y=\pi$	$y=\frac{-\pi}{4}$

Putting $y=\tan^{-1}x$

$\tan^{-1}x=\pi$	$\tan^{-1}x=\frac{-\pi}{4}$
$x=\tan\pi$	$x=\tan\left(\frac{-\pi}{4}\right)$
$x=0$	$x=-1$

S.S $=\{0,-1\}$

(xi) $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\frac{\pi}{4}-\frac{x}{2}$, where $x\in(0,\pi)$

$\because 1+\cos x=2\cos^2\left(\frac{x}{2}\right)$ $\quad\because 1-\cos x=2\sin^2\left(\frac{x}{2}\right)$

$\tan^{-1}\left[\frac{\sqrt{2\cos^2\left(\frac{x}{2}\right)}+\sqrt{2\sin^2\left(\frac{x}{2}\right)}}{\sqrt{2\cos^2\left(\frac{x}{2}\right)}-\sqrt{2\sin^2\left(\frac{x}{2}\right)}}\right]=\frac{\pi}{4}-\frac{x}{2}$

$\frac{\sqrt{2}\cos\left(\frac{x}{2}\right)+\sqrt{2}\sin\left(\frac{x}{2}\right)}{\sqrt{2}\cos\left(\frac{x}{2}\right)-\sqrt{2}\sin\left(\frac{x}{2}\right)}=\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$\frac{\cancel{\sqrt{2}}\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\cancel{\sqrt{2}}\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}=\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}=\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$\frac{\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos x/2}}{\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}}}=\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$ $\times$ & $\div\ \cos\frac{x}{2}$

$$\frac{\frac{\cos x/2}{\cos x/2}+\frac{\sin x/2}{\cos x/2}}{\frac{\cos x/2}{\cos x/2}-\frac{\sin x/2}{\cos x/2}} = \tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} = \tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\left(\tan\frac{\pi}{4}\tan\frac{x}{2}\right)} = \tan\left(\frac{\pi}{4}-\frac{x}{2}\right) \quad \because \tan\frac{\pi}{4} = 1$$

$$\cancel{\tan}\left(\frac{\pi}{4}+\frac{x}{2}\right) = \cancel{\tan}\left(\frac{\pi}{4}-\frac{x}{2}\right) \quad \because \tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\frac{\pi}{4}+\frac{x}{2} = \frac{\pi}{4}-\frac{x}{2}$$

$$\frac{x}{2}+\frac{x}{2} = \cancel{\frac{\pi}{4}}-\cancel{\frac{\pi}{4}}$$

$$\frac{2x}{2} = 0$$

$$\boxed{x = 0}$$

Q4 Find the values of x:

(i) $\Rightarrow \frac{\pi}{3} + \cos^{-1}(x) = \frac{\pi}{2}$

$$\cos^{-1}(x) = \frac{\pi}{2} - \frac{\pi}{3}$$

$$x = \cos\left(\frac{\pi}{6}\right)$$

$$\boxed{x = \frac{\sqrt{3}}{2}}$$

(ii) $\frac{\pi}{6} + \cos^{-1}(2x) = \frac{\pi}{6}$

$$\cos^{-1}(2x) = \frac{\pi}{6} - \frac{\pi}{6}$$

$$2x = \cos(0)$$

$$2x = 1$$

$$\boxed{x = \frac{1}{2}}$$

(iii) $\sin^{-1}x = \pi - \cos^{-1}\left(\frac{4}{5}\right)$

$$x = \sin\left(\pi - \cos^{-1}\left(\frac{4}{5}\right)\right)$$

$\because \sin(\pi-\theta) = \sin\theta$ $\because$ where sin is + in II-Q

$$x = \sin\left[\cos^{-1}\left(\frac{4}{5}\right)\right]$$

let $\theta = \cos^{-1}\left(\frac{4}{5}\right)$

$$\cos\theta = \frac{4}{5} = \frac{B}{H}$$

$$(H)^2 = (B)^2 + (P)^2$$

$$(5)^2 - (4)^2 = P^2$$

$$\sqrt{P^2} = \sqrt{9}$$

$$P = 3$$

$$\sin\theta = \frac{P}{H}$$

$$\sin\theta = \frac{3}{5}$$

$$x = \sin\theta$$

$$x = \frac{3}{5}$$

(iv) $\tan^{-1}(x) - \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{4}$

~~$\tan^{-1}(x) - \tan$~~ ~~$\tan^{-1}(x) - \tan^{-1}(y)$~~ =

$\because \tan^{-1}(x) - \tan^{-1}(y) = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

$$\tan^{-1}\left[\frac{x-\frac{1}{x}}{1+(x)\left(\frac{1}{x}\right)}\right] = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{\left(\frac{x^2-1}{x}\right)}{2}\right) = \frac{\pi}{4}$$

$$\tan^{-1}\left[\left(\frac{x^2-1}{x}\right)\times\frac{1}{2}\right] = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{x^2-1}{2x}\right) = \frac{\pi}{4}$$

$$\frac{x^2-1}{2x} = \tan\frac{\pi}{4}$$

$$\frac{x^2-1}{2x} = 1$$

$$x^2-1 = 2x$$

$$x^2-2x-1 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = \frac{\cancel{2}(1 \pm \sqrt{2})}{\cancel{2}}$$

$$x = 1 \pm \sqrt{2}$$

Q5 (i) $\sin^{-1}\left(\frac{5}{13}\right) - \sin^{-1}\left(\frac{63}{65}\right) = \cos^{-1}\left(\frac{3}{5}\right)$

we can use the formula's here but that will be very lengthy so we will do it the shorter way

L.H.S =

let $\alpha = \sin^{-1}\left(\frac{5}{13}\right)$, $\beta = \sin^{-1}\left(\frac{63}{65}\right)$

$\boxed{\sin\alpha = \frac{5}{13}}$, $\boxed{\sin\beta = \left(\frac{63}{65}\right)}$

$\because \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Finding $\cos\alpha$ & $\cos\beta$.

$$\sin\alpha = \frac{5}{13} = \frac{P}{H}$$

$$(H)^2 = (B)^2 + (P)^2$$

$(13)^2 = (5)^2 + B^2$

$169 - 25 = B^2$

$B = 12$

$\cos\alpha = \frac{B}{H}$

$\boxed{\cos\alpha = \frac{12}{13}}$

$\rightarrow \sin\beta = \frac{63}{65} = \frac{P}{H}$

$(H)^2 = (B)^2 + (P)^2$

$(65)^2 - (63)^2 = B^2$

$\sqrt{B^2} = \sqrt{256}$

$B = 16$

$\cos\beta = \frac{B}{H}$

$\boxed{\cos\beta = \frac{16}{65}}$

Putting the values in,

$\cos(\alpha-\beta) = \left(\frac{12}{13}\right)\left(\frac{16}{65}\right) + \left(\frac{5}{13}\right)\left(\frac{63}{65}\right)$

$\cos(\alpha-\beta) = \frac{192}{845} + \frac{63}{169}$

$\alpha - \beta = \cos^{-1}\left(\frac{3}{5}\right)$

$\sin^{-1}\left(\frac{5}{13}\right) - \sin^{-1}\left(\frac{63}{65}\right) = \cos^{-1}\left(\frac{3}{5}\right)$ Proved.

(ii) L.H.S

$\alpha = \sin^{-1}\left(\frac{8}{17}\right)$ $\quad \beta = \sin^{-1}\left(\frac{3}{5}\right)$

$\boxed{\sin\alpha = \frac{8}{17}}$ $\quad \boxed{\sin\beta = \frac{3}{5}}$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

Finding $\tan\alpha$ & $\tan\beta$,

$\sin\alpha = \frac{8}{17} = \frac{P}{H}$

$(H)^2 = (P)^2 + (B)^2$

$(17)^2 - (8)^2 = B^2$

$\sqrt{B^2} = \sqrt{225}$

$B = 15$

$\tan\alpha = \frac{P}{B}$

$\boxed{\tan\alpha = \frac{8}{15}}$

$\sin\beta = \frac{3}{5} = \frac{P}{H}$

$(H)^2 = (P)^2 + (B)^2$

$(5)^2 = (3)^2 + B^2$

$\sqrt{B^2} = \sqrt{16}$

$B = 4$

$\tan\beta = \frac{P}{B}$

$\boxed{\tan\beta = \frac{3}{4}}$

Putting in the values,

$\tan(\alpha+\beta) = \frac{\frac{8}{15} + \frac{3}{4}}{1 - \left(\frac{8}{15}\right)\left(\frac{3}{4}\right)}$

$\tan(\alpha+\beta) = \frac{\frac{77}{60}}{\frac{3}{5}}$

$\alpha + \beta = \tan^{-1}\left(\frac{77}{36}\right)$

$\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{77}{36}\right)$ Proved

(iii) L.H.S

let $\alpha = \sin^{-1}\left(\frac{3}{5}\right)$ $\quad \beta = \cos^{-1}\left(\frac{12}{13}\right)$

$\boxed{\sin\alpha = \frac{3}{5}}$ $\quad \boxed{\cos\beta = \frac{12}{13}}$

$\because \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin\alpha = \frac{3}{5} = \frac{P}{H}$

$(H)^2 = (B)^2 + (P)^2$

$(5)^2 - (3)^2 = B^2$

$\sqrt{B^2} = \sqrt{16}$

$B = 4$

$\cos\alpha = \frac{B}{H}$

$\boxed{\cos\alpha = \frac{4}{5}}$

$\cos\beta = \frac{12}{13} = \frac{B}{H}$

$(13)^2 - (12)^2 = P^2$

$\sqrt{P^2} = \sqrt{25}$

$P = 5$

$\sin\beta = \frac{P}{H}$

$\boxed{\sin\beta = \frac{5}{13}}$

Putting in the values,

$\sin(\alpha+\beta) = \left(\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)$

$\alpha + \beta = \sin^{-1}\left(\frac{56}{65}\right)$

$\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$ Proved

$(iv)\ Tan^{-1}\left(\frac{1}{4}\right) + Tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2} Tan^{-1}\left(\frac{4}{3}\right)$

L.H.S

let $\alpha = Tan^{-1}\left(\frac{1}{4}\right)$ $\qquad \beta = Tan^{-1}\left(\frac{2}{9}\right)$

$\tan\alpha = \frac{1}{4}$ $\qquad \tan\beta = \frac{2}{9}$

$\because \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$\tan(\alpha+\beta) = \frac{\frac{1}{4} + \frac{2}{9}}{1 - \left(\frac{1}{4}\right)\left(\frac{2}{9}\right)}$

$\tan(\alpha+\beta) = \frac{\frac{17}{36}}{\frac{17}{18}}$

$\alpha+\beta = Tan^{-1}\left(\frac{1}{2}\right)$

$Tan^{-1}\left(\frac{1}{4}\right) + tan^{-1}\left(\frac{2}{9}\right) = Tan^{-1}\left(\frac{1}{2}\right)$

let $x = Tan^{-1}\left(\frac{1}{2}\right)$ — eq(i)

$\tan x = \left(\frac{1}{2}\right)$

$\because \tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

$\tan 2x = \frac{2\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2}\right)^2}$

$\tan 2x = \frac{1}{1 - \frac{1}{4}}$

$\tan 2x = \frac{1}{3/4}$

$\tan 2x = \frac{4}{3}$

$2x = Tan^{-1}\left(\frac{4}{3}\right)$

$x = \frac{1}{2} Tan^{-1}\left(\frac{4}{3}\right)$

from eq (i),

$Tan^{-1}\left(\frac{1}{4}\right) + Tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2} Tan^{-1}\left(\frac{4}{3}\right)$ Proved.

$(v)\ Cos^{-1}\left(\frac{4}{5}\right) + Cos^{-1}\left(\frac{12}{13}\right) = Cos^{-1}\left(\frac{33}{65}\right)$

let $\alpha = Cos^{-1}\left(\frac{4}{5}\right)$ $\qquad \beta = Cos^{-1}\left(\frac{12}{13}\right)$

$\boxed{\cos\alpha = \frac{4}{5}}$ $\qquad \boxed{\cos\beta = \frac{12}{13}}$

$\because \cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

$\cos\alpha = \frac{4}{5} = \frac{B}{H}$

$(5)^2 = (4)^2 + (P)^2$

$\sqrt{P^2} = \sqrt{9}$

$P = 3$

$\sin\alpha = \frac{P}{H}$

$\boxed{\sin\alpha = \frac{3}{5}}$

$\cos\beta = \frac{12}{13} = \frac{B}{H}$

$(13)^2 = (12)^2 + P^2$

$\sqrt{P^2} = \sqrt{25}$

$P = 5$

$\sin\beta = \frac{P}{H}$

$\boxed{\sin\beta = \frac{5}{13}}$

Putting in the values,

$\cos(\alpha+\beta) = \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$

$\alpha+\beta = Cos^{-1}\left(\frac{33}{65}\right)$

$Cos^{-1}\left(\frac{4}{5}\right) + Cos^{-1}\left(\frac{12}{13}\right) = Cos^{-1}\left(\frac{33}{65}\right)$

$(vi)\ 2\,Tan^{-1}\left(\frac{3}{4}\right) - Tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$

L.H.S

$= 2\,Tan^{-1}\left(\frac{3}{4}\right) - Tan^{-1}\left(\frac{17}{31}\right)$

$\because Tan^{-1}(x) + Tan^{-1}(x) = 2Tan^{-1}(x)$

So, $2Tan^{-1}(x) = Tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= Tan^{-1}\left(\frac{2\left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2}\right) - Tan^{-1}\left(\frac{17}{31}\right)$

$= Tan^{-1}\left(\frac{\frac{3}{2}}{1 - \frac{9}{16}}\right) - Tan^{-1}\left(\frac{17}{31}\right)$

$= Tan^{-1}\left(\frac{3}{2} \div \frac{7}{16}\right) - Tan^{-1}\left(\frac{17}{31}\right)$

$= Tan^{-1}\left(\frac{3}{2} \times \frac{16}{7}\right) - Tan^{-1}\left(\frac{17}{31}\right)$

$= Tan^{-1}\left(\frac{24}{7}\right) - Tan^{-1}\left(\frac{17}{31}\right)$

$\because Tan^{-1}(x) - Tan^{-1}(y) = Tan^{-1}\left(\frac{x-y}{1+xy}\right)$

$= Tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \left(\frac{24}{7}\right)\left(\frac{17}{31}\right)}\right)$

$= Tan^{-1}\left(\frac{\frac{625}{217}}{\frac{625}{217}}\right)$

$= Tan^{-1}(1)$

$= \frac{\pi}{4}$ Proved

first we solve these two then the ans of that with the 3rd one

(vii) $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$

$\because \tan^{-1}(x)+\tan^{-1}(y)=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$=\tan^{-1}\left(\dfrac{\frac{1}{2}+\frac{1}{5}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{5}\right)}\right)+\tan^{-1}\left(\frac{1}{8}\right)$

$=\tan^{-1}\left(\dfrac{\frac{7}{10}}{\frac{9}{10}}\right)+\tan^{-1}\left(\frac{1}{8}\right)$

$=\tan^{-1}\left(\frac{7}{9}\right)+\tan^{-1}\left(\frac{1}{8}\right)$

$\because \tan^{-1}(x)+\tan^{-1}(y)=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$=\tan^{-1}\left(\dfrac{\frac{7}{9}+\frac{1}{8}}{1-\left(\frac{7}{9}\right)\left(\frac{1}{8}\right)}\right)$

$=\tan^{-1}\left(\dfrac{\frac{65}{72}}{\frac{65}{72}}\right)$

$=\tan^{-1}(1)$

$=\frac{\pi}{4}$ Proved

(viii) $\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$

$\because \sin^{-1}(x)+\sin^{-1}(y)=\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$

$=\sin^{-1}\left[\left(\frac{4}{5}\right)\sqrt{1-\left(\frac{5}{13}\right)^2}+\left(\frac{5}{13}\right)\sqrt{1-\left(\frac{4}{5}\right)^2}\right]+\sin^{-1}\left(\frac{16}{65}\right)$

$=\sin^{-1}\left[\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)+\left(\frac{5}{13}\right)\left(\frac{3}{5}\right)\right]+\sin^{-1}\left(\frac{16}{65}\right)$

$=\sin^{-1}\left(\frac{63}{65}\right)+\sin^{-1}\left(\frac{16}{65}\right)$

$\because \sin^{-1}(x)+\sin^{-1}(y)=\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$

$=\sin^{-1}\left[\left(\frac{63}{65}\right)\sqrt{1-\left(\frac{16}{65}\right)^2}+\left(\frac{16}{65}\right)\sqrt{1-\left(\frac{63}{65}\right)^2}\right]$

$=\sin^{-1}\left[\left(\frac{63}{65}\right)\left(\frac{63}{65}\right)+\left(\frac{16}{65}\right)\left(\frac{16}{65}\right)\right]$

$=\sin^{-1}(1)$

$=\frac{\pi}{2}$ Proved.

(ix) $\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{8}{19}\right)=\frac{\pi}{4}$

$\because \tan^{-1}(x)+\tan^{-1}(y)=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$=\tan^{-1}\left(\dfrac{\frac{3}{4}+\frac{3}{5}}{1-\left(\frac{3}{4}\right)\left(\frac{3}{5}\right)}\right)-\tan^{-1}\left(\frac{8}{19}\right)$

$=\tan^{-1}\left(\dfrac{\frac{27}{20}}{\frac{11}{20}}\right)-\tan^{-1}\left(\frac{8}{19}\right)$

$=\tan^{-1}\left(\frac{27}{11}\right)-\tan^{-1}\left(\frac{8}{19}\right)$

$\because \tan^{-1}(x)-\tan^{-1}(y)=\tan^{-1}\left(\frac{x-y}{1+xy}\right)$

$=\tan^{-1}\left(\dfrac{\frac{27}{11}-\frac{8}{19}}{1+\left(\frac{27}{11}\right)\left(\frac{8}{19}\right)}\right)$

$=\tan^{-1}\left(\dfrac{\frac{425}{209}}{\frac{425}{209}}\right)$

$=\tan^{-1}(1)$

$=\frac{\pi}{4}$

(x) $2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\left(\frac{1}{8}\right)=$

L.H.S

$=2\tan^{-1}\left(\frac{1}{5}\right)+2\tan^{-1}\left(\frac{1}{8}\right)+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)$

$=2\left[\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)\right]+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)$

$\because \tan^{-1}(x)+\tan^{-1}(y)=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$=2\left[\tan^{-1}\left(\dfrac{\frac{1}{5}+\frac{1}{8}}{1-\left(\frac{1}{5}\right)\left(\frac{1}{8}\right)}\right)\right]+\cos^{-1}\left(\frac{7}{5\sqrt{2}}\right)$

$=2\left[\tan^{-1}\left(\dfrac{\frac{13}{40}}{\frac{39}{40}}\right)\right]+\cos^{-1}\left(\frac{7}{5\sqrt{2}}\right)$

$=2\left[\tan^{-1}\left(\frac{1}{3}\right)\right]+\cos^{-1}\left(\frac{7}{5\sqrt{2}}\right)$

$\because 2\tan^{-1}(x)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$=\tan^{-1}\left(\dfrac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}\right)+\cos^{-1}\left(\frac{7}{5\sqrt{2}}\right)$

$=\tan^{-1}\left(\dfrac{\frac{2}{3}}{\frac{8}{9}}\right)+\cos^{-1}\left(\frac{7}{5\sqrt{2}}\right)$

$=\tan^{-1}\left(\frac{3}{4}\right)+\cos^{-1}\left(\frac{7}{5\sqrt{2}}\right)$

we need to convert the value of $\cos^{-1}$ into $\tan^{-1}$, for that we will need the value $\sin^{-1}$ so we can do $\frac{\sin}{\cos}$ is

let $\alpha=\cos^{-1}\left(\frac{7}{5\sqrt{2}}\right)$

$\cos\alpha=\frac{7}{5\sqrt{2}}$: $\frac{B}{H}$

$(H)^2=(P)^2+(B)^2$

$(5\sqrt{2})^2-(7)^2=P^2$

$P^2=1$

$P=1$

$\sin\alpha=\frac{P}{H}$

$\sin\alpha=\frac{1}{5\sqrt{2}}$

So $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}$

$\tan\alpha=\left(\dfrac{\frac{1}{5\sqrt{2}}}{\frac{7}{5\sqrt{2}}}\right)$

$\tan\alpha=\frac{1}{7}$

$\alpha = \tan^{-1}\left(\frac{1}{7}\right)$

Hence,

$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$\because \tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\left(\frac{3}{4}\right)\left(\frac{1}{7}\right)}\right)$

$= \tan^{-1}\left(\frac{\frac{25}{28}}{\frac{25}{28}}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$ Proved.

(xi) $\sin^{-1}(x) + \cos^{-1}(-x) = \frac{\pi}{2}$, where $-1 < x < 1$

we have to make the $\cos^{-1}$ into $\sin^{-1}$.

let $\alpha = \cos^{-1}(-x)$

$\cos\alpha = -x$

$\cos\alpha = \frac{-x}{1} = \frac{B}{H}$

$(H)^2 = (P)^2 + (B)^2$

$(1)^2 - (-x)^2 = \cancel{B}^2\ P$

$1 - x^2 = \cancel{B}^2\ P$

$P\ \cancel{B} = \sqrt{1-x^2}$

$\sin\alpha = \frac{P}{H}$

$\sin\alpha = \frac{\sqrt{1-x^2}}{1}$

$\alpha = \sin^{-1}\sqrt{1-x^2}$

So, $\sin^{-1}\sqrt{1-x^2} = \cos^{-1}(-x)$

Hence,

$= \sin^{-1}(x) + \sin^{-1}\left(\sqrt{1-x^2}\right)$

$\because \sin^{-1}(x) + \sin^{-1}(y) = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

$= \sin^{-1}\left[(x)\sqrt{1-\left(\sqrt{1-x^2}\right)^2} + \left(\sqrt{1-x^2}\right)\sqrt{1-(x)^2}\right]$

$= \sin^{-1}\left[x\sqrt{1-1+x^2} + \left(\sqrt{1-x^2}\right)\sqrt{1-x^2}\right]$

$= \sin^{-1}\left[x\sqrt{x^2} + \left(\sqrt{1-x^2}\right)^2\right]$

$= \sin^{-1}\left(x(x) + (1-x^2)\right)$

$= \sin^{-1}\left(\cancel{x^2} + 1 - \cancel{x^2}\right)$

$= \sin^{-1}(1)$

$= \frac{\pi}{2}$ Hence Proved

L.H.S = R.H.S