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*Where Minds Pollinate*

## Class 12
## Mathematics

Chapter 7

# Exercise 7.1

Handwritten Solution

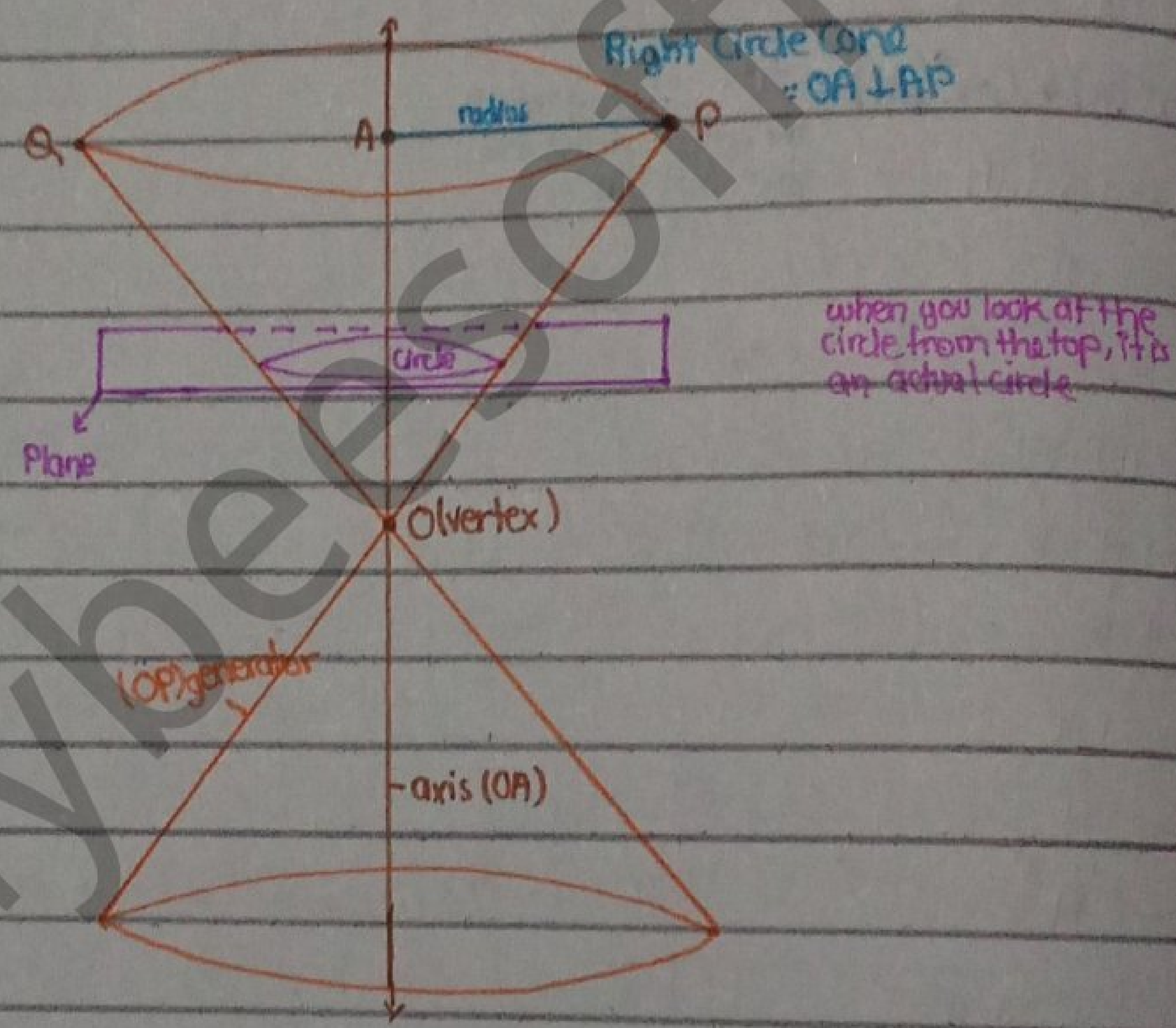
# Unit 7

## Conic Section

Conic = Cone (shape)

- The name means making sections of the shape cone.

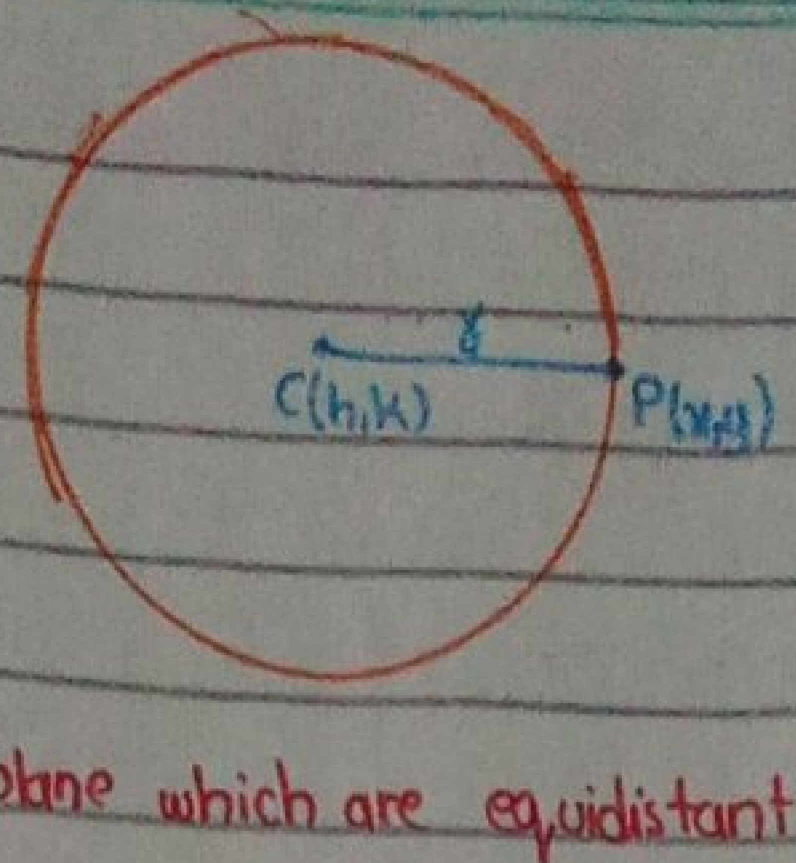
- The diagram is made with the help of a fixed axis and a generator.
- The cone is called Right Circular Cone, because the radius of the generator with the fixed axis is a $90^\circ$ angle which means that the fixed axis and the generator are perpendicular. $OA \perp AP$




Lets only consider the top half of the cone, The making of the circle:
We cut the cone at any point on a plane, and when we look at it from above its a circle. Now there ~~are~~ is a condition for cutting this cone,

The condition is,

When we cut a cone with a plane. The plane must be perpendicular to the axis of the cone and not passing through the vertex.




**Defination:** Set of all points in the plane which are equidistant (equal distance) from fixed point, then the fixed point is called Centre. And the distance of any point, on the circle, from the centre is called radius of a circle.

## Equation of a Circle:

~~Using the distance formula bcs we learnt that the distance on the circle is equal to the~~

Using the distance formula bcs we learnt that the distance from any point on the circle to the centre of the circle is equal.

$$\text{Distance} = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$

$$CP = \sqrt{(y-k)^2+(x-h)^2}$$

$\because CP = \text{radius}$ $\quad r = \sqrt{(y-k)^2+(x-h)^2}$

$$(r)^2 = \left(\sqrt{(y-k)^2+(x-h)^2}\right)^2$$

$$\boxed{r^2 = (y-k)^2+(x-h)^2} \quad — (1)$$

This is the standard equation of a circle

There is another equation of a circle,

if $(h,k) = (0,0)$

$$r^2 = (y-0)^2+(x-0)^2$$

$$r^2 = y^2+x^2$$

$$x^2+y^2 = r^2$$

But this equation is only for when the center of the circle is at the origin

Now, using the standard equation of a circle, finding the general equation of a circle.

$$r^2 = (y-k)^2+(x-h)^2$$

$$r^2 = y^2 - 2yk + k^2 + x^2 - 2hx + h^2$$

$$x^2+y^2-2hx-2yk+h^2+k^2-r^2 = 0$$

let, $h^2+k^2-r^2 = C$, $-h=g$, $-k=f$ ($h^2+k^2-r^2$ are all constant) (we also switch all the negative terms)

$$\boxed{x^2+y^2+2xg+2fy+C=0} \quad — (2)$$

This is the general equations of a circle.

Finding the center and the radius of the general equation ②,

$(x^2+2gx)+(y^2+2fy) = -c$

Do the completing square method.

$+ (g)^2$ & $(f)^2$ on b/s,

$[x^2+2gx+(g)^2]+[y^2+2fy+(f)^2] = -c+g^2+f^2$

$(x+g)^2+(y+f)^2 = g^2+f^2-c$

let $g^2+f^2-c = r^2$

$r = \sqrt{g^2+f^2-c}$ - is the radius

$(x-(-g))^2+(y-(-f))^2 = r^2$

So $(-g,-f)$ is the centre

**Ex 7.1**

**Q1 Find the equation....**

**(i) center at $(\overset{h}{3},\overset{k}{-1})$ and radius $\overset{r}{2}$**

$\because (x-h)^2+(y-k)^2=r^2$

$(x-3)^2+(y+1)^2=(2)^2$

$x^2-6x+9+y^2+2y+1=4$

$x^2+y^2-6x+2y+10-4=0$

$\boxed{x^2+y^2-6x+2y+6=0}$

**(ii) Center at $\left(\frac{-1}{2},\frac{-1}{3}\right)$ & radius $\frac{1}{5}$**

$\because (x-h)^2+(y-k)^2=r^2$

$\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{3}\right)^2=\left(\frac{1}{5}\right)^2$

$\left(\frac{2x+1}{2}\right)^2+\left(\frac{3y+1}{3}\right)^2=\frac{1}{25}$

$\frac{4x^2+4x+1}{4}+\frac{9y^2+6y+1}{9}=\frac{1}{25}$

×36 on b/s,

$36\left(\frac{4x^2+4x+1}{4}\right)+36\left(\frac{9y^2+6y+1}{9}\right)=36\left(\frac{1}{25}\right)$

$9(4x^2+4x+1)+4(9y^2+6y+1)=\frac{36}{25}$

$36x^2+36x+9+36y^2+24y+4=\frac{36}{25}$

$25(36x^2+36y^2+36x+24y+13)=36$

$900x^2+900y^2+900x+600y+325-36=0$

$\boxed{900x^2+900y^2+900x+600y+289=0}$

(iii) center at $\left(\frac{1}{a}, a\right)$ & radius is $a$ ; $a \neq 0$

$\because (x-h)^2+(y-k)^2=r^2$

$\left(x-\frac{1}{a}\right)^2+(y-a)^2=a^2$

$\left(\frac{ax-1}{a}\right)^2+(y^2-2ay+a^2)=a^2$

$\frac{a^2x^2-2ax+1}{a^2}+y^2-2ay+a^2-a^2=0$

$\frac{a^2x^2-2ax+1+a^2(y^2-2ay)}{a^2}=0$

$a^2x^2-2ax+1+a^2y^2-2a^3y=0(a^2)$

$\boxed{a^2x^2+a^2y^2-2ax-2a^3y+1=0}$

**Q2 Convert the following...**

(i) $x^2+y^2-4x+6y-36=0$

$\because$ Standard form of circle,

$(x-h)^2+(y-k)^2=r^2$ , center are $(h,k)$ radius is $r$

$(x^2-4x)+(y^2+6y)=36$ (using completing square method)

$+(2)^2$ & $(3)^2$ on b/s,

$(x^2-4x+(2)^2)+(y^2-6y+(3)^2)=36+(2)^2+(3)^2$

$(x-2)^2+(y+3)^2=36+4+9$

$(x-2)^2+(y-(-3))^2=49$

$(x-2)^2+(y-(-3))^2=(7)^2$

Standard form of the equation

Center $=(2,-3)$

radius $=7$

(ii) $5x^2+5y^2-2x+4y-27=0$ make sure no value or number is being multiplied with $x^2$ & $y^2$.

$\div 5$ on all sides,

$x^2+y^2-\frac{2}{5}x+\frac{4}{5}y-\frac{27}{5}=0$

$\left(x^2-\frac{2}{5}x\right)+\left(y^2+\frac{4}{5}y\right)=\frac{27}{5}$

$+\left(\frac{1}{5}\right)^2$ & $\left(\frac{2}{5}\right)^2$ on all sides, $\left(\frac{2}{5}\right)^2$

$\left[x^2-\frac{2}{5}x+\left(\frac{1}{5}\right)^2\right]+\left[y^2+\frac{4}{5}y+\left(\frac{2}{5}\right)^2\right]=\frac{27}{5}+\left(\frac{1}{5}\right)^2+$

$\left(x-\frac{1}{5}\right)^2+\left(y+\frac{2}{5}\right)^2=\frac{27}{5}+\frac{1}{25}+\frac{4}{25}$

$\left(x-\frac{1}{5}\right)^2+\left(y-\left(-\frac{2}{5}\right)\right)^2=\left(\sqrt{\frac{28}{5}}\right)^2$

Center $=\left(\frac{1}{5}, \frac{-2}{5}\right)$

Radius $=\sqrt{\frac{28}{5}}$

(iii) $4x^2+4y^2+2ax+by-a^2=0$

$\div 4$ on b/s,

$x^2+y^2+\frac{1}{4}ax+\frac{1}{4}by=\frac{a^2}{4}$

$x^2+y^2+\frac{ax}{2}+\frac{by}{4}=\frac{a^2}{4}$

$\left(x^2+\frac{ax}{2}\right)+\left(y^2+\frac{by}{4}\right)=\frac{a^2}{4}$

$+\left(\frac{a}{4}\right)^2$ & $\left(\frac{b}{8}\right)^2$ on b/s, $\left(\frac{b}{8}\right)^2$

$\left[x^2+\frac{ax}{2}+\left(\frac{a}{4}\right)^2\right]+\left[y^2+\frac{by}{4}+\left(\frac{b}{8}\right)^2\right]=\frac{a^2}{4}+\left(\frac{a}{4}\right)^2+$

$\left(x+\frac{a}{4}\right)^2+\left(y+\frac{b}{8}\right)^2=\frac{a^2}{4}+\frac{a^2}{16}+\frac{b^2}{64}$

$\left(x+\frac{a}{4}\right)^2+\left(y+\frac{b}{8}\right)=\frac{16a^2+4a^2+b^2}{64}$

$\left(x-\left(\frac{-a}{4}\right)\right)^2+\left(y-\left(\frac{-b}{8}\right)\right)^2=\left(\sqrt{\frac{20a^2+b^2}{64}}\right)^2$

Center $=\left(\frac{-a}{4}, \frac{-b}{8}\right)$

Radius $=\sqrt{\frac{20a^2+b^2}{64}}$

**Q3 Find the equation...**

(i) $(0,2),(2,0),(1,3)$

therefore given non-collinear points lie on the circle.

Equation of circle, (general)

$x^2+y^2+2gx+2fy+C=0$

$\Rightarrow (0,2)$ pass through the equation of a circle

$(0)^2+(2)^2+2g(0)+2f(2)+C=0$

$0+4+0+4f+C=0$

$4f+C=-4$ — eq(i)

$\Rightarrow (2,0)$

$(2)^2+(0)^2+2g(2)+2f(0)+C=0$

$4+4g+C=0$

$4g+C=-4$ — eq(ii)

=> (1,3)

$(1)^2+(3)^2+2g(1)+2f(3)+C=0$

$1+9+2g+6f+C=0$

$2g+6f+C=-10$ — eq(iii)

Using eq(i) & eq(ii) and subtracting them,

$4f+\cancel{C}=-4$

$\pm 4g \pm \cancel{C}=\mp 4$

$4f-4g=0$

$\cancel{4}f=\cancel{4}g$

$f=g$

Using eq value in eq(iii),

$2g+6f+C=-10$

$2g+6(g)+C=-10$

$2g+6g+C=-10$

$8g+C=-10$ — eq(iv)

Solving eq(i) & eq(iv),

$4g+\cancel{C}=-4$

$\pm 8g \pm \cancel{C}=\mp 10$

$-4g=6$

$\boxed{g=\frac{-3}{2}}$

As, $f=g$

$\boxed{f=\frac{-3}{2}}$

Putting values of f & g in eq(iii),

$2g+6f+C=-10$

$2\left(\frac{-3}{2}\right)+6\left(\frac{-3}{2}\right)+C=-10$

$-3-9+C=-10$

$-12+C=-10$

$C=-10+12$

$\boxed{C=2}$

Hence,

$x^2+y^2+2gx+2fy+C=0$

$x^2+y^2+2\left(\frac{-3}{2}\right)x+2\left(\frac{-3}{2}\right)y+2=0$

$x^2+y^2-3x-3y+2=0$

This is the required equation of a circle passing through 3 collinear points.

(ii) (1,3), (3,6), (5,7)

$x^2+y^2+2gx+2fy+C=0$

=> (1,3)

$(1)^2+(3)^2+2g(1)+2f(3)+C=0$

$1+9+2g+6f+C=0$

$2g+6f+C=-10$ — eq(i)

=> (3,6)

$(3)^2+(6)^2+2g(3)+2f(6)+C=0$

$9+36+6g+12f+C=0$

$6g+12f+C=-45$ — eq(ii)

=> (5,7)

$(5)^2+(7)^2+2g(5)+2f(7)+C=0$

$25+49+10g+14f+C=0$

$10g+14f+C=-74$ — eq(iii)

× eq(i) with 5 and subtracting with eq(iii),

$5(2g)+5(6f)+5(C)=5(-10)$

$\cancel{10g}+30f+5C=-50$

$\pm \cancel{10g} \pm 14f \pm C=\mp 74$

$16f+4C=24$

$16f=24-4C$

$f=\frac{24-4C}{16}$

× eq(i) with 2 and subtracting with eq(ii),

$2(2g)+2(6f)+2(C)=-30$

$4g+\cancel{12f}+2C=-30$

$\pm 6g \pm \cancel{12f} \pm C=\mp 45$

$-2g+C=15$

$2g=C-15$

$g=\frac{C-15}{2}$

Putting the values of f & g in eq(i)

$2\left(\frac{C-15}{2}\right)+6\left(\frac{24-4C}{16}\right)+C=-10$

$C-15+3\left(\frac{24-4C}{8}\right)+C=-10$

$C-15+\frac{72-12C}{8}+C=-10$

$2C=-10+15-\frac{72-12C}{8}$

$2C = 25 - \frac{72-12C}{8}$

$2C = \frac{200-72-12C}{8}$

$8\times 2C = 200-72-12C$

$16C+12C = 128$

$28C = 128$

$C = \frac{128}{28}$

$C$

$2C = 5 - \frac{72+12C}{8}$

$2C = \frac{40-72-12C}{8}$

$8\times 2C = -32-12C$

$16C+12C = -32$

× eq(i) with 3 & subtracting with eq(ii)

$3(2g)+3(6f)+3(C) = 3(-10)$

$6g+18f+3C = -30$

$\pm 6g \pm 12f \pm C = \mp 45$

$6f+2C = 15$ — eq(iv)

$6f = 15-2C$

$f = \frac{15-2C}{6}$

× eq(i) with 2 & subtracting with eq(ii),

$2(2g)+2(6f)+2(C) = 2(-10)$

$4g+12f+2C = -20$

$\pm 6g \pm 12f \pm C = \mp 45$

$-2g+C = 25$

$2g = C-25$

$g = \frac{C-25}{2}$

Putting values of f & g in eq(i),

$2g+6f+C = -10$

$2\left(\frac{C-25}{2}\right)+6\left(\frac{15-2C}{6}\right)+C = -10$

$C-25+15-2C+C = -10$

$C-2C+C = -10$

× eq(i) with 5 & subtracting with eq(iii),

$5(2g)+5(6f)+5(C) = 5(-10)$

$10g+30f+5C = -50$

$\pm 10g \pm 14f \pm C = \mp 74$

$16f+4C = 24$ — eq(v)

× eq(iv) with 2 & then subtracting with eq(v).

$2(6f)+2(2C) = 2(15)$

$12f+4C = 30$

$\pm 16f \pm 4C = \pm 24$

$-4f = 6$

$f = -\frac{6}{4}$

$\boxed{f = \frac{-3}{2}}$

Putting value of f in eq(iv),

$6f+2C = 15$

$6\left(\frac{-3}{2}\right)+2C = 15$

$-9+2C = 15$

$2C = 15+9$

$2C = 24$

$C = \frac{24}{2}$

$\boxed{C = 12}$

Putting the values of C & f in eq(i),

$2g+6f+C = -10$

$2g+6\left(\frac{-3}{2}\right)+12 = -10$

$2g = -10-12+9$

$\boxed{g = \frac{-13}{2}}$

Putting in the values,

$x^2+y^2+2gx+2fy+C = 0$

$x^2+y^2+2\left(\frac{-13}{2}\right)x+2\left(\frac{-3}{2}\right)y+12 = 0$

$\boxed{x^2+y^2-13x-3y+12=0}$

(iii) $(0,0), (a,0), (0,b);\ a\neq 0, b\neq 0$

$x^2+y^2+2gx+2fy+C=0$

$\Rightarrow (0,0)$

$(0)^2+(0)^2+2g(0)+2f(0)+C=0$

$\boxed{C=0}$

$\Rightarrow (a,0)$

$a^2+(0)^2+2g(a)+2f(0)+C=0$

$a^2+2ag+C=0$

$a^2+2ag+0=0$ ∵ Putting in $C=0$

$a^2+2ga=0$

$2ga=-a^2$

$g=\frac{-a^2}{2a}$

$\boxed{g=\frac{-a}{2}}$

$\Rightarrow (0,b)$

$(0)^2+(b)^2+2g(0)+2f(b)+C=0$

$b^2+2fb+C=0$

$b^2+2fb+0=0$

$2fb=-b^2$

$f=\frac{-b^2}{2b}$

$\boxed{f=\frac{-b}{2}}$

Putting in the values,

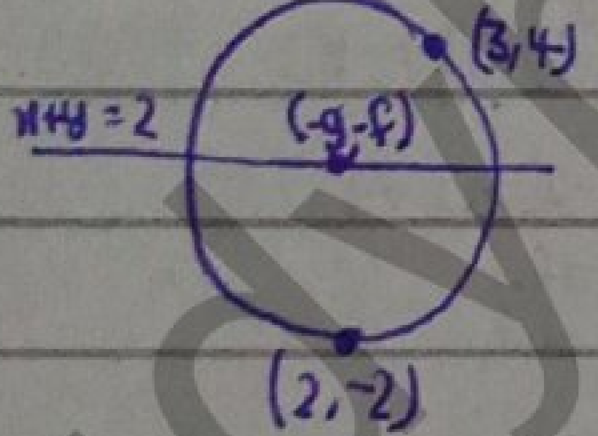
$x^2+y^2+2gx+2fy+C=0$

$x^2+y^2+2\left(\frac{-a}{2}\right)x+2\left(\frac{-b}{2}\right)y+0=0$

$x^2+y^2-ax-by+0=0$

$\boxed{x^2+y^2-ax-by=0}$

**Q4 Find the equation of circle with..**




General Equation of Circle,

$x^2+y^2+2gx+2fy+C=0$ — ①

$-g-f=2$ — ②

{this is bcs, the line x+y=2 is passing through the center and the values of the center is (-g,-f), so the values of the line will be the same}

We know the points (3,4) & (2,-2) are also passing through the circle so they will satisfy the equation of a circle ①,

$\Rightarrow (3,4)$

$(3)^2+(4)^2+2g(3)+2f(4)+C=0$

$9+16+6g+8f+C=0$

$6g+8f+C=-25$ — eq(i)

$\Rightarrow (2,-2)$

$(2)^2+(-2)^2+2g(2)+2f(-2)+C=0$

$4+4+4g-4f+C=0$

$4g-4f+C=-8$ — eq(ii)

Finding the values of g, f, C,

Subtracting eq(ii) from eq(i)

$6g+8f+\cancel{C}=-25$

$\underline{\pm 4g \mp 4f \pm \cancel{C} = \pm 8}$

$2g+12f=-17$ — eq(iii)

~~x eq(ii) with 2 & solve with eq(iii),~~

~~$4g+24f=-34$~~

~~$\pm 4g \mp 4f \pm$~~

x eq② with 2 and add with eq(iii)

$-\cancel{2g}-2f=4$

$\underline{\cancel{2g}+12f=-17}$

$10f=-13$

$\boxed{f=\frac{-13}{10}}$

Putting the value of f in eq(iii)

$2g+12\left(\frac{-13}{10}\right)=-17$

$2g-\frac{78}{5}=-17$

$2g=-17+\frac{78}{5}$

$2g=\frac{-7}{5}$

$g=\frac{-7}{5}\times\frac{1}{2}$

$\boxed{g=\frac{-7}{10}}$

Putting values of f & g in eq(i),

$6\left(\frac{-7}{10}\right)+8\left(\frac{-13}{10}\right)+C=-25$

$-\frac{21}{5}-\frac{52}{5}+C=-25$

$C=-25+\frac{73}{5}$

$\boxed{C=\frac{-52}{5}}$

Putting the values in eq①,

$x^2+y^2+\cancel{2}\left(\frac{-7}{\cancel{10}5}\right)x+\cancel{2}\left(\frac{-13}{\cancel{10}5}\right)y+\left(\frac{-52}{5}\right)=0$

$x^2+y^2-\frac{7}{5}x-\frac{13}{5}y-\frac{52}{5}=0$

x 5 on b/s,

$5x^2+5y^2-7x-13y-52=0$

∵ we just want to remove the fraction.

## Q5 Find the equation...

$x^2+y^2+2gx+2fy+C=0$ — ①

So,

$3x+4y=7$

$-3g-4f=7$ — eq(i)

$\Rightarrow (1,-2)$

$(1)^2+(-2)^2+2g(1)+2f(-2)+C=0$

$1+4+2g-4f+C=0$

$2g-4f+C=-5$ —eq(ii)

$\Rightarrow (4,-3)$,

$(4)^2+(-3)^2+2g(4)+2f(-3)+C=0$

$16+9+8g-6f+C=0$

$8g-6f+C=-25$ —eq(iii)

Sub eq(iii) from eq(ii),

$2g-4f+\cancel{C}=-5$

$\pm 8g \mp 6f \pm \cancel{C} = \mp 25$

$-6g+2f=20$ —eq(iv)

×eq(i) with 2 & sub with eq(iv),

$-\cancel{6g}-8f=14$

$\mp \cancel{6g} \pm 2f = \pm 20$

$-10f=-6$

$f=\frac{-6}{-10}=\frac{3}{5}$

$\boxed{f=\frac{3}{5}}$

Putting value of f in eq(iv),

$-6g+2\left(\frac{3}{5}\right)=20$

$-6g+\frac{6}{5}=20$

$-6g=20-\frac{6}{5}$

$-6g=\frac{94}{5}$

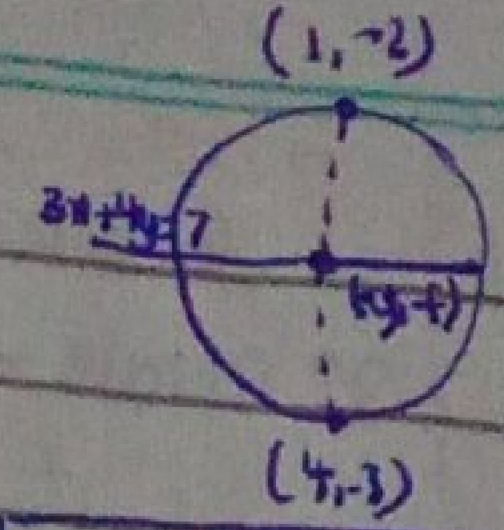
$g=\frac{94}{5}\times\frac{-1}{6}$

$\boxed{g=\frac{-47}{15}}$

Putting value of f & g in eq(ii),

$2g-4f+C=-5$

$2\left(\frac{-47}{15}\right)-4\left(\frac{3}{5}\right)+C=-5$




$\frac{-94}{15}-\frac{12}{5}+C=-5$

$C=-5+\frac{26}{3}$

$\boxed{C=\frac{11}{3}}$

Putting the values of f, C, g in eq ①,

$x^2+y^2+2\left(\frac{-47}{15}\right)x+2\left(\frac{3}{5}\right)y+\frac{11}{3}=0$

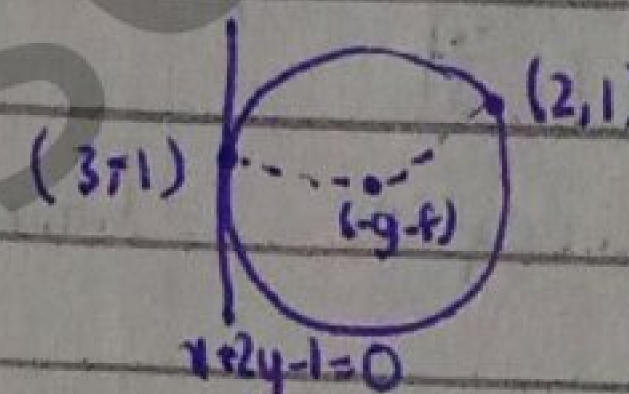
$x^2+y^2-\frac{94}{15}x+\frac{6}{5}y+\frac{11}{3}=0$

× 15 on all sides,

$(15)x^2+(15)y^2-(15)\frac{94}{15}x+(15)\frac{6}{5}y+(15)\frac{11}{3}=(15)0$

$\boxed{15x^2+15y^2-94x+18y+55=0}$

## Q6 Find the ...




∵ Tangent line, is the line that cuts a circle at one point. So line $x+2y-1=0$ is a tangent line.

$x^2+y^2+2gx+2fy+C=0$ — ①

$\Rightarrow (2,1)$:

$(2)^2+(1)^2+2g(2)+2f(1)+C=0$

$4+1+4g+2f+C=0$

$4g+2f+C=-5$ —eq(i)

$\Rightarrow (3,1)$:

$(3)^2+(1)^2+2g(3)+2f(1)+C=0$

$9+1+6g+2f+C=0$

$6g+2f+C=-10$ —eq(ii)

Solving eq(i) & eq(ii),

$4g+2f+\cancel{C}=-5$

$\pm 6g \mp 2f \pm \cancel{C} = \mp 10$

$-2g+4f=5$ —eq(iii)

Slope of line:

$m_1=\frac{-a}{b}=\frac{-1}{2}$

Slope of radius:

$m_2=\frac{y_2-y_1}{x_2-x_1}$

$m_2=\frac{-1+f}{3+g}$

We know that if $l_1 \perp l_2$ then

for MCQ
$\boxed{\& \text{ if } l_1 \parallel l_2,\ m_1 = m_2}$

$(m_1)(m_2) = -1$

$\left(\frac{-1}{2}\right)\left(\frac{f-1}{g+3}\right) = -1$

$\frac{f-1}{g+3} = \frac{-1\times 2}{-1}$

$\frac{f-1}{g+3} = 2$

$f-1 = 2(g+3)$

$f-1 = 2g+6$

$f-2g = 6+1$

$f-2g = 7$ — eq(iv)

Sub eq(iii) & eq(iv),

$-\cancel{2g}+4f = 5$

$\mp\cancel{2g} \pm f = \pm 7$

$3f = -2$

$\boxed{f = \frac{-2}{3}}$

Putting value of f in eq(iv),

$f-2g = 7$

$\frac{-2}{3} - 2g = 7$

$-2g = 7 + \frac{2}{3}$

$-2g = \frac{23}{3}$

$g = \frac{23}{3} \times \frac{-1}{2}$

$\boxed{g = \frac{-23}{6}}$

Putting the values of f & g in eq(i),

$4g+2f+C = -5$

$4\left(\frac{-23}{6}\right)+2\left(\frac{-2}{3}\right)+C = -5$

$\frac{-46}{3} - \frac{4}{3} + C = -5$

$C = -5 + \frac{50}{3}$

$\boxed{C = \frac{35}{3}}$

Putting the values of f, g, C in eq ①,

$x^2+y^2+2gx+2fy+C=0$

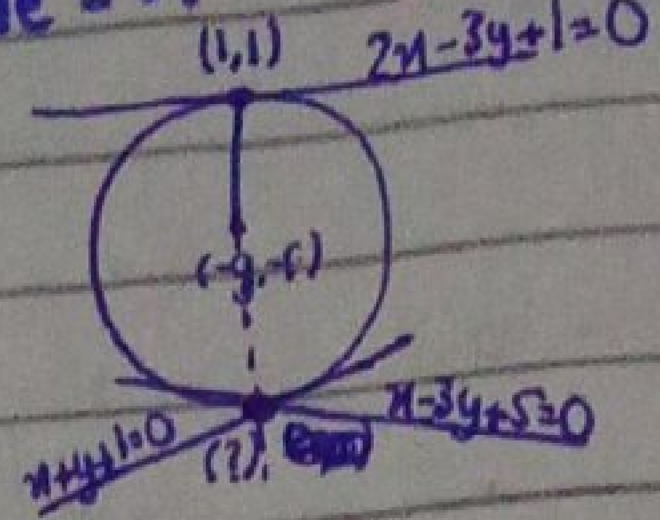
$x^2+y^2+2\left(\frac{-23}{6}\right)x + 2\left(\frac{-2}{3}\right)y + \left(\frac{35}{3}\right) = 0$

$x^2+y^2 - \frac{23}{3}x - \frac{4}{3}y + \frac{35}{3} = 0$

× 3 on all sides

$\boxed{3x^2+3y^2-23x-4y+35=0}$

**Q7 A circle - - -**




To find the second point we will solve the two equation $x+y+1=0$ & $x-3y+5=0$ Subtracting them,

$\cancel{x}+y+1=0$

$\pm\cancel{x}\mp 3y \pm 5=0$

$4y-4=0$

$4y=4$

$y=\frac{4}{4}$

$\boxed{y=1}$

Putting value of y in eq $x-3y+5=0$

$x-3(1)+5=0$

$x-3+5=0$

$x+2=0$

$\boxed{x=-2}$

So 2nd point (2) is $(-2,1)$

General Equation of a circle,

$x^2+y^2+2gx+2fy+C=0$ — ①

⇒ (1,1)

$(1)^2+(1)^2+2g(1)+2f(1)+C=0$

$1+1+2g+2f+C=0$

$2g+2f+C=-2$ — eq(i)

⇒ (-2,1)

$(-2)^2+(1)^2+2g(-2)+2f(1)+C=0$

$4+1-4g+2f+C=0$

$-4g+2f+C=-5$ — eq(ii)

Solving eq(i) & eq(ii),

$2g+\cancel{2f}+\cancel{C} = -2$

$\mp 4g \pm \cancel{2f} \pm \cancel{C} = \mp 5$

$6g = 3$

$g = \frac{3}{6}$

$\boxed{g = \frac{1}{2}}$

Slope of line $(2x-3y+1)$:

$m_1 = \frac{-b}{a}$

$m_1 = \frac{-2}{-3}$

$m_1 = \frac{2}{3}$

Slope of radius:

$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_2 = \frac{-f-1}{-g-1}$

$m_2 = \frac{\cancel{-}(f+1)}{\cancel{-}(g+1)}$

$m_2 = \frac{1+f}{1+g}$

We know,

$l_1 \perp l_2$ then $m_1 \cdot m_2 = -1$

$\frac{2}{3} \cdot \frac{1+f}{1+g} = -1$

$\frac{1+f}{1+g} = \frac{-3}{2}$

$2(1+f) = 1+g\left(\frac{-3}{2}\right)$

$2+2f = -3-3g$

$2f+3g = -3-2$

$3g+2f = -5$ – eq (iii)

Putting value of g in eq (iii)

$3\left(\frac{1}{2}\right)+2f = -5$

$\frac{3}{2}+2f = -5$

$2f = -5 - \frac{3}{2}$

$2f = \frac{-13}{2}$

$f = \frac{-13}{2} \times \frac{1}{2}$

$\boxed{f = \frac{-13}{4}}$

Putting values of g & f in eq (i),

$2g+2f+C = -2$

$2\left(\frac{1}{2}\right)+\cancel{2}\left(\frac{-13}{\cancel{4}2}\right)+C = -2$

$1 - \frac{13}{2} + C = -2$

$C = -2 + \frac{11}{2}$

$\boxed{C = \frac{7}{2}}$

Putting values of g, f, C in eq ①,

$x^2+y^2+2gx+2fy+C = 0$

$x^2+y^2+2\left(\frac{1}{2}\right)x+2\left(\frac{-13}{4}\right)y+\left(\frac{7}{2}\right) = 0$

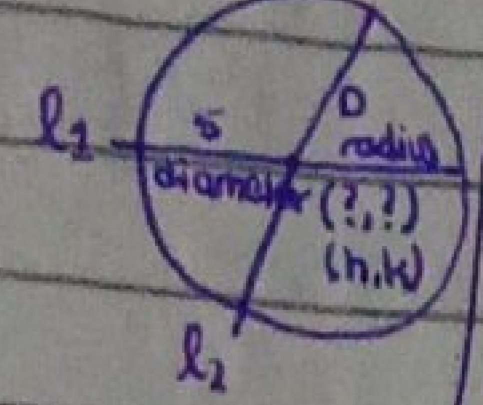
$x^2+y^2+x-\frac{13y}{2}+\frac{7}{2} = 0$

× 2 on all sides,

$\boxed{2x^2+2y^2+2x-13y+7=0}$

Equations

## Q8 ~~Part~~ of two ...




Radius = r = 5

$l_1 \Rightarrow x-y=3$ – eq (i)

$l_2 \Rightarrow 3x+y=5$ – eq (ii)

Solving eq (i) & eq (ii),

$x - \cancel{y} = 3$

$+3x + \cancel{y} = +5$

$4x = 8$

$x = \frac{8^2}{4_1}$

$x = 2$

Putting x in eq (i),

$x-y=3$

$2-y=3$

$-y=3-2$

$-y=1$

$y=-1$

So center is (2,-1)

Putting the radius & center of the circle in,

$(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-(-1))^2=(5)^2$

$x^2-4x+4+(y+1)^2=25$

$x^2-4x+4+y^2+2y+1=25$

$x^2+y^2-4x+2y=25-5$

$\boxed{x^2+y^2-4x+2y-20=0}$

Need to find radius.

Area of a circle = $\pi r^2$

$13\cancel{\pi} = \cancel{\pi} r^2$

$\sqrt{13} = \sqrt{r^2}$

$\boxed{r = \sqrt{13}}$

Used can never be negative

$\Rightarrow (x,y) = (\sqrt{13}, \sqrt{13})$ h k

$(x-h)^2+(y-k)^2=r^2$

$(x-\sqrt{13})^2+(y-\sqrt{13})^2=(\sqrt{13})^2$

$(x^2-2\sqrt{13}x+13)+(y^2-2\sqrt{13}y+13)=13$

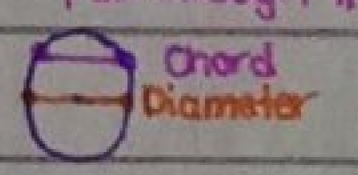
$x^2-2\sqrt{13}x+13+y^2-2\sqrt{13}y+13=13$

$x^2+y^2-2\sqrt{13}x-2\sqrt{13}y=13-13-13$

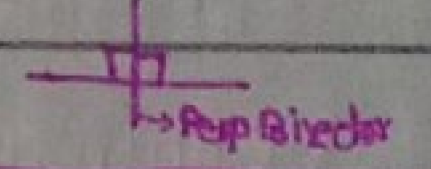
$\boxed{x^2+y^2-2\sqrt{13}x-2\sqrt{13}y+13=0}$

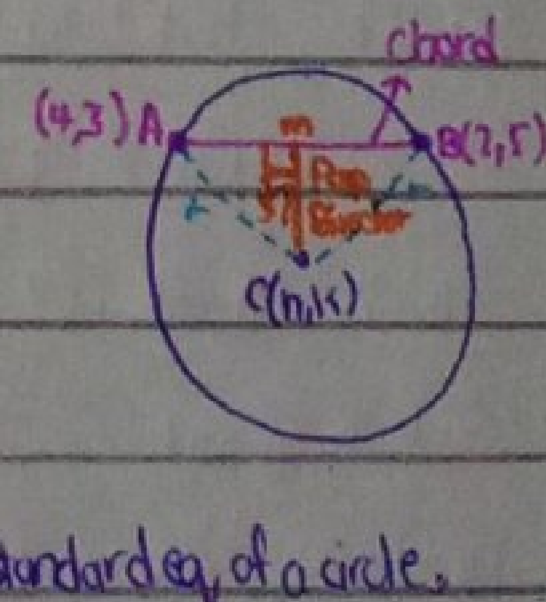
## Q10 The two points ......

Chord: it is a line that connects two points in a circle but ~~those par~~ that line does not pass through the center.




Perpendicular Bisector: A line that cuts the line, any line/chord, from the middle and makes a 90° angle with it.

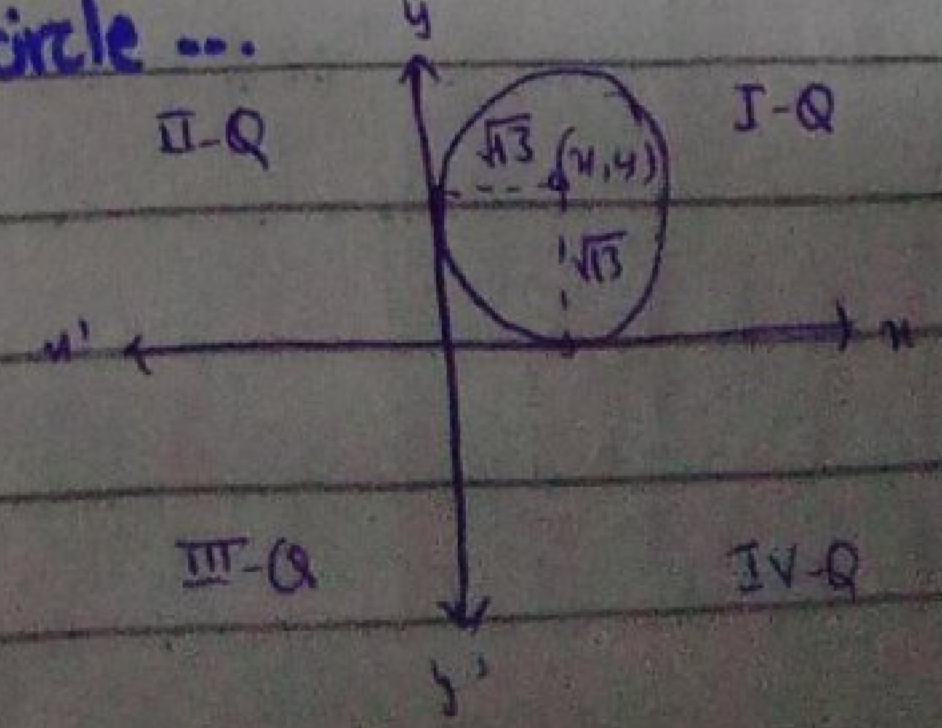



Standard eq. of a circle,

$(x-h)^2+(y-k)^2=r^2$ — ①

## Q9 A circle ...



Finding distance,

Distance $|CA|$ = Distance $|CB|$

Distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$\sqrt{(h-4)^2+(k-3)^2} = \sqrt{(h-2)^2+(k-5)^2} \quad \text{-eq(ii)}$$

$$\sqrt{h^2-8h+16+k^2-6k+9} = \sqrt{h^2-4h+4+k^2-10k+25}$$

$$\left(\sqrt{h^2+k^2-8h-6k+25}\right)^2 = \left(\sqrt{h^2+k^2-4h-10k+29}\right)^2$$

$$\cancel{h^2}-\cancel{h^2}+\cancel{k^2}-\cancel{k^2}-8h+4h-6k+10k+25-29=0$$

$$-4h+4k-4=0$$

$$-4(h-k+1)=0$$

$$h-k+1=0$$

$$h=k-1 \quad \text{-eq(i)}$$

Finding midpoint (m) of chord AB,

$$m=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$m=\left(\frac{4+2}{2}, \frac{3+5}{2}\right)$$

$$m=(3,4)$$

Distance of center to midpoint m.

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\sqrt{7}=\sqrt{(3-h)^2+(4-k)^2}$$

$$(\sqrt{7})^2=\left(\sqrt{9-6h+h^2+16-8k+k^2}\right)^2$$

$$7=h^2+k^2-6h-8k+25$$

$$0=h^2+k^2-6h-8k+25-7$$

$$h^2+k^2-6h-8k+18=0$$

Putting eq(i),

$$(k-1)^2+k^2-6(k-1)-8k+18=0$$

$$k^2-2k+1+k^2-6k+6-8k+18=0$$

$$2k^2-16k+25=0$$

$$k=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$k=\frac{-(-16)\pm\sqrt{(-16)^2-4(2)(25)}}{2(2)}$$

$$k=\frac{16\pm 2\sqrt{14}}{4}$$

$$k=\frac{2(8\pm\sqrt{14})}{4}$$

$$\boxed{k=\frac{8+\sqrt{14}}{2}} \qquad \boxed{k=\frac{8-\sqrt{14}}{2}}$$

Case I: $k=\frac{8+\sqrt{14}}{2}$

Putting value of k in eq(i)

$$h=\frac{8+\sqrt{14}}{2}-1$$

$$\boxed{h=\frac{6+\sqrt{14}}{2}}$$

Case II:

$$k=\frac{8-\sqrt{14}}{2}$$

$$h=\frac{8-\sqrt{14}}{2}-1$$

$$\boxed{h=\frac{6-\sqrt{14}}{2}}$$

Case I:

Putting the values of h & k in eq(i),

$$r=\sqrt{(h-4)^2+(k-3)^2}$$

$$r=\sqrt{\left(\frac{6+\sqrt{14}}{2}-4\right)^2+\left(\frac{8+\sqrt{14}}{2}-3\right)^2}$$

$$r=\sqrt{\frac{9-2\sqrt{14}}{2}+\frac{9+2\sqrt{14}}{2}}$$

$$r=\sqrt{\frac{9-2\sqrt{14}+9+2\sqrt{14}}{2}}$$

$$r=\sqrt{\frac{18}{2}}$$

$$r=\sqrt{9}$$

$$r=3$$

Putting the values in eq (1),

$$(x-h)^2+(y-k)^2=r^2$$

$$\left(x-\frac{6+\sqrt{14}}{2}\right)^2+\left(y-\frac{8+\sqrt{14}}{2}\right)^2=(3)^2$$

$$\boxed{\left(x-\frac{6+\sqrt{14}}{2}\right)^2+\left(y-\frac{8+\sqrt{14}}{2}\right)^2=9}$$

Case II:

$$r=\sqrt{(h-4)^2+(k-3)^2}$$

$$r=\sqrt{\left(\frac{6-\sqrt{14}}{2}-4\right)^2+\left(\frac{8-\sqrt{14}}{2}-3\right)^2}$$

$$r=\sqrt{\frac{9+2\sqrt{14}}{2}+\frac{9-2\sqrt{14}}{2}}$$

$$r=\sqrt{\frac{9+2\sqrt{14}+9-2\sqrt{14}}{2}}$$

$$r=\sqrt{\frac{18}{2}}$$

$$r=\sqrt{9}$$

$$r=3$$

Putting values in eq (1),

$$(x-h)^2+(y-k)^2=r^2$$

$$\left(x-\frac{6-\sqrt{14}}{2}\right)^2+\left(y-\frac{8-\sqrt{14}}{2}\right)^2=(3)^2$$

$$\boxed{\left(x-\frac{6-\sqrt{14}}{2}\right)^2+\left(y-\frac{8-\sqrt{14}}{2}\right)^2=9}$$

## Q11 Find the equation of ....

$C_1 + \lambda C_2 = 0$

$(x^2+y^2-8x-2y+7) + \lambda(x^2+y^2-4x+10y+8) = 0$ — eq(i)

$((-1)^2+(-2)^2-8(-1)-2(-2)+7) + \lambda((-1)^2+(-2)^2-4(-1)+10(-2)+8) = 0$

$(1+4+8+4+7) + \lambda(1+4+4-20+8) = 0$

$(24) + \lambda(-3) = 0$

$-3\lambda = -24$

$\lambda = \frac{-24}{3}$

$\boxed{\lambda = 8}$

Put the value of $\lambda = 8$ in eq(i).

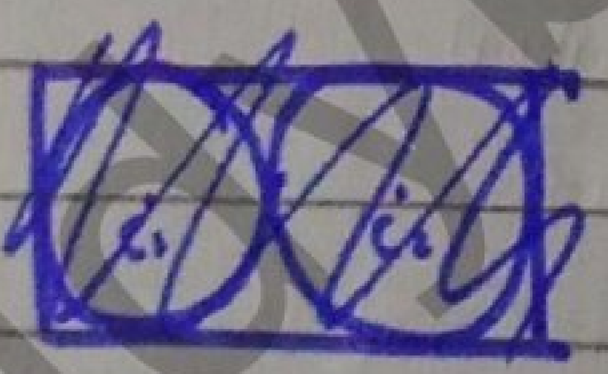
$(x^2+y^2-8x-2y+7) + 8(x^2+y^2-4x+10y+8) = 0$

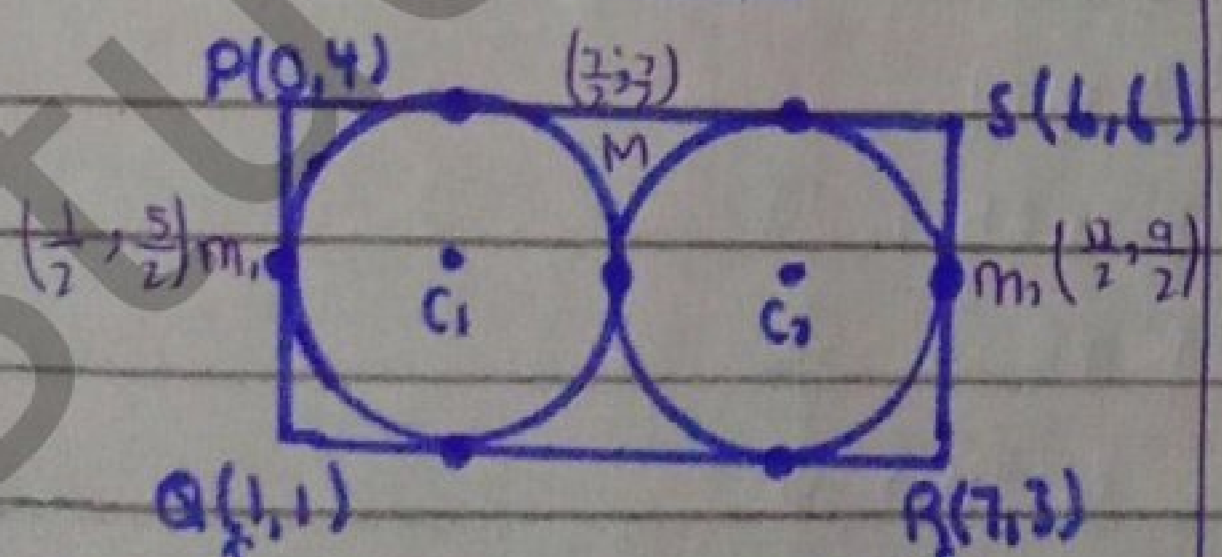
$x^2+y^2-8x-2y+7+8x^2+8y^2-32x+80y+64 = 0$

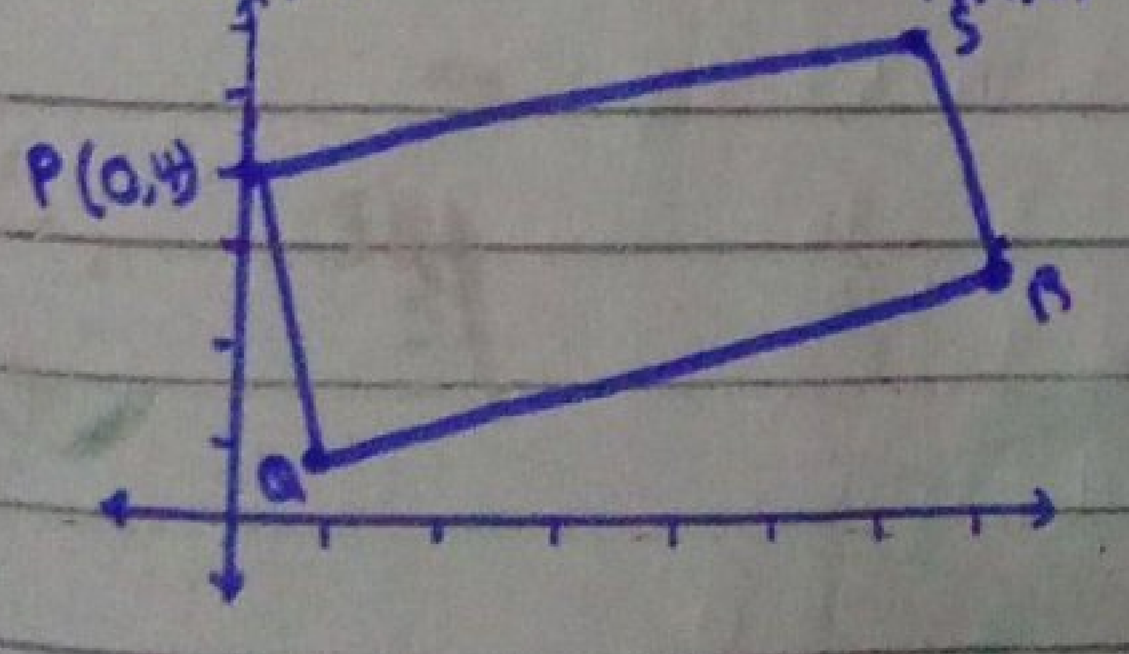
$8x^2+x^2+8y^2+y^2-8x-32x-2y+80y+7+64 = 0$

$\boxed{9x^2+9y^2-40x+78y+71=0}$

## Q12 The diagram - - -


Equation of a circle,

$(x-h)^2+(y-k)^2 = r^2$

→ Finding Distance, to find radius:

Distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|PQ| = \sqrt{(1-0)^2+(1-4)^2}$

$|PQ| = \sqrt{(1)^2+(-3)^2}$

$|PQ| = \sqrt{1+9}$

$|PQ| = \sqrt{10}$

$|SR| = \sqrt{(7-6)^2+(3-6)^2}$

$|SR| = \sqrt{(1)^2+(-3)^2}$

$|SR| = \sqrt{1+9}$

$|SR| = \sqrt{10}$

So we know Diameter $= \sqrt{10}$

Radius $= \frac{\text{Diameter}}{2}$

Radius $= \frac{\sqrt{10}}{2}$

$\boxed{r = \frac{\sqrt{10}}{2}}$

radius is same for circle 1 & Circle 2.

→ Finding Center:

Midpoint of PQ $= \left(\frac{0+1}{2}, \frac{4+1}{2}\right)$

$m_1 = \left(\frac{1}{2}, \frac{5}{2}\right)$

Midpoint of SR $= \left(\frac{6+7}{2}, \frac{6+3}{2}\right)$

$m_2 = \left(\frac{13}{2}, \frac{9}{2}\right)$

Finding M $= \left(\frac{\frac{1}{2}+\frac{13}{2}}{2}, \frac{\frac{5}{2}+\frac{9}{2}}{2}\right)$ using $m_1$ & $m_2$

$M = \left(\frac{7}{2}, \frac{7}{2}\right)$

Finding the center of $C_1$ using $m_1$ & M,

Center $C_1 = \left(\frac{\frac{1}{2}+\frac{7}{2}}{2}, \frac{\frac{5}{2}+\frac{7}{2}}{2}\right)$

$\boxed{\text{Center } C_1 = (2,3)}$

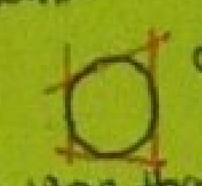
Now for $C_2$ using $m_2$ & M,

Center $C_2 = \left(\frac{\frac{13}{2}+\frac{7}{2}}{2}, \frac{\frac{9}{2}+\frac{7}{2}}{2}\right)$

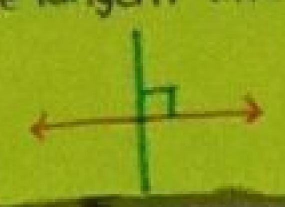
$\boxed{\text{Center } C_2 = (5,4)}$

→ Finding the equation:

**Tangent line:** It is ... a curve at just one point.




**Normal Line:** It is a line that is perpendicular and cuts the tangent line and makes a 90° angle.



Equation fo[r $C_1$,]

$(x-h)^2+(y-k)^2=(r)^2$

$(x-2)^2+(y-3)^2=\left(\frac{\sqrt{10}}{2}\right)^2$

$x^2-4x+4+y^2-6y+9=\frac{10}{4}$

$x^2+y^2-4x-6y+13=\frac{10}{4}$

$4x^2+4y^2-16x-24y+52-10=0$

$\boxed{4x^2+4y^2-16x-24y+42=0}$

Equation for $C_2$,

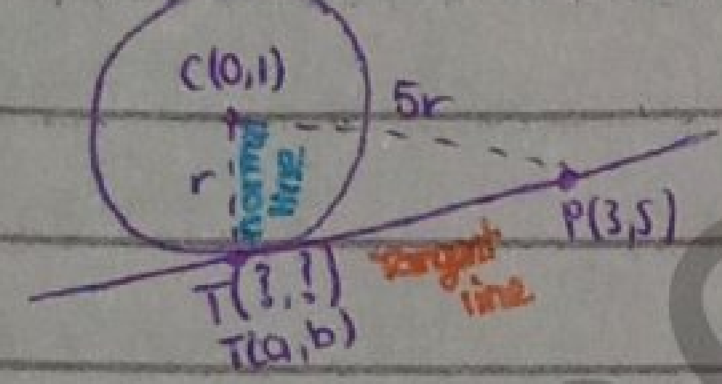
$(x-5)^2+(y-4)^2=\left(\frac{\sqrt{10}}{2}\right)^2$

$x^2-10x+25+y^2-8y+16=\frac{10}{4}$

$4x^2+4y^2-40x-32y+164-10=0$

$\boxed{4x^2+4y^2-40x-32y+154=0}$

## Q13 A Circle ...




→ **Finding equation of the circle:**

$(x-h)^2+(y-k)^2=r^2$ —eq(i)

finding radius,

$|PC|=\sqrt{(0-3)^2+(1-5)^2}$

$5r=\sqrt{(-3)^2+(-4)^2}$

$5r=\sqrt{9+16}$

$5r=\sqrt{25}$

$\cancel{5}r=\cancel{5}$

$\boxed{r=1}$

Center (0,1) (we know).

Putting center & radius in eq(i)

$(x-h)^2+(y-k)^2=r^2$

$(x-0)^2+(y-1)^2=(1)^2$

$x^2+y^2-2y+1=1$

$x^2+y^2-2y+1-1=0$

$\boxed{x^2+y^2-2y=0}$

(equation of the circle)

...oint where the line touches the circle:

$\Rightarrow (a,b)$ satisfies the equation of a circle because T(a,b) lies on the circle.

~~$a^2+b^2-2b=0$~~

$x^2+y^2-2y=0$

$(a)^2+(b)^2-2(b)=0$

$a^2+b^2-2b=0$ —eq(ii)

We know if $l_1 \perp l_2$ then $m_1m_2=-1$

So finding the slope of both lines. $\frac{y_2-y_1}{x_2-x_1}$

Slope of (TP) $= m_1 =$

$m_1=\frac{5-b}{3-a}$

Slope of (TC) $= m_2 = \frac{y_2-y_1}{x_2-x_1}$

$m_2=\frac{b-1}{a-0}$

$m_2=\frac{b-1}{a}$

So,

$m_1m_2=-1$

$\left(\frac{5-b}{3-a}\right)\left(\frac{b-1}{a}\right)=-1$

$\frac{5b-5-b^2+b}{3a-a^2}=-1$

$6b-b^2-5=-1(3a-a^2)$

$6b-b^2-5=-3a+a^2$

$a^2+b^2-3a-6b+5=0$ —eq(iii)

Sub eq(ii) & eq(iii),

$a^2+b^2-3a-6b+5=0$

$\pm a^2 \pm b^2 \mp 2b = 0$

$-3a-4b+5=0$

$3a+4b=5$ —eq(iv)

$3a=5-4b$

$a=\frac{5-4b}{3}$

Putting value of a in eq(ii),

$\left(\frac{5-4b}{3}\right)^2+b^2-2b=0$

$\frac{25-40b+16b^2}{9}+b^2-2b=0$

$25-40b+16b^2=9(-b^2+2b)$

$25-40b+16b^2=-9b^2+18b$

~~$16b^2+3b^2-40b-6b+25=0$~~

~~$19B^2-50B+25=0$~~

~~$B=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$~~

~~$B=\frac{(-50)\pm\sqrt{(-50)^2-4(19)(25)}}{2(19)}$~~

~~$B=\frac{50\pm10\sqrt{6}}{38}$~~

$16b^2+9b^2-40b-18b+25=0$

$25b^2-58b+25=0$

$b=\frac{-(-58)\pm\sqrt{(-58)^2-4(25)(25)}}{2(25)}$

$b=\frac{58\pm12\sqrt{6}}{50}$

$b=\frac{58+12\sqrt{6}}{50}$ $\quad b=\frac{58-12\sqrt{6}}{50}$

$b=\frac{29\pm6\sqrt{6}}{25}$

Case I: $b=\frac{29+6\sqrt{6}}{25}$ $\boxed{\approx 1.75}$

Putting value of b in eq(iv)

$3a+4b=5$

$3a+4(1.75)=5$

$3a+7=5$

$3a=5-7$

$3a=-2$

$\boxed{a=\frac{-2}{3}}$ or $\boxed{a=-0.67}$

Putting a & b in $T(a,b)$

$\boxed{T(a,b)=(-0.67,1.75)}$

Verification:

Putting values in eq iv

$a^2+b^2-2b=0$

$(-0.67)^2+(1.75)^2-2(1.75)=0$

$0\approx0$

Case II:

$b=\frac{29-6\sqrt{6}}{25}$

$\boxed{b\approx0.57}$

Putting in eq(iv)

$3a+4(0.57)=5$

$3a=5-2.28$

$3a=2.72$

$a=\frac{2.72}{3}$

$\boxed{a\approx0.91}$

$\boxed{T(a,b)=(0.91,0.57)}$

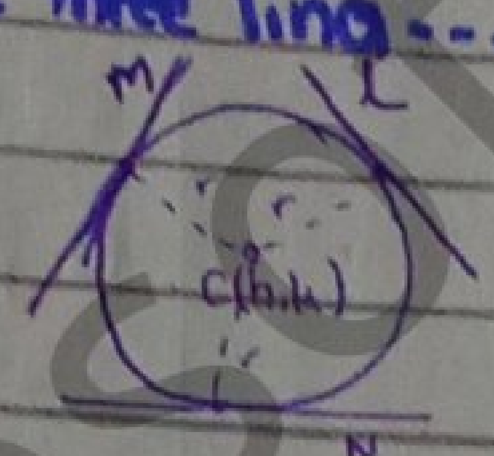
Verification:

Putting values in eq(ii)

$(0.91)^2+(0.57)^2-2(0.57)=0$

$0\approx0$

## Q14 The three line---

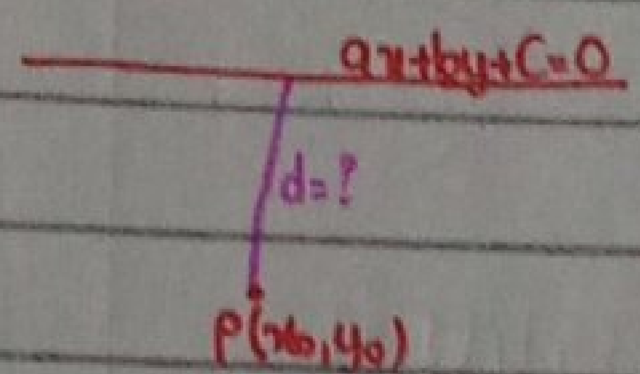


$l \Rightarrow 2x-y+1=0$

$M \Rightarrow 2x+y-3=0$

$N \Rightarrow x-2y+4=0$

## IMP Concept:



Some questions in entry test ask to find the distance from a line to a point.

$d=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$ This is the formula for finding that.

The reason why we used absolute is so that the ans cannot be negative as we already know distance can never be negative. The reason for why we didn't use absolute in the denominator is because the square $a^2$ & $b^2$ will already make the value positive.

bcs radius is same

$|Cl|=|CN|$ & $|CM|=|Cl|$

$\because d=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$

$|Cl|=|CN|$

$\frac{|2(h)-1(k)+1|}{\sqrt{(2)^2+(-1)^2}}=\frac{|1(h)-2(k)+4|}{\sqrt{(1)^2+(-2)^2}}$

$\frac{|2h-k+1|}{\sqrt{4+1}}=\frac{|h-2k+4|}{\sqrt{1+4}}$

$2h-k+1=h-2k+4$

Whenever the question talks about, the point of intersection of a line and a circle. Do remember, the points could be one or two.

This way there is only one point.

But this way, there are two points.

$\boxed{\underline{\text{For Ex 7.2 Q1}}}$

$2h-k+1-h+2k-4=0$

$h+k-3=0 \quad$ - eq(i)

Now $|CM| = |CL|$

$\because d = \dfrac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$

$\dfrac{|2(h)+1(k)-3|}{\sqrt{(2)^2+(1)^2}} = \dfrac{|2(h)-1(k)+1|}{\sqrt{(2)^2+(-1)^2}}$

$\dfrac{|2h+k-3|}{\cancel{\sqrt{4+1}}} = \dfrac{|2h-k+1|}{\cancel{\sqrt{4+1}}}$

$2h+k-3 = 2h-k+1$

$2h-2h+k+k-3-1=0$

$2k-4=0$

$2k=4$

$\boxed{k=2}$

Putting k in eq.(i),

$h+k-3=0$

$h+2-3=0$

$h-1=0$

$\boxed{h=1}$

So Center$(1,2)$

→ <u>Finding equation of circle:</u>

$|CL| = r$

$\dfrac{|2(h)-1(k)+1|}{\sqrt{(2)^2+(-1)^2}} = r$

$\dfrac{2h-k+1}{\sqrt{4+1}} = r$

$\dfrac{2(1)-(2)+1}{\sqrt{5}} = r$

$\dfrac{2-2+1}{\sqrt{5}} = r$

$\boxed{r=\dfrac{1}{\sqrt{5}}}$

Putting the values in the eq,

$(x-h)^2+(y-k)^2 = r^2$

$(x-1)^2+(y-2)^2 = \left(\dfrac{1}{\sqrt{5}}\right)^2$

$5(x^2+y^2-2x-4y+5) = 1$

$5x^2+5y^2-10x-20y+25-1=0$

$\boxed{5x^2+5y^2-10x-20y+24=0}$