Studybeesofficial

Where Minds Pollinate

Class 12

Mathematics

Chapter 7

Exercise 7.5

Handwritten Solution

Ex 7.5

Q1 Find the points of intersections - - - - - - - - - -

$\rightarrow$ (i) Finding point of intersection

From equation of line

$$x-2y+3=0$$

$$x=2y-3 \quad -eq(i)$$

Putting eq(i) in parabola,

$$y^2=8x+1$$

$$y^2=8(2y-3)+1$$

$$y^2=16y-24+1$$

$$y^2-16y+23=0$$

$$y=\frac{-(-16)\pm\sqrt{(-16)^2-4(1)(23)}}{2(1)}$$

$$y=\frac{16\pm2\sqrt{41}}{2}$$

$$y=\frac{\cancel{2}(8\pm\sqrt{41})}{\cancel{2}}$$

$$\boxed{y=8\pm\sqrt{41}}$$

Putting y in eq(i),

$$x=2(8\pm\sqrt{41})-3$$

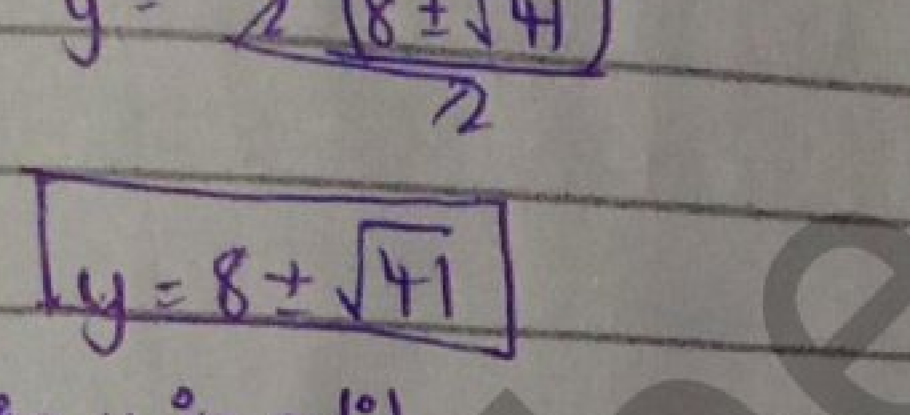
$$x=16\pm2\sqrt{41}-3$$

$$\boxed{x=13\pm2\sqrt{41}}$$

Hence point of intersection is,

$$(13\pm2\sqrt{41},\ 8\pm\sqrt{41})$$

(ii) Finding the cord intercepted



we have to find the length of this cord.

$$|P_1P_2|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$l=\sqrt{(13-2\sqrt{41}-13-2\sqrt{41})^2+(8-\sqrt{41}-8-\sqrt{41})^2}$$

$$l=\sqrt{(-4\sqrt{41})^2+(-2\sqrt{41})^2}$$

$$l=\sqrt{16(41)+4(41)}$$

$$l=\sqrt{20(41)}$$

$$\boxed{l=2\sqrt{205}}$$

(iii) Finding if the cord is focal cord

$\because$ focal cord is when the cord passes through the focus point.

For that we need to find focus point,

So we will consider the equation of parabola

$$y^2=8x+1$$

$$y^2=8\left(x+\frac{1}{8}\right)$$

let $Y=y$ & $X=\left(x+\frac{1}{8}\right)$

$$Y^2=8X \quad -eq (A)$$

The standard equation of a parabola for this shape is,

$$Y^2=4aX \quad -eq (B)$$

So comparing the eq (A) and eq (B),

$$\cancel{8}^{2}=\cancel{4}a$$

$$a=2$$

So we know $f(a,0)$, so,

$$X=a,\ Y=0$$

Converting these back,

$$x+\frac{1}{8}=2,\ y=0$$

$$x=2-\frac{1}{8},\ y=0$$

$$x=\frac{15}{8},\ y=0$$

So $f\left(\frac{15}{8},0\right)$

Putting value of focus in line,

$$x-2y+3=0$$

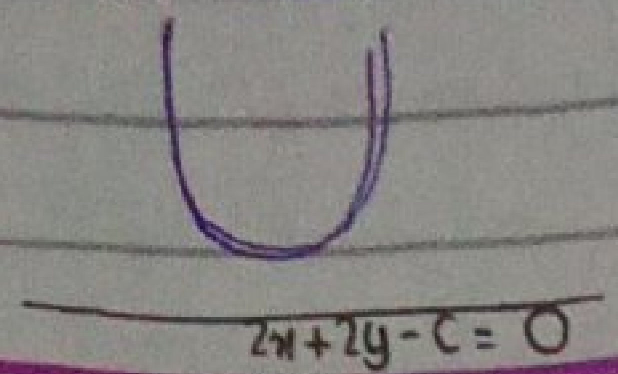
$$x\left(\frac{15}{8}\right)-2(0)+3=0$$

$$\frac{15}{8}+3=0$$

$$\frac{39}{8}\neq0$$

Hence focus does not lie on the line, so the line is just a cord.

Q2 For what values of c line . . . .



(i) if disc <0 (b^2-4ac) then the line never touches or intersects

(ii) if disc $=0$ then the line is tangent

(iii) if disc >0 then the line intersects

we have to make a quadratic equation to find the discriminate.

$x^2-x+2y+3=0$

From equation of line

$$2x+2y-C=0$$

$$2y=C-2x$$

$$y=\frac{C-2x}{2}$$

Putting y in the equation of parabola,

$$x^2-x+\cancel{2}\left(\frac{C-2x}{\cancel{2}}\right)+3=0$$

$$x^2-x+C-2x+3=0$$

$$x^2-3x+(3+C)=0$$

$a=1$, $b=-3$, $c=C+3$

$$\text{disc}=b^2-4ac$$

$$\text{disc}=(-3)^2-4(1)(C+3)$$

$$\text{disc}=9-4C-12$$

$$\text{disc}=-3-4C$$

for this condition we know,

$$\text{disc}<0$$

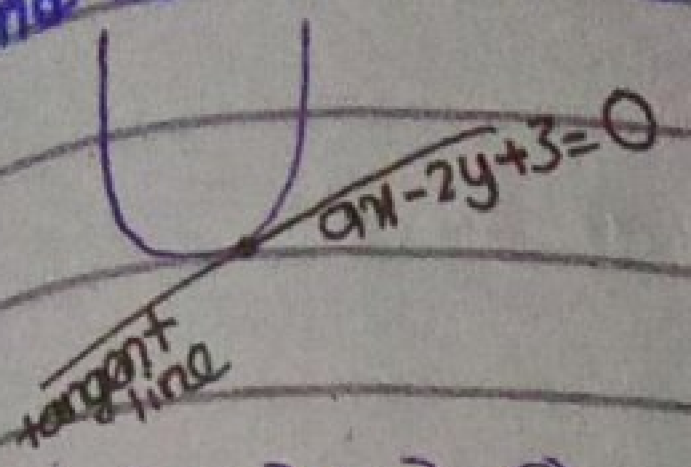
$$-3-4C<0$$

$$-4C<3$$

Divide by -4,

$$\boxed{C>\frac{-3}{4}}$$

Find the value of "a" so that - - - - -



$y^2 - 2y + 3x + 7 = 0$

from equation of the line,

$ax - 2y + 3 = 0$

$ax = 2y - 3$

$x = \frac{2y-3}{a}$

Putting x in the eq of the parabola,

$y^2 - 2y + 3x + 7 = 0$

$y^2 - 2y + 3\left(\frac{2y-3}{a}\right) + 7 = 0$

x a on all sides

$ay^2 - 2ay + 3(2y-3) + 7a = 0$

$ay^2 - 2ay + 6y - 9 + 7a = 0$

$ay^2 + y(-2a+6) + (7a-9) = 0$

$Disc = B^2 - 4AC$

$disc = (-2a+6))^2 - 4(a)(7a-9)$

$disc = (6-2a)^2 - 4a(7a-9)$

$disc = 36 - 24a + 4a^2 - 28a^2 + 36$

$disc = -24a^2 - 24a + 72$

as in condition (ii),

$disc = 0$

$-24a^2 - 24a + 72 = 0$

$-24(a^2 + a + 3) = 0$

$a^2 + a + 3 = 0$

$a = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$a = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(3)}}{2(1)}$

$a = \frac{-1 \pm \sqrt{1-12}}{2}$

$a = \frac{-1 \pm \sqrt{-11}}{2}$

Q4 Prove that the line - - - -

From the equation of line,

$$3x+y-5=0$$
$$3x=5-y$$
$$x=\frac{5-y}{3} \quad -eq\ (1)$$

Putting this in the eq of parabola,

$$y^2-2y+6x-6=0$$
$$y^2-2y+6\left(\frac{5-y}{3}\right)-6=0$$
$$y^2-2y+10-2y-6=0$$
$$y^2-4y+4=0 \quad -eq\ (*)$$

$$Disc = b^2-4ac$$
$$Disc=(-4)^2-4(1)(4)$$
$$Disc=16-16$$
$$Disc=0$$

Hence proved given line is tangent

from eq (*),

$$y^2-4y+4=0$$
$$(y)^2-2(2)(y)+(2)^2=0$$
$$(y-2)^2=0$$
$$\sqrt{(y-2)^2}=\sqrt{0}$$
$$y-2=0$$
$$\boxed{y=2}$$

Putting in eq (1),

$$x=\frac{5-2}{3}$$
$$x=\frac{3}{3}$$
$$\boxed{x=1}$$

Hence point of contact is,

(1,2)

Q5 Find the equation - - -

$$2y^2-3y+11x-16=0$$

Differentiate w.r.t x,

$$2\frac{d}{dx}(y^2)-3\frac{d}{dx}(y)+11\frac{d}{dx}(x)-\frac{d}{dx}(16)=\frac{d}{dx}(0)$$
$$4y\frac{dy}{dx}-3\frac{dy}{dx}+11-0=0$$
$$4y\frac{dy}{dx}-3\frac{dy}{dx}+11=0$$
$$\frac{dy}{dx}(4y-3)=-11$$
$$\frac{dy}{dx}=\frac{-11}{4y-3}$$
$$\frac{dy}{dx}=\frac{-11}{-(3-4y)}$$
$$\frac{dy}{dx}=\frac{11}{3-4y}$$
$$\left.\frac{dy}{dx}\right|_{(1,-1)}=\frac{11}{3-4(-1)}$$
$$\left.\frac{dy}{dx}\right|_{(1,-1)}=\frac{11}{3+4}$$
$$\left.\frac{dy}{dx}\right|_{(1,-1)}=\frac{11}{7}$$

Slope of tangent $= m_1 = \frac{11}{7}$

Slope of normal $= m_2 = \frac{-7}{11}$

Equation of tangent,

$$y-y_1=m(x-x_1)$$
$$y-(-1)=\frac{11}{7}(x-1)$$
$$y+1=\frac{11}{7}(x-1)$$
$$7y+7=11x-11$$
$$11x-7y-11-7=0$$
$$\boxed{11x-7y-18=0}$$

Equation of normal,

$$y-y_1=m(x-x_1)$$
$$y-(-1)=\frac{-7}{11}(x-1)$$
$$11(y+1)=-7(x-1)$$
$$11y+11=-7x+7$$
$$7x+11y+11-7=0$$
$$\boxed{7x+11y+4=0}$$

Find the equation of . . . .

Given.

$x=1$

$\boxed{\text{x-axis is called Abscissa} \quad \text{y-axis is called Ordinate}}$

Putting x in equation of parabola,

$x^2-5x+2y+6=0$

$(1)^2-5(1)+2y+6=0$

$1-5+2y+6=0$

$2y=-2$

$y=\frac{-2}{2}$

$y=-1$

point $(1,-1)$

$x^2-5x+2y+6=0$

differentiate w.r.t x,

$\frac{d}{dx}(x^2)-5\frac{d}{dx}(x)+2\frac{d}{dx}(y)+\frac{d}{dx}(6)=\frac{d}{dx}(0)$

$2x-5+2\frac{dy}{dx}+0=0$

$2x-5+2\frac{dy}{dx}=0$

$2\frac{dy}{dx}=5-2x$

$\frac{dy}{dx}=\frac{5-2x}{2}$

$\left.\frac{dy}{dx}\right|_{(1,-1)}=\frac{5-2(1)}{2}$

$\left.\frac{dy}{dx}\right|_{(1,-1)}=\frac{5-2}{2}$

$\left.\frac{dy}{dx}\right|_{(1,-1)}=\frac{3}{2}$

Slope of tangent $= m_1 = \frac{3}{2}$

Slope of normal $= m_2 = \frac{-2}{3}$

Equation of tangent,

$y-y_1=m(x-x_1)$

$y-(-1)=\frac{3}{2}(x-1)$

$y+1=\frac{3}{2}(x-1)$

$2y+2=3x-3$

$3x-2y-3-2=0$

$\boxed{3x-2y-5=0}$

Equation of normal,

$y-y_1=m(x-x_1)$

$y-(-1)=\frac{-2}{3}(x-1)$

$3(y+1)=-2(x-1)$

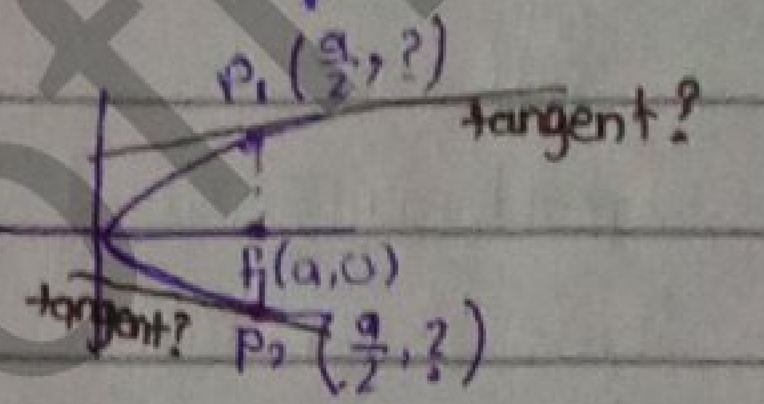
$3y+3=-2x+2$

$2x+3y+3-2=0$

$\boxed{2x+3y+1=0}$

Q7 Find the equation of tangent ...

...



let $y^2=4ax$ — eq(i)

$y^2=18x$ — eq(ii)

Comparing eq(i) & eq(ii),

$4a=18$

$a=\frac{18}{4}$

$a=\frac{9}{2}$

So $F=\left(\frac{9}{2},0\right)$

we know $x=\frac{9}{2}$, Putting it in the eq of the parabola,

$y^2=18x$

$y^2=18\left(\frac{9}{2}\right)$

$\sqrt{y^2}=\sqrt{81}$

$y=\pm 9$

So $P_1\left(\frac{9}{2},9\right)$ & $P_2\left(\frac{9}{2},-9\right)$

Finding slope,

$\frac{d}{dx}(y^2)=18\frac{d}{dx}(x)$

$2y\frac{dy}{dx}=18$

$\frac{dy}{dx}=\frac{18}{2y}$

$\frac{dy}{dx}=\frac{9}{y}$

lets take $P_1(\frac{9}{2}, 9)$ first,

$$\left.\frac{dy}{dx}\right|_{(\frac{9}{2},9)} = \frac{9}{y}$$

$$\left.\frac{dy}{dx}\right|_{(\frac{9}{2},9)} = \frac{9}{9}$$

$$\left.\frac{dy}{dx}\right|_{(\frac{9}{2},9)} = 1$$

Slope of tangent $= m_1 = 1$

Slope of normal $= m_2 = \frac{-1}{1} = -1$

lets take $P_2(\frac{9}{2}, -9)$,

$$\left.\frac{dy}{dx}\right|_{(\frac{9}{2},-9)} = \frac{9}{y}$$

$$\left.\frac{dy}{dx}\right|_{(\frac{9}{2},-9)} = \frac{9}{-9}$$

$$\left.\frac{dy}{dx}\right|_{(\frac{9}{2},-9)} = \frac{1}{-1}$$

$$\left.\frac{dy}{dx}\right|_{(\frac{9}{2},-9)} = -1$$

Slope of tangent $= m_3 = -1$

Slope of normal $= m_4 = \frac{1}{1} = 1$

Equation of tangent for $P_1(\frac{9}{2}, 9)$

$$y - y_1 = m_1(x - x_1)$$
$$y - 9 = 1(x - \frac{9}{2})$$
$$y - 9 = x - \frac{9}{2}$$
$$x - y - \frac{9}{2} + 9 = 0$$
$$x - y + \frac{9}{2} = 0$$
$$\boxed{2x - 2y + 9 = 0}$$

Equation of normal for $P_1(\frac{9}{2}, 9)$

$$y - y_1 = m_2(x - x_1)$$
$$y - 9 = -1(x - \frac{9}{2})$$
$$y - 9 = -x + \frac{9}{2}$$
$$x + y - 9 - \frac{9}{2} = 0$$
$$x + y - \frac{27}{2} = 0$$
$$\boxed{2x + 2y - 27 = 0}$$

Equation of tangent for $P_2(\frac{9}{2}, -9)$

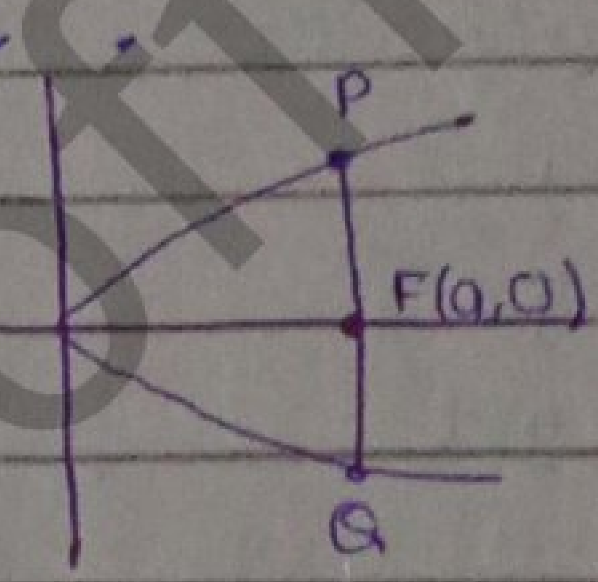
$$y - y_1 = m_3(x - x_1)$$
$$y - (-9) = -1(x - \frac{9}{2})$$
$$y + 9 = -x + \frac{9}{2}$$
$$x + y + 9 - \frac{9}{2} = 0$$
$$x + y + \frac{9}{2} = 0$$
$$\boxed{2x + 2y + 9 = 0}$$

Equation of normal $P_2(\frac{9}{2}, -9)$

$$y - y_1 = m_4(x - x_1)$$
$$y - (-9) = 1(x - \frac{9}{2})$$
$$y + 9 = x - \frac{9}{2}$$
$$x - y - 9 - \frac{9}{2} = 0$$
$$x - y - \frac{27}{2} = 0$$
$$\boxed{2x - 2y - 27 = 0}$$

Q8 Let $P(at_1^2, 2at_1)$ & $Q(at_2^2, 2at_2)$

- - - .



Finding the equation of ~~a~~ the cord:

$$y - y_1 = m(x - x_1) \quad \text{—} \quad (1)$$

So we need slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$P(at_1^2, 2at_1)$ & $Q(at_2^2, 2at_2)$

$$m = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}$$

$$m = \frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)}$$

$$m = \frac{2(t_2 - t_1)}{t_2^2 - t_1^2}$$

$$m = \frac{2\cancel{(t_2 - t_1)}}{\cancel{(t_2 - t_1)}(t_2 + t_1)}$$

$$m = \frac{2}{t_2 + t_1}$$

Putting values in eq (1),

$P(at_1^2, 2at_1)$

$y - 2at_1 = \frac{2}{t_2 - t_1}(x - at_1^2)$ -eq(i) ∴ u can use either point P or point Q, both r points of the same line so any will do.

this is the equation of chord.

if this line passes through focus $(a, 0)$ then it is called focal chord.

So putting $f(a,0)$ in eq(i)

$$0 - 2at_1 = \frac{2}{t_2 - t_1}(a - at_1^2)$$

$$(t_2 - t_1)(-2at_1) = 2(a - at_1^2)$$

$$-2at_1(t_2 - t_1) = 2a(1 - t_1^2)$$

$$-t_1(t_2 - t_1) = (1 - t_1^2)$$

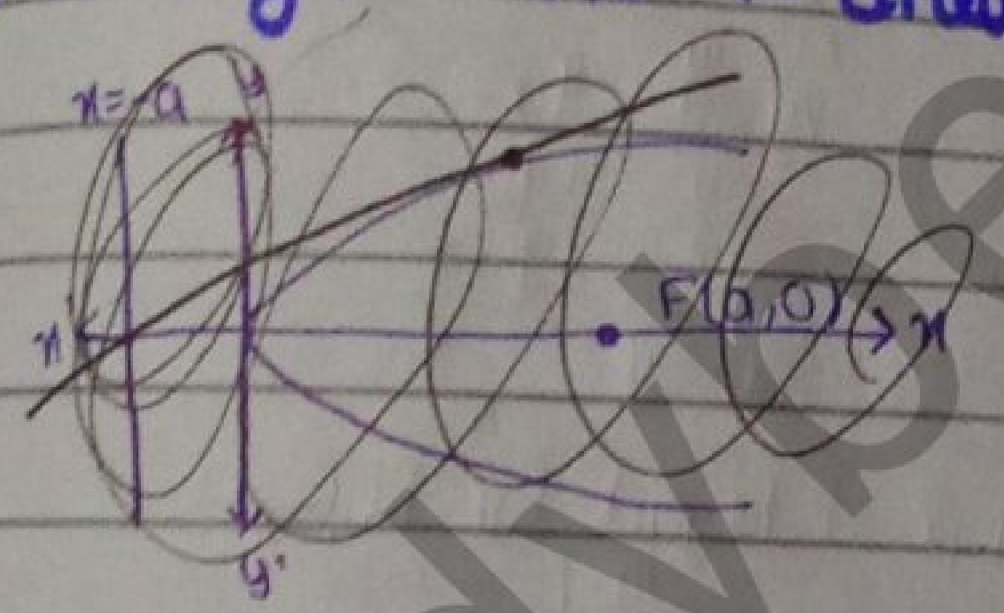
$$-t_1 t_2 - \cancel{t_1^2} = 1 - \cancel{t_1^2}$$

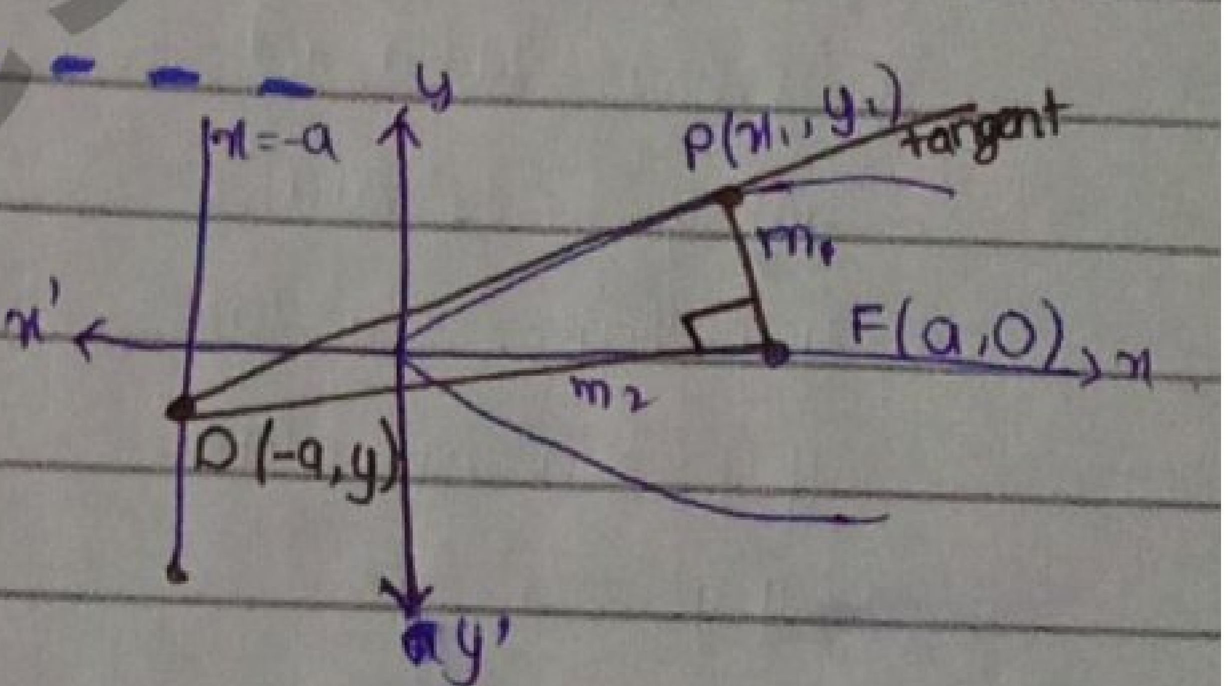
$$-t_1 t_2 = 1$$

$$t_1 t_2 = -1$$

Hence condition is satisfied, so the chord is focal chord.

Q A tangent line is drawn - - - - -





To prove that it is 90° we know that if the product of two slopes of two lines are equal to -1 then we can prove that the two lines are perpendicular. $m_1 m_2 = -1$ then $l_1 \perp l_2$

Finding slope m_1,

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{0 - y}{a - x_1}$$

$$\boxed{m_1 = \frac{-y}{a - x}}$$

Finding slope m_2,

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

We know $D(-a, y)$ so we need to find the value of y. For that we need to find the equation of tangent line,

Consider the equation of a parabola.

$$y^2 = 4ax$$

Diff w.r.t x,

$\frac{d}{dx}(y^2) = 4a\frac{d}{dx}(x)$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{4a}{2y}$

$\frac{dy}{dx} = \frac{2a}{y}$

$\left.\frac{dy}{dx}\right|_{(x_1,y_1)} = \frac{2a}{y_1}$

Slope of tangent $= m_T = \frac{2a}{y_1}$

Equation of tangent at $P(x_1,y_1)$,

$y-y_1 = m_T(x-x_1)$

$y-y_1 = \frac{2a}{y_1}(x-x_1)$

This is the equation of tangent

Putting $x=-a$ in the equation of tangent,

$y-y_1 = \frac{2a}{y_1}(x-x_1)$

$y-y_1 = \frac{2a}{y_1}(-a-x_1)$

$y_1(y-y_1) = 2a(-a-x_1)$

$y_1y-y_1^2 = -2a^2-2ax_1$

$y_1y-4ax_1 = -2a^2-2ax_1 \quad \because y_1^2 = 4ax_1$

$y_1y = -2a^2-2ax_1+4ax_1$

$y_1y = -2a^2-2ax_1$

$y = \frac{-2a^2+2ax_1}{y_1}$

$y = \frac{2ax_1-2a^2}{y_1}$

$y = \frac{2a(x_1-a)}{y_1}$

Hence $D\left(-a, \frac{2a(x_1-a)}{y_1}\right)$

Now finding the slope of DF.

$m_2 = \frac{y_2-y_1}{x_2-x_1}$

$m_2 = \frac{\left(\frac{2a(x_1-a)}{y_1}\right)-0}{-a-a}$

$m_2 = \frac{2a(x_1-a)}{y_1} \times \frac{1}{-2a}$

$m_2 = \frac{x_1-a}{-y_1}$

$\boxed{m_2 = \frac{a-x_1}{y_1}}$

Now seeing if it satisfies the condition $m_1m_2=-1$

$= m_1m_2$

$= \frac{-y_1}{a-x_1}\left(\frac{a-x_1}{y_1}\right)$

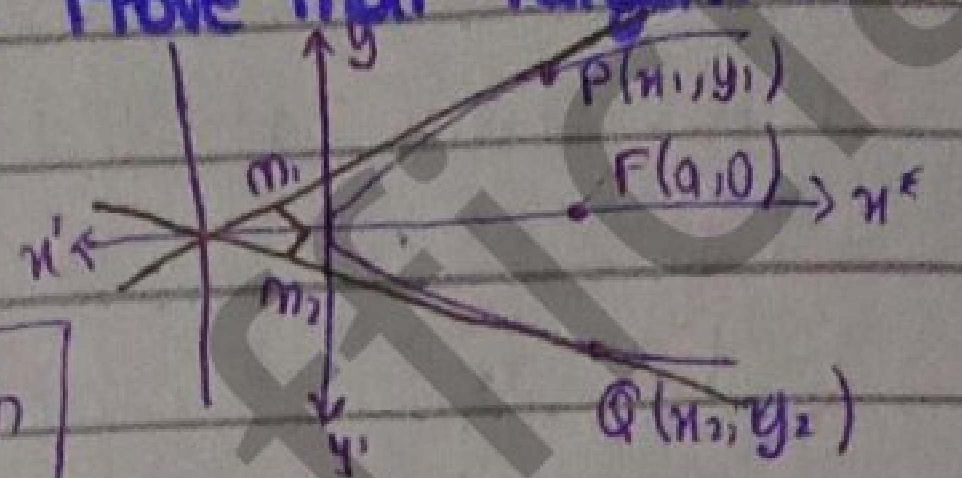
$= -1$

Hence $m_1m_2=-1$

So $D\perp F$

Hence proved the angle is 90°

Q10 Prove that tangent - - - -



$y^2=4ax$

Diff w.r.t x,

$\frac{d}{dx}(y^2) = 4a\frac{d}{dx}(x)$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{4a}{2y}$

$\frac{dy}{dx} = \frac{2a}{y}$

slope of tangent for $P(x_1, y_1)$

$\left.\frac{dy}{dx}\right|_{(x_1,y_1)} = \frac{2a}{y}$

$\left.\frac{dy}{dx}\right|_{(x_1,y_1)} = \frac{2a}{y_1}$

So Slope $= m_1 = \frac{2a}{y_1}$

Slope of tangent for $Q(x_2,y_2)$.

$\left.\frac{dy}{dx}\right|_{(x_2,y_2)} = \frac{2a}{y_2}$

So Slope $= m_2 = \frac{2a}{y_2}$

So, $m_1 . m_2 = -1$

$\left(\frac{2a}{y_1}\right)\left(\frac{2a}{y_2}\right) = -1$

$\frac{4a^2}{y_1y_2} = -1$ — eq (A)

Finding equation of focal cord.

$y-y_1 = m(x-x_1)$

Considering point $P(x_1, y_1)$.

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$ equation of chord

This equation pass through focus so that satisfy the points of focus.

eq $x=a, y=0$,

$0-y_1 = \frac{y_2-y_1}{x_2-x_1}(a-x_1)$ ← ★

$-y_1 = \frac{y_2-y_1}{x_2-x_1}(a-x_1)$

equation of focal card

$-y_1(x_2-x_1) = (y_2-y_1)(a-x_1)$

$-x_2y_1 + \cancel{x_1y_1} = ay_2 - x_1y_2 - ay_1 + \cancel{x_1y_1}$

$-x_2y_1 = ay_2 - x_1y_2 - ay_1$ --eq(i)

from equation of parabola,

$y^2 = 4ax$

$P(x_1,y_1) \Rightarrow y_1^2 = 4ax_1 \Rightarrow x_1 = \frac{y_1^2}{4a}$

$Q(x_2,y_2) \Rightarrow y_2^2 = 4ax_2 \Rightarrow x_2 = \frac{y_2^2}{4a}$

Putting values of x_1 & x_2 in eq(i),

$-\left(\frac{y_2^2}{4a}\right)y_1 = ay_2 - \left(\frac{y_1^2}{4a}\right)y_2 - ay_1$

$-\frac{y_2^2}{4a}y_1 = ay_2 - ay_1 - \frac{y_1^2}{4a}y_2$

$-\frac{y_2^2}{4a}y_1 + \frac{y_1^2}{4a}y_2 = a(y_2-y_1) \Rightarrow \frac{y_1y_2}{4a}\cancel{(-y_2+y_1)} = -a\cancel{(-y_2+y_1)}$

$\cancel{\frac{y_1y_2}{4a}(-y_2+y_1) = a(y_2-y_1)}$

$\frac{y_1y_2}{4a} = -a$

$y_1y_2 = -4a^2$

Putting in eq ★

$\frac{4a^2}{y_1y_2} = -1$

$\frac{4a^2}{-4a^2} = -1$

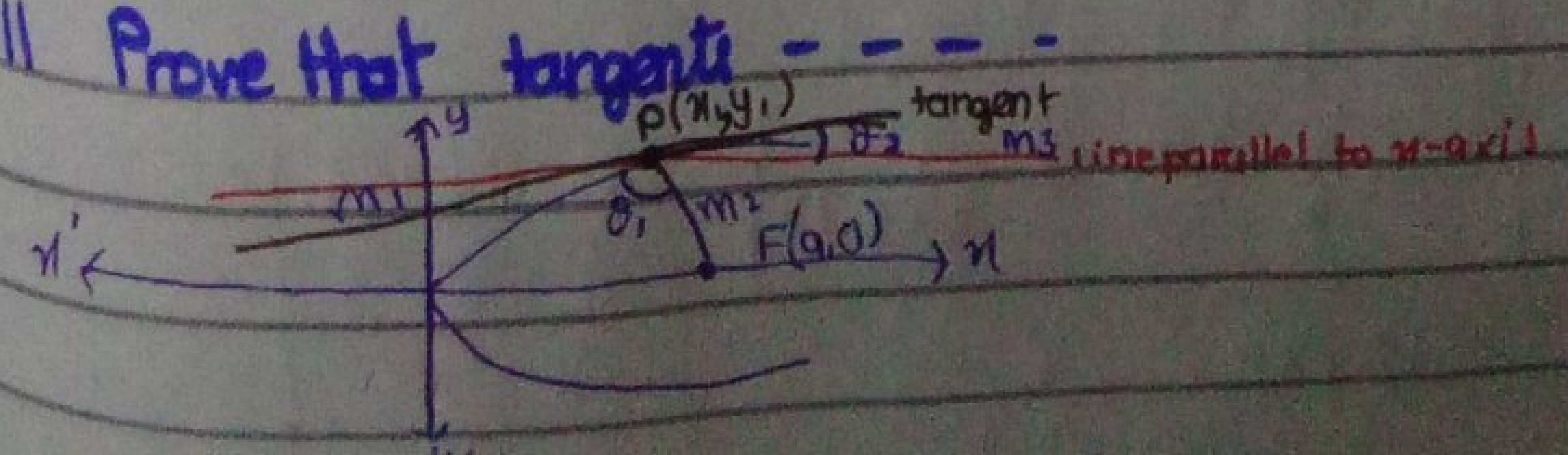
$-1 = -1$

Hence $m_1m_2 = -1$

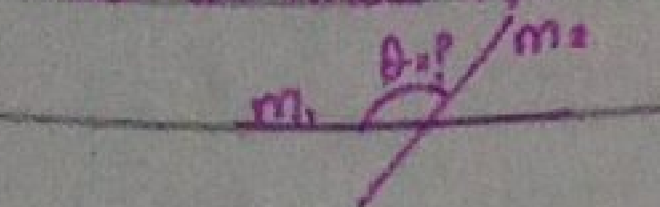
Proved !

Q11 Prove that tangents - - - -

we need to show that $\theta_1 = \theta_2$



Now we know if,



$\tan\theta = \left(\frac{m_1-m_2}{1+m_1m_2}\right)$

Consider equation of parabola,

$y^2 = 4ax$

Diff w.r.t x,

$\frac{d}{dx}(y^2) = 4a\frac{d}{dx}(x)$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{4a}{2y}$

$\frac{dy}{dx} = \frac{2a}{y}$

at point $P(x_1, y_1)$,

$\left.\frac{dy}{dx}\right|_{(x_1,y_1)} = \frac{2a}{y}$

$\left.\frac{dy}{dx}\right|_{(x_1,y_1)} = \frac{2a}{y_1}$

So slope $= \boxed{m_1 = \frac{2a}{y_1}}$

Slope of PF $= m_2 = \frac{y_2-y_1}{x_2-x_1}$

$m_2 = \frac{0-y_1}{a-x_1}$

$\boxed{m_2 = \frac{-y}{a-x_1}}$

$\cancel{m_2 = \frac{y}{x_1-a}}$

Slope of m_3 which is a horizontal line so,

$m_3 = \frac{0}{x}$

$\boxed{m_3 = 0}$

So now,

$\tan\theta_1 = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\tan\theta_1 = \left|\frac{\frac{2a}{y_1}+\frac{y_1}{a-x_1}}{1+\left(\frac{2a}{y_1}\right)\left(\frac{y_1}{a-x_1}\right)}\right|$

$\tan\theta_1 = \left|\frac{2a^2-2ax_1+y_1^2}{y_1(a-x_1)} \div \left(1-\frac{2a}{a-x_1}\right)\right|$

$\tan\theta_1 = \left|\frac{2a^2-2ax_1+y_1^2}{y_1(\cancel{a-x_1})} \div \left(\frac{a-x_1-2a}{\cancel{a-x_1}}\right)\right|$

$\tan\theta_1 = \left|\frac{2a^2-2ax_1+y_1^2}{y_1} \times \frac{1}{a-x_1-2a}\right|$

$\tan\theta_1 = \left|\frac{2a^2-2ax_1+4ax_1}{y_1} \times \frac{1}{-x_1+a-2a}\right| \quad \because y_1^2 = 4ax_1$

$\tan\theta_1 = \left|\frac{2a^2+2ax_1}{y_1} \times \frac{1}{-x-a}\right|$

$\tan\theta_1 = \left|\frac{2a^2+2ax_1}{y_1} \times \frac{1}{-x-a}\right|$

$\tan\theta_1 = \left|\frac{2a^2+2ax_1}{-y_1(x+a)}\right| \Rightarrow \tan\theta_1 = \frac{2a^2+2ax_1}{y_1(x+a)}$ — eq(i)

Now for θ_2

$\tan\theta_2 = \left|\frac{m_1-m_3}{1+m_1m_3}\right|$

$\tan\theta_2 = \left|\frac{\frac{2a}{y_1}-0}{1+\left(\frac{2a}{y_1}\right)(0)}\right|$

$\tan\theta_2 = \left|\frac{\frac{2a}{y_1}}{1+0}\right|$

$\tan\theta_2 = \frac{2a}{y_1}$ — eq(ii)

from eq(i),

$\tan\theta_1 = \frac{2a(\cancel{a+x_1})}{y_1(\cancel{x+a})}$

$\tan\theta_1 = \frac{2a}{y_1}$ — eq(iii)

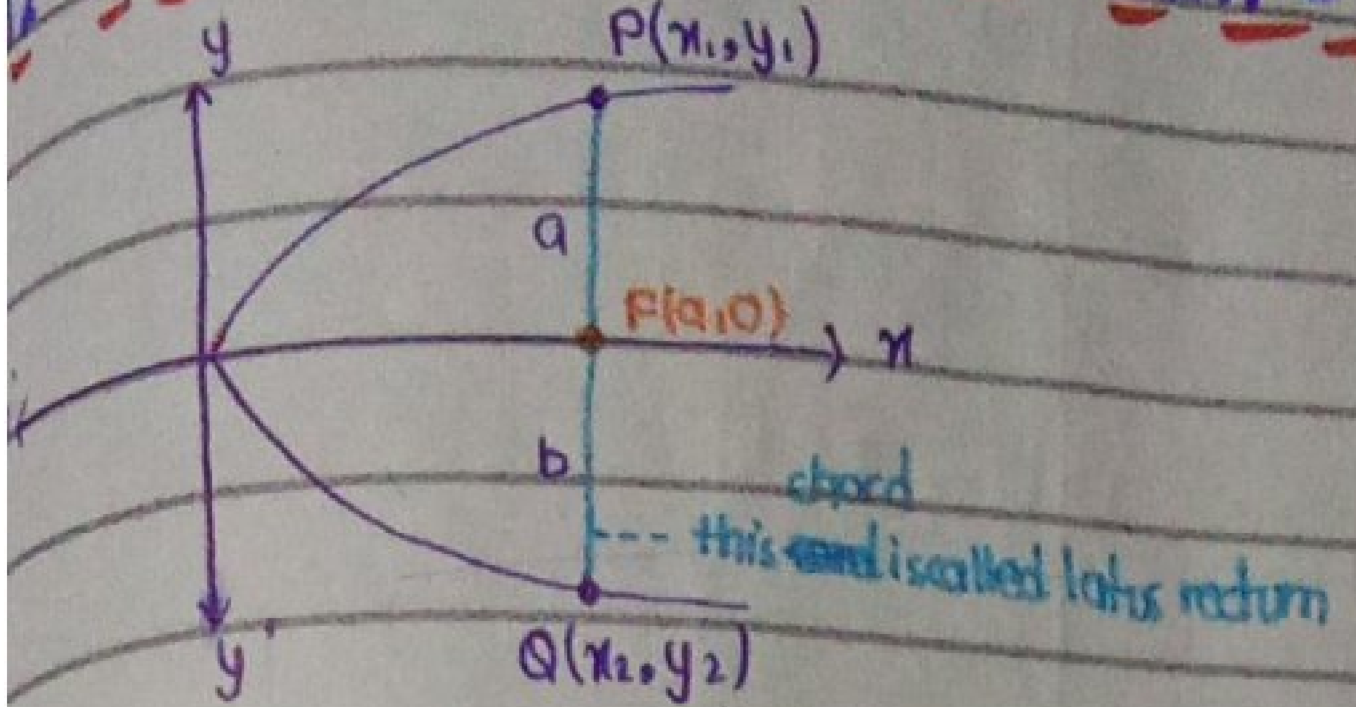
from eq(ii) & eq(iii),

$\tan\theta_1 = \tan\theta_2$

$\theta_1 = \theta_1$

Hence Proved.

Q12 Prove that the semi latus rectum - - -



Equation of the parabola, $y^2 = 4ax$

Latus rectum = 4a

So Semi Latus Rectum = $\frac{4a}{2}$

Semi Latus Rectum = 2a

Harmonic Mean = $\frac{2ab}{a+b}$ —eq(☆)

So, finding value of a,

$PF = a = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$a = \sqrt{(0-y_1)^2+(a-x_1)^2}$

$a = \sqrt{(-y_1)^2+[(a)^2-2(a)(x_1)+(x_1)^2]}$

$a = \sqrt{y_1^2+a^2-2ax_1+x_1^2}$

$a = \sqrt{4ax_1-2ax_1+a^2+x_1^2}$ $\because y_1^2 = 4ax_1$ —eq(a)

$a = \sqrt{2ax_1+a^2+x_1^2}$

$a = \sqrt{(a+x_1)^2}$

$\boxed{a = a+x_1}$

finding value of b,

$FQ = b = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$b = \sqrt{(y_2-0)^2+(x_2-a)^2}$

$b = \sqrt{(y_2)^2+[(x_2)^2-2(x_2)(a)+(a)^2]}$

$b = \sqrt{y_2^2+x_2^2-2x_2a+a^2}$

$b = \sqrt{4ax_2-2ax_2+x_2^2+a^2}$ $\because y_2^2 = 4ax_2$ —eq(b)

$b = \sqrt{2ax_2+x_2^2+a^2}$

$b = \sqrt{(x_2+a)^2}$

$\boxed{b = x_2+a}$

Now putting the values in eq(☆),

$H.M = \frac{2(a+x_1)(a+x_2)}{(a+x_1)+(a+x_2)}$

$H.M = \frac{2(a^2+ax_2+ax_1+x_1x_2)}{a+x_1+a+x_2}$

$H.M = \frac{2(a^2+ax_2+ax_1+x_1x_2)}{2a+x_1+x_2}$ —eq(i)

As we have proved in Q10

$y_1y_2 = -4a^2$

~~$y_1^2y_2^2 = 16a^2x_1x_2$~~

$y_1^2 \cdot y_2^2 = (4ax_1)(4ax_2)$

$y_1^2y_2^2 = 16a^2x_1x_2$

from eq(a) & eq(b)

$(-4a^2)^2 = 16a^2x_1x_2$

$16a^4 = 16a^2x_1x_2$

$\frac{\cancel{16}a^{\cancel{4}2}}{\cancel{16a^2}} = x_1x_2$

$a^2 = x_1x_2$

$x_1x_2 = a^2$

Putting in eq(i),

$H.M = \frac{2(a^2+ax_2+ax_1+a^2)}{2a+x_1+x_2}$

$H.M = \frac{2(2a^2+ax_2+ax_1)}{2a+x_1+x_2}$

$H.M = \frac{2a\cancel{(2a+x_1+x_2)}}{\cancel{2a+x_1+x_2}}$

$H.M = 2a$ = Semi Latus Rectum

Hence Proved.