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Where Minds Pollinate

Class 12
Mathematics

Chapter 8

Exercise 8.1

Handwritten Solution

Unit 8 — Ex 8.1

MUST MEMORISE

Functions	Domain	Range
$y=\sin^{-1}x$	$-1 \le x \le 1$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y=\cos^{-1}x$	$-1 \le x \le 1$	$[0, \pi]$
$y=\tan^{-1}x$	$\mathbb{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y=\cot^{-1}x$	$\mathbb{R}$	$(0, \pi)$
$y=\sec^{-1}x$	$\lvert x\rvert \ge 1$	$[0, \pi]-\left\{\frac{\pi}{2}\right\}$
$y=\csc^{-1}x$	$\lvert x\rvert \ge 1$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$

$\underbrace{|x| \ge 1}$ This opens up to these

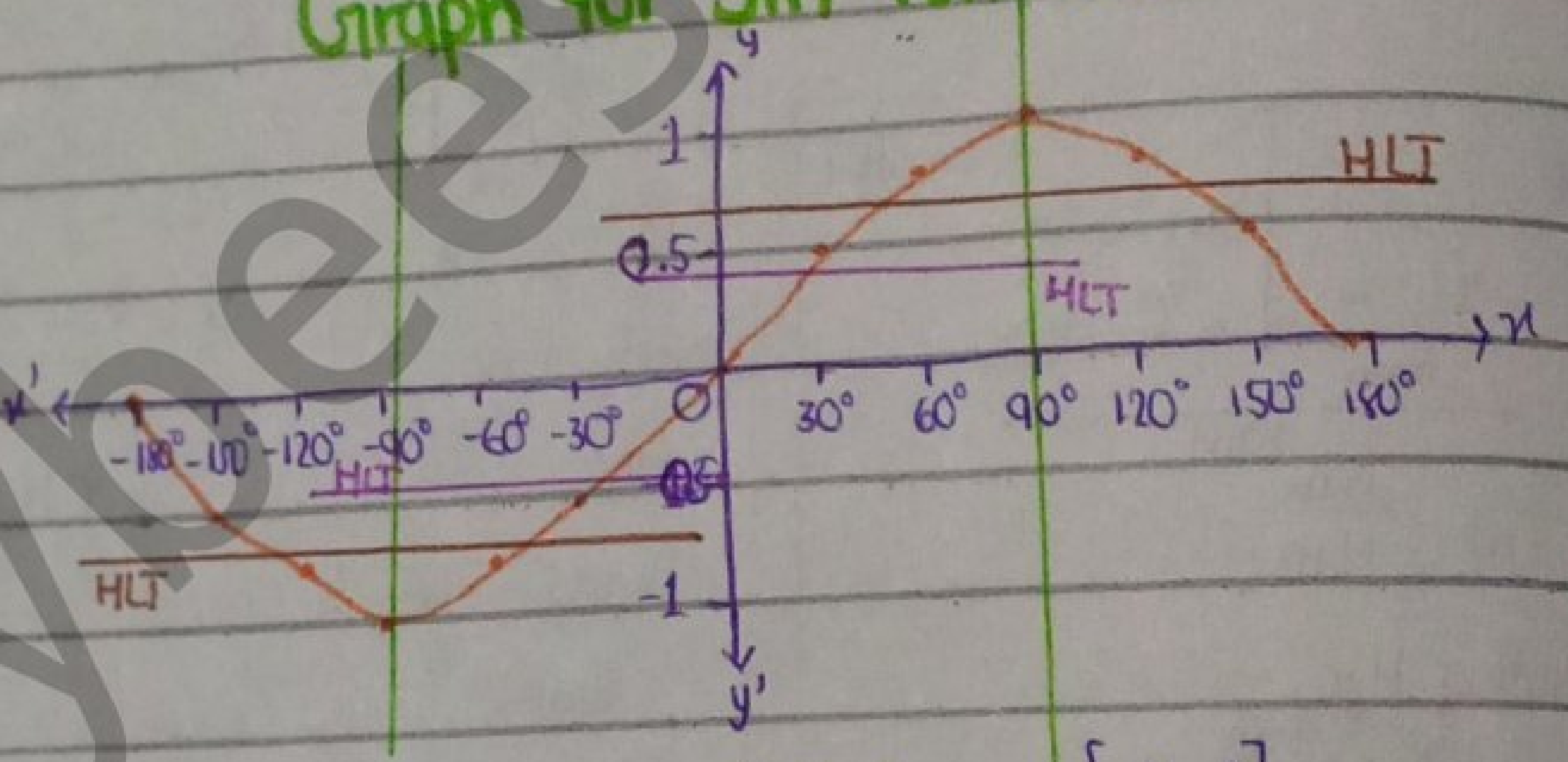
$\Rightarrow x \ge 1$

$-x \ge 1$

$\Rightarrow x \le -1$

The Domain was the Range of the normal trignometric functions

Graph for Sin Function



Sin = ■ So the Domain is $\mathbb{R}$ & Range is $[-1, 1]$

Now we know that if we ever want to find the inverse of the function (Sin) the function must satisfy 2 conditions the function must be one-one and onto. Now to find one-one we do the horizontal line test and that line must pass through only one point. But here if we do the HLT it passes through 2 points. So the question is how can we make it a one-one function. It's simple, we will $\boxed{\text{bound}}$ the graph to 90° or $\frac{\pi}{2}$. And now when we do the HLT, it passes through only one point in the graph so it satisfies one-one. And for onto, the condition was that the Range of the normal function is exactly equal to the Domain of the trignometric function, and that condition is satisfied. So for $\sin^{-1}$,

Domain = $[-1, 1]$

Principle Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

> The range of $\sin^{-1}$ is called principle range, bcs we restricted the graph at a value

$y = Sec^{-1}x$ for its range. We know $Sec^{-1}x$ is $\frac{1}{Cos x^{-1}}$, now we need to see at what value will it become undefined at. We know that if we put $x = 90^\circ$ or $x = \frac{\pi}{2}$ the $Cos x^{-1}$ will become zero $\frac{1}{0} = \infty$ so we know to not include $\frac{\pi}{2}$ and as $Sec^{-1}x = \frac{1}{Cos^{-1}x}$ so the range will be same $[0, \pi]$ with just $\frac{\pi}{2}$ not in it so $[0, \pi] - \{\frac{\pi}{2}\}$.

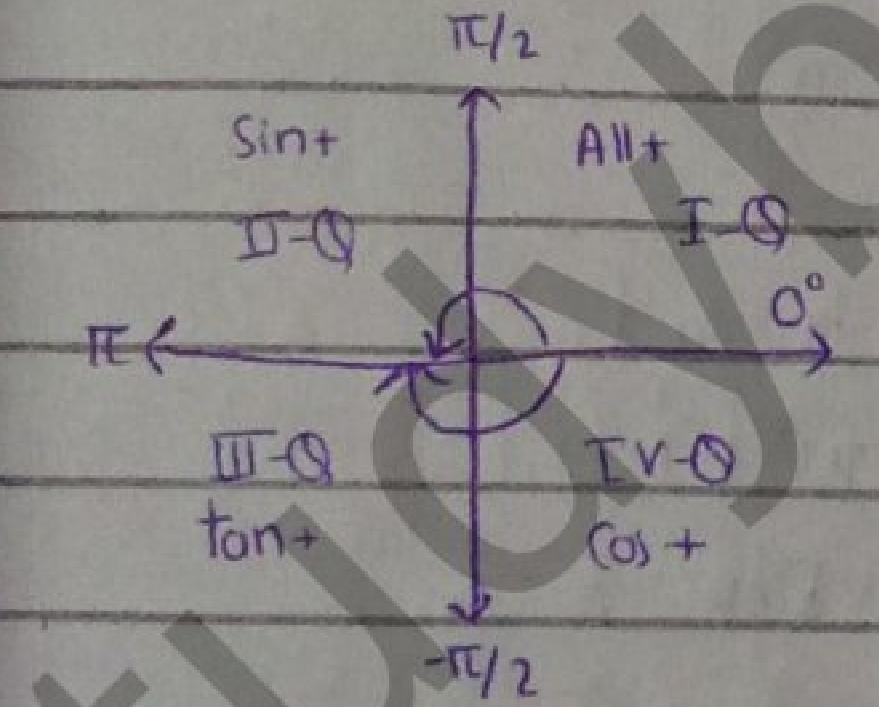
$y = Cosec^{-1}x$ for its range. We know $Cosec^{-1}x = \frac{1}{Sin^{-1}x}$ and $Sin^{-1}x$ is become zero at 0. So the range will be same $[-\frac{\pi}{2}, \frac{\pi}{2}]$ with 0 not includes so $[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

IMPORTANT POINT When you are writting the inverse functions make sure that their first letter is always capital. So, $Sin^{-1}, Cos^{-1}, Tan^{-1}, Cot^{-1}, Sec^{-1}, Cosec^{-1}$ for normal functions you can write them in small letters if you want sin^{-1}, cos^{-1}

In the question of our exercise it does say to solve it without a calc but u can use the calc. But for entry tests its important so here is how to make a table.

x	0°	30°	45°	60°	90°
	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
Sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Sin: Write 0-4 first, then put square root and divide by 2 the ans we get is the value.
Cos: Write the value of Sin but backward.
Tan: for tan we know $\tan = \frac{\sin}{\cos}$ so just divide the values and the ans will be value of tan



I-Q → No Change

II-Q → $\pi - x$ → if interval is $[0, 2\pi]$

III-Q → $\pi + x$ / $-\pi + x$ → if interval is $[-\pi, \pi]$

IV-Q → $-x$

Q1 Find—

(i) $Cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

let $x = Cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$x \in [0, \pi]$ — x should belong in the range of Cos^{-1}

$Cos x = \frac{\sqrt{3}}{2}$

$x = 30^\circ$

$x = \frac{30\pi}{180}$

$x = \frac{\pi}{6}$ — belongs in our interval $[0, \pi]$

~~We know that the value of Cos $Cos x = \frac{\sqrt{3}}{2}$ is + and our range $[0, \pi]$ so we will only see I-Q and II-Q and we know that Cos is positive at I-Q. so~~

Cos is positive in I-Q.

$S.S = \{\frac{\pi}{6}\}$

(ii) $Sin^{-1}(1)$

let $x = Sin^{-1}(1)$

$x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$Sin x = 1$

$x = 90^\circ$

$x = \frac{\pi}{2} \quad \because \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

∵ Consider I-Q and IV-Q

Sin is + at I-Q

$S.S = \{\frac{\pi}{2}\}$

(iii) $Tan^{-1}(\sqrt{3})$

let $x = Tan^{-1}(\sqrt{3})$

$x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\tan x = \sqrt{3}$

$x = 60^\circ$

$x = \frac{\pi}{3} \quad \because \in (-\frac{\pi}{2}, \frac{\pi}{2})$

∵ Consider I-Q & IV-Q

tan is + at I-Q

$SS = \{\frac{\pi}{3}\}$

(iv) $\cot^{-1}\left(\frac{\sqrt{3}}{3}\right)$

let $x = \cot^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$x \in (0, \pi)$

$\cot x = \frac{1}{\sqrt{3}}$

$\tan x = \sqrt{3}$

$x = 60^\circ$

$x = \frac{\pi}{3} \because \in (0, \pi)$

∵ Considering I-Q & II-Q,

tan + in I-Q

$S.S = \left\{\frac{\pi}{3}\right\}$

(v) $\sec^{-1}\left(\frac{2\sqrt{3}}{3}\right)$

let $x = \sec^{-1}\left(\frac{2\sqrt{3}}{3}\right)$

$x \in (0, \pi) - \left\{\frac{\pi}{2}\right\}$

$\sec x = \frac{2}{\sqrt{3}}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = 30^\circ$

$x = \frac{\pi}{6} \because \in (0,\pi) - \left\{\frac{\pi}{2}\right\}$

∵ Considering I-Q & II-Q

Cos + in I-Q

$S.S = \left\{\frac{\pi}{6}\right\}$

(vi) $\csc^{-1}(-\sqrt{2})$

let $x = \mathrm{cosec}^{-1}(-\sqrt{2})$

$x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$\mathrm{cosec}\, x = -\sqrt{2}$

$\sin x = \frac{-1}{\sqrt{2}}$

$x = \frac{\pi}{4}$ ∵ we neglect "–" sign

∵ Considering I-Q & IV-Q

Sin is negative at IV-Q

$x = \frac{-\pi}{4}$

$S.S = \left\{\frac{-\pi}{4}\right\}$

(vii) $\cos^{-1}\left(-\frac{1}{2}\right)$

let $x = \cos^{-1}\left(\frac{-1}{2}\right)$

$x \in [0, \pi]$

$\cos x = \frac{-1}{2}$

$x = 60^\circ$

$x = \frac{\pi}{3}$

∵ Considering I-Q & II-Q

Cos is negative at II-Q

$x = \pi - x$

$x = \pi - \frac{\pi}{3}$

$x = \frac{2\pi}{3}$

$S.S = \left\{\frac{2\pi}{3}\right\}$

(viii) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

let $x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\tan x = \frac{1}{\sqrt{3}}$

$x = 30^\circ$

$x = \frac{\pi}{6}$

∵ I-Q & IV-Q

tan is + in I-Q

$x = \frac{\pi}{6}$

$S.S = \left\{\frac{\pi}{6}\right\}$

(ix) $\csc^{-1}(-2)$

let $x = \csc^{-1}(-2)$

$x \in \boxed{\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}}$

$\csc x = -2$

$\sin x = \frac{-1}{2}$

$x = 30^\circ$

$x = \frac{\pi}{6}$

∵ I-Q & IV-Q

Sin is – at IV-Q,

$x = \frac{-\pi}{6}$

$S.S = \left\{\frac{-\pi}{6}\right\}$

Q2 Find sum of ....

(i) $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

let $x = \tan^{-1}(1)$

$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\tan x = 1$

$x = 45^\circ$

$x = \frac{\pi}{4}$

∵ I-Q & IV-Q

tan + in I-Q

$x = \frac{\pi}{4}$

let $y = \cos^{-1}\left(\frac{-1}{2}\right)$

$y \in [0, \pi]$

$\cos y = \frac{-1}{2}$

$y = 60^\circ$

$y = \frac{\pi}{3}$

∵ I-Q & II-Q

Cos is – at II-Q

$y = \pi - \frac{\pi}{3}$

$y = \frac{2\pi}{3}$

let $z = \sin^{-1}\left(\frac{-1}{2}\right)$

$z \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin x = \frac{-1}{2}$

$z = 30^\circ$

$z = \frac{\pi}{6}$

∵ I-Q & IV-Q

Sin is – at IV-Q

$z = \frac{-\pi}{6}$

$x + y + z = \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$= \frac{3\pi + 8\pi - 2\pi}{12}$

$= \frac{9\pi}{12}$

$\boxed{= \frac{3\pi}{4}}$

(ii) $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

let $x=\cos^{-1}\left(\frac{1}{2}\right)$

$x\in[0,\pi]$

$\cos x=\frac{1}{2}$

$x=60^\circ$

$x=\frac{\pi}{3}$

∵ I-Q & II-Q

Cos is + at I-Q

$\boxed{x=\frac{\pi}{3}}$

not necessary to make solution set

let $y=\sin^{-1}\left(\frac{1}{2}\right)$

$y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\sin y=\frac{1}{2}$

$y=30^\circ$

$y=\frac{\pi}{6}$

∵ I-Q & IV-Q

Sin is + at I-Q

$y=\frac{\pi}{6}$

So,

$x+2y=\frac{\pi}{3}+2\left(\frac{\pi}{6}\right)$

$=\frac{\pi}{3}+\frac{\pi}{3}$

$=\frac{2\pi}{3}$

$\boxed{=\frac{2\pi}{3}}$

(iii) $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)$

let $x=\tan^{-1}(\sqrt{3})$

$x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\tan x=\sqrt{3}$

$x=60^\circ$

$x=\frac{\pi}{6}$

∵ I-Q & IV-Q

tan is + at I-Q

$x=\frac{\pi}{3}$

let $y=\sec^{-1}(-2)$

$y\in[0,\pi]-\left\{\frac{\pi}{2}\right\}$

$\sec y=-2$

$\cos y=\frac{-1}{2}$

$y=60^\circ$

$y=\frac{\pi}{3}$

∵ I-Q & II-Q

Cos is - at II-Q

$y=\pi-y$

$y=\pi-\frac{\pi}{3}$

$y=\frac{2\pi}{3}$

So now

$=x-y$

$=\frac{\pi}{3}-\frac{2\pi}{3}$

$\boxed{=\frac{-\pi}{3}}$

(iv) $\cot^{-1}(-\sqrt{3})+\csc^{-1}(-2)-\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

let $x=\cot^{-1}(-\sqrt{3})$

$x\in(0,\pi)$

$\cot x=-\sqrt{3}$

$\tan x=-\frac{1}{\sqrt{3}}$

$x=30^\circ$

$x=\frac{\pi}{6}$

∵ I-Q & II-Q

tan is - at II-Q,

$x=\pi-x$

$x=\pi-\frac{\pi}{6}$

$\boxed{x=\frac{5\pi}{6}}$

let $y=\csc^{-1}(-2)$

$y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$

$\csc y=-2$

$\sin y=-\frac{1}{2}$

$y=30^\circ$

$y=\frac{\pi}{6}$

∵ I-Q & IV-Q

Sin is - at IV-Q

$y=-y$

$\boxed{y=-\frac{\pi}{6}}$

let $z=\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$z\in[0,\pi]$

$\cos z=\frac{1}{\sqrt{2}}$

$z=45^\circ$

$z=\frac{\pi}{4}$

∵ I-Q & II-Q

Cos is + at I-Q

$\boxed{z=\frac{\pi}{4}}$

$=x+y-z$

$=\frac{5\pi}{6}-\frac{\pi}{6}-\frac{\pi}{4}$

$=\frac{10\pi-2\pi-3\pi}{12}$

$\boxed{=\frac{5\pi}{12}}$

Q3 Find the exact ----

(i) $\cos^{-1}\left[\sin\left(\frac{\pi}{4}\right)\right]$

$=\cos^{-1}(\sin(45^\circ))$

$=\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

let $x=\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$x\in[0,\pi]$

$\cos x=\frac{1}{\sqrt{2}}$

$x=45^\circ$

$x=\frac{\pi}{4}$

∵ I-Q & II-Q

Cos is + at I-Q

$\boxed{x=\frac{\pi}{4}}$

(ii) $\sin^{-1}\left[\cos\left(\frac{-2\pi}{3}\right)\right]$

$=\sin^{-1}\left[\cos\left(\frac{2\pi}{3}\right)\right]$

$=\sin^{-1}(\cos 120)$

$=\sin^{-1}\left(\frac{-1}{2}\right)$

let $x=\sin^{-1}\left(-\frac{1}{2}\right)$

$x\in\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$

$\sin x=-\frac{1}{2}$

∴ $x=30^\circ$

$x=\frac{\pi}{6}$

∵ I-Q & IV-Q

Sin is - at IV-Q

$x=-x$

$\boxed{x=-\frac{\pi}{6}}$

(iii) $\cos^{-1}\left[\sin\left(\frac{11\pi}{6}\right)\right]$

$=\cos^{-1}\left(-\frac{1}{2}\right)$

let $x=\cos^{-1}\left(-\frac{1}{2}\right)$

$x\in[0,\pi]$

$\cos x=\frac{-1}{2}$

$x=60^\circ$

$x=\frac{\pi}{3}$

∵ I-Q & II-Q

Cos is - at II-Q

$x=\pi-x$

$x=\pi-\frac{\pi}{3}$

$\boxed{x=\frac{2\pi}{3}}$

(iv) $\cos^{-1}\left(\cos\frac{\pi}{6}\right)$

$=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

let $x=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$x\in[0,\pi]$

$\cos x=\frac{\sqrt{3}}{2}$

$x=30^\circ$

$x=\frac{\pi}{6}$

∵ I-Q & II-Q

Cos is + at I-Q

$\boxed{x=\frac{\pi}{6}}$

(v) $\sin\left(\tan^{-1}\frac{3}{4}\right)$

~~let $x = \tan^{-1}\left(\frac{3}{4}\right)$, $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $\tan x = \frac{3}{4}$, $x =$~~

let $x = \tan^{-1}\left(\frac{3}{4}\right)$

$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\tan x = \frac{3}{4}$

Now as we have let $x = \tan^{-1}\left(\frac{3}{4}\right)$ so technically it means $\sin(x)$

$\sin(x) = \frac{Perp}{Hyp}$

Finding Hypotenuse,

We know $\tan x = \frac{3}{4} = \frac{Perp}{Base}$

So using the Pythagoras formula,

$(Hyp)^2 = (Perp)^2 + (Base)^2$

$(Hyp)^2 = (3)^2 + (4)^2$

$(Hyp)^2 = 9 + 16$

$\sqrt{(Hyp)^2} = \sqrt{25}$

$Hyp = 5$

Now,

$\sin x = \frac{Perp}{Hyp}$

$\boxed{\sin x = \frac{3}{5}}$

(vi) $\cos\left(2\sin^{-1}\frac{\sqrt{2}}{2}\right)$

let $x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin x = \frac{1}{\sqrt{2}}$

$x = 45^\circ$

$x = \frac{\pi}{4}$

Now,

$= \cos\left(2\left(\frac{\pi}{4}\right)\right)$

$= \cos\left(\frac{\pi}{2}\right)$

$\boxed{= 0}$

(vii) $\sin\left[2\sin^{-1}\left(\frac{4}{5}\right)\right]$

let $x = \sin^{-1}\left(\frac{4}{5}\right)$

$x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin x = \frac{4}{5} = \frac{Perp}{Hyp}$

$\Rightarrow \sin 2x$

$= 2\sin x \cos x$ — eq(i)

Using pythagoras theorem,

$(H)^2 = (P)^2 + (B)^2$

$(5)^2 = (4)^2 + (B)^2$

$25 - 16 = (B)^2$

$\sqrt{(Base)^2} = \sqrt{9}$

$Base = 3$

$\cos x = \frac{Base}{Hyp}$

$\cos x = \frac{3}{5}$

Putting value of $\sin x$ & $\cos x$ in eq(i)

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= 2\left(\frac{12}{25}\right)$

$\boxed{= \frac{24}{25}}$

(viii) $\cos\left(\sin^{-1}\frac{5}{13}\right)$

let $x = \sin^{-1}\left(\frac{5}{13}\right)$ — eq(i)

$x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin x = \frac{5}{13} = \frac{Perp}{Hyp}$

Using eq(i) in the question,

$= \cos x$

$= \frac{Base}{Hyp}$ — eq(ii)

Finding Base using the pythagoras theorem

$(H)^2 = (P)^2 + (B)^2$

$(13)^2 = (5)^2 + (B)^2$

$169 - 25 = (B)^2$

$\sqrt{(B)^2} = \sqrt{144}$

$B = 12$

Putting in eq(ii)

$= \cos x$

$= \frac{Base}{Hyp}$

$\boxed{= \frac{12}{13}}$

(ix) $\sin\left[\cos^{-1}\left(\frac{-3}{5}\right)\right]$

let $x = \cos^{-1}\left(\frac{-3}{5}\right)$ — eq(i)

$x \in [0, \pi]$

$\cos x = \frac{-3}{5}$

Using eq(i) in the question

$= \sin(x)$

Using the identity to find $\sin x$,

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\sqrt{\sin^2 x} = \sqrt{1 - \cos^2 x}$

$\sin x = \sqrt{1 - \left(\frac{-3}{5}\right)^2}$

$\sin x = \sqrt{1 - \frac{9}{25}}$

$\sin x = \sqrt{\frac{16}{25}}$

$\sin x = \frac{\sqrt{16}}{\sqrt{25}}$

$\boxed{\sin x = \frac{4}{5}}$

(x) $\sin\left[\sin^{-1}\left(\frac{2}{3}\right) + \cos^{-1}\left(\frac{1}{2}\right)\right]$

let $\alpha = \sin^{-1}\left(\frac{2}{3}\right)$ — eq(i)

U can let them x & y too, upto

$\alpha \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin\alpha = \frac{2}{3}$ — eq(iii)

let $\beta = \cos^{-1}\left(\frac{1}{2}\right)$ — eq(ii)

$\beta \in [0, \pi]$

$\cos\beta = \frac{1}{2}$ — eq(iv)

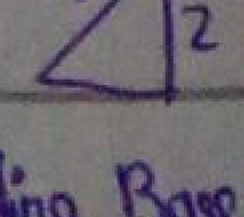
Putting eq(i) & (ii) in the question,

$\sin[\alpha + \beta]$

$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$ — ①

Finding $\cos\alpha$, using pythagoras theorem from eq(iii)

$\sin\alpha = \frac{2}{3} = \frac{Perp}{Hyp}$



finding Base

$(H)^2=(P)^2+(B)^2$

$(3)^2=(2)^2+(B)^2$

$9-4=(B)^2$

$\sqrt{(B)^2}=\sqrt{5}$

$B=\sqrt{5}$

So $\cos\alpha=\dfrac{\text{Base}}{\text{Hyp}}$

$\cos\alpha=\dfrac{\sqrt{5}}{3}$ — eq(v)

Finding $\sin\beta$ using the pythagoras theorem,

from eq(iv) $\cos\beta=\dfrac{1}{2}=\dfrac{\text{Base}}{\text{Hyp}}$

finding Perpendicular

$(H)^2=(P)^2+(B)^2$

$(2)^2=(P)^2+(1)^2$

$4-1=(P)^2$

$\sqrt{(P)^2}=\sqrt{3}$

$P=\sqrt{3}$

So $\sin\beta=\dfrac{\text{Perp}}{\text{Hyp}}$

$\sin\beta=\dfrac{\sqrt{3}}{2}$ — eq(vi)

Putting eq(iii),(iv),(v),(vi) in eq①

$=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$=\left(\dfrac{2}{3}\right)\left(\dfrac{1}{2}\right)+\left(\dfrac{\sqrt{5}}{3}\right)\left(\dfrac{\sqrt{3}}{2}\right)$

$=\dfrac{2}{6}+\dfrac{\sqrt{15}}{6}$

$\boxed{=\dfrac{2+\sqrt{15}}{6}}$

(xi) $\cos\left(\sin^{-1}\dfrac{3}{4}+\cos^{-1}\dfrac{5}{13}\right)$

let $x=\sin^{-1}\left(\dfrac{3}{4}\right)$ — eq(i)

$x\in\left[\dfrac{-\pi}{2},\dfrac{\pi}{2}\right]$

$\boxed{\sin x=\left(\dfrac{3}{4}\right)}$ — eq(iii)

let $y=\cos^{-1}\left(\dfrac{5}{13}\right)$ — eq(ii)

$y\in[0,\pi]$

$\boxed{\cos y=\left(\dfrac{5}{13}\right)}$ — eq(iv)

Putting eq(i) & eq(ii) in the question,

$\cos(x+y)=\cos x\cos y-\sin x\sin y$ — eq★

Finding $\cos x$ using the pythagoras theorem,

from eq(iii),

$\sin x=\dfrac{3}{4}=\dfrac{\text{Perp}}{\text{Hyp}}$

finding base,

$(H)^2=(P)^2+(B)^2$

$(4)^2=(3)^2+(B)^2$

$16-9=(B)^2$

$\sqrt{(B)^2}=\sqrt{7}$

$B=\sqrt{7}$

So, $\cos x=\dfrac{\text{Base}}{\text{Hyp}}$

$\boxed{\cos x=\dfrac{\sqrt{7}}{4}}$ — eq(v)

Finding $\sin y$ using the P.T,

from eq(iv),

$\cos y=\dfrac{5}{13}=\dfrac{\text{Base}}{\text{Hyp}}$

finding perpendicular

$(H)^2=(P)^2+(B)^2$

$(13)^2-(5)^2=(P)^2$

$\sqrt{(P)^2}=\sqrt{144}$

$P=12$

So $\sin y=\dfrac{\text{Perp}}{\text{Hyp}}$

$\boxed{\sin y=\dfrac{12}{13}}$ — eq(vi)

Putting eq(iii),(iv),(v),(vi) in eq★

$=\cos x\cos y-\sin x\sin y$

$=\left(\dfrac{\sqrt{7}}{4}\right)\left(\dfrac{5}{13}\right)-\left(\dfrac{3}{4}\right)\left(\dfrac{12}{13}\right)$

$=\dfrac{5\sqrt{7}}{52}-\dfrac{9}{13}$

$\boxed{=\dfrac{-36+5\sqrt{7}}{52}}$

(xii) $\cos\left[\sec^{-1}(3)+\tan^{-1}(2)\right]$

let $x=\sec^{-1}(3)$ — eq(i)

$x\in[0,\pi]-\left\{\dfrac{\pi}{2}\right\}$

$\sec x=3$

$\boxed{\cos x=\dfrac{1}{3}}$ — eq(iii)

let $y=\tan^{-1}(2)$ — eq(ii)

$y\in\left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)$

$\tan y=2$ — eq(iv)

Putting eq(i) & (ii) in the question,

$\cos(x+y)=\cos x\cos y-\sin x\sin y$ — ★

Finding $\sin x$ using pythagoras theorem,

from eq(iii) $\cos x=\dfrac{1}{3}=\dfrac{\text{Base}}{\text{Hyp}}$

finding perpendicular,

$(H)^2=(P)^2+(B)^2$

$(3)^2-(1)^2=(P)^2$

$\sqrt{P^2}=\sqrt{8}$

$P=\sqrt{8}$

So, $\sin x=\dfrac{\text{Perp}}{\text{Hyp}}$

$\boxed{\sin x=\dfrac{\sqrt{8}}{3}}$ — eq(v)

Finding $\cos y$ & $\sin y$ using P.T;

from eq(iv) $\tan y=\dfrac{2}{1}=\dfrac{\text{Perp}}{\text{Base}}$

finding Hypotenuse,

$(H)^2=(P)^2+(B)^2$

$(H)^2=(2)^2+(1)^2$

$\sqrt{(H)^2}=\sqrt{5}$

$H=\sqrt{5}$

So, $\sin y=\dfrac{P}{H}$

$\boxed{\sin y=\dfrac{2}{\sqrt{5}}}$ — eq(vi)

$\cos y=\dfrac{B}{H}$

$\boxed{\cos y=\dfrac{1}{\sqrt{5}}}$ — eq(vii)

Putting eq(iii),(v),(vi),(vii) in eq★,

$\cos(x+y)=\cos x\cos y-\sin x\sin x$

$=\left(\dfrac{1}{3}\right)\left(\dfrac{1}{\sqrt{5}}\right)-\left(\dfrac{\sqrt{8}}{3}\right)\left(\dfrac{2}{\sqrt{5}}\right)$

$=\dfrac{1-2\sqrt{8}}{3(\sqrt{5})}$

$=\dfrac{1-2(2\sqrt{2})}{3\sqrt{5}}$

$\boxed{=\dfrac{1-4\sqrt{2}}{3\sqrt{5}}}$

Q4 Find the unknown...

(i) $\cos^{-1}\left(\frac{3}{5}\right) = \sin^{-1}(\ )$

let $x = \cos^{-1}\left(\frac{3}{5}\right)$

$x \in [0, \pi]$

$\cos x = \frac{3}{5} = \frac{\text{Base}}{\text{Hyp}}$

finding Perpendicular

$(H)^2 = (P)^2 + (B)^2$

$(5)^2 = (P)^2 + (3)^2$

$25 - 9 = P^2$

$\sqrt{P^2} = \sqrt{16}$

$P = 4$

So $\sin x = \frac{P}{H}$

$\sin x = \frac{4}{5}$

$x = \sin^{-1}\left(\frac{4}{5}\right)$

As $x = x$ so

Hence $\cos^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{4}{5}\right)$

$\cos^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{4}{5}\right)$

u can find either of the two's value $= 53.130$

(ii) $\sin^{-1}\left(\frac{2}{3}\right) = \cos^{-1}(\ )$

let $x = \sin^{-1}\left(\frac{2}{3}\right)$

$x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin x = \frac{2}{3} = \frac{P}{H}$

Finding Base,

$(H)^2 = (P)^2 + (B)^2$

$(3)^2 = (2)^2 + (B)^2$

$9 - 4 = (B)^2$

$\sqrt{(B)^2} = \sqrt{5}$

$B = \sqrt{5}$

So $\cos x = \frac{B}{H}$

$\cos x = \frac{\sqrt{5}}{3}$

$x = \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$

Hence

$\sin^{-1}\left(\frac{2}{3}\right) = \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$

$= 41.810$

(iii) $\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right) = -\cos^{-1}(\ )$

let $-x = \sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)$

$-x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin(-x) = \frac{-1}{\sqrt{5}}$

$-\sin x = -\frac{1}{\sqrt{5}}$

$\sin x = \frac{1}{\sqrt{5}} = \frac{P}{H}$

Finding Base,

$(H)^2 = (P)^2 + (B)^2$

$(\sqrt{5})^2 = (1)^2 + (B)^2$

$5 - 1 = (B)^2$

$\sqrt{(B)^2} = \sqrt{4}$

$B = 2$

So $\cos x = \frac{B}{H}$

$\cos x = \frac{2}{\sqrt{5}}$

$x = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

x with (-1),

$-x = -\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Hence,

$\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right) = -\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

$= -26.565$

(iv) $\tan^{-1}\left(\frac{1}{4}\right) = \cos^{-1}(\ )$

let $x = \tan^{-1}\left(\frac{1}{4}\right)$

$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\tan x = \frac{1}{4} = \frac{P}{B}$

finding Hypotenuse

$(H)^2 = (P)^2 + (B)^2$

$(H)^2 = (1)^2 + (4)^2$

$\sqrt{(H)^2} = \sqrt{17}$

$H = \sqrt{17}$

So $\cos x = \frac{B}{H}$

$\cos x = \frac{4}{\sqrt{17}}$

$x = \cos^{-1}\left(\frac{4}{\sqrt{17}}\right)$

Hence, $\tan^{-1}\left(\frac{1}{4}\right) = \cos^{-1}\left(\frac{4}{\sqrt{17}}\right)$

$= 14.036$

(v) $\tan^{-1}(-1.2) = -\cos^{-1}(\ )$

let $-x = \tan^{-1}(-1.2)$

$-x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\tan(-x) = -1.2$

$\cancel{-}\tan x = \cancel{-}1.2$

$\tan x = 1.2$

$\tan x = \frac{12}{10}$

$\tan x = \frac{6}{5} = \frac{P}{B}$

finding H,

$(H)^2 = (P)^2 + (B)^2$

$(H)^2 = (6)^2 + (5)^2$

$(H)^2 = 36 + 25$

$\sqrt{(H)^2} = \sqrt{61}$

$H = \sqrt{61}$

So $\cos x = \frac{B}{H}$

$\cos x = \frac{5}{\sqrt{61}}$

$x = \cos^{-1}\left(\frac{5}{\sqrt{61}}\right)$

x(-1)

$-x = -\cos^{-1}\left(\frac{5}{\sqrt{61}}\right)$

Hence,

$\tan^{-1}(-1.2) = -\cos^{-1}\left(\frac{5}{\sqrt{61}}\right)$

$= -50.194$

(vi) $\cot^{-1}\left(\frac{-3}{4}\right) = -\sin^{-1}(\ )$

let $-x = \cot^{-1}\left(\frac{-3}{4}\right)$

$-x \in (0, \pi)$

$\cot(-x) = \frac{-3}{4}$

$-\cot x = \frac{-3}{4}$

$\cot x = \frac{3}{4}$

$\tan x = \frac{4}{3} = \frac{P}{B}$

finding Hypotenuse,

$(H)^2 = (P)^2 + (B)^2$

$(H)^2 = (4)^2 + (3)^2$

$(H)^2 = 16 + 9$

$\sqrt{(H)^2} = \sqrt{25}$

$H = 5$

So $\sin x = \frac{P}{H}$

$\sin x = \frac{4}{5}$

$x = \sin^{-1}\left(\frac{4}{5}\right)$

$\times(-1)$ on b/s

$-x = -\sin^{-1}\left(\frac{4}{5}\right)$

Hence,

$\cot^{-1}\left(\frac{-3}{4}\right) = -\sin^{-1}\left(\frac{4}{5}\right)$

$= 53.130$

(vii) $\sec^{-1}(2.041) = \tan^{-1}(\ )$

let $x = \sec^{-1}(2.041)$

$x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$ ∵ I-Q & II-Q Sec is + in I-Q So all values are same.

$\sec x = \frac{2041}{1000}$

$\cos x = \frac{1000}{2041} = \frac{B}{H}$

Finding Perp,

$(H)^2 = (P)^2 + (B)^2$

$(2041)^2 = (P)^2 + (1000)^2$

$(P)^2 = \sqrt{3165681}$

$P = 1779.2$

So $\tan x = \frac{P}{B}$

$\tan x = \frac{1779.2}{1000}$

$\tan x = 1.779$

$x = \tan^{-1}(1.779)$

Hence

$\sec^{-1}(2.041) = \tan^{-1}(1.779)$

$= 60.659$

(viii) $\sec^{-1}(-\sqrt{5}) = \cot^{-1}(\ )$

let $x = \sec^{-1}(-\sqrt{5})$

$x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

∵ I & II-Q

Sec is - at II-Q

Cosx =

$\sec x = -\sqrt{5}$

Using identity,

$\sec^2 x + \tan^2 x = 1$

$\tan^2 x = 1 - \sec^2 x$

$\sqrt{\tan^2 x} = \sqrt{1 - (-\sqrt{5})^2}$

$\tan x = \sqrt{1-5}$

$\tan x = \sqrt{4}$

$\tan x = -2$ ∵ (-) bcs in II-Q

So $\cot x = -\frac{1}{2}$

$x = \cot^{-1}\left(\frac{-1}{2}\right)$

Hence,

$\sec^{-1}(-\sqrt{5}) = \cot^{-1}\left(\frac{-1}{2}\right)$ ∵ Put $\tan^{-1}(-2)$ in calc

$= -63.435$

(ix) $\csc^{-1}(1.172) = \sin^{-1}(\ )$

let $x = \operatorname{cosec}^{-1}(1.172)$

$x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

∵ I-Q & IV-Q

Cosec is + at I-Q

$\operatorname{cosec} x = 1.172$

$\sin x = \frac{1}{1.172}$

$x = \sin^{-1}\left(\frac{1}{1.172}\right)$

Hence

$\operatorname{cosec}^{-1}(1.172) = \sin^{-1}\left(\frac{1}{1.172}\right)$

$= 58.566$

(x) $\csc^{-1}\left(\frac{-5}{3}\right) = \tan^{-1}(\ )$

let $x = \operatorname{cosec}^{-1}\left(\frac{-5}{3}\right)$

$x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

∵ I-Q & IV-Q

Cosec is - at IV-Q

$\operatorname{cosec} x = \frac{-5}{3}$

Using identity

$\operatorname{cosec}^2 x - \cot^2 x = 1$

$\cot^2 x = -1 + \operatorname{cosec}^2 x$

$\sqrt{\cot^2 x} = \sqrt{-1 + \left(\frac{-5}{3}\right)^2}$

$\cot x = \sqrt{-1 + \frac{25}{9}}$

$\cot x = -\sqrt{\frac{16}{9}}$

$\cot x = \frac{-4}{3}$ ∵ Cot is - at IV-Q

$\tan x = \frac{-3}{4}$

$x = \tan^{-1}\left(\frac{-3}{4}\right)$

Hence,

$\operatorname{cosec}^{-1}\left(\frac{-5}{3}\right) = \tan^{-1}\left(\frac{-3}{4}\right)$

$= -36.870$