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Where Minds Pollinate

Class 12

Mathematics

Chapter 6

Exercise 6.2

Handwritten Solution

Exercise # 6.2

- Homogenous equation: $f(x,y)=0$

$$x=kx \quad , y=ky$$

$$f(kx,ky)=k^n f(x,y)$$

Homogenous equation of degree n.

- Linear Homogenous eq in two variable.

$$ax+by=0$$

- Non Homogenous eq :- $ax+by+c=0$

- Quadratic Homogenous eq in variables or (Joint Eq).

$$ax^2+2hxy+by^2=0$$

Q.1 (i) $x^2+5xy+6y^2=0$

Sol:

$$x^2+3xy+2xy+6y^2=0$$

$$x(x+3y)+2y(x+3y)=0$$

$$(x+2y)(x+3y)=0$$

$x+2y=0$; $x+3y=0$

are the required lines.

(vi) $x^2-3xy-y^2=0$

Sol: $\div$ing eq both sides by y^2

$$\frac{x^2}{y^2}-\frac{3xy}{y^2}-\frac{y^2}{y^2}=\frac{0}{y^2}$$

$$\left(\frac{x}{y}\right)^2-3\left(\frac{x}{y}\right)-1=0$$

or also can let $\frac{x}{y}=t$ & use Quadratic formula

$a=1 \quad b=-3 \quad c=-1$.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{x}{y} = \frac{+3 \pm \sqrt{9-4(1)(-1)}}{2(1)}$$

$$\frac{x}{y} = \frac{3 \pm \sqrt{13}}{2}$$

$$\frac{x}{y} = \frac{3+\sqrt{13}}{2} \quad ; \quad \frac{x}{y} = \frac{3-\sqrt{13}}{2}$$

$$x = \frac{3+\sqrt{13}}{2} y \quad ; \quad x = \frac{3-\sqrt{13}}{2} y$$

OR

$x - \left(\frac{3+\sqrt{13}}{2}\right)y = 0$; $x - \left(\frac{3+\sqrt{13}}{2}\right)y$ are required lines.

Concept: • $ax^2+2hxy+by^2=0$ represents a pai of lines passing through origian. The lines are.

i) Real and distinct if $h^2 > ab$

ii) Real & concident if $h^2 = ab$

iii) imaginary if $h^2 < ab$.

∴ distinct means different

∴ concident means similar

Q3) (ii) $4x^2-12xy+9y^2=0$.

Compare with

$$ax^2+2hxy+by^2=0$$

$\boxed{a=4}$ $2h=-12$, $b=9$.

$\boxed{h=-6}$

$\boxed{h^2=36}$ (1) & $ab=(4)(9) \Rightarrow \boxed{ab=36}$ (2) as $h^2=ab$ so lines are real & concident

∴ Trick:- if sum of coefficients of x^2 & y^2 is 0 then angle b/w lines will be $90°$.

(v) $x^2-9y^2=0$

Solution:- $x^2+0xy-9y^2=0$

$a=1$, $2h=0$, $b=-9$

$\boxed{h=0}$ $\qquad ab=-9$ — (2)

$h^2=0$ — (1)

so $h^2>ab$ real & distinct

Concept:- Angle b/w lines represented by $ax^2+2hxy+by^2=0$

also IMP

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$$

∴ for acute angle:

$\tan\theta = |x|$

∴ $l_1 \parallel l_2$ if $h^2=ab$.

$l_1 \perp l_2$ if $a+b=0$. ($\theta=90°$)

Q2) IMP

(vi) $7x^2+2xy-9y^2=0$.

Compare with $ax^2+2hxy+by^2=0$.

$\boxed{a=7}$ $\quad 2h=2$ $\quad \boxed{b=-9}$

$\boxed{h=1}$

As $\tan\theta = \dfrac{2\sqrt{h^2-ab}}{a+b}$

$= \dfrac{2\sqrt{1^2-(7)(-9)}}{7-9}$

$\boxed{\tan\theta = -8}$

for acute angle $\tan\theta=|-8|$

∴ $\theta = \tan^{-1}(+8)$

$\boxed{\theta = 82.87°}$

Q4 (ii) $x^2-2\tan\theta\, xy - y^2=0$

Sol:- Comparing with $ax^2+2hxy+by^2=0$.

$a=1$, $2h=-2\tan\theta$, $\boxed{b=-1}$

$\boxed{h=-\tan\theta}$

$m_1 . m_2 = -1$

$$\boxed{m_1 = \frac{-1}{m_2}}$$

Let α be the angle b/w lines -

$$\tan\alpha = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$= \frac{2\sqrt{(-\tan\theta)^2-(1)(-1)}}{1-1}$$

$\tan\alpha = \infty$

$\alpha = \tan^{-1}(\infty) \Rightarrow \boxed{\alpha = 90^\circ}$ Ans

V. IMPORTANT

Q5) i) $2x^2 - 7xy + 6y^2 = 0$

Sol:- $2x^2 - 4xy - 3xy + 6y^2 = 0$

$2x(x - 2y) - 3y(x - 2y) = 0$

$(x-2y)(2x-3y) = 0$

$x - 2y = 0$ — (1); $2x - 3y = 0$ — (2).

$x = 2y$

$y = \frac{x}{2}$ $\therefore y = mx + c$.

so $y = mx$ for homogenous eq.

$\boxed{m = \frac{1}{2}}$

Eq of line passing through origin & $\perp$ to eq (1).

$y = -\frac{1}{m}x$

$y = -\frac{1}{+\frac{1}{2}}x$

$\boxed{y = -2x}$

$\therefore y = mx$

$\boxed{y = -\frac{1}{m}x}$

from ②

$3y = 2x$

$y = \frac{2}{3}x$ $\boxed{m = \frac{2}{3}}$

Eq of line passing through origin & $\perp$ to ②

$$y = -\frac{1}{m}x$$

$$y = -\frac{1}{2/3}x$$

$$\boxed{y = -\frac{3}{2}x}$$ or [scribbled out]

Joint equation:-

$$2x + y = 0 \quad ; \quad 3x + 2y = 0.$$

$$(2x+y)(3x+2y) = 0$$

$$6x^2 + 4xy + 3xy + 2y^2 = 0$$

$$6x^2 + 7xy + 2y^2 = 0$$ is the

required joint equation.

Q1) (ii) $7x^2 - 2xy - 9y^2 = 0$

Sol:- $7x^2 - 9xy + 7xy - 9y^2 = 0$

$x(7x - 9y) + y(7x - 9y) = 0$

$(x+y)(7x-9y) = 0$

$\boxed{x+y=0}$ $\boxed{7x - 9y = 0}$ are required lines

(iii) $x^2 + 6xy = 0$

Solution: $x(x+6y) = 0$

$\boxed{x=0}$ $\boxed{x + 6y = 0}$ are required lines

(v) $5x^2+3xy-2y^2=0$

Solution:- $5x^2+5xy-2xy-2y^2=0$

$5x(x+y)-2y(x+y)=0$

$(5x-2y)(x+y)=0$

$\boxed{5x-2y=0}$ $\boxed{x+y=0}$ are given lines.

(iv) $x^2-8xy+12y^2=0$

Solution:- $x^2-6xy-2xy+12y^2=0$

$x(x-6y)-2y(x-6y)=0$

$\boxed{x-2y=0}$; $\boxed{x-6y=0}$ are given lines.

Q3 (i) $x^2+4xy-21y^2=0$

Sol:- $ax^2+2hxy+by^2=0$

$\boxed{a=1}$ $\boxed{b=-21}$ $2h=4$

$\boxed{h=2}$

$h^2=\boxed{4}$ $ab=(1)(-21)=\boxed{-21}$.

$h^2>ab$ lines are real and distinct.

(iii) $x^2+xy+y^2=0$.

Sol:- $a=1$ $h=1$ $b=1$.

$h^2=1^2$ so $h=ab$ lines are real and concident.

Q2) (i) $x^2-y^2=0$

Solution:- $x^2+0hxy-y^2=0$

$\boxed{a=1}$ $\boxed{b=-1}$ $\boxed{2h=0}$ $\boxed{h=0}$

$\tan\theta=\dfrac{2\sqrt{0^2-(1)(-1)}}{1-1}=\dfrac{1}{0}\Rightarrow\tan\theta=\infty$

$\theta=\tan^{-1}(\infty)=\boxed{90^\circ}$

(ii) $x^2+5xy+4y^2=0$

Sol:- $a=1$; $2h=5$ $\boxed{b=4}$

$\boxed{h=\frac{5}{2}}$

$\tan\theta = \frac{2\sqrt{(\frac{5}{2})^2-4}}{5} \Rightarrow \tan\theta = \frac{2\sqrt{\frac{25-16}{4}}}{5}$

$\tan\theta = \frac{3}{5}$

$\theta = \tan^{-1}\left(\frac{3}{5}\right)$

$\theta = 30.96°$ Ans

(iii) $15x^2-19xy+6y^2=0$

$\boxed{a=15}$ $2h=-19$ $\boxed{b=6}$

$\boxed{h=\frac{-19}{2}}$

$\tan\theta = \frac{2\sqrt{(\frac{-19}{2})^2-(15)(6)}}{15+6} \Rightarrow \frac{2\sqrt{(\frac{-19}{2})^2-(15)(6)}}{21}$

$\tan\theta = \frac{1}{21} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{21}\right) \Rightarrow \boxed{2.72°}$ Ans

(iv) $10x^2-xy-9y^2=0$

Sol:- $a=10$ $2h=-1$ $b=-9$

$\boxed{h=-\frac{1}{2}}$

$\tan\theta = \frac{2\sqrt{(\frac{-1}{2})^2-(10)(-9)}}{10-9}$

$\tan\theta = 19 \Rightarrow \theta = \tan^{-1}(19) \Rightarrow \boxed{\theta = 86.98°}$

(v) $5x^2-3xy-2y^2=0$

Sol:- $\boxed{a=5}$ $2h=-3$ $\boxed{b=-2}$

$\boxed{h=-\frac{3}{2}}$

$\tan\theta = \frac{2\sqrt{(-\frac{3}{2})^2-(5)(-2)}}{5-2}$

$\tan\theta = \frac{7}{3}$ $\theta = \tan^{-1}\left(\frac{7}{3}\right)$ $\boxed{\theta = 66.80°}$

Q4) i) $x^2+5xy-y^2=0$

Solution:- $a=1 \quad b=-1 \quad 2h=5$ $\quad \boxed{h=\frac{5}{2}}$

$\tan\alpha = \frac{2\sqrt{(\frac{5}{2})^2-(1)(-1)}}{1-1}$

$\tan\alpha = \frac{1}{0} \Rightarrow \tan\alpha = \infty \Rightarrow \alpha = \tan^{-1}(\infty)$

$\boxed{\alpha = 90^\circ}$ Ans

Q5) (ii) $x^2+17xy+60y^2=0$

Solution:- $x^2+5xy+12xy+60y^2=0$

$x(x+5y)+12y(x+5y)=0$

$(x+12y=0$ — ① ; $x+5y=0$ — ②

$x=-12y$

$y=\frac{-1}{12}x$

$\boxed{m=\frac{-1}{12}}$

now eq of line passing through origion and $\perp$ to eq ①

$y=\frac{-1}{m}x$

$y=\frac{-1}{-\frac{1}{12}}x$

$\boxed{y=12x}$

from eq ②

$x=-5y$

$y=\frac{-1}{5}x$

$\boxed{m=\frac{-1}{5}}$

now

eq of line passing through origion and $\perp$ to eq ②

$y=\frac{-1}{-\frac{1}{5}}x$

$\boxed{y=5x}$

$(y-12x)(y-5x)$

$y^2-5xy-12xy+60x^2$

$y^2-17xy+60x^2=0$ is the required equation Ans

(iii) $3x^2-13xy-10y^2=0$

Solution:- ~~$3x^2-3xy-10xy-10y^2=0$~~ ~~$3x(x-y)$~~

$a=3 \quad b=-13y \quad c=-10$

$x=\dfrac{-(-13y)\pm\sqrt{(-13y)^2-4(3)(-10y)^2}}{2(3)}$

$x=\dfrac{13y\pm\sqrt{289y^2}}{6}$

$x=\dfrac{13y+17y}{6}$ and $x=\dfrac{13y-17y}{6}$

$\boxed{x=5y}$ $\quad x=-\dfrac{2}{3}y$

$\boxed{x-5y=0}$ (1) $\quad \boxed{3x+2y=0}$ (2)

$y=\dfrac{1}{5}x$

$\boxed{m=\dfrac{1}{5}}$

now eq of line at origin $\perp$ to eq (1)

$y=-\dfrac{1}{1/5}x$

$y=-5x$

$\boxed{y+5x=0}$

using (2)

$3x=-2y$

$y=-\dfrac{3}{2}x$

$\boxed{m=-\dfrac{3}{2}}$

$y=-\dfrac{1}{-3/2}x$

$y=\dfrac{2}{3}x$

$2x-3y=0$

$(y+5x)(2x-3y)=0$

$10x^2-15xy+2xy-3y^2=0$

$10x^2-3xy-3y^2=0$

(iv) $x^2+3xy-28y^2=0$

Sol: $x^2+7xy-4xy-28y^2=0$

$x(x+7y)-4y(x+7y)=0$

$(x-4y)(x+7y)=0$

$x-4y=0$ —(1) ; $x+7y=0$ —(2)

$y=\frac{1}{4}x$

$\boxed{m=\frac{1}{4}}$

eq of line passing through origin & ⊥ to (1)

$y=\frac{-1}{1/4}x$

$y=-4x$

$\boxed{y+4x=0}$

from (2)

$x=-7y$

$y=-\frac{1}{7}x$

$\boxed{m=-\frac{1}{7}}$

eq of line passing through origin & ⊥ to (2)

$y=-\frac{1}{-1/7}x$

$y=+7x$

$\Rightarrow \boxed{7x-y=0}$

$(4x+y)(-y+7x)=0$

$28x^2-4xy+7xy-y^2=0$

$28x^2+3xy-y^2=0$