## Studybeesofficial

*Where Minds Pollinate*

**Class 12**

**Mathematics**

Chapter 7

# Exercise 7.2

Handwritten Solution

$= \frac{|2(h)-1(k)+1|}{\sqrt{(2)^2+(-1)^2}}$

$\frac{|2h-k+1|}{\sqrt{4+1}}$

1

$=0$

nter (1,2)

cle:

r

# Ex 7.2

**Q1 Find the point of intersection...**

**(i) $x-y+1=0$ ; $x^2+y^2-3x-8=0$**

$x-y+1=0$ - eq(i)    $x^2+y^2-3x-8=0$ - eq(ii)

Solving eq(i),

$-y=-1-x$

$y=1+x$

Putting value of y in eq(ii)

$x^2+(1+x)^2-3x-8=0$

$x^2+1+2x+x^2-3x-8=0$

$2x^2-x-7=0$

$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(-7)}}{2(2)}$

$x=\frac{1\pm\sqrt{1+56}}{4}$

$x=\frac{1\pm\sqrt{57}}{4}$

$\boxed{x_1=\frac{1+\sqrt{57}}{4}}$    $\boxed{x_2=\frac{1-\sqrt{57}}{4}}$

Putting $x_1$ in eq(i)

$x-y+1=0$

$\frac{1+\sqrt{57}}{4}-y+1=0$

$-y=-1-\frac{1+\sqrt{57}}{4}$

$-y=-\frac{5+\sqrt{57}}{4}$

$\boxed{y_1=\frac{5+\sqrt{57}}{4}}$

Putting $x_2$ in eq(i)

$x-y+1=0$

$\frac{1-\sqrt{57}}{4}-y+1=0$

$-y=-1-\frac{1-\sqrt{57}}{4}$

$-y=-\frac{5-\sqrt{57}}{4}$

$y_2=\frac{5-\sqrt{57}}{4}$

$(x,y)=\left(\frac{1\pm\sqrt{57}}{4},\frac{5\pm\sqrt{57}}{4}\right)$

(ii) $2x+y+4\sqrt{5}=0$ —eq(i) ; $x^2+y^2-2x+4y-11=0$ —eq(ii)

Solving eq(i)

$$2x+y+4\sqrt{5}=0$$

$$y=-4\sqrt{5}-2x$$

Putting y in eq(ii),

$$x^2+y^2-2x+4y-11=0$$

$$x^2+(-4\sqrt{5}-2x)^2-2x+4(-4\sqrt{5}-2x)-11=0$$

$$x^2+(-(4\sqrt{5}+2x))^2-2x-16\sqrt{5}-8x-11=0$$

$$x^2+(4\sqrt{5}+2x)^2-10x-11-16\sqrt{5}=0$$

$$x^2+\left[(4\sqrt{5})^2+2(4\sqrt{5})(2x)+(2x)^2\right]-10x-11-16\sqrt{5}=0$$

$$x^2+\left[16(5)+16\sqrt{5}x+4x^2\right]-10x-11-16\sqrt{5}=0$$

$$x^2+80+16\sqrt{5}x+4x^2-10x-11-16\sqrt{5}=0$$

$$5x^2+x(16\sqrt{5}-10)+(69-16\sqrt{5})=0$$

$a=5$, $b=16\sqrt{5}-10$, $c=69-16\sqrt{5}$

$$x=\frac{-(16\sqrt{5}-10)\pm\sqrt{(16\sqrt{5}-10)^2-4(5)(69-16\sqrt{5})}}{2(5)}$$

$$x=\frac{(-16\sqrt{5}+10)\pm 0}{10}$$

$$\cancel{\boxed{x=\frac{10-16\sqrt{5}}{10}}} \qquad \boxed{x=\frac{5-8\sqrt{5}}{5}}$$

Putting x in eq(i),

$$2x+y+4\sqrt{5}=0$$

$$2\left(\frac{5-8\sqrt{5}}{5}\right)+y+4\sqrt{5}=0$$

$$\frac{10-16\sqrt{5}}{5}+y+4\sqrt{5}=0$$

$$y=-\left(\frac{10+4\sqrt{5}}{5}\right)$$

$$\boxed{y=\frac{-10-4\sqrt{5}}{5}}$$

$$(x,y)=\left(\frac{5-8\sqrt{5}}{5},\ \frac{-10-4\sqrt{5}}{5}\right)$$

(iii) $3x+2y+1=0$ —eq(i) ; $x^2+y^2-x+y+2=0$ —eq(ii)

Solving eq(i),

$$3x+2y+1=0$$

$$3x=-1-2y$$

$$x=\frac{-1-2y}{3}$$

Putting x in eq(ii),

$$\left(\frac{-1-2y}{3}\right)^2+y^2-\left(\frac{-1-2y}{3}\right)+y+2=0$$

$$\frac{(-(1+2y))^2}{9}+y^2-\left(\frac{-1-2y}{3}\right)+y+2=0$$

$$\frac{1+4y+4y^2}{9}+y^2+\frac{1}{3}+\frac{2y}{3}+y+2=0$$

$$\frac{1}{9}+\frac{4y}{9}+\frac{4y^2}{9}+y^2+\frac{1}{3}+\frac{2y}{3}+y+2=0$$

$$\frac{13}{9}y^2+\frac{19}{9}y+\frac{22}{9}=0$$

$\times 9$ on all sides,

$$13y^2+19y+22=0$$

$$y=\frac{-(19)\pm\sqrt{(19)^2-4(13)(22)}}{2(13)}$$

$$y=\frac{-19\pm\sqrt{361-1144}}{26}$$

$$y=\frac{-19\pm\sqrt{-783}}{26}$$

$$y=\frac{-19\pm\sqrt{783}\,i}{26}$$

$$\boxed{y=\frac{-19\pm3\sqrt{87}\,i}{26}}$$

Putting y in eq(i),

$$3x+2\left(\frac{-19\pm3\sqrt{87}\,i}{26}\right)+1=0$$

$$3x=-1-\cancel{2}\left(\frac{-19\pm3\sqrt{87}\,i}{\cancel{26}\,13}\right)$$

$$3x=-1-\left(\frac{-19\pm3\sqrt{87}\,i}{13}\right)$$

$$3x=\frac{6\pm3\sqrt{87}}{13}\,i$$

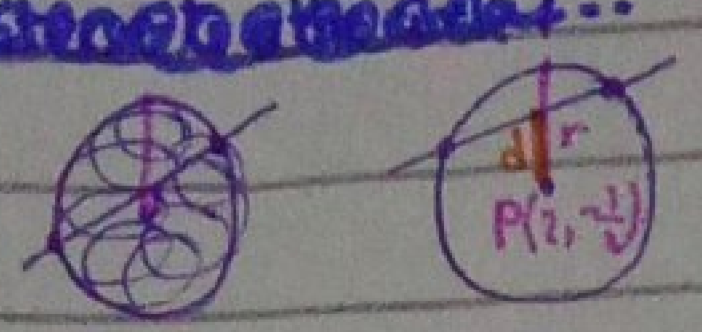
$$x=\frac{6\pm3\sqrt{87}\,i}{39}$$

$$x=\frac{3(2\pm\sqrt{87}\,i)}{39}$$

$$\boxed{x=\frac{2\pm\sqrt{87}\,i}{13}}$$

$$(x,y)=\left(\frac{2\pm\sqrt{87}\,i}{13},\ \frac{-19\pm3\sqrt{87}\,i}{26}\right)$$

## Q2 The equation of a circle ---




Equation of a circle,

$x^2+y^2-4x+y-7=0$

Completing square method

$(x^2-4x)+(y^2+y)-7=0$

$[x^2-4x+(2)^2]+[y^2+y+(\frac{1}{2})^2]=7+(2)^2+(\frac{1}{2})^2$

$(x-2)^2+(y+\frac{1}{2})^2=7+4+\frac{1}{4}$

$(x-2)^2+(y-(\frac{-1}{2}))^2=(\sqrt{\frac{45}{4}})^2$

$\because (x-h)^2+(y-k)^2=r^2$

center $(h,k)=(2,\frac{-1}{2})=P$

Radius $= \boxed{r=\sqrt{\frac{45}{4}}}$

Finding distance of line from point

$$d=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$$

line of equation $l=2x-y+C=0$ (as given)

$P(2,\frac{-1}{2})$

$$d=\frac{|2(2)+(-1)(\frac{-1}{2})+C|}{\sqrt{(2)^2+(-1)^2}}$$

$$d=\frac{|4+\frac{1}{2}+c|}{\sqrt{4+1}}$$

$$\boxed{d=\frac{|\frac{9}{2}+c|}{\sqrt{5}}}$$

(i). Case I: $d<r$

$$\frac{|\frac{9}{2}+C|}{\sqrt{5}}<\frac{\sqrt{45}}{2}$$

$$|\frac{9}{2}+C|<\frac{3(\sqrt{5})^2}{2}$$

$$\boxed{|\frac{9}{2}+C|<\frac{15}{2}} \text{ — eq } \Delta$$

$\frac{9}{2}+C<\frac{15}{2}$ | $-(\frac{9}{2}+C)<\frac{15}{2}$

inequality changes after x '-' on all sides

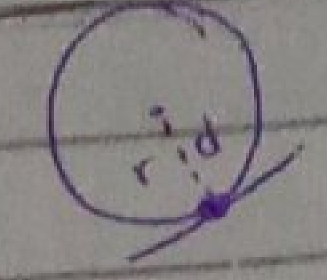
$\frac{9}{2}+C>\frac{-15}{2}$

$C<\frac{15}{2}-\frac{9}{2}$ | $-C<\frac{15}{2}+\frac{9}{2}$

$\boxed{C<3}$ | $\boxed{C>-12}$

$$\boxed{-12<C<3}$$

(ii) Case II:




$d=r$

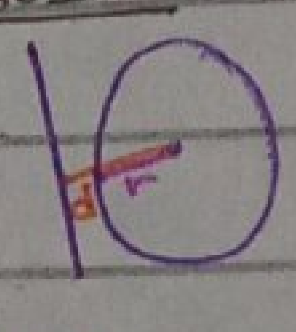
$|\frac{9}{2}+C|=\frac{15}{2}$ ∵ used the simplified version in eq $\Delta$

$\frac{9}{2}+C=\frac{15}{2}$ | $\frac{9}{2}+C=\frac{-15}{2}$

$C=\frac{15}{2}-\frac{9}{2}$ | $C=\frac{-15}{2}-\frac{9}{2}$

$\boxed{C=3}$ | $\boxed{C=-12}$

(iii) Case III:




$d>r$

$|\frac{9}{2}+C|>\frac{15}{2}$

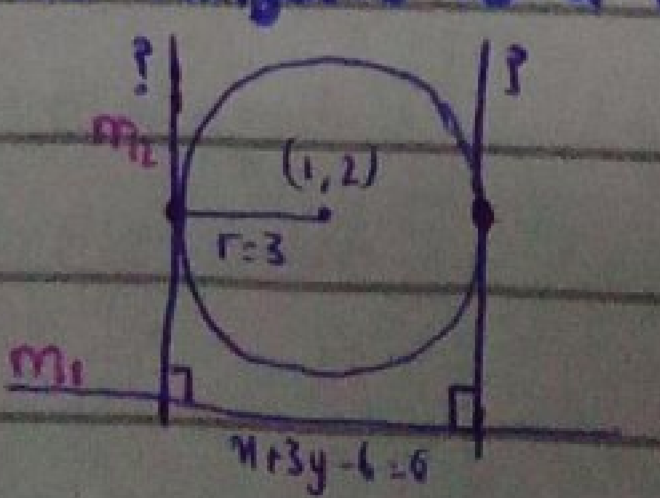
$\frac{9}{2}+C>\frac{15}{2}$ | $\frac{9}{2}+C<\frac{-15}{2}$

$C>\frac{15}{2}-\frac{9}{2}$ | $C<\frac{-15}{2}-\frac{9}{2}$

$C>3$ | $C<-12$

$\boxed{-\infty<C<-12}$ , $\boxed{3<C<\infty}$

## Q3 The tangents to a circle -..




Equation of a line,

$y=mx+C$ — ①

tangent line

Finding slope of tangent,

$$\boxed{\begin{aligned} &ax+by+C=0 \quad \because \text{General eq of line} \\ &by=-ax-C \\ &y=\frac{-a}{b}x-\frac{C}{b} \\ &y=mx+C \end{aligned}}$$

$m_1 = \cancel{\frac{-a}{b}} \quad \frac{-a}{b}$ ∵ Slope of line $x+3y-6=0$

$m_1 = \frac{-1}{3}$

So we know,

$m_1 \cdot m_2 = -1$

$m_2 = \frac{-1}{m_1}$

$m_2 = \frac{-1}{-1/3}$

$m_2 = 3$

Now putting $m_2$ in eq ①

$y = mx + C$

$\boxed{y = 3x + C}$ — eq (1)

Now to find C.

(rearranged eq(i)) $3x-y+C=0$, $P(1,2)$

$$d = \frac{|3(1)-1(2)+C|}{\sqrt{(3)^2+(-1)^2}}$$

$$3 = \frac{|3-2+C|}{\sqrt{10}}$$

$3\sqrt{10} = |1+C|$

$\pm 3\sqrt{10} = 1 + C$

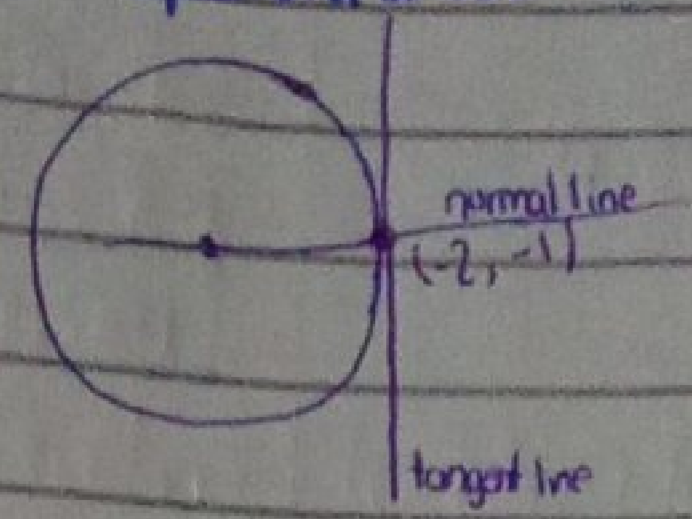
$\boxed{-1 \pm 3\sqrt{10} = C}$

So,

$y = 3x + C$

$\boxed{y = 3x - 1 \pm 3\sqrt{10}}$

## Q4 Find the equations of...




$x^2+y^2+3x+2y+3=0$

Differentiate w.r.t $x$,

$$\frac{d}{dx}(x^2)+\frac{d}{dx}(y^2)+3\frac{d}{dx}(x)+2\frac{d}{dx}(y)+\frac{d}{dx}(3)=0$$

$$2x+2y\frac{dy}{dx}+3+2\frac{dy}{dx}+0=0$$

$$2y\frac{dy}{dx}+2\frac{dy}{dx}=-2x-3$$

$$\frac{dy}{dx}(2y+2)=-2x-3$$

$$\frac{dy}{dx}=\frac{-2x-3}{2y+2}$$

This is the general slope of the tangent line

$$\left.\frac{dy}{dx}\right|_{(-2,-1)}=\frac{-2(-2)-3}{2(-1)+2}$$

$m = \frac{4-3}{-2+2}$

$m = \frac{1}{0}$

$m = \infty$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y-(-1) = \frac{1}{0}(x-(-2))$

$y+1 = \frac{1}{0}(x+2)$

$0(y+1) = 1(x+2)$

$0 = x+2$

$\boxed{x+2=0}$ equation of tangent line

Slope of normal line equation:

$m = \frac{-0}{1}$

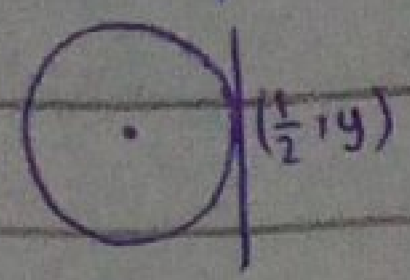
$m = 0$

$y - y_1 = m(x - x_1)$

$y+1 = 0(x+2)$

$\boxed{y+1=0}$ equation of normal line

## Q5 Find the equation...




$36x^2+36y^2-72y+11=0$ ; $(\frac{1}{2}, y)$

Putting this in this

$36(\frac{1}{2})^2+36(y)^2-72(y)+11=0$

$36(\frac{1}{4})+36y^2-72y+11=0$

$36y^2-72y+11+9=0$

$36y^2-72y+20=0$

$4(9y^2-18y+5)=0$

$9y^2-18y+5=0$

$9y^2-3y-15y+5=0$

$3y(3y-1)-5(3y-1)=0$

$(3y-1)(3y-5)=0$

$3y-1=0$, $\boxed{y=\frac{1}{3}}$ | $3y-5=0$, $\boxed{y=\frac{5}{3}}$

$(\frac{1}{2},\frac{1}{3})$ , $(\frac{1}{2},\frac{5}{3})$

Finding slope:

$36x^2+36y^2-72y+11=0$

$36\frac{d}{dx}(x^2)+36\frac{d}{dx}(y^2)-72\frac{d}{dx}(y)+\frac{d}{dx}(11)=0$

$72x+72y\frac{dy}{dx}-72\frac{dy}{dx}+0=0$

$72(x+y\frac{dy}{dx}-\frac{dy}{dx})=0$

$x+y\frac{dy}{dx}-\frac{dy}{dx}=0$

$y\frac{dy}{dx}-\frac{dy}{dx}=-x$

$\frac{dy}{dx}(y-1)=-x$

$\frac{dy}{dx}=\frac{-x}{y-1}$

$\frac{dy}{dx}=\frac{x}{1-y}$

$\frac{dy}{dx}\Big|_{(\frac{1}{2},\frac{5}{3})}=\frac{1/2}{1-5/3}$

$=\frac{\frac{1}{2}}{-2/3}$

$m_1=\frac{-3}{4}$

So now

$y-y_1=m(x-x_1)$

$y-\frac{5}{3}=\frac{-3}{4}(x-\frac{1}{2})$

$\boxed{18x+24y-49=0}$ eq of tangent at point $(\frac{1}{2},\frac{5}{3})$

For, $(\frac{1}{2},\frac{1}{3})$

$\frac{dy}{dx}\Big|_{(\frac{1}{2},\frac{1}{3})}=\frac{1/2}{1-1/3}$

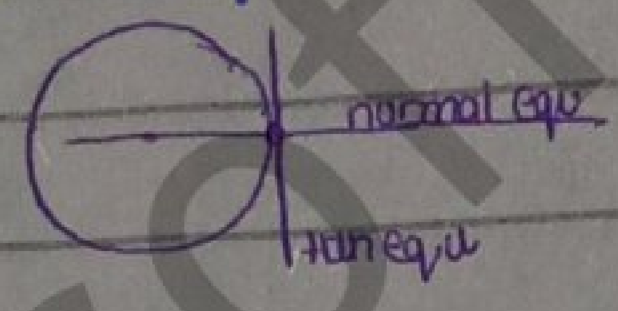
$m_2=\frac{3}{4}$

So, $y-y_1=m(x-x_1)$

$y-\frac{1}{3}=\frac{3}{4}(x-\frac{1}{2})$

$\boxed{18x-24y-1=0}$ eq of tan at point $(\frac{1}{2},\frac{1}{3})$

## Q6 Find the equation...




$y=-4$

$x^2+(-4)^2-2x+2(-4)-11=0$

$x^2+16-2x-8-11=0$

$x^2-2x-3=0$

$x^2-3x+x-3=0$

$x(x-3)+1(x-3)=0$

$(x+1)(x-3)=0$

$\boxed{x_1=-1}$ | $\boxed{x_2=3}$

$(-1,-4)$ | $(3,-4)$

Finding slope:

$x^2+y^2-2x+2y-11=0$

$\frac{d}{dx}(x^2)+\frac{d}{dx}(y^2)-2\frac{d}{dx}(x)+2\frac{d}{dx}(y)-\frac{d}{dx}(11)=0$

$2x+2y\frac{dy}{dx}-2+2\frac{dy}{dx}-0=0$

$2y\frac{dy}{dx}+2\frac{dy}{dx}=2-2x$

$\frac{dy}{dx}(2y+2)=2-2x$

$\frac{dy}{dx}=\frac{2-2x}{2y+2} \Rightarrow \frac{dy}{dx}=\frac{2(1-x)}{2(y+1)} \Rightarrow \frac{dy}{dx}=\frac{1-x}{y+1}$

Slope of normal,

$\frac{dx}{dy}=-\left(\frac{y+1}{1-x}\right)$

$\frac{dx}{dy}=\frac{-y-1}{1-x}$

$\frac{dx}{dy}=\frac{-(y+1)}{-(x-1)}$

$\boxed{\frac{dx}{dy}=\frac{y+1}{x-1}}$

(i) for $(-1,-4)$

$$\frac{dy}{dx}\Big|_{(-1,-4)} = \frac{-4+1}{-1-1}$$

$$m_1 = \frac{-3}{-2}$$

$$m_1 = \frac{3}{2}$$

Eq of normal at $(-1,-4)$,

$$y - y_1 = m(x - x_1)$$

$$y-(-4) = \frac{3}{2}(x-(-1))$$

$$y+4 = \frac{3}{2}(x+1)$$

$$2y+8 = 3x+3$$

$$\boxed{3x-2y-5=0}$$

(ii) for $(3,-4)$

$$\frac{dy}{dx}\Big|_{(3,-4)} = \frac{-4+1}{3-1}$$

$$m_2 = \frac{-3}{2}$$

Eq of normal at $(3,-4)$,

$$y-y_1 = m(x-x_1)$$

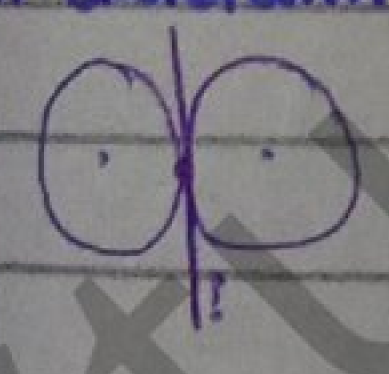
$$y-(-4) = \frac{-3}{2}(x-3)$$

$$y+4 = \frac{-3}{2}(x-3)$$

$$2y+8 = -3x+9$$

$$\boxed{3x+2y-1=0}$$

**Q7 Circles with ---**



$x^2+y^2-2y-3=0$ — eq(i)

$x^2+y^2-8x+4y+11=0$ — eq(ii)

Subtracting eq(ii) & eq(i),

$$\cancel{x^2}+\cancel{y^2}-8x+4y+11=0$$

$$\cancel{\pm x^2}\cancel{\pm y^2} \mp 2y \mp 3=0$$

$$-8x+6y+14=0$$

$$6y-8x+14=0$$

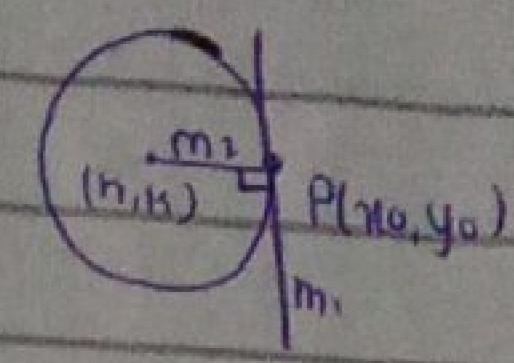
$$2(3y-4x+7)=0$$

$$3y-4x+7=0$$

$$-4x+3y+7=0$$

$$\boxed{4x-3y-7=0}$$

**Q8 Show that the ---**




Equ of a circle.

$$(x-h)^2+(y-k)^2=r^2$$

$$\frac{d}{dx}(x-h)^2+\frac{d}{dx}(y-k)^2=\frac{d}{dx}(r^2)$$

$$2(x-h)\left(\frac{d}{dx}(x)-\frac{d}{dx}(h)\right)+2(y-k)\left(\frac{d}{dx}(y)-\frac{d}{dx}(k)\right)=0$$

$$2(x-h)(1-0)+2(y-k)\left(\frac{dy}{dx}-0\right)=0$$

$$2(x-h)+2(y-k)\frac{dy}{dx}=0$$

$$2\left[(x-h)+(y-k)\frac{dy}{dx}\right]=0$$

$$(y-k)\frac{dy}{dx}=-(x-h)$$

$$\boxed{m_1=\frac{dy}{dx}=\frac{-(x-h)}{(y-k)}}$$ slope of tangent

Slope of radial line,

$\underset{x_1\ y_1}{(h,k)}$, $\underset{x_2\ y_2}{(x_0,y_0)}$

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m_2=\frac{y_0-k}{x_0-h}$$

$$\boxed{m_2=\frac{y-k}{x-h}}$$

Now,

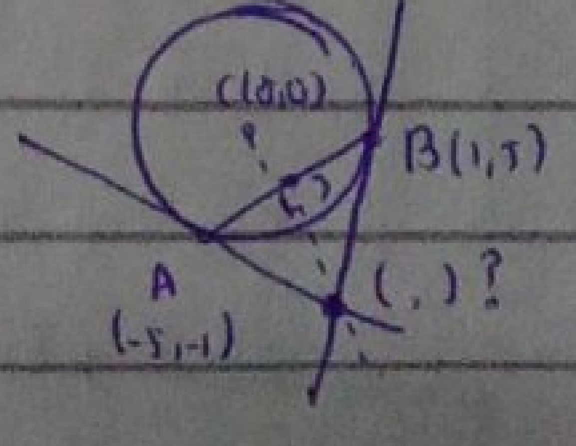
$$m_1 \cdot m_2=-1$$

$$\left(\frac{-(x-h)}{(y-k)}\right)\cdot\left(\frac{(y-k)}{(x-h)}\right)=-1$$

$$-1=-1$$

Hence proved $m_1 \perp m_2$

**Q9 A(-5,-1) & B(1,5) are two points . . .**




$$\text{Midpoint of } \overline{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-5+1}{2}, \frac{-1+5}{2}\right)$$

$$= (-2,2)$$

$x^2+y^2=26$

$2x+2y\frac{dy}{dx}=0$ derivative liya idhar

$\frac{dy}{dx}=\frac{-2x}{2y}$

$\frac{dy}{dx}=\frac{-x}{y}$

for A(-5,-1),

$\left.\frac{dy}{dx}\right|_{(-5,-1)}=\frac{5}{-1}$

$m=-5$

Equ of tangent A

$y-y_1=m(x-x_1)$

$y+1=-5(x+5)$

$y+1=-5x-25$

$\boxed{5x+y+26=0}$ — eq(i)

for B(1,5),

$\left.\frac{dy}{dx}\right|_{(1,5)}=\frac{-1}{5}$

$m=\frac{-1}{5}$

Equ of tangent B

$y-5=-\frac{1}{5}(x-1)$

$5y-25=-x+1$

$\boxed{x+5y-26=0}$ — eq(ii)

~~Solving eq(i) & eq(ii)~~ × eq(i) with 5 & subtr with eq(ii)

$25x+\cancel{5y}+130=0$

$\pm x \cancel{\pm 5y} \mp 26=0$

$24x+156=0$

$\boxed{x=\frac{-13}{2}}$

Put in eq(i),

$5\left(-\frac{13}{2}\right)+y+26=0$

$\boxed{y=\frac{13}{2}}$

Point of intersection of tangent is $\left(\frac{-13}{2},\frac{13}{2}\right)$

for Collinear,

$$=\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$=\begin{vmatrix} 0 & 0 & 1 \\ -2 & 2 & 1 \\ \frac{-13}{2} & \frac{13}{2} & 1 \end{vmatrix}$$

Expanding by $R_1$:

$$=1\begin{vmatrix} -2 & 2 \\ -13/2 & 13/2 \end{vmatrix}$$

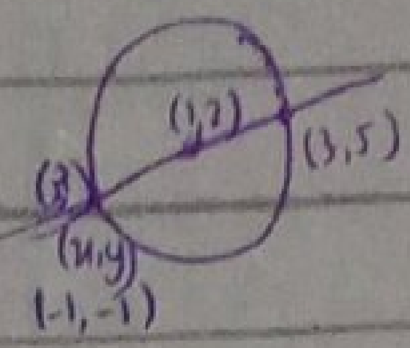
$=-2\left(\frac{13}{2}\right)-\left(-\frac{13}{2}\right)(2)$

$=-13+13$

$=0$

Hence proved.

**Q10 A normal line cuts ...**




→ Finding point of intersection:

Midpoint of chord:

$(1,2)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$(1,2)=\left(\frac{x+3}{2},\frac{y+5}{2}\right)$

$1=\frac{x+3}{2}$, $2=\frac{y+5}{2}$

$2=x+3$, $4=y+5$

$\boxed{x=-1}$, $\boxed{y=-1}$

$\boxed{(-1,-1)}$

→ Finding equation of a circle;

radius $=r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$r=\sqrt{(1+1)^2+(2+1)^2}$

$r=\sqrt{4+9}$

$\boxed{r=\sqrt{13}}$

$(x-h)^2+(y-k)^2=r^2$

$(x-1)^2+(y-2)^2=(\sqrt{13})^2$

$x^2-2x+1+y^2-4y+4=13$

$x^2+y^2-2x-4y+5-13=0$

$\boxed{x^2+y^2-2x-4y-8=0}$