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Where Minds Pollinate

Class 12
Mathematics

Chapter 7
Exercise 7.3
Handwritten Solution

Ex 7.3

Q1 Check whether ...

(i) (3,5) ; $x^2+y^2-6x+2y-18=0$

$=(3)^2+(5)^2-6(3)+2(5)-18$

$=9+25-18+10-18$

$=44-36$

$=8>0$ if the value of the eqn is greater than 0, then it lies

given point (3,5) is outside of the circle

(ii) (1,6) ; $2x^2+2y^2-4x-16y+5=0$

$\div 2$ on all sides,

$=x^2+y^2-2x-8y+\frac{5}{2}$

$=(1)^2+(6)^2-2(1)-8(6)+\frac{5}{2}$

$=1+36-2-48+\frac{5}{2}$

$=-\frac{21}{2}<0$

Given point (1,6) is inside the circle

(iii)

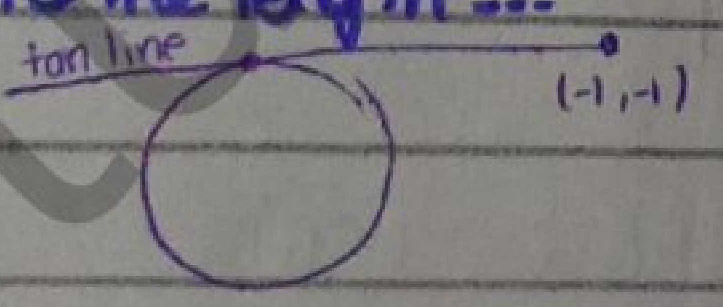
$=(-1)^2+(-2)^2+6(-1)+4(-2)+9$

$=1+4-6-8+9$

$=0$

Hen Given point lies on the circle

Q2 Find the length ...



length of tangent $=\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$

$=\sqrt{x_1^2+y_1^2-18x_1+16y_1+10}$

$=\sqrt{(-1)^2+(-1)^2-18(-1)+16(-1)+10}$

$=\sqrt{1+1+18-16+10}$

$\boxed{=\sqrt{14}}$

Q3 ...

$\div 3$ on all sides,

$x^2+y^2+6x-8y+\frac{50}{3}=0$

length $=\sqrt{x_1^2+y_1^2+6x_1-8y_1+\frac{50}{3}}$

$=\sqrt{(-3)^2+(1)^2+6(-3)-8(1)+\frac{50}{3}}$

$=\sqrt{9+1-18-8+\frac{50}{3}}$

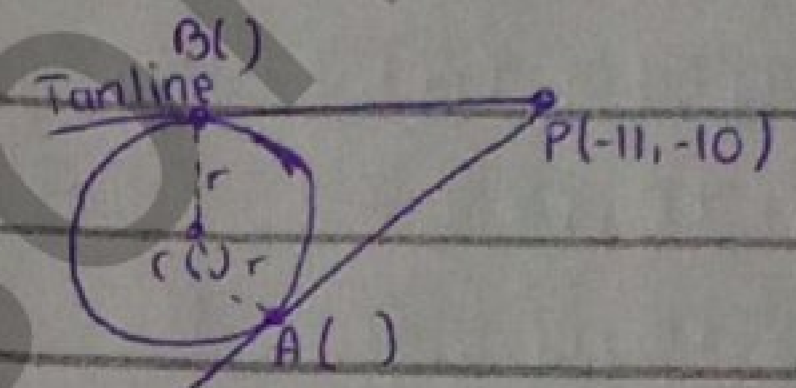
$=\sqrt{-8-8+\frac{50}{3}}$

$=\sqrt{-16+\frac{50}{3}}$

$=\sqrt{\frac{-48+50}{3}}$

$\boxed{=\sqrt{\frac{2}{3}}}$

Q4 ...



$x^2+y^2+6x+8y+5=0$ — eq(i)

$(x^2+6x)+(y^2+8y)+5=0$

$[x^2+6x+(3)^2]+[y^2+8y+(4)^2]=-5+(3)^2+(4)^2$

$(x+3)^2+(y+4)^2=-5+9+16$

$(x-(-3))^2+(y-(-4))^2=(\sqrt{20})^2$

Center $=C(-3,4)$

Radius $=\sqrt{20}$

$y-y_1=m(x-x_1)$

$y+10=m(x+11)$

$y+10=mx+11m$

$mx-y+11m-10=0$

Distance of line from point

$D=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$\sqrt{20}=\frac{|m(-3)+(-1)(-4)+11m-10|}{\sqrt{m^2+(-1)^2}}$

$\sqrt{20(m^2+1)}=|-3m+4+11m-10|$

$(\sqrt{20m^2+20})^2=(8m-6)^2$

$20m^2+20=64m^2-96m+36$

$64m^2-20m^2-96m+36-20=0$

$44m^2-96m+16=0$

$2(22m^2-48m+8)=0$

$22m^2-48m+8=0$

$4(11m^2-24m+4)=0$

$11m^2-24m+4=0$

$11m^2-22m-2m+4=0$

$11m(m-2)-2(m-2)=0$

$11m-2=0$ | $m-2=0$

$\boxed{m=\frac{2}{11}}$ | $\boxed{m=2}$

$y+10=\frac{2}{11}(x+11)$ | $y+10=2(x+11)$

$\boxed{2x-11y-88=0}$ | $\boxed{2x-y+12=0}$

→ Finding the point of contact,

$$\boxed{x=\frac{88+11y}{2}}$$

Put in eq(i)

$$\left(\frac{88+11y}{2}\right)^2+y^2+6\left(\frac{88+11y}{2}\right)+8y+5=0$$

$$\frac{1}{4}\left[7744+121y^2+1936y\right]+y^2+264+33y+8y+5=0$$

× 4 on all sides

$$7744+121y^2+1936y+4y^2+1056+132y+32y+20=0$$

$$125y^2+2100y+8820=0$$

$$5(25y^2+420y+1764)=0$$

$$25y^2+420y+1764=0$$

$$(5y)^2+420y+(42)^2=0$$

$$(5y)^2+2(5y)(42)+(42)^2=0$$

$$\sqrt{(5y+42)^2}=\sqrt{0}$$

$$5y+42=0$$

$$5y=-42$$

$$\boxed{y=\frac{-42}{5}}$$

Putting in $x=\frac{88+11y}{2}$

$$x=\frac{88+11\left(-\frac{42}{5}\right)}{2}$$

$$\boxed{x=\frac{-11}{5}}$$

First point of intersection $\boxed{\left(-\frac{11}{5},-\frac{42}{5}\right)}$

Now for point B,

$y=2x+12$ — eq(ii)

Put in eq(i)

$$x^2+(2x+12)^2+6x+8(2x+12)+5=0$$

$$x^2+4x^2+48x+144+6x+16x+96+5=0$$

$$5x^2+70x+245=0$$

$$5(x^2+14x+49)=0$$

$$x^2+14x+49=0$$

$$(x)^2+2(x)(7)+(7)^2=0$$

$$\sqrt{(x+7)^2}=\sqrt{0}$$

$$x+7=0$$

$$\boxed{x=-7}$$

Putting x in eq(ii)

$$y=2x+12$$

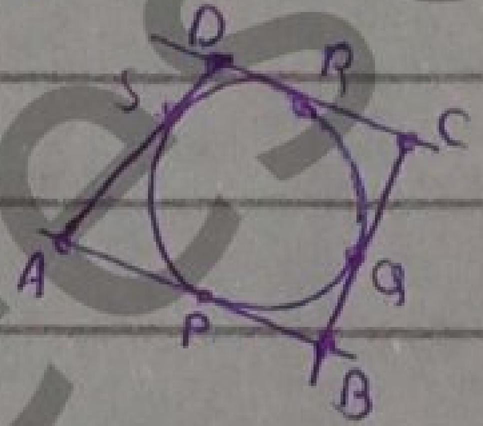
$$y=2(-7)+12$$

$$y=-14+12$$

$$\boxed{y=-2}$$

$$\boxed{B(-7,-2)}$$

Q5 A quadrilateral - - -



Point A:

length of tangent $=\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$

$$|\overline{AP}|=\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}=|\overline{AS}|$$

Point B:

$$|\overline{BP}|=\sqrt{x_2^2+y_2^2+2gx_2+2fy_2+c}=|\overline{BQ}|$$

Point C:

$$|\overline{CQ}|=\sqrt{x_3^2+y_3^2+2gx_3+2fy_3+c}=|\overline{CR}|$$

Point D:

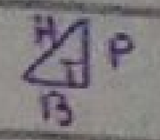
$$|\overline{DS}|=\sqrt{x_4^2+y_4^2+2gx_4+2fy_4+c}=|\overline{DR}|$$

$$\underbrace{|\overline{AP}|+|\overline{PB}|}+\underbrace{|\overline{CR}|+|\overline{RD}|}=|\overline{AS}|+|\overline{BQ}|+|\overline{CQ}|+|\overline{DS}|$$

$$|\overline{AB}|+|\overline{CD}|=|\overline{AD}|+|\overline{BC}|$$

Hence Proved.

Q6 Two tangents . . .



$x^2+y^2-8x-2y+14=0$

$x^2-8x+y^2-2y+14=0$

$[x^2-8x+(4)^2]+[y^2-2y+(1)^2]=-14+(4)^2+(1)^2$

$(x-4)^2+(y-1)^2=-14+16+1$

$(x-4)^2+(y-1)^2=(\sqrt{3})^2$

Center $C=(4,1)$

Radius $=\sqrt{3}$

So $C(4,1)$, $P(6,1)$

$|CP|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|CP|=\sqrt{(6-4)^2+(1-1)^2}$

$|CP|=\sqrt{(2)^2+0}$

$|CP|=2$ = hypotenuse

Area of triangle $=\frac{1}{2}\times$ base $\times$ height

need to find height,

$(H)^2=(B)^2+(P)^2$

$(2)^2=(\sqrt{3})^2+P^2$

$4-3=P^2$

$\sqrt{P^2}=\sqrt{1}$

$P=1$ = height

$=\frac{1}{2}\times\sqrt{3}\times 1$

$\boxed{=\frac{\sqrt{3}}{2}}$

Sector of Circle $=\frac{1}{2}r^2\theta$

finding θ

$\sin\theta=\frac{P}{H}$

$\sin\theta=\frac{1}{2}$

$\theta=\sin^{-1}\left(\frac{1}{2}\right)$

$\theta=\frac{\pi}{6}$

$=\frac{1}{2}(\sqrt{3})^2\left(\frac{\pi}{6}\right)$

$=\frac{\pi}{4}$

Area of shaded — Area of sector of Circle

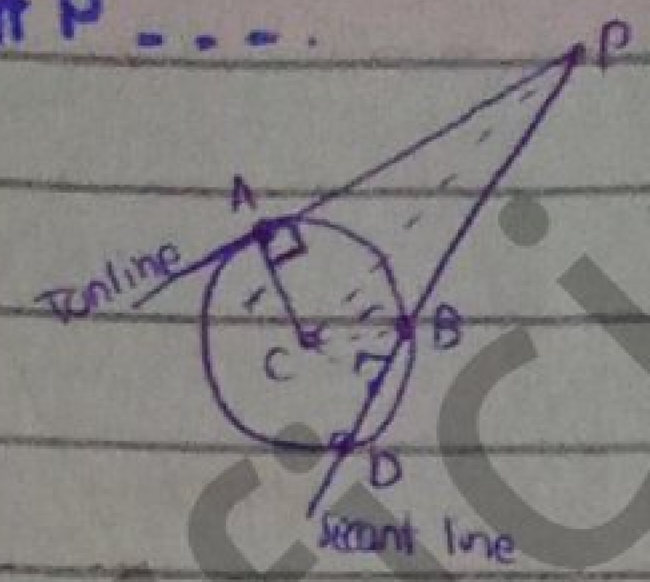
$=\left(\frac{\sqrt{3}}{2}-\frac{\pi}{4}\right)$

Total shaded area $=2\left(\frac{\sqrt{3}}{2}-\frac{\pi}{4}\right)$

$=\frac{2\sqrt{3}}{2}-\frac{2\pi}{4}$

$\boxed{=\left(\sqrt{3}-\frac{\pi}{2}\right)}$

Q7 From a point P . . .



Join C to P and draw $C\perp A$.

And draw $\perp$ bisector on BD.

In ΔACP

$|CP|^2=|CA|^2+|AP|^2$ — eq(i) $\because$ By pythagoras theorem

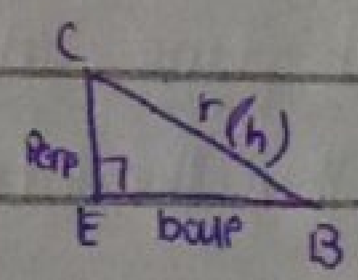
And ΔECP

$|CP|^2=|CE|^2+|EP|^2$ — eq(ii) $\because$ By pythagoras theorem

Compare eq(i) & eq(ii), $|CP|^2=|CP|^2$

$|CA|^2+|AP|^2=|CE|^2+|EP|^2$ — eq(iii)

Join C to B.



In ΔECB

$|CB|^2=|BE|^2+|EC|^2$ $\because$ Pythagoras theorem

$|CB|^2-|BE|^2=|CE|^2$ — eq(iv)

Using eq(iv) in eq(iii),

$|CA|^2+|AP|^2=(|CB|^2-|BE|^2)+|EP|^2$

$\cancel{r^2}+|AP|^2=\cancel{r^2}-|BE|^2+|EP|^2$

$|AP|^2=|EP|^2-|BE|^2$

$\because |EP|=|EB|+|BP|$

$|AP|^2=(|EB|+|BP|)^2-|BE|^2$

$|AP|^2=\cancel{|EB|^2}+2|EB||BP|+|BP|^2-\cancel{|BE|^2}$

$|AP|^2=2|EB||BP|+|BP|^2$

$|AP|^2=|BP|\cdot[|BP|+2|EB|]$

$|AP|^2=|PB|\cdot[|BP|+|BD|]$ $\because 2|\overline{EB}|=|\overline{BD}|$

$|\overline{AP}|^2=|\overline{PB}|\cdot|\overline{PD}|$ $\because \overline{BP}+\overline{BD}=\overline{PD}$ Hence Proved