Studybeesofficial

Where Minds Pollinate

Class 12
Mathematics

Chapter 10

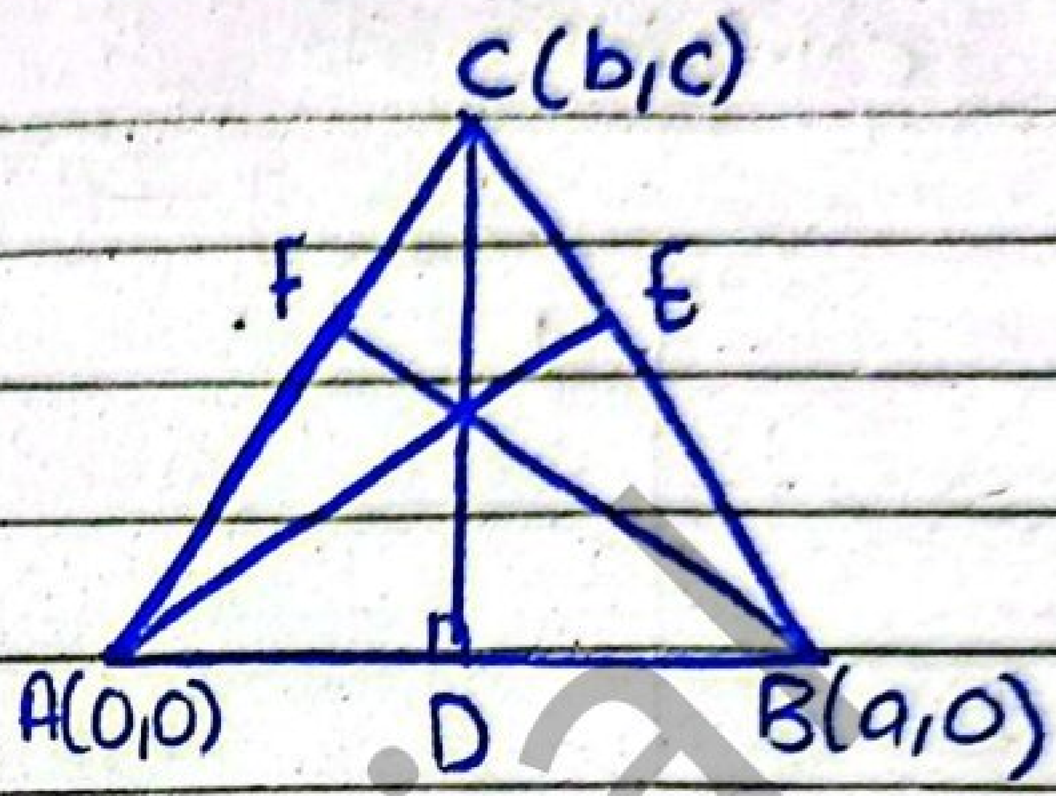
Review Exercise 6

Handwritten Solution

Review # 6.

Q2

(i)



Slope of side AB $= \frac{0-0}{a-0} = 0$.

Slope of $\overline{BC} = \frac{c-0}{b-a} = \frac{c}{b-a}$

Slope of $\overline{AC} = \frac{c-0}{b-0} = \frac{c}{b}$.

~~As~~ CD is ⊥ to AB then

Slope of CD $= \frac{-1}{\text{slope of } AB} = \frac{-1}{0} = \infty$.

So equation of line CD ⊥ to AB is:

$$y - c = -\frac{1}{0}(x-b)$$

$$\boxed{x-b=0} \quad \text{—— (i)}$$

~~Now equation of Altitude AE~~ : Hence $\overline{AE}$ is ⊥ BC.

slope of AE $= \frac{-1}{\left(\frac{c}{b-a}\right)} = \boxed{\frac{a-b}{c}}$

So equation of Altitude AE.

$$y - 0 = \left(\frac{a-b}{c}\right)(x-0)$$

$$yc = (a-b)x$$

$$\boxed{(a-b)x - yc = 0} \quad \text{—— (ii)}$$

Now $\overline{BF}$ is ⊥ to CA thus

Slope of $\overrightarrow{BF} = \frac{-1}{\text{slope of } \overline{AC}} = \frac{-1}{\left(\frac{c}{b}\right)} = \frac{-b}{c}$

The equation of altitude $\overrightarrow{BF}$ is.

$$y - 0 = \frac{-b}{c}(x-a)$$

$$cy = -bx + ab$$

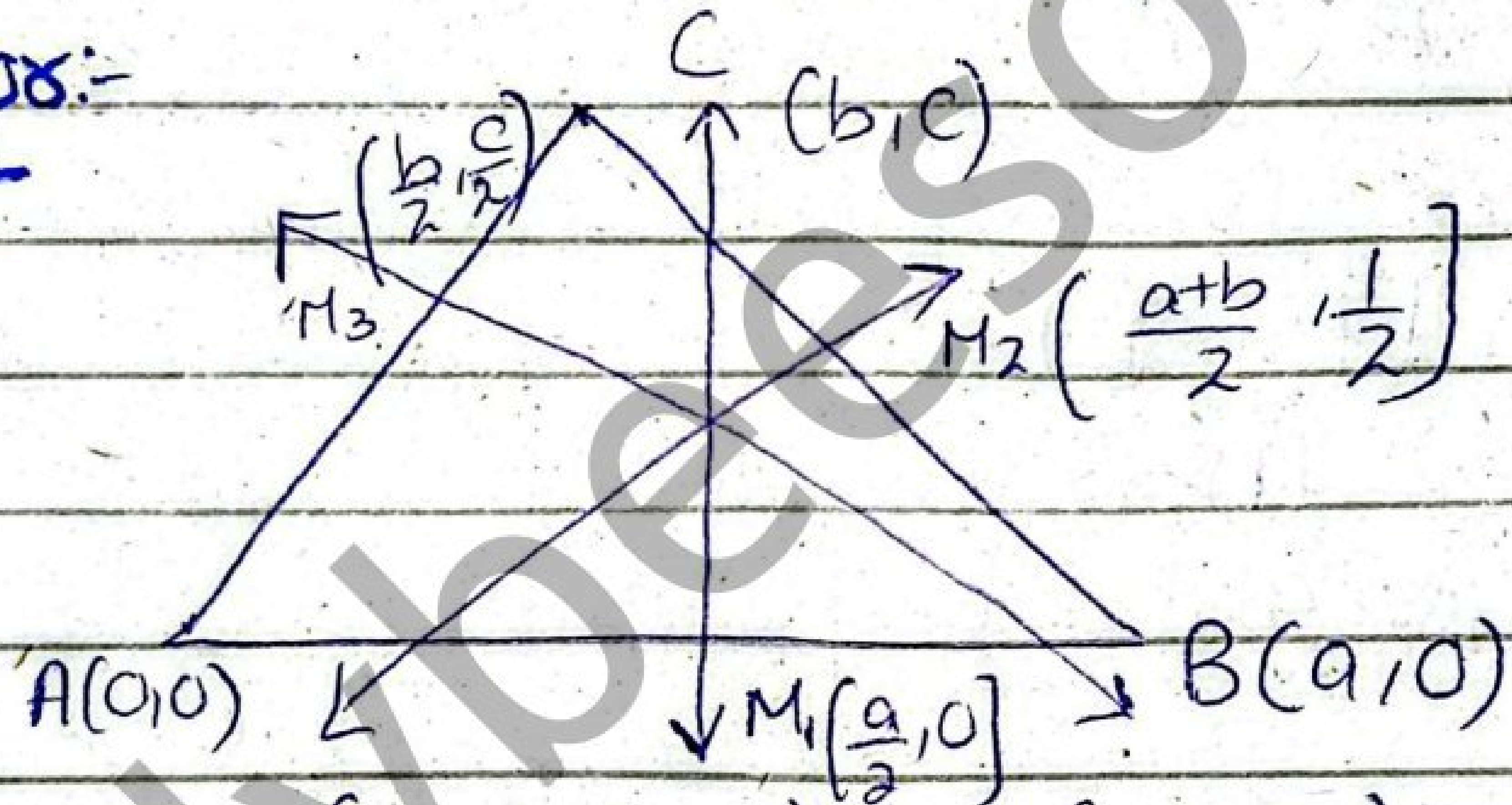
$$\boxed{bx + cy - ab = 0} \text{ --- (iii)}$$

Now
$$\begin{vmatrix} 1 & 0 & -b \\ a-b & -c & 0 \\ b & c & -ab \end{vmatrix}$$

$$= 1\begin{vmatrix} -c & 0 \\ c & -ab \end{vmatrix} - 0\begin{vmatrix} a-b & 0 \\ b & -ab \end{vmatrix} - b\begin{vmatrix} a-b & -c \\ b & c \end{vmatrix}$$

$$= abc - abc = \boxed{0}$$ Hence altitudes are concurrent.

Right bisector:-



Therefore $M_1 = \left(\frac{a+0}{2}, \frac{0+a}{2}\right) = \left(\frac{a}{2}, 0\right)$

$M_2 = \left(\frac{a+b}{2}, \frac{0+c}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$

$M_3 = \left(\frac{b+0}{2}, \frac{c+0}{2}\right) = \left(\frac{b}{2}, \frac{c}{2}\right)$

slope of $AB = \frac{-1}{0} = \infty$

equation of right bisector passing through point $\left[\frac{a}{2}, 0\right]$ having slope ∞.

$$y - 0 = \frac{1}{0}\left(x - \frac{a}{2}\right) \Rightarrow \boxed{2x - a = 0} \text{ --- (i)}$$

slope of BC = $\dfrac{-1}{\left(\frac{c}{b-a}\right)} = \boxed{\dfrac{a-b}{c}}$

equation of right bisecter $\overline{BC}$ is:

$$y - \frac{c}{2} = \frac{a-b}{2}\left(x - \frac{a+b}{2}\right).$$

$$y - \frac{c}{2} = \left(\frac{a-b}{2}\right)x - \frac{a^2-b^2}{2c}$$

multiply both sides by $2c$

$$2cy - c^2 = 2(a-b)x - (a^2-b^2)$$

$$2(a-b)x - 2cy - \{(a^2-b^2) + c^2 = 0$$

$$2(a-b)x - 2cy - (a^2-b^2-c^2) = 0 \quad \text{— ii}$$

slope of AC = $\dfrac{-1}{\left(\frac{c}{b}\right)} = \boxed{\dfrac{-b}{c}}$.

equation of right bisecter of $\overline{AC}$ is

$$y - \frac{c}{2} = \frac{-b}{c}\left(x - \frac{b}{2}\right)$$

$$y - \frac{c}{2} = \frac{-b}{c}x + \frac{b^2}{2c}$$

multiply both sides by $2c$

$$2cy - c^2 = -2bx + b^2$$

$$2bx + 2cy - (b^2+c^2) = 0 \quad \text{— iii}$$

Now
$$\begin{vmatrix} 2 & 0 & -a \\ 2(a-b) & -2c & -(a^2-b^2-c^2) \\ 2b & 2c & -(b^2+c^2) \end{vmatrix}$$

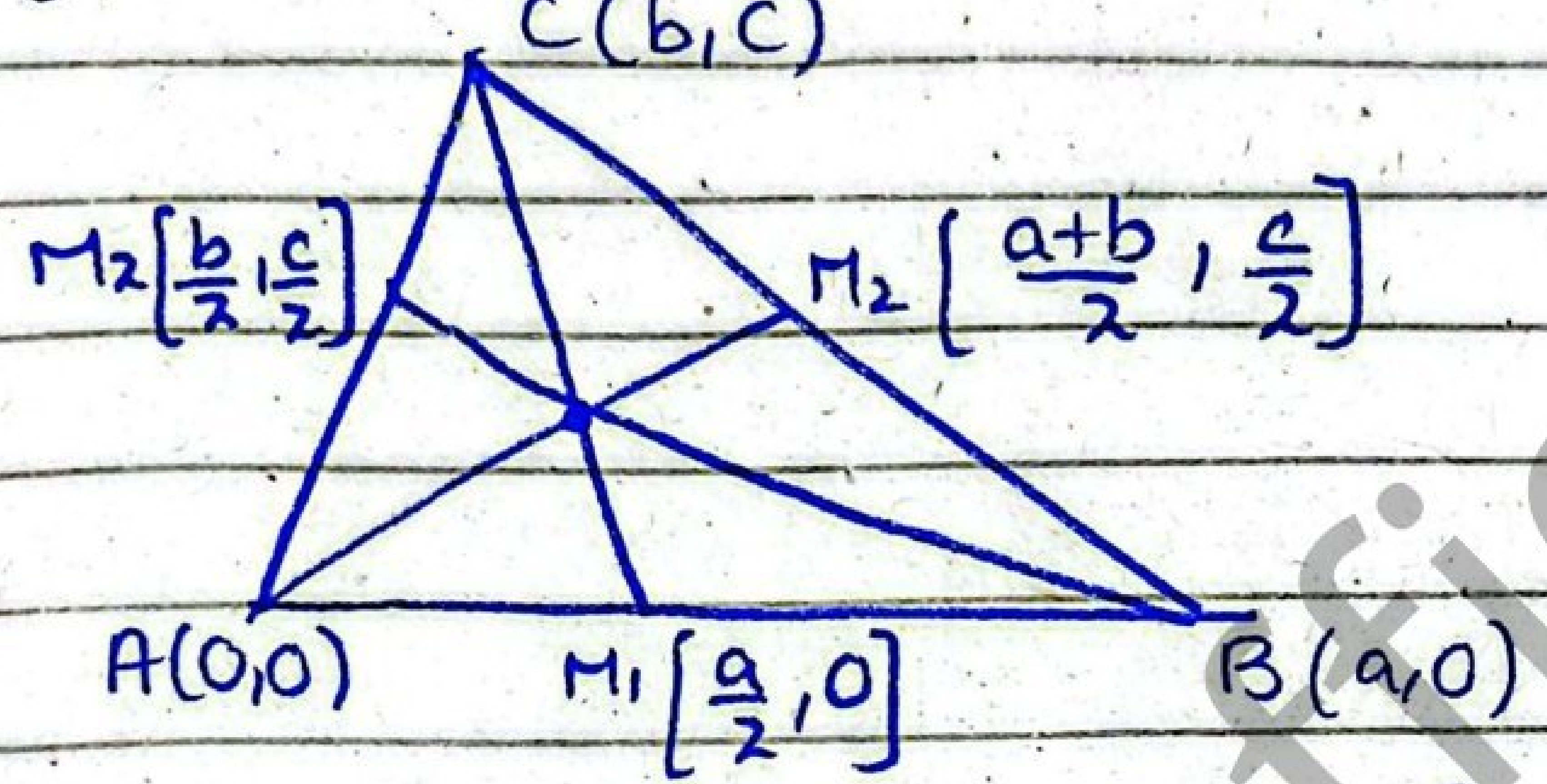
$$2\begin{vmatrix} -2c & -(a^2-b^2-c^2) \\ 2c & -(b^2+c^2) \end{vmatrix} - 0 - a\begin{vmatrix} 2(a-b) & -2c \\ 2b & 2c \end{vmatrix}$$

$$= 2\left(2c(b^2+c^2) + 2c(a^2-b^2-c^2)\right) - a\left[4c(a-b) + 4bc\right]$$

$$4c\left(b^2+c^2+a^2-b^2-c^2\right)-4ac(a-b+b)$$
$$4c(a^2)-4ac(a) \Rightarrow 4a^2c-4a^2c=\boxed{0}$$

Hence righ bisecters are concurrent.

Median:-



Slope of $CM_1 = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{0-c}{\frac{a}{2}-b} = \dfrac{-c}{\frac{a-2b}{2}} = \boxed{\dfrac{-2c}{a-2b}}$

Equation of line CM_1 -

$$y-c = \frac{-2c}{a-2b}(x-b)$$
$$(a-2b)y-(a-2b)c = -2cx+2bc$$
$$2cx+(a-2b)y-ac+2bc-2bc=0$$
$$2cx+(a-2b)y-ac=0 \quad \text{--- (1)}$$

Equation of median AM_2

$$y-0=\frac{c}{a+b}(x-0)$$
$$y(a+b)=cx$$
$$cx-y(a+b)=0 \quad \text{--- (2)}$$

Equation of median BM_3

$$y-0=\frac{c}{b-2a}(x-a)$$
$$(b-2a)y = cx-ca$$
$$cx-(b-2a)y-ca=0 \quad \text{--- (3)}$$

Now $\begin{vmatrix} c & -(a+b) & 0 \\ c & -(b-2a) & -ac \\ 2c & (a-2b) & -ac \end{vmatrix}$

$= c\begin{vmatrix} -(b-2a) & -ac \\ a-2b & -ac \end{vmatrix} - \left(-(a+b)\right)\begin{vmatrix} c & -ac \\ 2c & -ac \end{vmatrix}$

$= c\left[ac(b-2a) + ac(a-2b)\right] + (a+b)\left[-ac^2 + 2ac^2\right]$

$= c[ac]\left[b-2a+a-2b\right] + (a+b)(ac^2)$

$= -a^2c^2\left(-a-b\right) + (a+b)ac^2$

$= -a^2c^2 - abc^2 + a^2c^2 + abc^2 = \boxed{0}$ so medians are concurrent

Q3

A(6, 1) B(-2, -3) C(8, 2m) are collinear. find m.

Sol slope of $AB = \frac{-3-1}{-2-6} = \boxed{\frac{1}{2}}$

Slope of $BC = \frac{2m-(-3)}{8-(-2)} = \frac{2m+3}{8+2} = \boxed{\frac{2m+3}{10}}$

collinear slope of AB = slope of BC

$\frac{1}{2} = \frac{2m+3}{10}$

$5-3 = 2m \Rightarrow \boxed{m=1}$

Q4.

~~slope of~~ $A(x, y)$ $B(-4, 6)$ & $C(-2, 3)$

Slope of $AB = \frac{6-y}{-4-x}$

Slope of BC $= \boxed{\dfrac{-3}{2}}$

$$\frac{6-4}{-4-x} = \frac{-3}{2}$$

$$2(6-y) = -3(4-x)$$

$$12-2y = -12+3x$$

$3x+2y=0$ is required relation

S

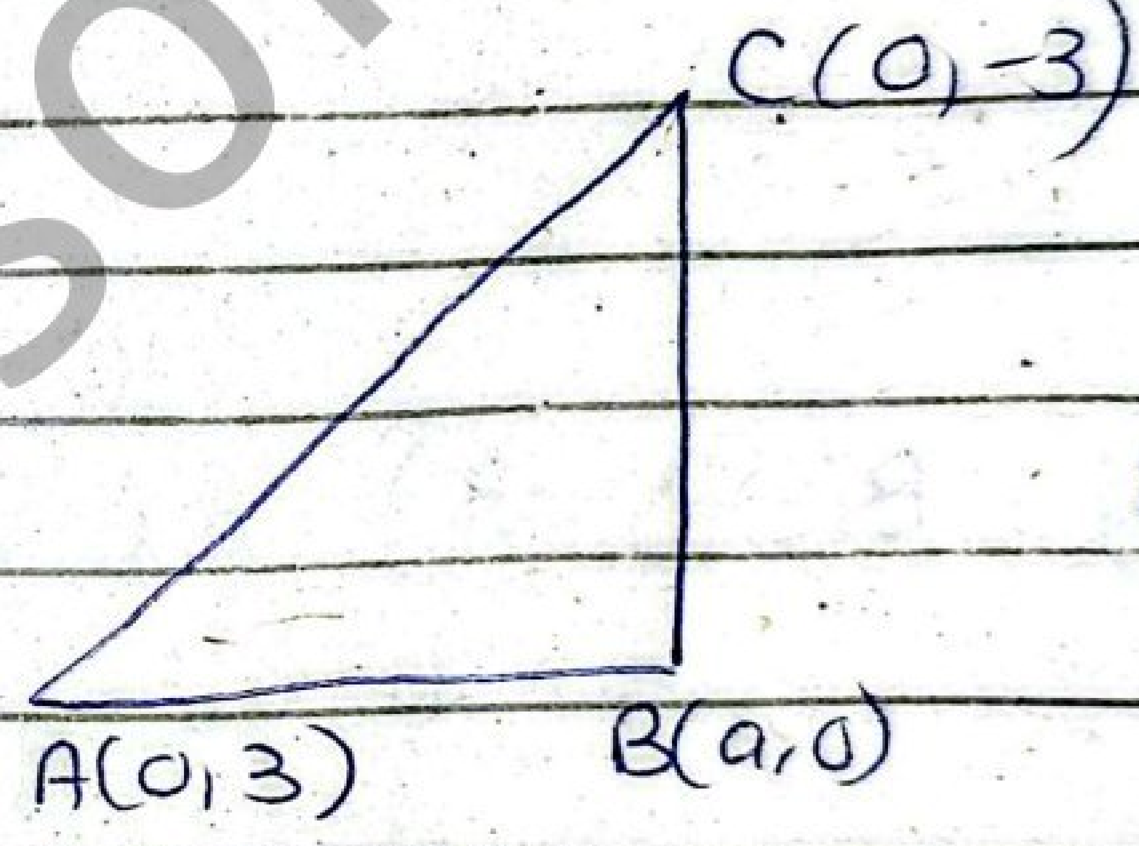
Sol (Slope of $\overline{AB}$)(slope of $\overline{BC}$) $= -1$

$$\left(\frac{0-3}{a-0}\right)\left(\frac{-3-0}{0-a}\right) = -1$$

$$\frac{9}{-a^2} = -1$$

$$a^2 = 9$$

$$\boxed{a = \pm 3}$$



Thus, vertex B is $(a,0) = (-3,0)$ or $(3,0)$

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & 3 & 1 \\ -3 & 0 & 1 \\ 0 & -3 & 1 \end{vmatrix}$$

$= \boxed{9 \text{ sq. unit}}$

Q6

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 3 & 1 \\ 1 & 0 & 1 \\ 9 & 0 & 1 \end{vmatrix}$$

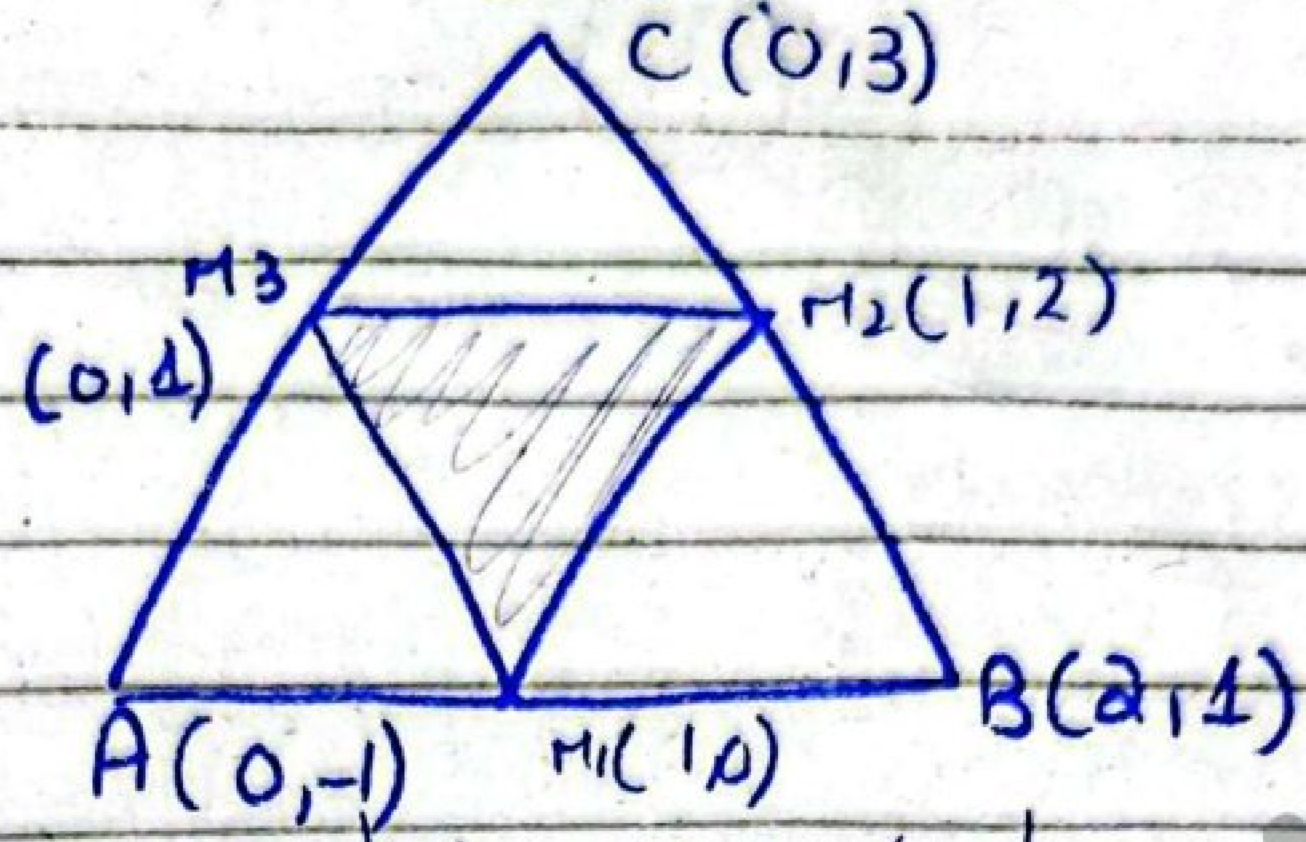
A(0,3)

B(1,0)

C(9,0)

$= \frac{15}{2}$ sq. units

(Q7)



Area of $M_1M_2M_3 = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix}$

$= \boxed{1 \text{ sq. unit}}$

Now area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & -1 & 1 \\ 2 & 1 & 1 \\ 0 & 3 & 1 \end{vmatrix}$

$= \boxed{4 \text{ sq. unit}}$ ratio $= \boxed{1 : 4}$

(Q8)

$l_1 : x - y - 1 = 0$ —(1)

$l_2 : 2x + y - 5 = 0$ —(2)

$l_3 : (2m-5)x - my - 3 = 0$ —(3)

$$\begin{vmatrix} 1 & -1 & -1 \\ 2 & 1 & -5 \\ 2m-5 & -m & -3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -5 \\ -m & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & -5 \\ 2m-5 & -3 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 2m-5 & -m \end{vmatrix} = 0$$

$$(-3-5m) + (-6+10m-25) - (-2m-2m+5) = 0$$

$$\boxed{m = \frac{13}{3}}$$

Q9

$$2x - y + 10 = 0.$$

x-Intercept. put $y = 0$.

$$2x - 0 + 10 = 0$$

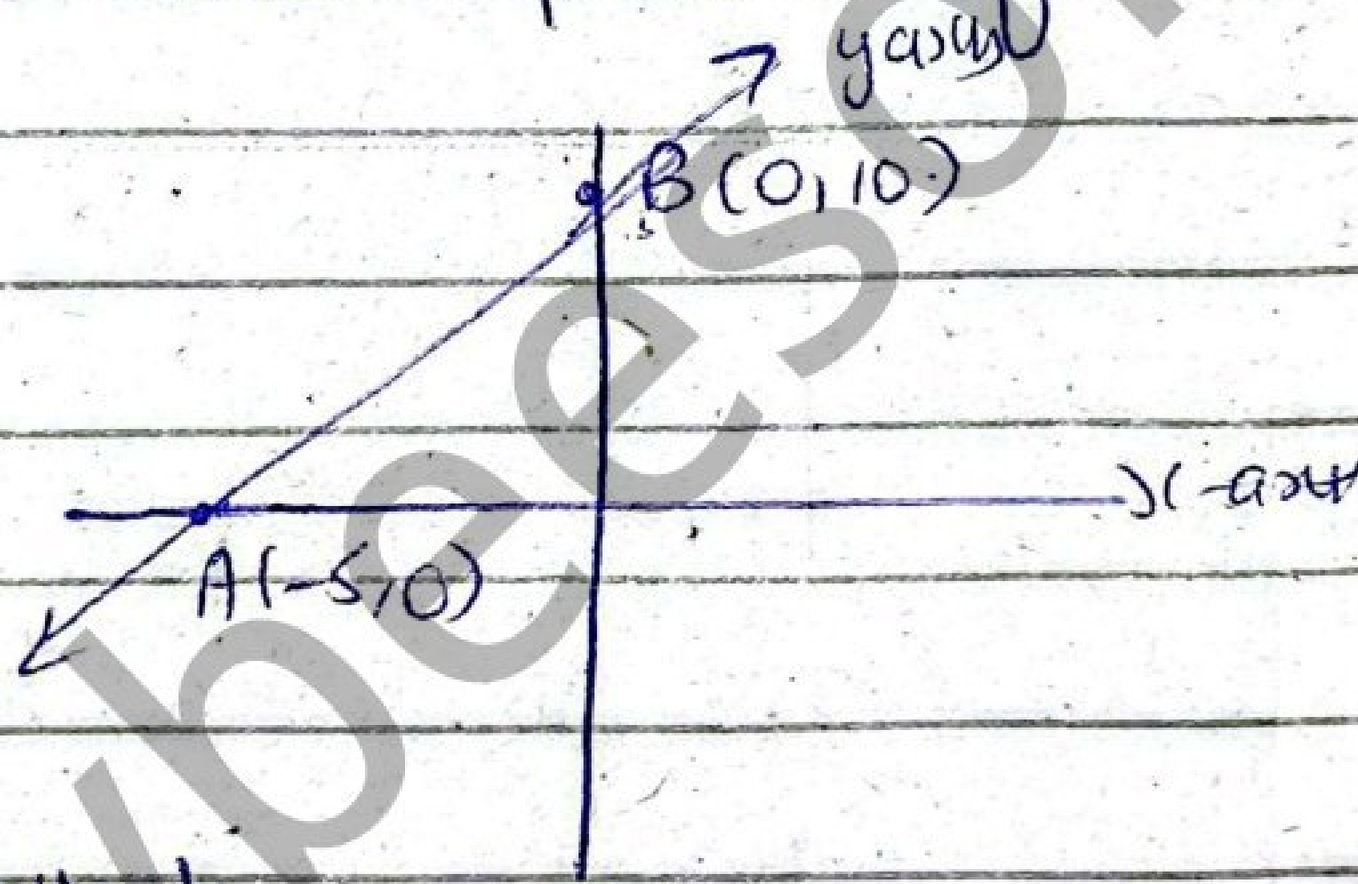
$$\boxed{x = -5}$$

y-Intercept put $x = 0$.

$$2(0) - y + 10 = 0$$

$$\boxed{y = 10}$$

Thus, $B(0,10)$ is the point of intersection on y-axis



$$\text{Area} = \frac{1}{2}|-5||10|$$

$$= 25 \text{ sq unit}$$

Q10

$$x^2 - xy - 6y^2 = 0 \; ; \quad x - y + 3 = 0.$$

$$x^2 - xy - 6y^2 = 0$$

$$x^2 - 3xy + 2xy - 6y^2 = 0$$

$$(x + 2y)(x - 3y) = 0$$

$$x + 2y = 0 \text{ — (1)} \quad ; \quad x - 3y = 0 \text{ — (2)}$$

$$x - y + 3 = 0 \text{ — (3)}$$

- point of ∩ of ① & ②.

$\cancel{x} + 2y = 0$

$\underline{\cancel{-x} \mp 3y \quad = 0}$

$5y = 0$

$\boxed{y = 0}$ put in ① $x + 2(0) = 0$

$\boxed{x = 0}$

- Point of ∩ of l_1 & l_3

$\cancel{x} + 2y = 0$

$\underline{\cancel{+x} - y + 3 = 0}$
$\quad\quad - \quad + \quad -$

$3y - 3 = 0$

$\boxed{y = 1}$ put in ① $x + 2 = 0$

$\boxed{x = -2}$

- Point of ∩ of l_2 & l_3

$\cancel{x} - 3y = 0$

$\underline{\cancel{+x} \mp y \pm 3 = 0}$

$-2y - 3 = 0$

$-2y = 3$

$\boxed{y = -\frac{3}{2}}$ put in ③

$x - \left(-\frac{3}{2}\right) + 3 = 0$

$x + \frac{3}{2} + 3 = 0$

$\boxed{x = -\frac{9}{2}}$

Area

$$= \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ -\frac{9}{2} & -\frac{3}{2} & 1 \\ -2 & 1 & 1 \end{vmatrix}$$

$= \boxed{-\frac{15}{4}}$ sq. unit

Q11.

Sol Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} -3 & 1 & 1 \\ -1 & 4 & 1 \\ 3 & 2 & 1 \end{vmatrix}$

$= \boxed{-8}$ sq. unit

Area of ADC: $\frac{1}{2}\begin{vmatrix} -3 & 1 & 1 \\ 1 & -2 & 1 \\ 3 & 2 & 1 \end{vmatrix}$

$= 11$ sq. unit

Area of quadrilateral $= 8 + 11 = 19$ sq. unit