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Where Minds Pollinate

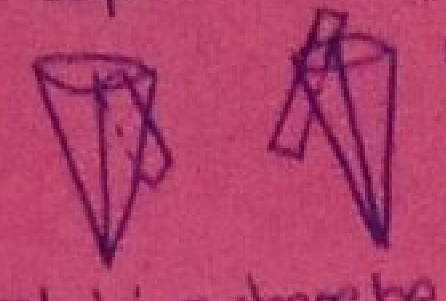
Class 12
Mathematics

Chapter 7
Exercise 7.4
Handwritten Solution

Ex7.4

PARABOLA

can be parallel to any generator

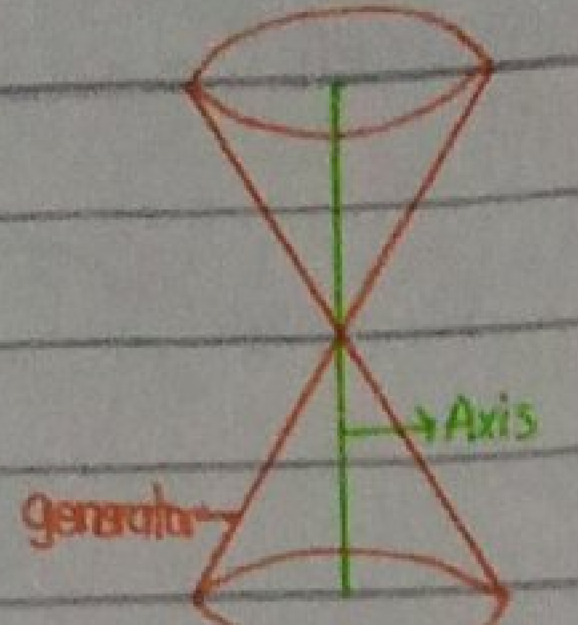


both are correct. make sure it isn't passing through the vertex

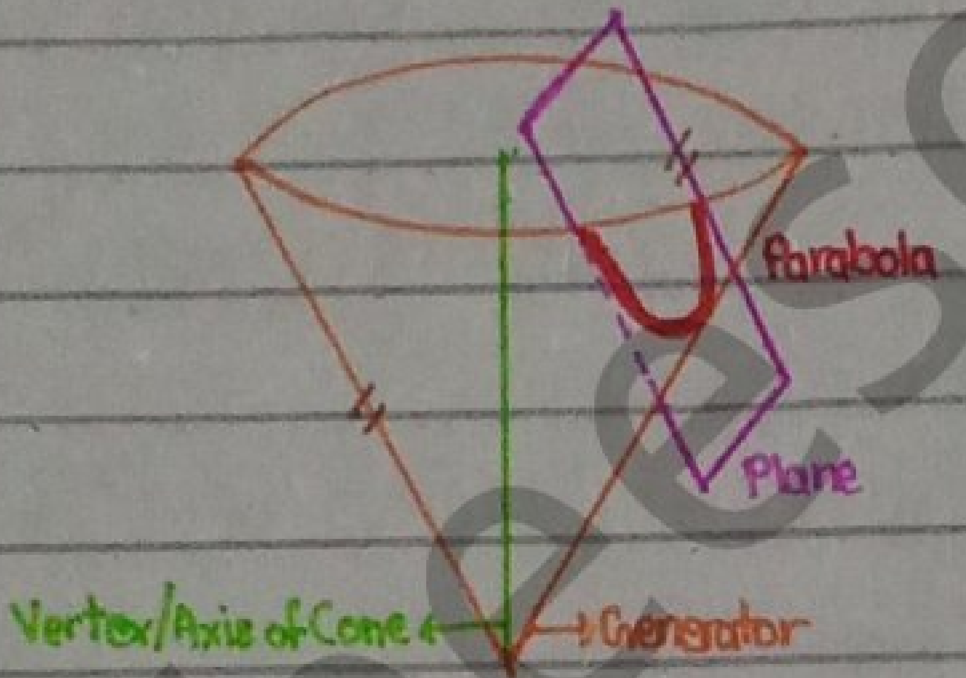
Parabola's shape be |U|∩|C|⊃|

doesn't matter it can even be made on the bottom half of the cone

A purpose of a parabola, is that it concentrates all the light to fall at one point.



Only consider the top half,



• Parallel

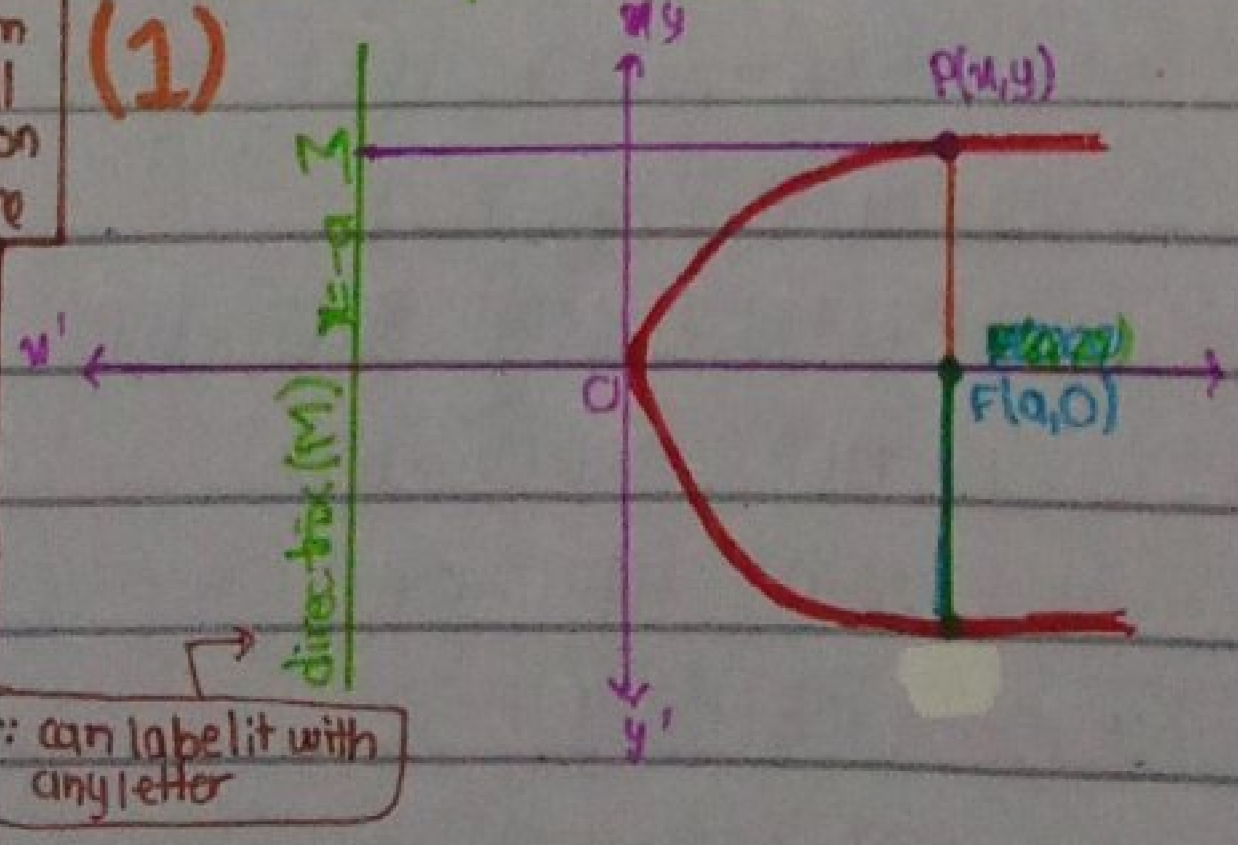
Defination of Parabola geometrically:

When we cut a cone with plane, such that plane is parallel to any generator and not passing through the vertex, then the intersection is a Parabola.

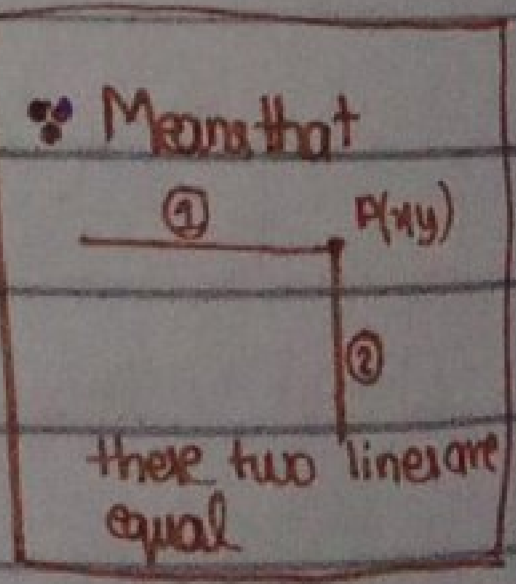
Parabola has 4 shapes so 4 equations, the shapes are U, ∩, C, ⊃. We only need to understand one shape C, and the rest will become easy to understand.

∵ how $x=-a$, bcs from the graph we can tell that the directrix is on the II & III-Q, and the value of x is negative so $x=-a$. But what about y. The value of y is 0 on x-axis so the final line equation become $x=-a$.

(1)



∵ can label it with any letter



Defination of Parabola mathematically:

Points on the parabola are equidistant from the focus and the directrix, thus eccentricity of the parabola is 1.

Eccentricity means that the point (any point on the figure) its distance from focal point and directrix, their ratio is 1.

Eccentricity of different shapes:

1. Eccentricity of a circle is 0
2. Eccentricity of parabola is 1
3. Eccentricity of Ellipse is less than 1
4. Eccentricity of Hyperbola is greater than 1

Standard Equation of Parabola (by Definition):

$|PF| = |PM|$ ∵ The distance from point P to the focal point A is the same as the distance from point P to the directrixM.

$\frac{|PF|}{|PM|} = 1$ Hence eccentricity is 1

The formula for finding the distance from a point to a line.

$$d = \frac{|ax_0 + by_0 + C|}{\sqrt{(a)^2 + (b)^2}}$$ if point is (x_0, y_0)

line is $ax+by+C=0$

line equation of directrix is,

$x = -a$

$x + a = 0$

Distance for |PM|:

$x + 0y + a = 0$, we know the point $P(x,y)$

$$d = |PM| = \frac{|1(x) + 0(y) + a|}{\sqrt{(1)^2 + (0)^2}}$$

$$|PM| = \frac{|x+a|}{\sqrt{1}}$$

$$|PM| = \frac{x+a}{1}$$

$|PM| = x + a$ — eq(i)

Distance for |PF|:

Point $F(a,0)$, Point $P(x,y)$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|PF| = \sqrt{(x-a)^2+(y-0)^2}$ --eq(i)

Now by the definition of parabola $|PF| = |PM|$

So from eq(i) and eq(ii),

$x+a = \sqrt{(x-a)^2+(y)^2}$

$(x+a)^2 = \left(\sqrt{x^2-2ax+a^2+y^2}\right)^2$

$x^2+2ax+\cancel{a^2} = x^2+a^2-2ax+y^2$

$2ax = -2ax+y^2$

$6 = y^2-2ax-2ax$

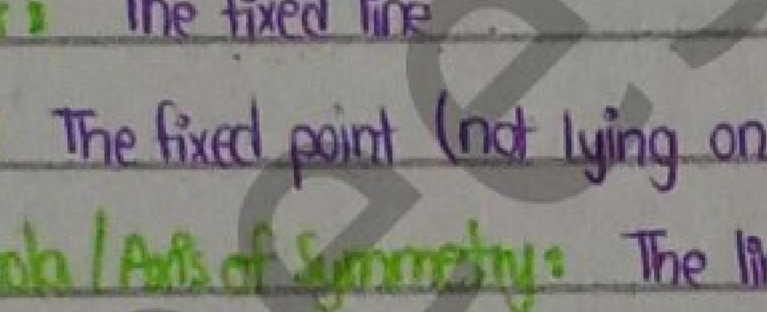
$0 = y^2-4ax$

$\boxed{y^2 = 4ax}$ Standard equation of Parabola

Elements of Parabola:

- Directrix: The fixed line
- Focus: The fixed point (not lying on the directrix)
- Axis of Parabola / Axis of Symmetry: The line that passes through the focus and is perpendicular to the directrix.
- Vertex: The point where the parabola cuts its axis.
- Chord: A line segment with its end points on the parabola (doesn't pass through the origin)
- Focal Chord: A chord of parabola which passes through the focus.
- Focal Distance: The distance of any point of the parabola from the focus.
- Latus Rectum: A focal chord of the parabola which is parallel to the directriex. And its length is 4a. ∵ 4a just for this case

* Proof of how the length of the Latus Rectum is 4a:



we know that total length of the directrix is l, so for the top half it will be $\frac{l}{2}$ (positive bcs y is positive) and for bottom half will be $-\frac{l}{2}$ (negative bcs y is negative).

Now as points A & B lie on the parabola so it means that they both will satisfy the standard equation of a parabola.

For $A(a, \frac{l}{2})$;

$y^2 = 4ax$

$(\frac{l}{2})^2 = 4a(a)$

$\frac{l^2}{4} = 4a^2$

$l^2 = 16a^2$

$\sqrt{l^2} = \sqrt{16a^2}$

$l = \pm 4a$

$l = 4a$

neglect the minus bcs length can never be negative

For $B(a, \frac{-l}{2})$

$y^2 = 4ax$

$(\frac{-l}{2})^2 = 4a(a)$

$\frac{l^2}{4} = 4a^2$

$l^2 = 16a^2$

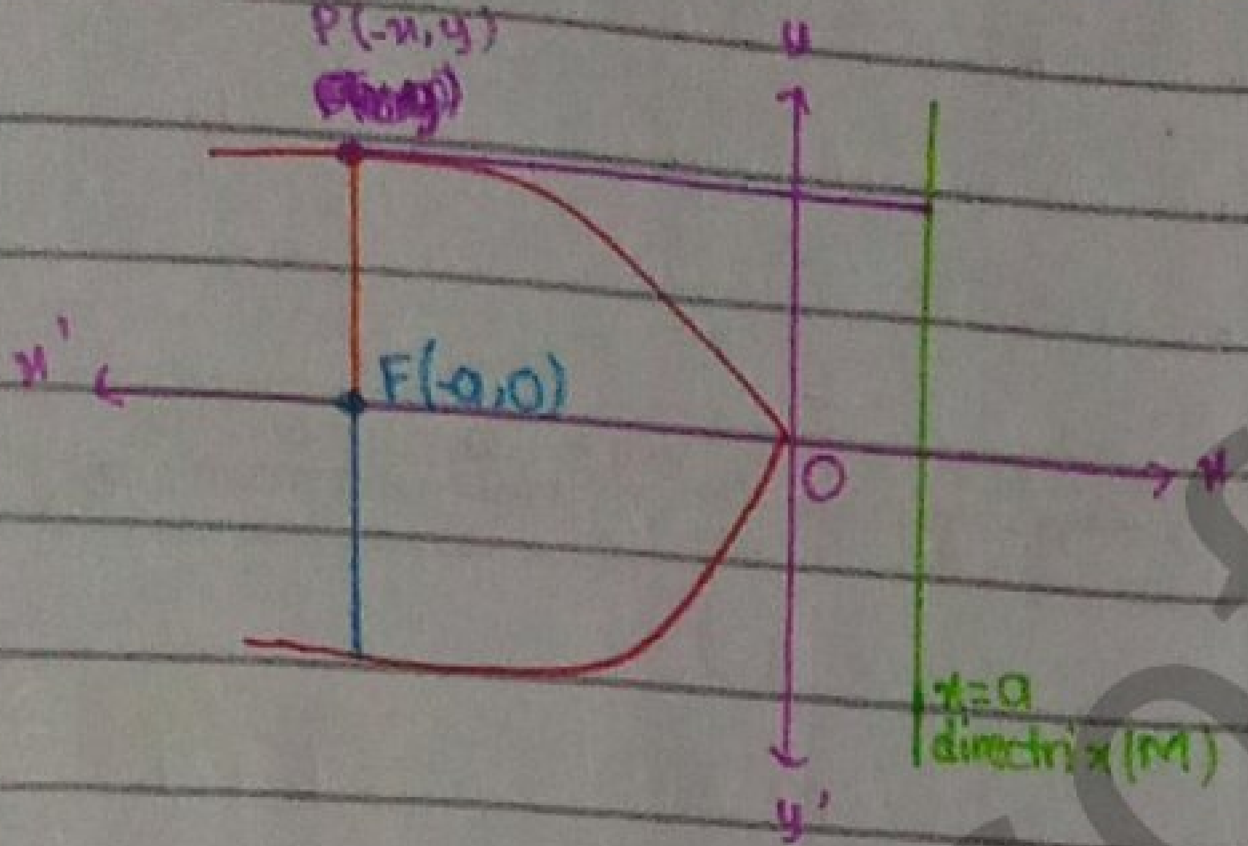
$\sqrt{l^2} = \sqrt{16a^2}$

$l = \pm 4a$

$l = 4a$

Standard Equation of Parabola for the rest 3 shapes:

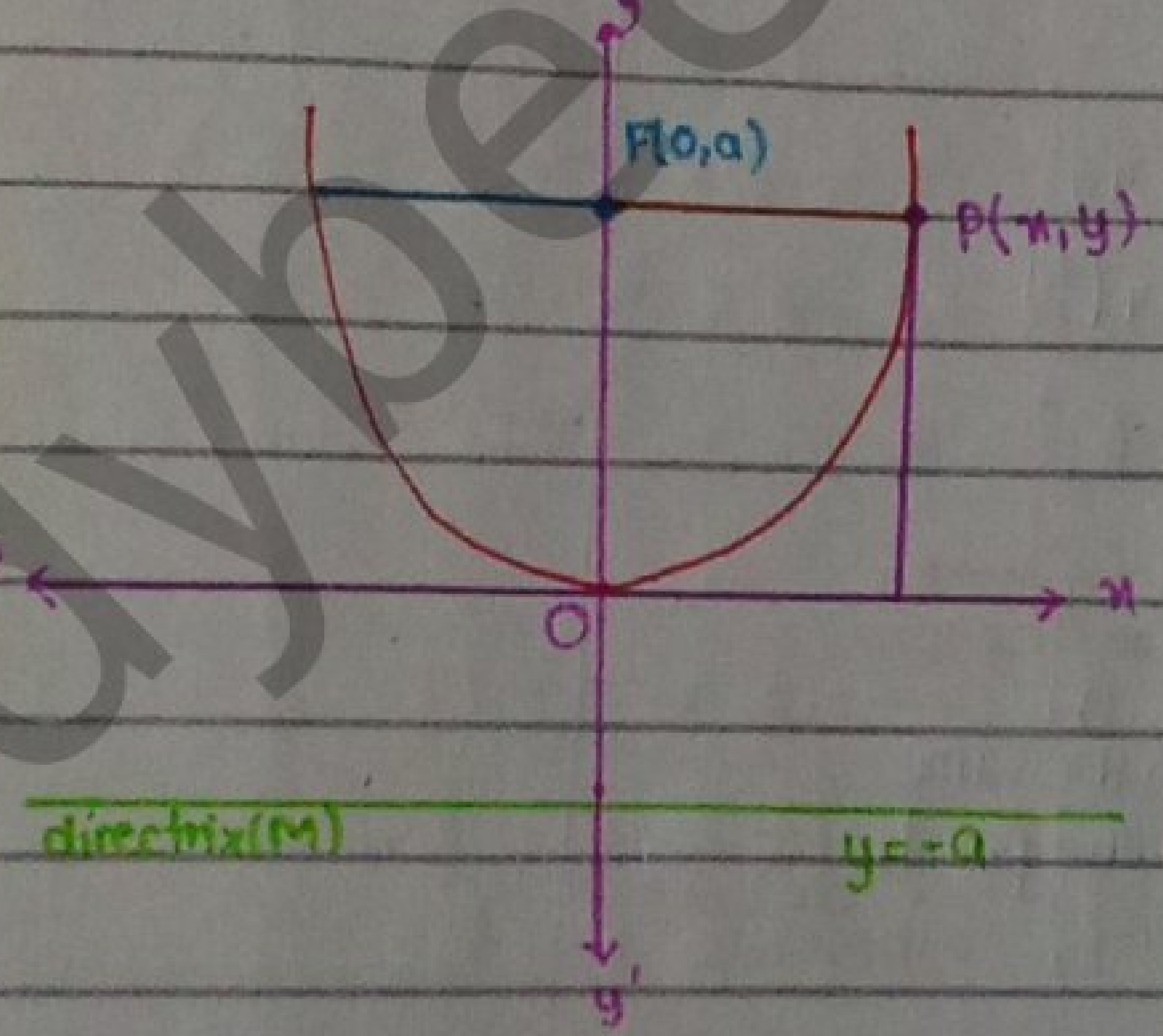
(2)



IMP point:
How do you differentiate between a parabola equation and a circle equation?
The equation of a parabola has only one square term. Where as, the equation of a parabola has more than one squared term.

$$\boxed{y^2 = -4ax}$$

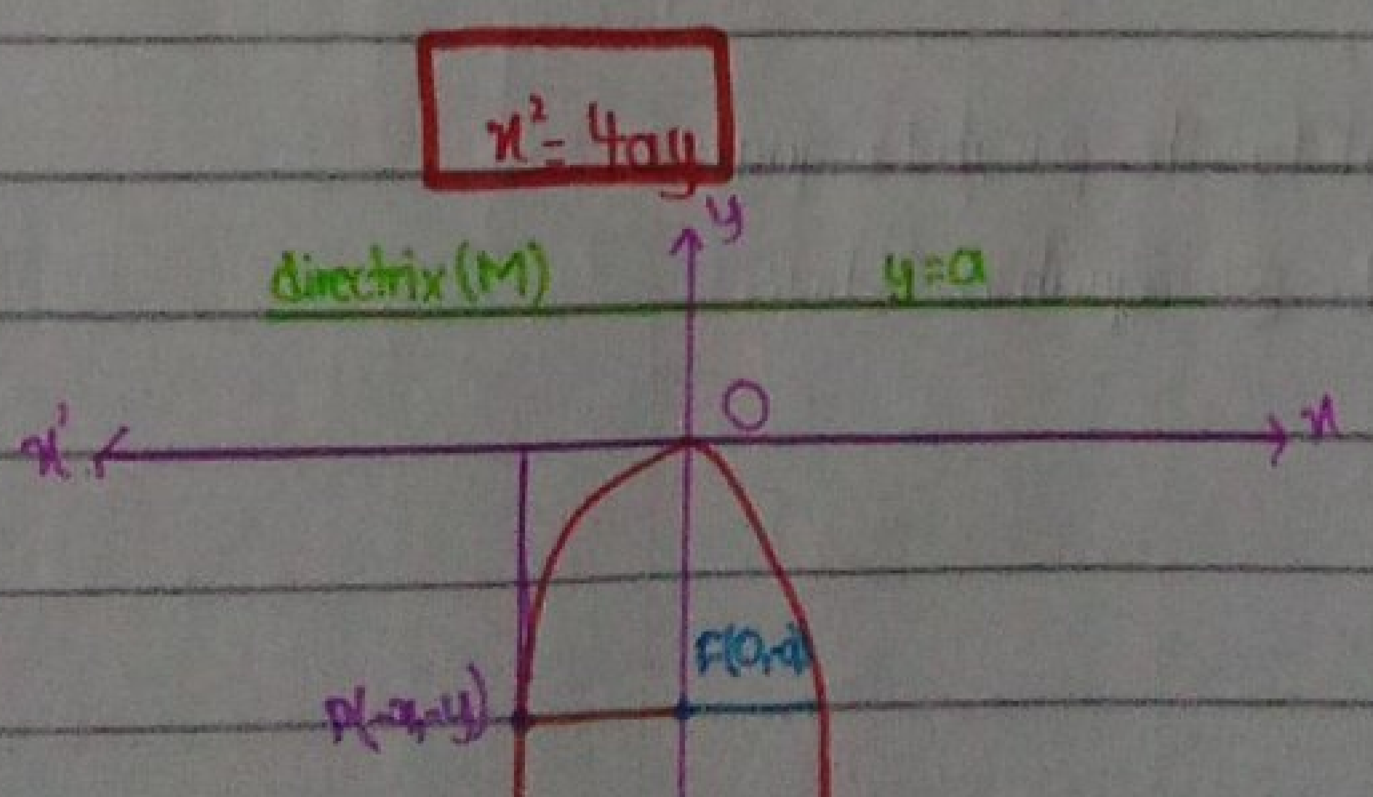
(3)



you can take the value of P anywhere but the value of f and the directrix has to stay where they are.

$$\boxed{x^2 = 4ay}$$

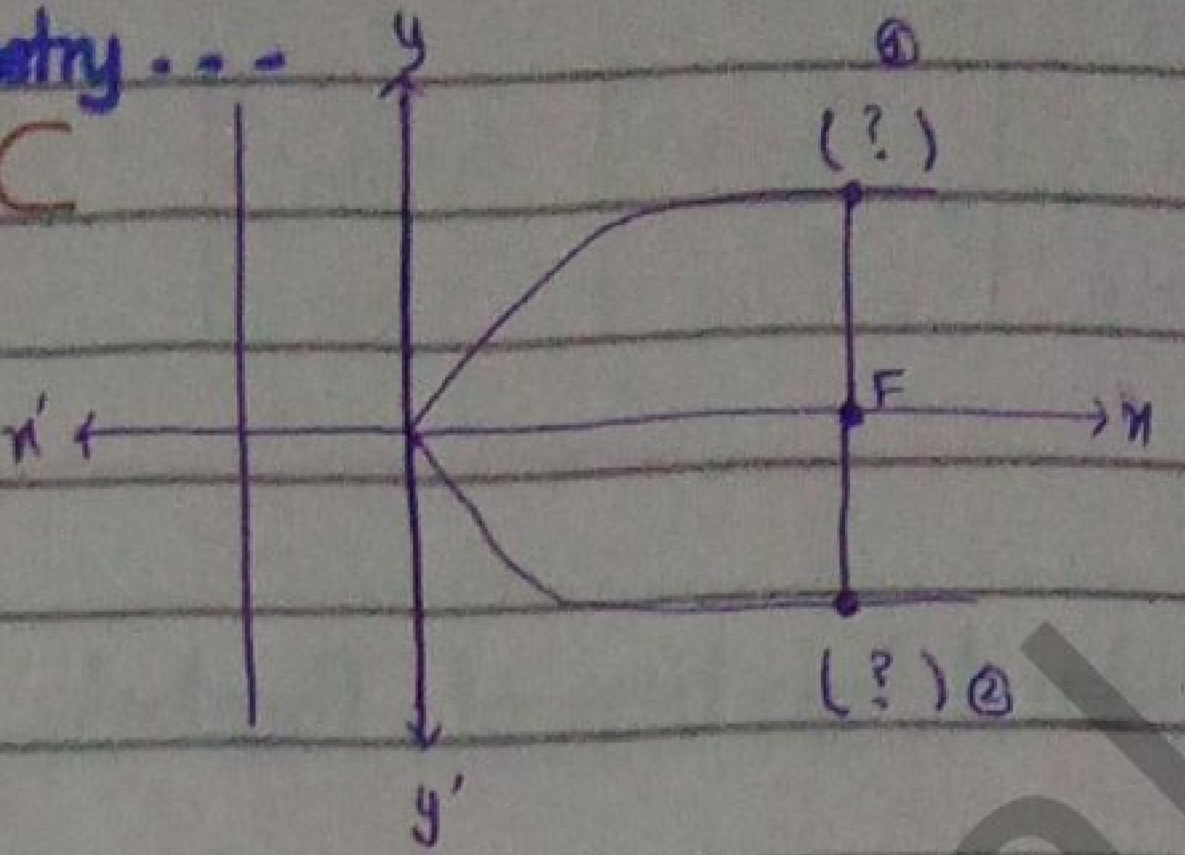
(4)



$$\boxed{x^2 = -4ay}$$

Q1 Find Focus, Vertex, Axis of Symmetry . . .

(i) $y^2 = 6x$ matches with $y^2 = 4ax$ C



Compare

$y^2 = 6x$ and $y^2 = 4ax$

$4a = 6$

$a = \frac{6}{4}$

$a = \frac{3}{2}$

(1) $F(a,0)$

$\boxed{F = (\frac{3}{2}, 0)}$

(2) $\boxed{\text{Vertex is } 0}$

bcs, like remember for circle we did $(x-h)^2 + (y-k)^2$ and we will get the vertices (h,k). But here in this question, nothing is being subtracted for y and x, so vertex is 0

$\boxed{V = (0,0)}$

(3) Axis of symmetry = x-axis

$\Rightarrow \boxed{y = 0}$

(4) $x = -a$

$\boxed{x = \frac{-3}{2}}$ or $\boxed{2x+3=0}$

(5) Length of latus rectum $= 4a$

$= 4(\frac{3}{2})$

$\boxed{= 6}$

(6) For end points of the latus rectum we have to find ① & ② on the graph

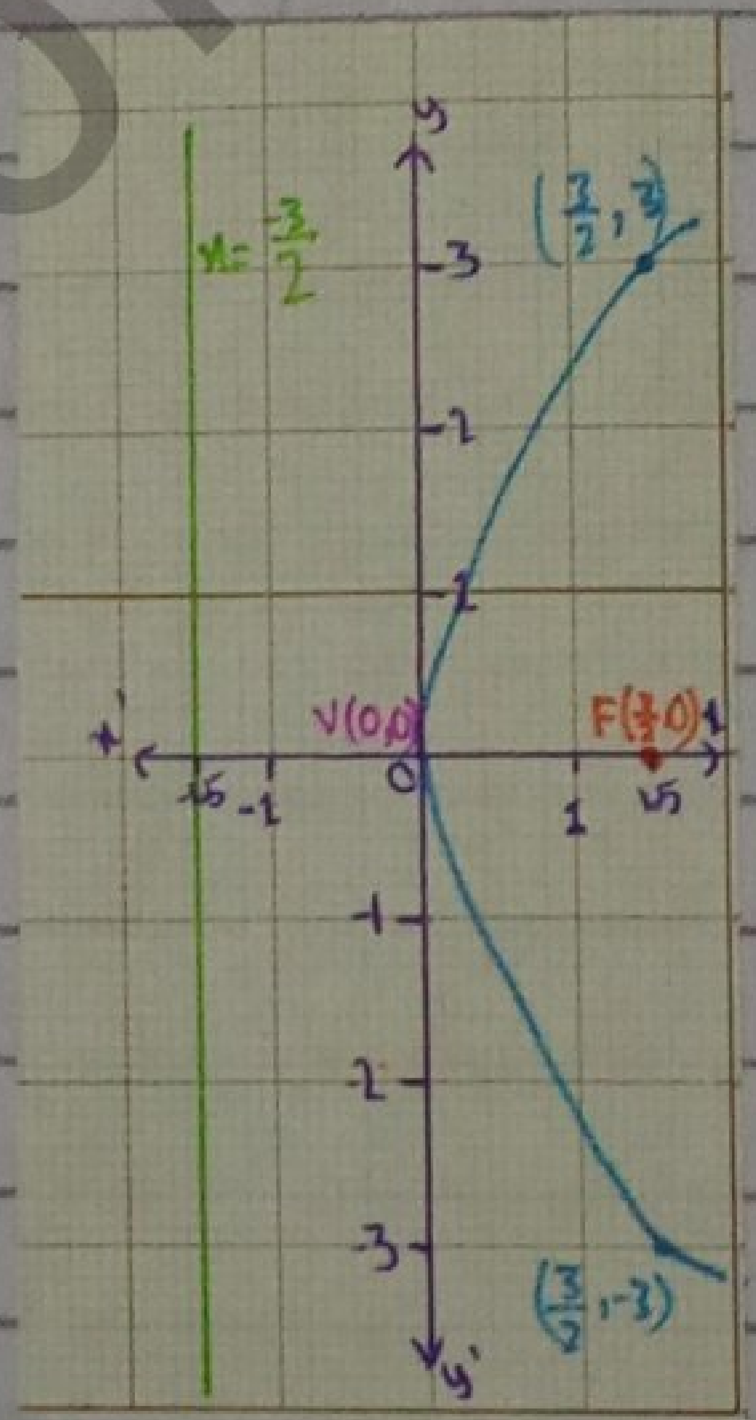
Now as we know $F = (\frac{3}{2}, 0)$

the value of x will remain the same

So for the two unknown end points

$(\frac{3}{2}, ?)$ & $(\frac{-3}{2}, -?)$ "–" bcs below ∵ the x-axis

(7)



Finding the value of y,

Since the points satisfy the parabola equation, so,

Put $x = \frac{3}{2}$ in the equation below,

$y^2 = 6x$

$y^2 = 6(\frac{3}{2})$

$\sqrt{y^2} = \sqrt{9}$

$y = 3$

So now we know the points

$\boxed{(\frac{3}{2}, 3) \ \& \ (\frac{3}{2}, -3)}$

Here these are the end point.

(ii) $y^2 = \frac{-3}{2}x$ — eq(i) ∵ $y^2 = -4ax$

Compare

$y^2 = -\frac{3}{2}x$, $y^2 = -4ax$

$\frac{-3}{2} = 4a$

$-\frac{3}{2 \times 4} = a$

$a = \frac{3}{8}$

(1) Focus:

$F = (-a, 0)$

$\boxed{F = (-\frac{3}{8}, 0)}$

(2) Vertex:

Vertex = $V(0,0)$

(3) Axis of Symmetry:

Axis of Symmetry = x-axis

$\boxed{\Rightarrow y = 0}$

(4) Directrix:

$x = +a$

$\boxed{x = \frac{3}{8}}$

(5) Length of latus rectum:

$l = 4a$

$l = 4\left(\frac{3}{8_2}\right)$

$\boxed{l = \frac{3}{2}}$

(6) End points of latus rectum:

The value of x will remain the same as in Focus so,

$(-\frac{3}{8}, y)$

Put $x = \frac{-3}{8}$ in eq(i),

$y^2 = \frac{-3}{2}x$

$y^2 = \frac{-3}{2}\left(\frac{-3}{8}\right)$

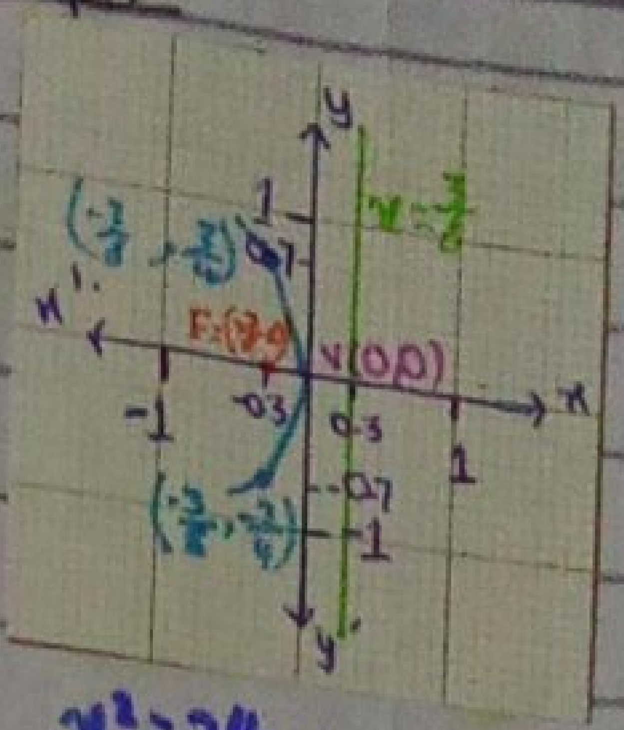
$y^2 = \frac{9}{16}$

$\sqrt{y^2} = \pm\sqrt{\frac{9}{4}}$

$y = \pm\frac{3}{4}$

Hence $(-\frac{3}{8}, \frac{3}{4})$ & $(\frac{-3}{8}, \frac{-3}{4})$ or $(-\frac{3}{8}, \pm\frac{3}{4})$

(7) Graph:



(iii) $x^2 = 24y$ — eq(i) ∵ $x^2 = 4ay$

Compare,

$x^2 = 24y$, $x^2 = 4ay$

$24 = 4a$

$a = 24/4$

$a = 6$

(1) Focus:

$F(0, a)$

$F = (0, 6)$

(2) Vertex:

Vertex = $V = (0,0)$

(3) Axis of Symmetry:

Axis of Symmetry = y-axis

$x = 0$

(4) Directrix:

$y = -a$

$y = -6$

(5) Length of latus rectum:

$l = 4a$

$l = 4(6)$

$l = 24$

(6) End points of latus rectum:

From focus $F = (0, 6)$

Putting $y = 6$ in eq(i),

$x^2 = 24y$

$x^2 = 24(6)$

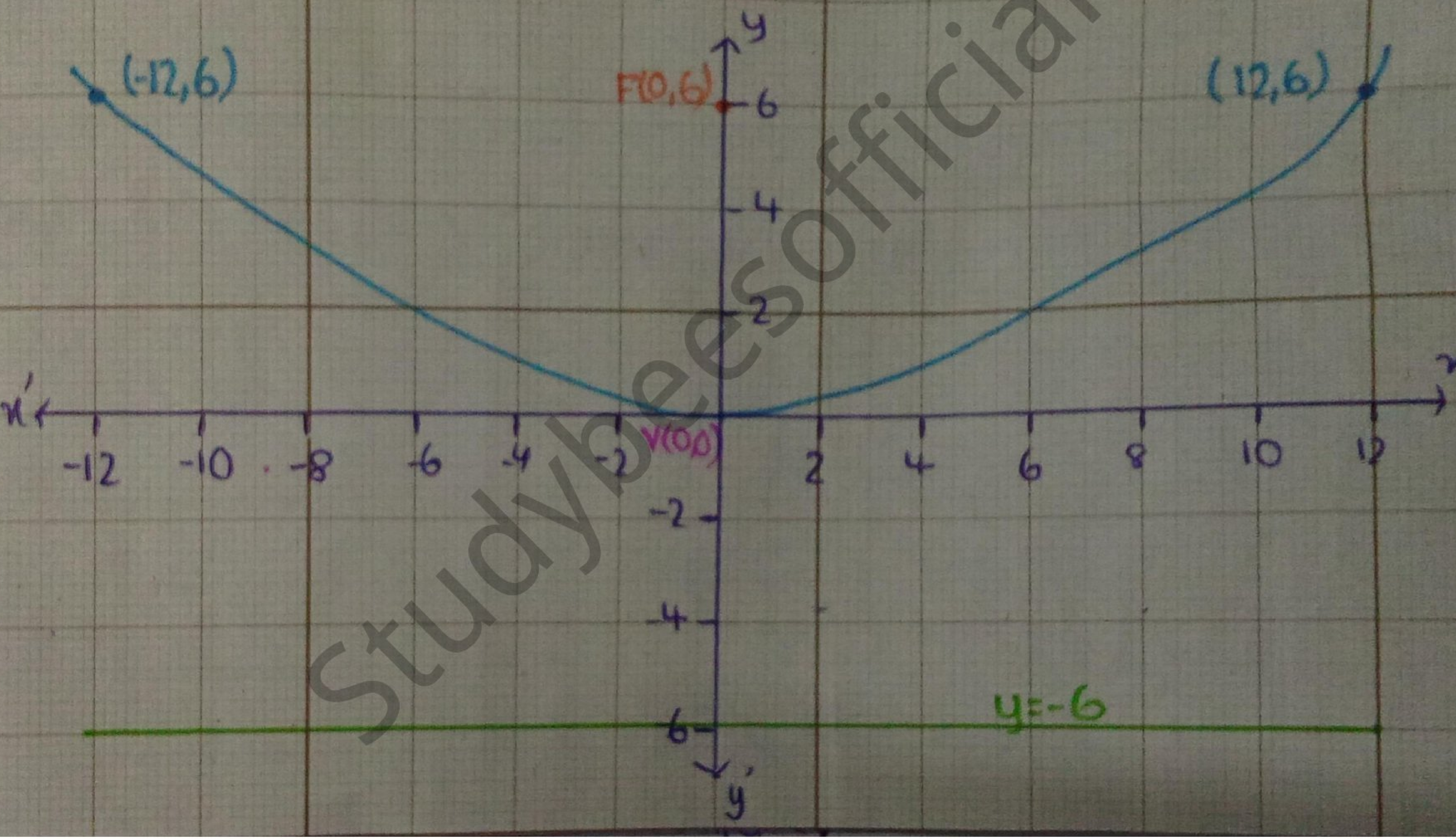
$\sqrt{x^2} = \sqrt{144}$

$x = \pm 12$

So, $(12, 6)$ and $(-12, 6)$ or $(\pm 12, 6)$

(7) Graph:

Q1(iii)



(iv) $x^2 = -5y$ $-eq(i)$ $\because x^2 = 4ay$

Compare.

$x^2 = -5y$, $x^2 = -4ay$

$-5 = -4a$

$a = \frac{5}{4}$

(1) Focus:

$F = (0, -a)$

$F = (0, \frac{-5}{4})$

(2) Vertex:

$V = (0,0)$

(3) Axis of Symmetry:

Axis of symmetry = y-axis

$x = 0$

(4) Directrix:

$y = a$

$y = \frac{5}{4}$

(5) length of latus rectum:

$l = 4a$

$l = 4(\frac{5}{4})$

$l = 5$

(6) End points of latus rectum:

Value of y from focus will be the same,

$(x, \frac{-5}{4})$

$y = \frac{-5}{4}$ Putting in $eq(i)$

$x^2 = -5y$

$x^2 = -5(\frac{-5}{4})$

$x^2 = \frac{25}{4}$

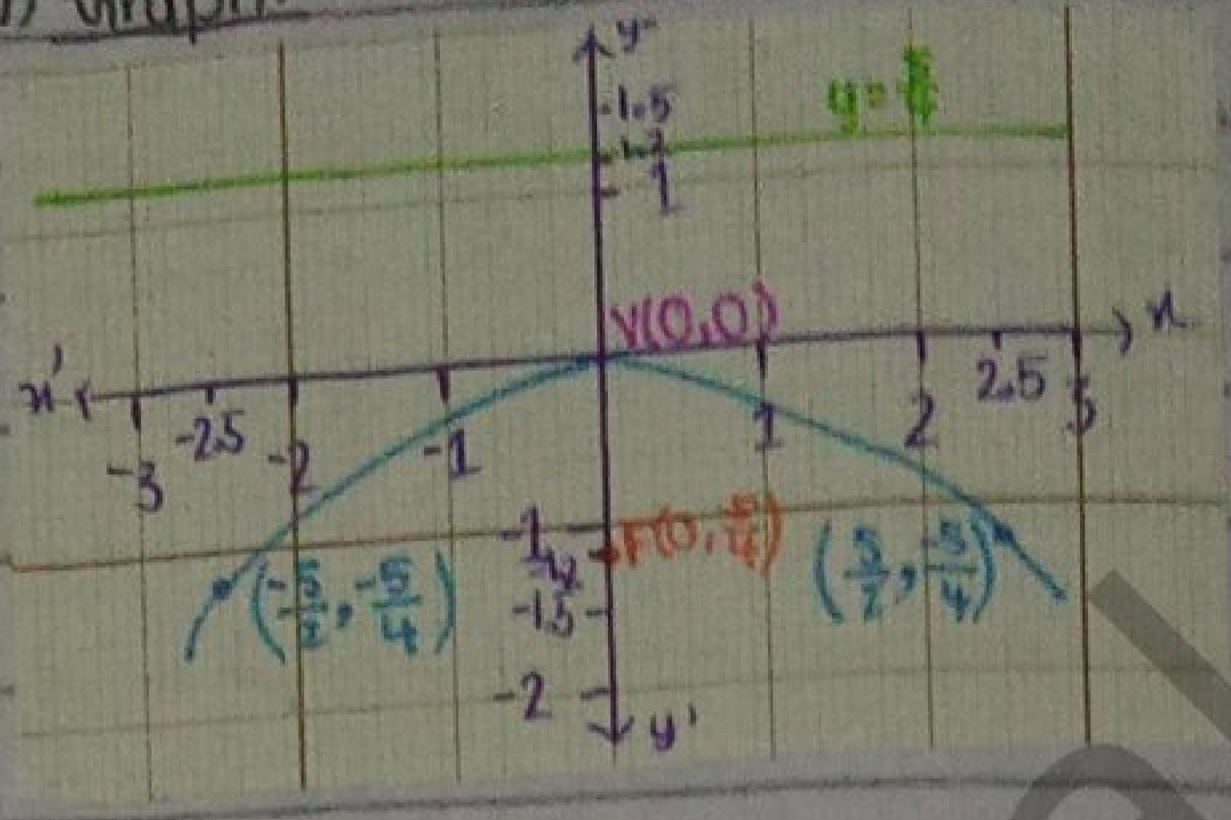
$\sqrt{x^2} = \pm\sqrt{\frac{25}{4}}$

$x = \pm\frac{5}{2}$

So, $(\frac{5}{2}, \frac{-5}{4})$ & $(\frac{-5}{2}, \frac{-5}{4})$ or

$(\pm\frac{5}{2}, \frac{-5}{4})$

(7) Graph:



(v) $y^2 - 2y - 12x - 71 = 0$

$y^2 - 2y = 12x + 71$

$(y)^2 - 2(y)(1) + (1)^2 = 12x + 71 + (1)^2$

$(y-1)^2 = 12x + 71 + 1$

$(y-1)^2 = 12x + 72$

$(y-1)^2 = 12(x+6)$ $-eq(i)$

let $Y = (y-1)$ & $X = (x+6)$

$Y^2 = 12X$

Compare,

$Y^2 = 12X$ & $Y^2 = 4aX$

$12 = 4a$

$a = \frac{12}{4}$

$a = 3$

(1) Focus:

$F = (a, 0)$

$X = a$, $Y = 0$

$x+6 = a$, $y-1 = 0$

$x+6 = 3$, $y = 1$

$x = 3-6$, $y = 1$

$x = -3$, $y = 1$

So $\boxed{F = (-3, 1)}$

(2) Vertex:

$V = (0,0)$

$X = 0$, $Y = 0$

$x+6 = 0$, $y-1 = 0$

$x = -6$, $y = 1$

So $\boxed{V = (-6, 1)}$

(3) **Axis of Symmetry:**

Along X-axis So

$Y = 0$

$y - 1 = 0$

$\boxed{y = 1}$

(4) **Directrix:**

$X = -a$

$x + 6 = -3$

$x = -3 - 6$

$\boxed{x = -9}$

(5) **Length of latus rectum:**

$l = 4a$

$l = 4(3)$

$\boxed{l = 12}$

(6) **End points of latus rectum:**

Put $x = -3$ in eq (i),

$(y-1)^2 = 12(x+6)$

$(y-1)^2 = 12(-3+6)$

$(y-1)^2 = 12(3)$

$(y-1)^2 = 36$

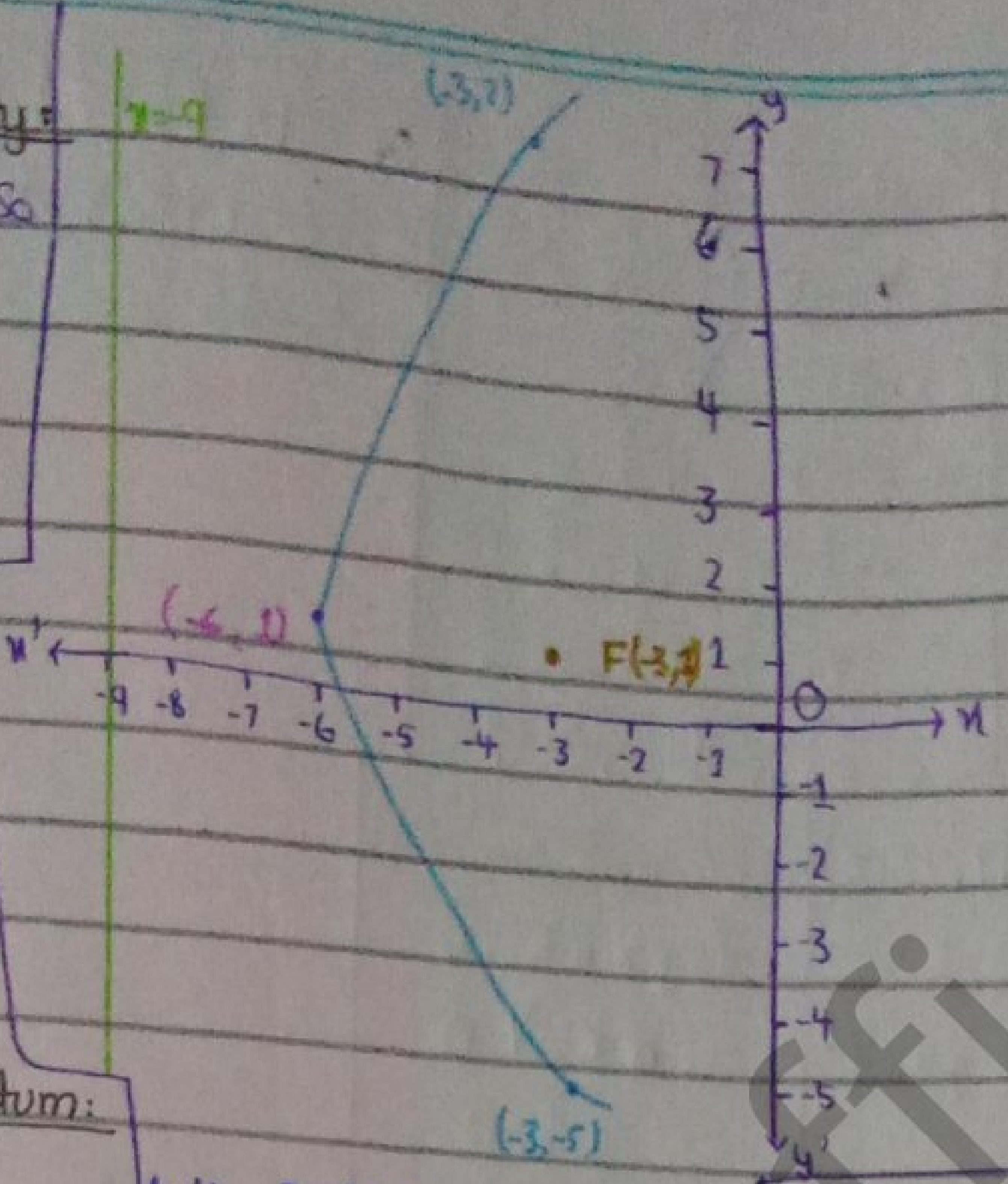
$\sqrt{(y-1)^2} = \pm\sqrt{36}$

$y - 1 = \pm 6$

$y = \pm 6 + 1$

$y = 6+1$	$y = -6+1$
$y = 7$	$y = -5$
$(-3,7)$	$(-3,-5)$

Graph:



(vi) $3x^2 + 42x + y + 149 = 0$

$3x^2 + 42x = -y - 149$

$\times\left(\frac{1}{3}\right)$ on allsides

$x^2 + 14x = -\frac{y}{3} - \frac{149}{3}$

$x^2 + 14x + (7)^2 = \frac{-y}{3} - \frac{149}{3} + (7)^2$

$(x+7)^2 = \frac{-y}{3} - \frac{2}{3}$

$(x+7)^2 = -\frac{1}{3}(y+2)$ — eq (i)

let $X = (x+7)$ & $Y = (y+2)$

$X^2 = -\frac{1}{3}Y$

Compare,

$X^2 = -\frac{1}{3}Y$, $X^2 = -4aY$

$\cancel{-}\frac{1}{3} = \cancel{-}4a$

$a = \frac{1}{12}$

(1) **Focus:**

$F = (0, -a)$

$X = 0$	$Y = -a$
$x + 7 = 0$	$y + 2 = -\frac{1}{12}$
$x = -7$	$y = -\frac{1}{12} - 2$
	$y = \frac{-25}{12}$

So $\boxed{F = \left(-7, \frac{-25}{12}\right)}$ or $(-7, -2.08)$

(2) **Vertex:**

$V = (0,0)$

$X = 0$	$Y = 0$
$x + 7 = 0$	$y + 2 = 0$
$x = -7$	$y = 2$

So $\boxed{V = (-7, 2)}$

(3) **Axis of Symmetry:**

along y-axis so

$X = 0$

$x + 7 = 0$

$\boxed{x = -7}$

(4) **Directrix:**

$Y = a$

$y + 2 = \frac{1}{12}$

$y = \frac{1}{12} - 2$

$\boxed{y = \frac{-23}{12}}$ or (1.92)

(5) **Length of latus rectum:**

$l = 4a$

$l = 4\left(\frac{1}{12}\right)$

$\boxed{l = \frac{1}{3}}$

(6) **End points of latus rectum:**

Put $y = \frac{-25}{12}$ in eq (i),

$(x+7)^2 = -\frac{1}{3}\left(\frac{-25}{12} + 2\right)$

$(x+7)^2 = -\frac{1}{3}\left(\frac{-1}{12}\right)$

$\sqrt{(x+7)^2} = \pm\sqrt{\frac{1}{36}}$

$x+7 = \pm\frac{1}{6}$

$x = \pm\frac{1}{6} - 7$

$x = \frac{1}{6} - 7$	$x = \frac{-1}{6} - 7$
$x = \frac{-41}{6}$	$x = \frac{-43}{6}$
$\left(\frac{-41}{6}, \frac{-25}{12}\right)$	$\left(\frac{-43}{6}, \frac{-25}{12}\right)$

or

$(-6.89, -2.08)$	$(-7.17, -2.08)$

(7) Graph:

You can plot the graph yourself!

(vii) $4y^2 + 4y = 15 - 32x$

$\times \frac{1}{4}$ on all sides

$y^2 + y = \frac{15}{4} - 8x$

$y^2 + y + \left(\frac{1}{2}\right)^2 = \frac{15}{4} - 8x + \left(\frac{1}{2}\right)^2$

$\left(y+\frac{1}{2}\right)^2 = 4 - 8x$

$\left(y+\frac{1}{2}\right)^2 = -8\left(x - \frac{1}{2}\right)$ — eq(i)

let $Y = \left(y+\frac{1}{2}\right)$ & $X = \left(x - \frac{1}{2}\right)$

$Y^2 = -8X$

: Compare,

$Y^2 = -8X$, $Y^2 = -4aX$

$-8 = -4a$

$a = \frac{-8}{-4}$

$a = 2$

(1) Focus:

$F = (-a, 0)$

$X = -a$	$Y = 0$
$x - \frac{1}{2} = -2$	$y + \frac{1}{2} = 0$
$x = -2 + \frac{1}{2}$	$y = \frac{-1}{2}$
$x = \frac{-3}{2}$	

So $F = \left(\frac{-3}{2}, \frac{-1}{2}\right)$ or

$F = (-1.5, -0.5)$

(2) Vertex:

$V = (0,0)$

$X = 0$	$Y = 0$
$x - \frac{1}{2} = 0$	$y + \frac{1}{2} = 0$
$x = \frac{1}{2}$	$y = \frac{-1}{2}$

$V = \left(\frac{1}{2}, \frac{-1}{2}\right)$ or

$V = (0.5, -0.5)$

(3) Axis of symmetr

along x-axis so,

$Y = 0$

$y + \frac{1}{2} = 0$

$y = \frac{-1}{2}$

or $y = -0.5$

(4) Directrix:

$X = a$

$x - \frac{1}{2} = 2$

$x = 2 + \frac{1}{2}$

$x = \frac{5}{2}$

or $x = 2.5$

(5) Length of Latus Rectum:

$l = 4a$

$l = 4(2)$

$l = 8$

(6) End points of Latus Rectum:

Put $x = \frac{-3}{2}$ in eq(i),

$\left(y+\frac{1}{2}\right)^2 = -8\left(\frac{-3}{2} - \frac{1}{2}\right)$

$\left(y+\frac{1}{2}\right)^2 = -8(-2)$

$\sqrt{\left(y+\frac{1}{2}\right)^2} = \pm\sqrt{16}$

$y + \frac{1}{2} = \pm 4$

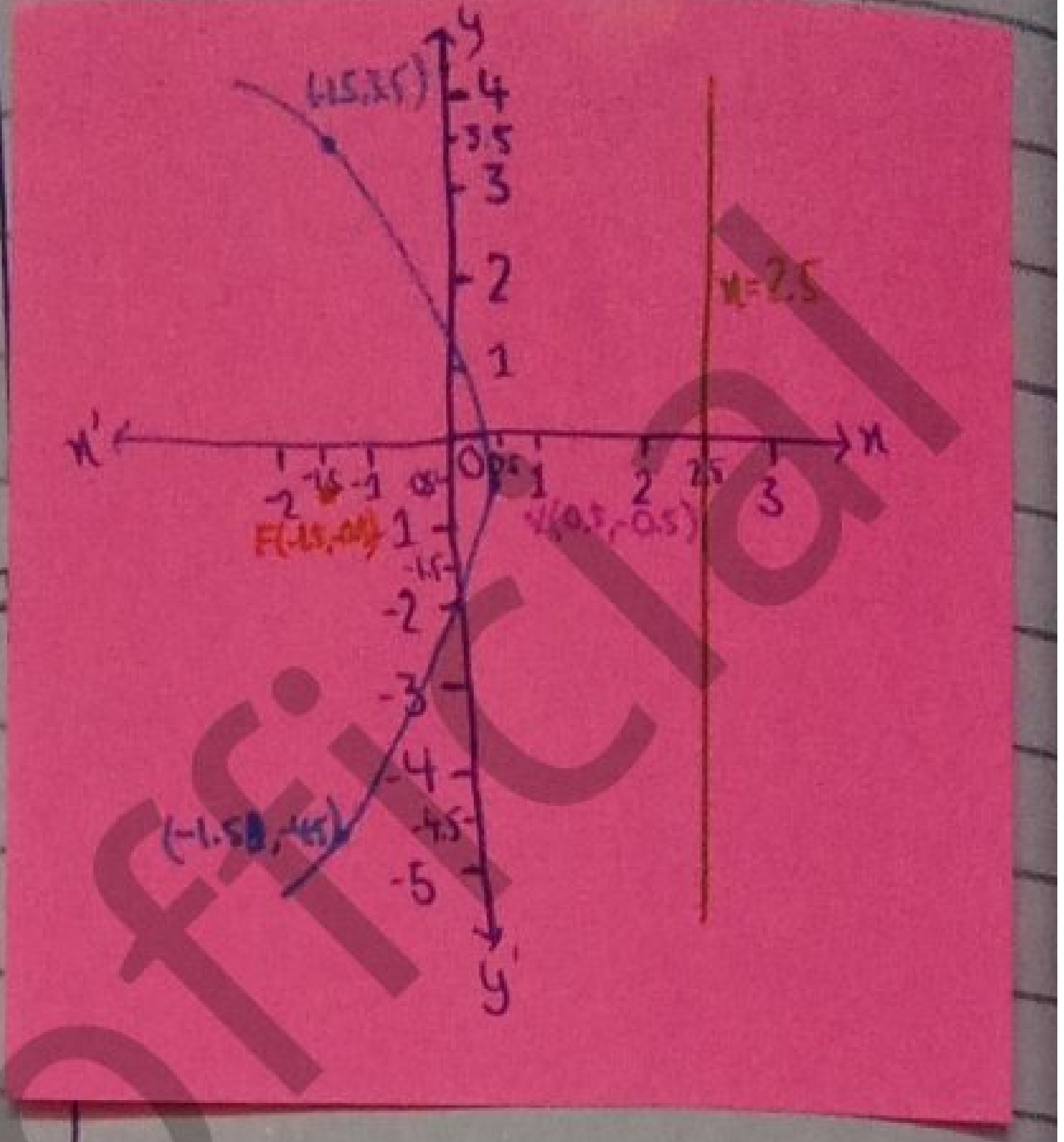
$y = \pm 4 - \frac{1}{2}$

$y = 4 - \frac{1}{2}$	$y = -4 - \frac{1}{2}$
$y = \frac{7}{2}$ or	$y = \frac{-9}{2}$
$y = 3.5$	or $y = -4.5$

So, $\left(\frac{-3}{2}, \frac{7}{2}\right)$ & $\left(\frac{-3}{2}, \frac{-9}{2}\right)$

or $(-1.5, 3.5)$ & $(-1.5, -4.5)$

(7) Graph:



(viii) $9x^2 - 6x = 108y + 26$

$\times \frac{1}{9}$ on b/s,

$x^2 - \frac{2}{3}x = 12y + \frac{26}{9}$

$x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2 = 12y + \frac{26}{9} + \left(\frac{1}{3}\right)^2$

$\left(x - \frac{1}{3}\right)^2 = 12y + 3$

$\rightarrow \left(x - \frac{1}{3}\right)^2 = 12\left(y + \frac{1}{4}\right) \leftarrow$ eq (i)

let $X = x - \frac{1}{3}$ & $Y = y + \frac{1}{4}$

$(X)^2 = 12Y$

Compare,

$X^2 = 12Y$, $X^2 = 4aY$

$12 = 4a$

$\frac{12}{4} = a$

$a = 3$

(1) Focus:

$F = (0, a)$

$X = 0$

$x - \frac{1}{3} = 0$

$x = \frac{1}{3}$

$Y = a$

$y + \frac{1}{4} = 3$

$y = 3 - \frac{1}{4}$

$y = \frac{11}{4}$

So $F\left(\frac{1}{3}, \frac{11}{4}\right)$

or $F(0.33, 2.75)$

(2) Vertex:

$V = (0, 0)$

$X = 0$

$x - \frac{1}{3} = 0$

$x = \frac{1}{3}$

$Y = 0$

$y + \frac{1}{4} = 0$

$y = \frac{-1}{4}$

So $V\left(\frac{1}{3}, \frac{-1}{4}\right)$

or $V(0.33, -0.25)$

(3) Axis of Symmetry:

along y-axis so

$X = 0$

$x - \frac{1}{3} = 0$

$x = \frac{1}{3}$

or $x = 0.33$

(4) Directrix:

$Y = -a$

$y + \frac{1}{4} = -3$

$y = -3 - \frac{1}{4}$

$y = \frac{-13}{4}$

or $y = -3.25$

(5) Length of latus rectum:

$l = 4a$

$l = 4(3)$

$l = 12$

(6) End points of L.R:

Put $y = \frac{11}{4}$ in eq (i),

$\left(x - \frac{1}{3}\right)^2 = 12\left(y + \frac{1}{4}\right)$

$\left(x - \frac{1}{3}\right)^2 = 12\left(\frac{11}{4} + \frac{1}{4}\right)$

$\sqrt{\left(x - \frac{1}{3}\right)^2} = \pm\sqrt{36}$

$x - \frac{1}{3} = \pm 6$

$x = \pm 6 + \frac{1}{3}$

$x = 6 + \frac{1}{3}$

$x = \frac{19}{3}$

or $x = 6.33$

$x = -6 + \frac{1}{3}$

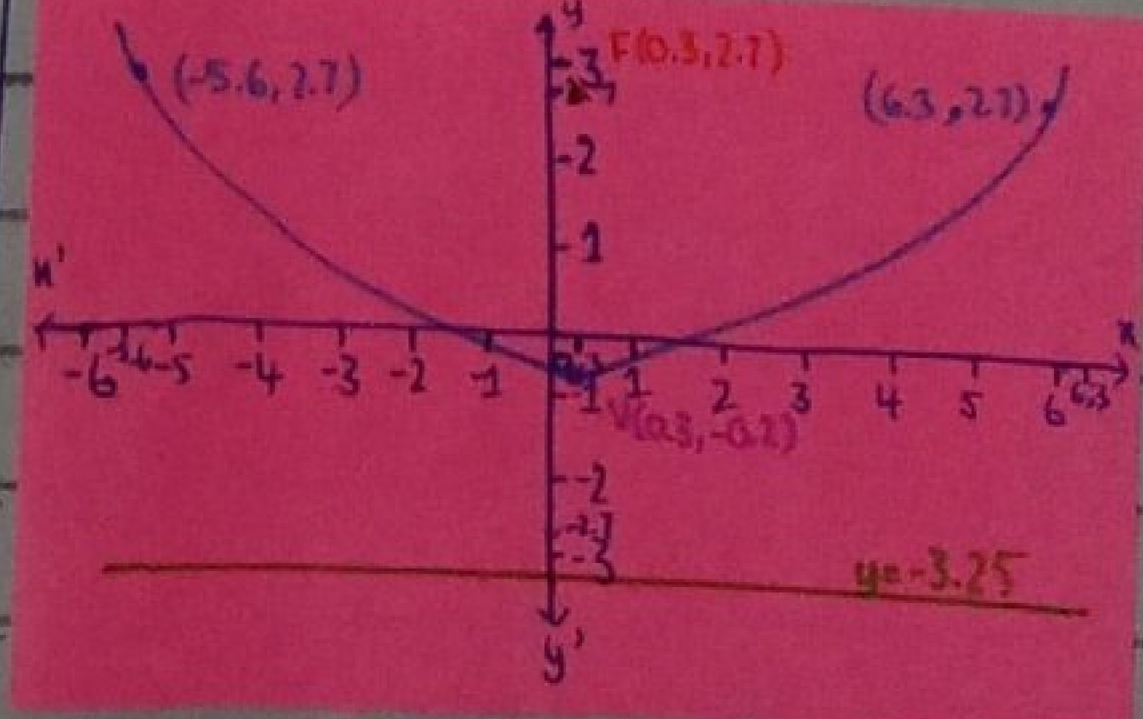
$x = \frac{-17}{3}$

or $x = -5.67$

So $\left(\frac{19}{3}, \frac{11}{4}\right)$ & $\left(\frac{-17}{3}, \frac{11}{4}\right)$

or $(6.33, 2.75)$ & $(-5.67, 2.75)$

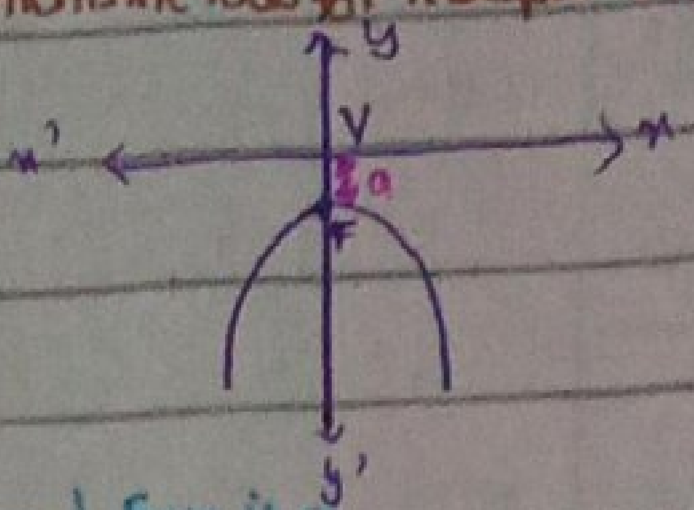
(7) Graph:



Q2 Find the equation of the parabola, ...

(i) Vertex at origin and focus $\left(0, \frac{-1}{32}\right)$

$V(\overset{h}{0}, \overset{k}{0})$, $F\left(0, \frac{-1}{32}\right)$ ∵ $f(0,-a)$ which is the focus for the equation $x^2 = -4ay$



$(x-h)^2 = -4a(y-k)$

$x^2 = -4ay$ — eq(i)

So we know that the distance b/w Vertex and Focus is a,

$|FV| = a$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = a$

$\sqrt{\left(0-\left(\frac{-1}{32}\right)\right)^2+(0-0)^2} = a$

$\sqrt{\left(\frac{1}{32}\right)^2} = a$

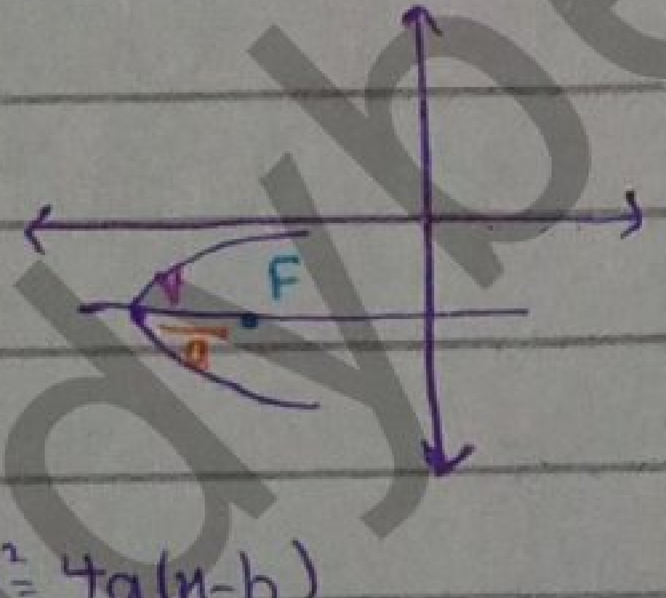
$a = \frac{1}{32}$

Putting the value of a in eq(i),

$x^2 = -4\left(\frac{1}{32}\right)y$

$\boxed{x^2 = \frac{-1}{8}y}$

(ii) Vertex $(\overset{h}{-8}, \overset{k}{-9})$ and Focus $\left(\frac{-31}{4}, -9\right)$



$(y-k)^2 = 4a(x-h)$

$(y+9)^2 = 4a(x+8)$ — eq(i)

Finding a,

$|FV| = a$

$\sqrt{\left(-8+\frac{31}{4}\right)^2+(-9+9)^2} = a$

$\sqrt{\left(\frac{-1}{4}\right)^2} = a$

$\sqrt{\frac{1}{16}} = a$

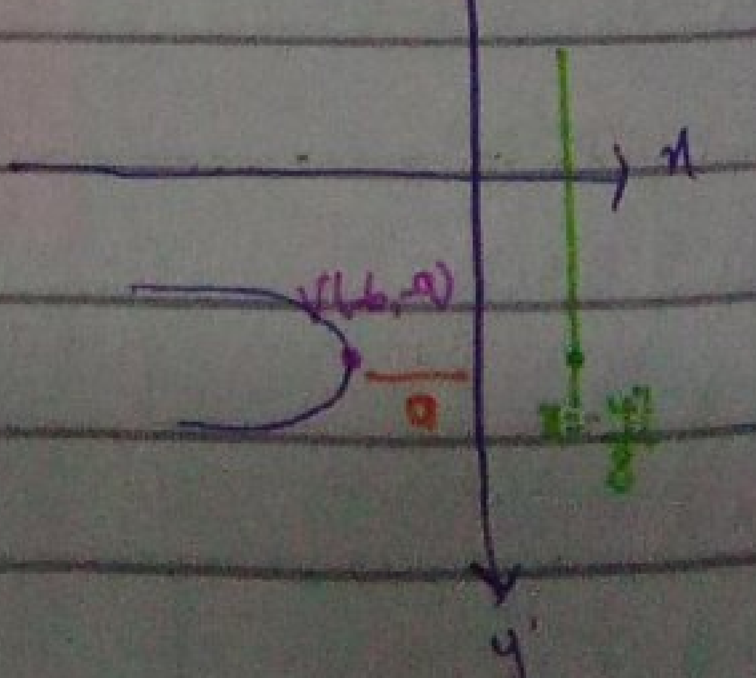
$a = \frac{1}{4}$

Putting a in eq(i),

$(y+9)^2 = 4\left(\frac{1}{4}\right)(x+8)$

$\boxed{(y+9)^2 = x+8}$

(iii) Vertex $(-6,-9)$ and directrix: $x = \frac{-47}{8}$



$\overset{h}{}\ \overset{k}{}$

$V(-6,-9)$, Directrix $x = -\frac{47}{8}$

$(y-k)^2 = -4a(x-h)$

$(y-(-9))^2 = -4a(x-(-6))$

$(y+9)^2 = -4a(x+6)$ — eq(i)

let, $x = -\frac{47}{8}$

$8x = -47$

$8x+47 = 0$

$8x+0y+47 = 0$ ($\because$ equation of line)

$8x+0y+47$, $V(-6,-9)$

$$d = \frac{|ax_0+by_0+C|}{\sqrt{a^2+b^2}}$$

$$a = \frac{|8(-6)+0(9)+47|}{\sqrt{(8)^2+(0)^2}}$$

$$a = \frac{|-48+47|}{\sqrt{8^2}}$$

$$a = \frac{|-1|}{8}$$

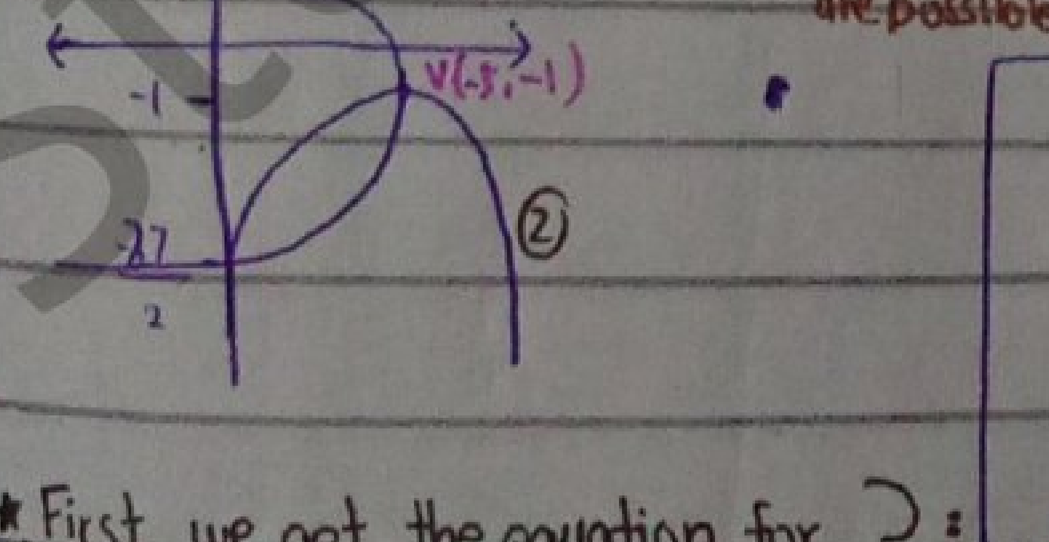
$$a = \frac{1}{8}$$

Putting a in eq(i),

$(y+9)^2 = -4\left(\frac{1}{8}\right)(x+6)$

$$\boxed{(y+9)^2 = -\frac{1}{2}(x+6)}$$

(iv) $V(5,-1)$ y-intercept : $\frac{-27}{2}$ $\because \left(0,-\frac{27}{2}\right)$



$\because$ so 2 parabola shapes are possible there

① ★ First we get the equation for ⊃ :

$(y-k)^2 = -4a(x-h)$

$(y+1)^2 = -4a(x-5)$

$(y+1)^2 = -4a(x-5)$ — eq(i)

Finding a,

Put $\left(0,\frac{27}{2}\right)$ in eq(i)

$\left(\frac{27}{2}+1\right)^2 = -4a(0-5)$

$\left(\frac{-25}{2}\right)^2 = -4a(-5)$

$\frac{625}{4} = 20a$

$\frac{625}{80} = a$

$a = \frac{125}{16}$

Putting value of a in eq(i),

$(y+1)^2 = -4\left(\frac{125}{16}\right)(x-5)$

$$\boxed{(y+1)^2 = \frac{-125}{4}(x-5)}$$

② Finding the equation for ∩ :

$(x-h)^2 = -4a(y-k)$

$(x-5)^2 = -4a(y+1)$ — eq(ii)

Finding a,

Put $\left(0,-\frac{27}{2}\right)$ in eq(ii)

$(0-5)^2 = -4a\left(-\frac{27}{2}+1\right)$

$(-5)^2 = -\overset{2}{\cancel{4}}a\left(-\frac{25}{\cancel{2}}\right)$

$25 = 50a$

$\frac{25}{50} = a$

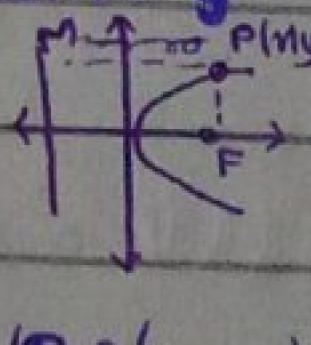
$a = \frac{1}{2}$

Putting a in eq(ii),

$(x-5)^2 = -4\left(\frac{1}{2}\right)(y+1)$

$$\boxed{(x-5)^2 = -2(y+1)}$$

(v) $F\left(-\frac{7}{4},-1\right)$ & Directrix $y = \frac{2}{5}$



We know,

$|PM| = |PF|$

let any point on the parabola P(x,y)

by definition of parabola,

$|PM| = |PF|$

Now line, $y = \frac{2}{5}$

$5y = 2$ $5y+2 = 0$ $0x+5y+2 = 0$

Sol ~~$0x+5y+2=0$, $P(x,y)$~~

~~$d = |ax_0+by_0+c|$~~

$|PM| = |PF|$

$$\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}} = \text{[illegible]} \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$

$$\frac{|0(x)+5(y)+(-2)|}{\sqrt{(0)^2+(5)^2}} = \sqrt{(y+1)^2+(x+\frac{3}{4})^2}$$

$$\frac{|5y-2|}{\sqrt{5^2}} = \sqrt{y^2+2y+1+x^2+\frac{3}{2}x+\frac{9}{16}}$$

$$\left(\frac{|5y-2|}{5}\right)^2 = \left(\sqrt{y^2+2y+x^2+\frac{3}{2}x+\frac{25}{16}}\right)^2$$

$$\frac{25y^2-20y+4}{25} = y^2+2y+x^2+\frac{3}{2}x+\frac{25}{16}$$

$$25y^2-20y+4 = 25y^2+50y+25x^2+25\left(\frac{3}{2}x\right)+25\left(\frac{25}{16}\right)$$

$$0 = \cancel{25y^2}-\cancel{25y^2}+50y+20y+25x^2+\frac{75}{2}x+\frac{625}{16}-4$$

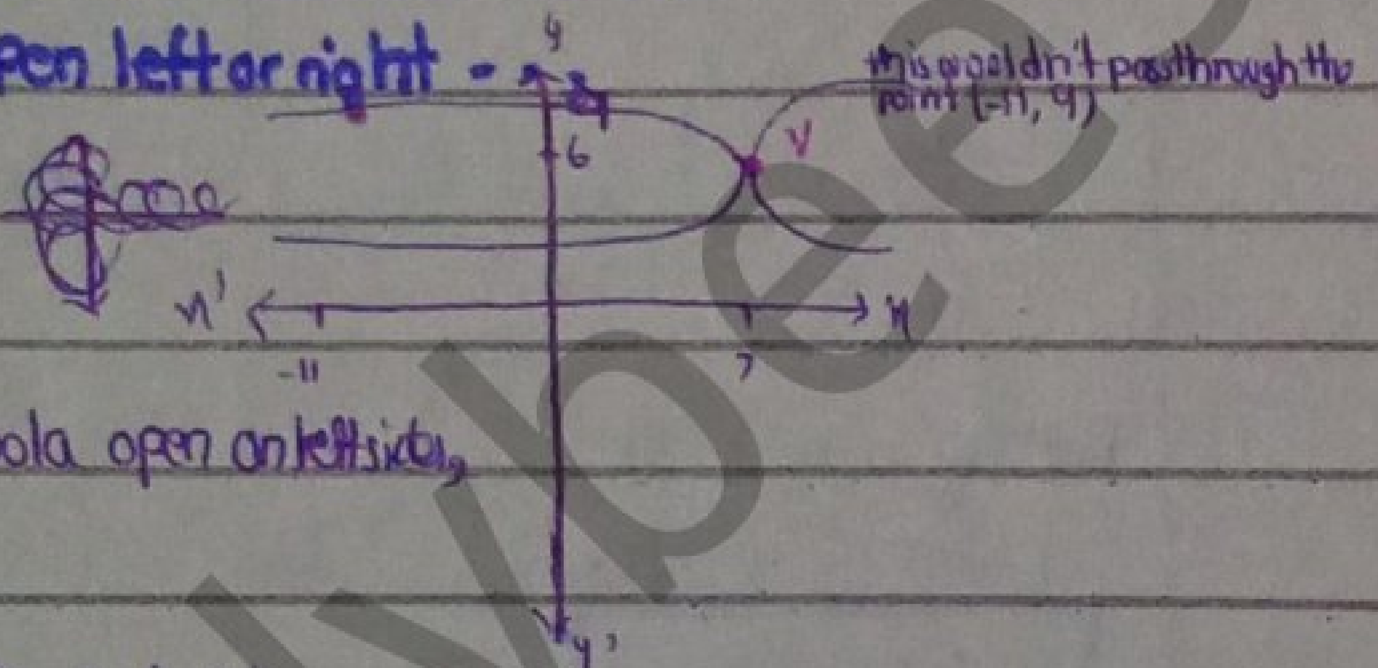
$$0 = 25x^2+\frac{75}{2}x+70y+\frac{561}{16}$$

$$25x^2+\frac{75}{2}x+70y+\frac{561}{16}=0$$

$\times\frac{1}{25}$ on allside

$$\boxed{x^2+\frac{3}{2}x+\frac{14}{5}y+\frac{561}{400}=0}$$

(vi) Open left or right -



So, Parabola open on leftside,

$(y-k)^2=-4a(x-h)$

$(y-6)^2=-4a(x-7)$ —eq(i)

Finding a,

Putting $(-11,9)$ in eq(i),

$(9-6)^2=-4a(-11-7)$

$(3)^2=-4a(-18)$

$9=72a$

$a=\frac{9}{72}$

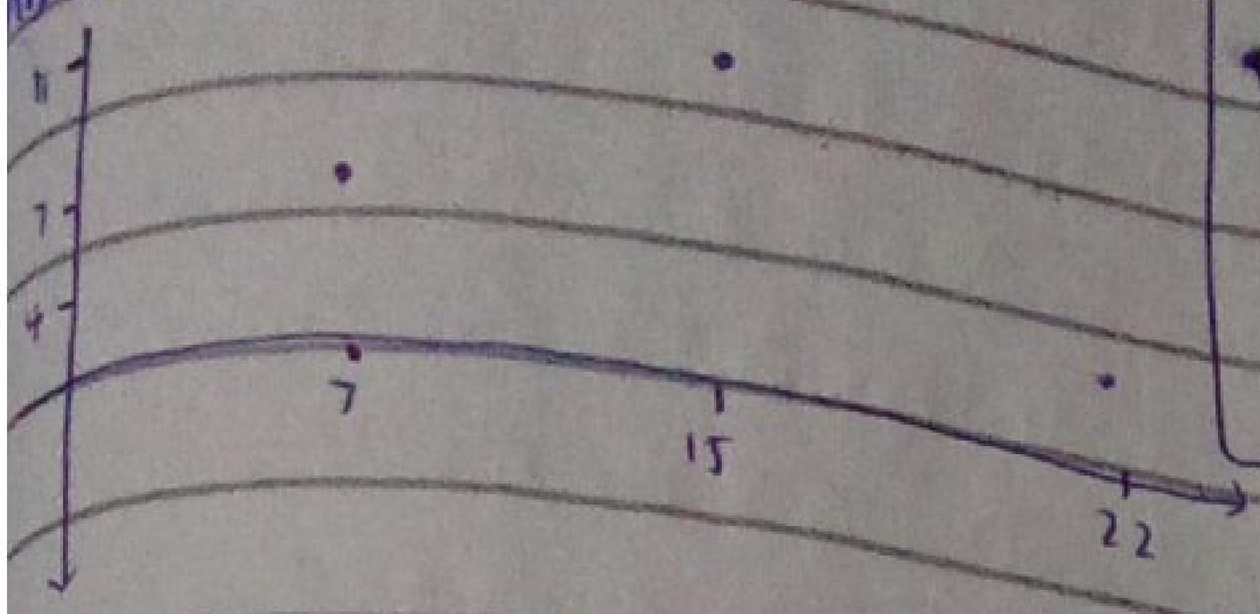
$a=\frac{1}{8}$

Putting value of a in eq(i),

$(y-6)^2=-4\left(\frac{1}{8}\right)(x-7)$

$$\boxed{(y-6)^2=\frac{-1}{2}(x-7)}$$

(vii) Opens upordown...



$y = ax^2+bx+C$ $-eq①$

can solve using $(x-h)^2 = 4a(y-k)$
∵ but this way is easier.

⇒ $(11,15)$;

$15 = a(11)^2+b(11)+C$

$15 = 121a+11b+C$ $-eq(i)$

⇒ $(7,7)$:

$7 = a(7)^2+b(7)+C$

$7 = 49a+7b+C$ $-eq(ii)$

⇒ $(4,22)$;

$22 = a(4)^2+b(4)+C$

$22 = 16a+4b+C$ $-eq(iii)$

Subtracting eq(i) & eq(ii),

$15 = 121a+11b+\cancel{c}$

$\underline{\pm 7 = \pm 49a \pm 7b \pm \cancel{c}}$

$8 = 72a+4b$

$\overset{2}{\cancel{8}} = \overset{1}{\cancel{4}}(18a+b)$

$2 = 18a+b$ $-eq(iv)$

Subtracting eq(ii) & eq(iii),

$7 = 49a+7b+\cancel{c}$

$\underline{\pm 22 = \pm 16a \pm 4b \pm \cancel{c}}$

$-15 = 33a+3b$

$\cancel{3}(-5) = \cancel{3}(11a+b)$

$-5 = 11a+b$ $-eq(v)$

Sub eq(iv) & eq(v),

$2 = 18a+\cancel{b}$

$\underline{\mp 5 = \pm 11a \pm \cancel{b}}$

$7 = 7a$

$\boxed{a = 1}$

Putting value of a in eq(v),

$-5 = 11a+b$

$-5 = 11(1)+b$

$-5 = 11+b$

$b = -5-11$

$\boxed{b = -16}$

Putting a & b in eq(i)

$15 = 121a+11b+C$

$15-121(1)-11(-16) = C$

$15-121+176 = C$

$C = 70$

Putting the values of a, b, c in eq①

$y = ax^2+bx+c$

$y = 1(x^2)+(-16)x+70$

$y = x^2-16x+70$

$y-70 = x^2-16x$

$y-70+(8)^2 = x^2-16x+(8)^2$

$y-70$ [illegible] $64 = (x-8)^2$

$(y-6) = (x-8)^2$

$\boxed{(x-8)^2 = (y-6)}$

(viii) V(10,0), Axis of symmetry ....

$(y-k)^2 = -4a(x-h)$ $-eq(i)$

length of Latus rectum = 1

$l = 4a$

$1 = 4a$

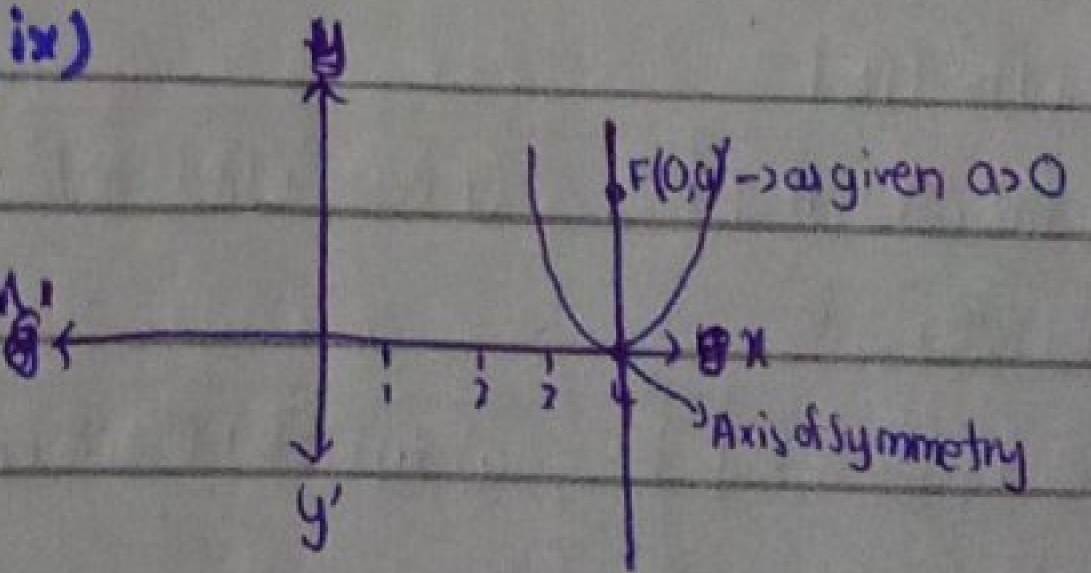
$a = \frac{1}{4}$

Putting a in eq(i),

$(y-0)^2 = -4\left(\frac{1}{4}\right)(x-10)$

$\boxed{y^2 = -(x-10)}$

(ix)



$(x-h)^2 = 4a(y-k)$ $-eq(i)$

finding a,

Length of latus rectum = $\frac{1}{3}$

$l = 4a$

$\frac{1}{3} = 4a$

$a = \frac{1}{12}$

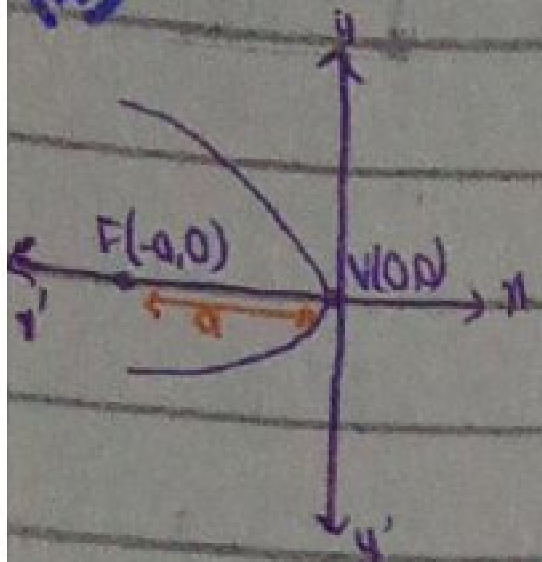
Putting value of a in eq(i)

$(x-h)^2 = 4a(y-k)$

$(x-4)^2 = \cancel{4}\left(\frac{1}{\cancel{12}_3}\right)(y-2)$

$\boxed{(x-4) = \frac{1}{3}(y-2)}$

(x)



$|FV| = a$

$\boxed{\frac{1}{8} = a}$

$(y-k)^2 = -4a(x-h)$

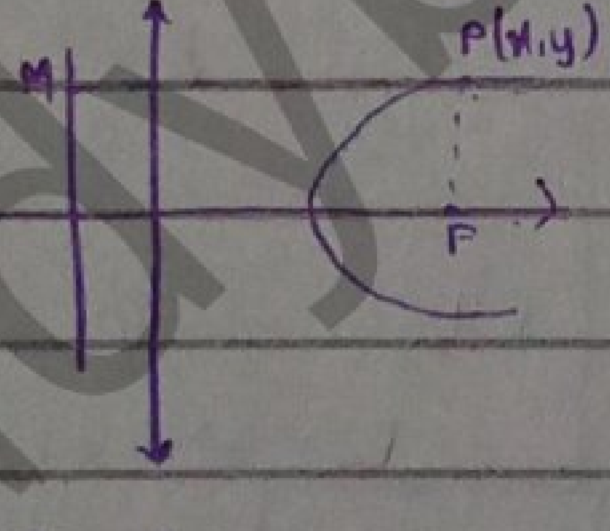
$(y-0)^2 = -4\left(\frac{1}{8}\right)(x-0)$

$(y)^2 = \frac{-1}{2}(x)$

$\boxed{y^2 = \frac{-1}{2}(x)}$

Q3 Find the equation of - - -

$F(PSin\theta, P(os\theta)$, Directrix; $x Cos\theta + y Sin\theta - P = 0$, let $P(x,y)$



by defination of parabola,

$|PM| = |PF|$

$$\frac{|Cos\theta(x) + Sin\theta(y) - P|}{\sqrt{Cos^2\theta + Sin\theta}} = \sqrt{(y-PCos\theta)^2 + (x-PSin\theta)^2}$$

$$\left(\frac{|xCos\theta + ySin\theta - P|}{\sqrt{1}}\right)^2 = \left(\cancel{\sqrt{}}\,y^2 - 2yPCos\theta + P^2Cos^2\theta + x^2 - 2xPSin\theta + Sin^2\theta\right)^{\cancel{2}} \quad \because Cos^2\theta + Sin^2\theta = 1$$

$$\frac{(xCos\theta + ySin\theta - P)^2}{(\sqrt{1})^{\cancel{2}}} = y^2 - 2yPCos\theta + P^2Cos^2\theta + x^2 - 2xPSin\theta + Sin^2\theta$$

$$x^2Cos^2\theta + y^2Sin^2\theta + P^2 + \overset{2xyCos\theta Sin\theta}{[illegible]} - 2PySin\theta - 2PxCos\theta = x^2 + y^2 + P^2(Cos^2\theta + Sin^2\theta) - 2yPCos\theta - 2xPSin\theta$$

$'' \quad + P^2 + \quad '' \qquad = \quad '' \quad + P^2(1) - \quad ''$

$'' \quad + \cancel{P^2} + \quad '' \qquad = \quad '' \quad \cancel{+P^2} - \quad ''$

$x^2\cos^2\theta+y^2\sin^2\theta+2xy\cos\theta\sin\theta-2Py\sin\theta-2Px\cos\theta=x^2+y^2-2yP\cos\theta-2xP\sin\theta$

$0=x^2+y^2-2Py\cos\theta-2Px\sin\theta-x^2\cos^2\theta-y^2\sin^2\theta-2xy\cos\theta\sin\theta+2Py\sin\theta+2Px\cos\theta$

$0=x^2-x^2\cos^2\theta+y^2-y^2\sin^2\theta-2xy\cos\theta\sin\theta-2Py\cos\theta-2Px\sin\theta+2Py\sin\theta+2Px\cos\theta$

$x^2(1-\cos^2\theta)+y^2(1-\sin^2\theta)-2xy\cos\theta\sin\theta-2P(y\cos\theta+x\sin\theta-y\sin\theta-x\cos\theta)=0$

$x^2\sin^2\theta+y^2\cos^2\theta-2xy\cos\theta\sin\theta-2P[\cos\theta(y-x)+\sin\theta(x-y)]=0$

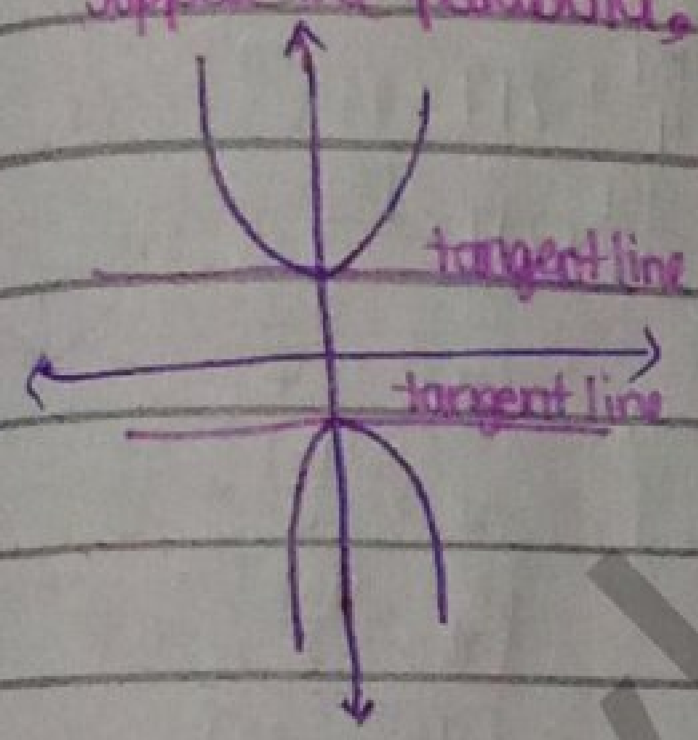
$(x\sin\theta)^2+(y\cos\theta)^2-2(x\sin\theta)(y\cos\theta)=2P[-\cos\theta(x-y)+\sin\theta(x-y)]$

$$\boxed{(x\sin\theta-y\cos\theta)^2=2P[(x-y)(\sin\theta-\cos\theta)]}$$

Q4 Find the coordinates ...

The first thing that we need to do, is to figure out the equation of parabola, by looking at the given equation.

Suppose the parabola.



Vertical Parabola:

∵ we know that the axis of symmetry has a single power. If the axis of symmetry is x-axis then x's power will be one. And if the axis of symmetry is on y-axis than y's power will be one.

So if the y is in single power then the shape is Vertical and if the x is in single power then the shape is Horizontal.

Tangent is, $\frac{dy}{dx}$ = slope

finding slope, $m=\tan\theta(0)$

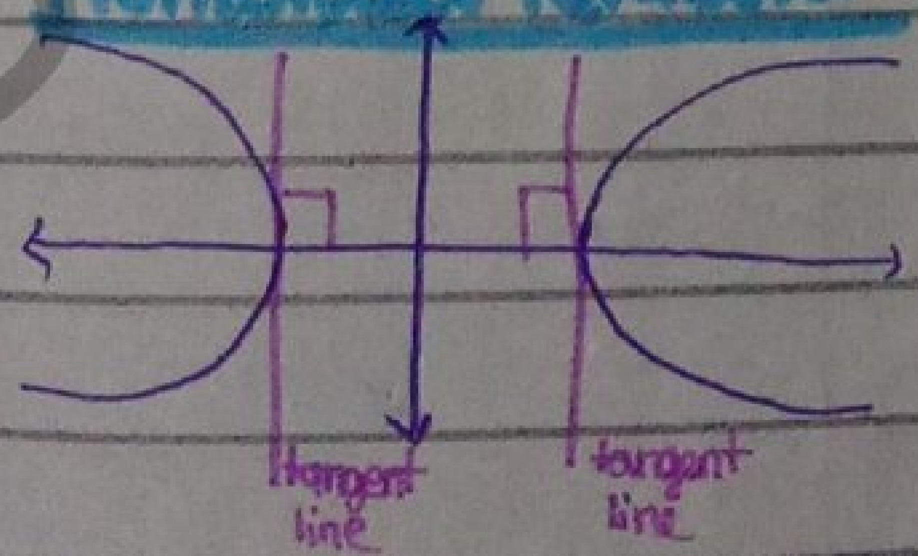
0 bcs, the tangent line makes
∵ no angle with the x-axis

$m=0$

So, $\boxed{\frac{dy}{dx}=0}$

$\boxed{y=x^2}$

Horizontal Parabola:



Tangent is, $\frac{dy}{dx}$ = slope

finding slope, $m=\tan 90^\circ$

90° bcs tangent line makes
∵ a 90° angle with the x-axis

$m=\frac{1}{0}=\infty$

So, $\boxed{\frac{d}{dx}=\infty}$

$\boxed{x=y^2}$

(i) $y = x^2 - 4x + 10$ — eq(i)

Differentiate w.r.t x,

$\frac{dy}{dx} = \frac{d}{dx}(x^2) - 4\frac{d}{dx}(x) + \frac{d}{dx}(10)$

$\frac{dy}{dx} = 2x - 4$

As its vertical shape so,

$\frac{dy}{dx} = 0$

So,

$2x - 4 = 0$

$x = \frac{4}{2}$

$\boxed{x = 2}$

Putting x in eq(i)

$y = (2)^2 - 4(2) + 10$

$y = 4 - 8 + 10$

$\boxed{y = 6}$

Hence V(2,6)

(ii) $4x^2 + 24x + 39 - 3y = 0$ — eq(i)

Diff w.r.t x,

$4\frac{d}{dx}(x^2) + 24\frac{d}{dx}(x) + \frac{d}{dx}(39) - 3\frac{d}{dx}(y) = \frac{d}{dx}(0)$

$8x + 24 + 0 - 3\frac{dy}{dx} = 0$

$3\frac{dy}{dx} = 8x + 24$

$\frac{dy}{dx} = \frac{8x}{3} + \frac{24}{3}$

$\frac{dy}{dx} = \frac{8}{3}x + 8$

As its vertical parabola,

$\frac{dy}{dx} = 0$

So, $\frac{8}{3}x + 8 = 0$

$\frac{8}{3}x = -8$

$x = -8 \times \frac{3}{8}$

$\boxed{x = -3}$

Putting x in eq(i),

~~$4(-3)^2 + 24(-3) + 39 - 3y = 0$~~

~~$-36 - 72 + 39 = 3y$~~

~~$-33 = 3y$~~

~~$3y = -141$~~

$4(-3)^2 + 24(-3) + 39 - 3y = 0$

$4(9) + 24(-3) + 39 = 3y$

$36 - 72 + 39 = 3y$

~~$-108 + 39 = 3y$~~

~~$-69 = 3y$~~

~~$y = -23$~~

$\cancel{3} = \cancel{3}y$

$y = 1$

Hence V(-3,1)

(iii) $y^2 - 10y + 4x + 28 = 0$ — eq(i)

Diff w.r.t x,

$\frac{d}{dx}(y^2) - 10\frac{d}{dx}(y) + 4\frac{d}{dx}(x) + \frac{d}{dx}(28) = \frac{d}{dx}(0)$

$2y\frac{dy}{dx} - 10\frac{dy}{dx} + 4 + 0 = 0$

$2y\frac{dy}{dx} - 10\frac{dy}{dx} = -4$

$\frac{dy}{dx}(2y - 10) = -4$

$\frac{dy}{dx} = \frac{-4}{2y-10}$

As its a horizontal parabola so,

$\frac{dy}{dx} = \frac{1}{0}$

$\frac{-4}{2y-10} = \frac{1}{0}$

$0(-4) = 1(2y - 10)$

$0 = 2y - 10$

$y = \frac{10}{2}$

$\boxed{y = 5}$

Putting y in eq(i),

$(5)^2 - 10(5) + 4x + 28 = 0$

$25 - 50 + 28 = -4x$

$8 = -4x$

$x = \frac{8}{-4}$

$\boxed{x = -2}$

Hence V(-2,5)

(iv) $y^2 - 14y = -3x - 45y$ — eq(i)

Diff w.r.t x,

$\frac{d}{dx}(y^2) - 14\frac{d}{dx}(y) = -3\frac{d}{dx}(x) - 45\frac{d}{dx}(y)$

$2y\frac{dy}{dx} - 14\frac{dy}{dx} = -3 - 45\frac{dy}{dx}$

$2y\frac{dy}{dx} - 14\frac{dy}{dx} + 45\frac{dy}{dx} = -3$

$\frac{dy}{dx}(2y - 14 + 45) = -3$

$\frac{dy}{dx} = \frac{-3}{2y+31}$

As its a horizontal parabola,

$\frac{dy}{dx} = \frac{1}{0}$

So, $\frac{-3}{2y+31} = \frac{1}{0}$

$0(-3) = 1(2y + 31)$

$0 = 2y + 31$

$\boxed{y = \frac{-31}{2}}$

Putting y in eq(i),

$\left(\frac{-31}{2}\right)^2 - 14\left(\frac{-31}{2}\right) = -3x - 45\left(\frac{-31}{2}\right)$

$\frac{961}{4} + 217 + 45\left(\frac{-31}{2}\right) = -3x$

$\frac{961}{4} + 217 - \frac{1395}{2} = -3x$

$-3x = -\frac{961}{4}$

$x = -\frac{961}{4} \times \frac{-1}{3}$

$\boxed{x = \frac{961}{2}}$

Hence $V\left(\frac{961}{2}, \frac{-31}{2}\right)$