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Where Minds Pollinate

Class 12

Mathematics

Chapter 5

Exercise 5.3

Handwritten Solution

Exercise 5.3.

Vector Valued function: A vector whose domain is real number but range is Vector is called vector valued function.

eg $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

$t = 1, 2, 3$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ $\boxed{t=1}$

Q3. (i) $\vec{r}(t) = (3t-7)\hat{i} + t\hat{j} - t^2\hat{k}$

Solution:- $|\vec{r}(t)| = \sqrt{(3t-7)^2 + t^2 + (-t^2)^2}$

$= \sqrt{9t^2 + 49 - 42t + t^2 + t^4}$

$|\vec{r}(t)| = \sqrt{t^4 + 10t^2 - 42t + 49}$ Answer Scalar number.

(iii) $r(t) = (\ln(t^2+1))\hat{i} + (\tan t)\hat{j} + \sec t\hat{k}$

Solution $|\vec{r}(t)| = \sqrt{[\ln(t^2+1)]^2 + \tan^2 t + \sec^2 t}$ Answer

(ii) $r(t) = \left(\frac{t}{\sqrt{2}}\right)\hat{i} + \left(\frac{t}{\sqrt{2}} + 16t^2\right)\hat{j} - 2t\hat{k}$

Solution: $|\vec{r}(t)| = \sqrt{\left(\frac{t}{\sqrt{2}}\right)^2 + \left(\frac{t}{\sqrt{2}} + 16t^2\right)^2 + (-2t)^2}$

$|\vec{r}(t)| = \sqrt{\frac{t^2}{2} + \frac{t^2}{2} + 2\left(\frac{t}{\sqrt{2}}\right)(16t^2) + 256t^4 + 4t^2}$

$r = \sqrt{256t^4 + 4t^2 + t^2 + 16\sqrt{2}t^3}$

$= \sqrt{256t^4 + 16\sqrt{2}t^3 + 5t^2}$ Answer

$\therefore \boxed{V = \frac{dr}{dt}} \qquad \boxed{a = \frac{dv}{dt}}$

Q2. (i) $r(t) = (3t+1)\hat{i} + \sqrt{3}t\hat{j} + t^2 K$

Solution $V = \frac{d\vec{r}}{dt} = 3\hat{i} + \sqrt{3}\hat{j} + 2tK$ —— (1)

At $t=0$.

$V = 3\hat{i} + \sqrt{3}\hat{j} + 0K$

$\boxed{V = 3\hat{i} + \sqrt{3}\hat{j}}$ A

Diff (1) w.r.t t.

$\frac{dv}{dt} = 0\hat{i} + 0\hat{j} + 2K$

$a = 0\hat{i} + 0\hat{j} + 2K$

At $t=0$.

$a = 0\hat{i} + 0\hat{j} + 2K$ Ans

(iii) $r(t) = (\ln(t^2+1))\hat{i} + (\tan^{-1} t)\hat{j} + \sqrt{t^2+1}\,K$

Diff w.r.t $\boxed{t}$.

Solution $\frac{d\vec{r}}{dt} = \frac{1}{t^2+1} \cdot \frac{d}{dt}(t^2+1)\hat{i} + \frac{1}{1+t^2}\hat{j} + \frac{1}{2}(t^2+1)^{\frac{1}{2}-1} \cdot \frac{d}{dt}(t^2+1)K$

$V = \frac{2t}{t^2+1}\hat{i} + \frac{1}{1+t^2}\hat{j} + \frac{1}{2\sqrt{t^2+1}} 2t K$ —— (1)

At $t=0$

$V = \frac{0}{0^2+1}\hat{i} + \frac{1}{1+0^2}\hat{j} + \frac{1}{\sqrt{0^2+1}}(0)K$

$V = 0\hat{i} + \hat{j} + 0K$

Again from (1)

$V = \frac{2t}{t^2+1}\hat{i} + (1+t^2)^{-1}\hat{j} + \frac{t}{\sqrt{t^2+1}}K$

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Diff w.r.t t

$$\frac{dv}{dt} = \frac{(t^2+1)\cdot\frac{d}{dt}(2t) - 2t\frac{d}{dt}(t^2+1)}{(t^2+1)^2}\,\underline{i} - (1+t^2)^{-2}\cdot\frac{d}{dt}(1+t^2)\,\underline{j}$$

$$+ \frac{\sqrt{t^2+1}\cdot\frac{d}{dt}(t) - t\frac{d}{dt}(t^2+1)^{1/2}}{(\sqrt{t^2+1})^2}\,K$$

$$\underline{a} = \frac{2(t^2+1) - 2t(2t)}{(t^2+1)^2}\,\hat{\underline{i}} - \frac{2t}{(1+t^2)^2}\,\underline{j} + \frac{\sqrt{t^2+1}\cdot(1) - t\cdot\frac{1}{2\sqrt{t^2+1}}\,2t}{t^2+1}\,\underline{K}$$

$$= \frac{2t^2+2-4t^2}{(t^2+1)^2}\,\underline{i} - \frac{2t}{(1+t^2)^2}\,\underline{j} + \frac{1}{t^2+1}\left[\frac{(\sqrt{t^2+1})^2 - t^2}{\sqrt{t^2+1}}\right]\underline{K}$$

$$\underline{a} = \frac{2-2t^2}{(t^2+1)^2}\,\hat{\underline{i}} - \frac{2t}{(1+t^2)^2}\,\underline{\hat{j}} + \frac{t^2+1-t^2}{(t^2+1)\sqrt{t^2+1}}\,\underline{K}$$

At $t=0$

$$\underline{a} = \frac{2-0}{(0+1)^2}\,\underline{\hat{i}} - \frac{0}{(1+0)^2}\,\hat{j} + \frac{1}{(0+1)\sqrt{0+1}}\,K$$

$$\boxed{\underline{a} = 2\underline{\hat{i}} - 0\underline{\hat{j}} + \underline{K}}$$ Answer

Q1) (i) $\vec{r}(t) = (t+1)\underline{\hat{i}} + (t^2-1)\underline{\hat{j}} + 5t\underline{K}$ — (1) ; $t=2$

Solution:- Dom $\vec{r}(t) = \mathbb{R}$ or $(-\infty, +\infty)$

Range = Vectors.

$\vec{r}(2) = 3\underline{\hat{i}} + 3\underline{\hat{j}} + 10\underline{k}$ at $t=2$

Diff (1) w.r.t "t"

$$\frac{d\vec{r}}{dt} = \frac{d}{dt}(t+1)\hat{i} + \frac{d}{dt}(t^2-1)\hat{j} + \frac{d}{dt}(5t)K$$

$\frac{d\vec{r}}{dt} = 1\hat{i} + 2t\hat{j} + 5\hat{k}$ —— ②

At $t=2$

$v = \hat{i} + 2(2)\hat{j} + 5\hat{k}$

$\boxed{\frac{d\vec{r}}{dt} = \hat{i} + 4\hat{j} + 5\hat{k}}$

Diff ② w.r.t t

$\frac{d^2\vec{r}}{dt^2} = 0\hat{i} + 2\hat{j} + 0\hat{k}$

$\frac{d^2\vec{r}}{dt^2} = 2\hat{j}$

At $t=2$

$\boxed{\frac{d^2\vec{r}}{dt^2} = 2\hat{j}}$ Answer.

(iii) $\vec{r}(t) = e^{t}\,\underline{\hat{i}} + \frac{2}{9}\, e^{2t}\,\underline{\hat{j}} + 5e^{-t}\,\underline{K}$; $t = \ln 2$

Solution:- Dom $\vec{r}(t) = \mathbb{R}$

Rang $\vec{r}(t)$ = vector

$\frac{dr}{dt} = e^{t}\hat{i} + \frac{2}{9} \cdot 2e^{2t}\hat{j} + 5(-1)e^{-t}\hat{K}$

$\frac{dr}{dt} = e^{t}\hat{i} + \frac{4}{9}e^{2t}\hat{j} - 5e^{-t}\hat{k}$ —— (2)

At $t = \ln 2$

$r = e^{\ln 2}\hat{i} + \frac{4}{9}e^{2\ln 2}\hat{j} - 5e^{-\ln 2}\hat{K}$

$= 2\hat{i} + \frac{4}{9}e^{\ln 2^{2}}\hat{j} - 5e^{\ln 2^{-1}}\hat{K}$

$= \boxed{2\hat{i} + \frac{4}{9}4\hat{j} - \frac{5}{2}\hat{K}}$

Diff w.r.t 2.

$\frac{d^2r}{dt} = e^{t}\hat{i} + \frac{4}{9} \times 2e^{2t}\hat{j} - 5(-1)e^{-t}\hat{K}$

$= e^{t}\hat{i} + \frac{8}{9}e^{2t}\hat{j} + 5e^{-t}\hat{k}$

At $t = \ln 2$

$\frac{d^2r}{dt} = e^{\ln 2}\hat{i} + \frac{8}{9}e^{2\ln 2}\hat{j} + 5e^{-\ln 2}\hat{k}$

$= 2\hat{i} + \frac{8}{9}e^{\ln 2^{2}}\hat{j} + 5e^{\ln 2^{-1}}\hat{K}$

$= 2\hat{i} + \frac{8}{9}4\hat{j} + \frac{5}{2}\hat{K}$

$= 2\hat{i} + \frac{32}{9}\hat{j} + \frac{5}{2}\hat{k}$ Answer.

Dom $= \mathbb{R}-[0,1]$ $\therefore \cos 0 = 1$

QR Rang = Vector $\therefore \sin 0 = 0$

(iv) $r(t) = (\cos 2t)\hat{i} + (3\sin 2t)\hat{j} + 5tk$, $t=0$

Solution: ~~differentiating~~ Dom $r(t) = \mathbb{R}$

Rang $r(t)$ = vector

$\frac{dr}{dt} = -\sin 2t \cdot \frac{d}{dt}(2t)\hat{i} + 3\cos 2t \frac{d}{dt}(2t)\hat{j} + 5 \cdot 1\hat{k}$

$= -2\sin 2t\hat{i} + 6\cos 2t\hat{j} + 5\hat{k}$ — (2)

At t=0

$r = -2\sin 0\hat{i} + 6\cos 0\hat{j} + 5\hat{k}$

$= 6\hat{j} + 5\hat{k}$

Diff w.r.t in eq (2)

$\frac{d^2r}{dt} = -2\cos 2t \times 2\hat{i} + 6(-\sin 2t \times 2)\hat{j} + 0\hat{k}$

$= -4\cos 2t\hat{i} - 12\sin 2t\hat{j} + 0$

At t=0

$r'' = -4\cos 0^\circ\hat{i} - 12\sin 0\hat{j}$

$= \boxed{-4\hat{i}}$

3(iv) $r(t) = \frac{4}{9}(t+1)^{3/2}\hat{i} + \frac{4}{9}(1-t)^{3/2}\hat{j} + \frac{1}{3}t^{3/2}\hat{k}$

Solution:

$|\vec{r}(t)| = \sqrt{\left\{\frac{4}{9}(t+1)^{3/2}\right\}^2 + \left\{\frac{4}{9}(1-t)^{3/2}\right\}^2 + \left(\frac{1}{3}t^{3/2}\right)^2}$

$= \sqrt{\frac{16}{81}(t+1)^3 + \frac{16}{81}(1-t)^3 + \frac{1}{9}t^3}$

$\frac{4}{9}\sqrt{t^2+3t^2+3t+1+1-3t+3t^2-t^2+9t^3}$

$= \frac{4}{9}\sqrt{9t^3+6t^2+2}$ Answer.

Q1) (ii) $r(t) = \frac{t}{t+1}\hat{i} + \frac{1}{t}\hat{j} + t^3\hat{k}$, $t = -\frac{1}{2}$

Solution: Diff w.r.t t

$$r' = \frac{(1+t)\frac{d}{dt}(t) - t\frac{d}{dt}(1+t)}{(1+t)^2}\hat{i} + \frac{t\frac{d}{dt}(1) - 1\frac{d}{dt}(t)}{t^2}\hat{j} + 3t^2\hat{k}$$

$$= \frac{1+t-t}{(1+t)^2}\hat{i} + \frac{t\times 0 - 1}{t^2}\hat{j} + 3t^2\hat{k}$$

$$= \frac{1}{(1+t)^2}\hat{i} - \frac{1}{t^2}\hat{j} + 3t^2\hat{k} \quad \text{--- (2)}$$

At $t = -\frac{1}{2}$

$$r' = \frac{1}{\left(1-\frac{1}{2}\right)^2}\hat{i} - \frac{1}{\left(\frac{-1}{2}\right)^2}\hat{j} + 3\left(\frac{-1}{2}\right)^2\hat{k}$$

$$= \frac{1}{\left(\frac{2-1}{2}\right)^2}\hat{i} - \frac{1}{\frac{1}{4}}\hat{j} + 3\left(\frac{1}{4}\right)\hat{k}$$

$$= 4\hat{i} - 4\hat{j} + \frac{3}{4}\hat{k}$$

Diff with from (2) Diff w.r.t t.

$$r'' = \frac{d}{dt}(1+t)^{-2}\hat{i} - \frac{d}{dt}(t^{-2})\hat{j} + 3\frac{d}{dt}(t^2)\hat{k}$$

$$= -2(1+t)^{-3}\frac{d}{dt}(1+t)\hat{i} - (-2)t^{-3}\frac{d}{dt}(t)\hat{j} + 3\cdot 2t\hat{k}$$

$$= \frac{-2}{(1+t)^3}\hat{i} + \frac{2}{t^3}\hat{j} + 6t\hat{k}$$

at $t = -\frac{1}{2}$

$$r'' = \frac{-2}{\left(1-\frac{1}{2}\right)^3}\hat{i} + \frac{2}{\left(\frac{-1}{2}\right)^3}\hat{j} + 6\left(\frac{-1}{2}\right)\hat{k} = -16\hat{i} - 16\hat{j} - 3\hat{k}$$

Answer

Q2 (iv) $r(t) = \frac{4}{9}(t+1)^{3/2}\hat{i} + \frac{4}{9}(1-t)^{3/2}\hat{j} + \frac{1}{3}tk$

Sol: Diff w.r.t t

$v = \frac{dr}{dt} = \frac{4}{9}\cdot\frac{3}{2}(1+t)^{\frac{3}{2}-1}\hat{i} + \frac{4}{9}\times\frac{3}{2}(1-t)^{\frac{3}{2}-1}(-1)\hat{j} + \frac{1}{3}\hat{k}$

$= \frac{2}{3}(1+t)^{1/2}\hat{i} - \frac{2}{3}(1-t)^{1/2}\hat{j} + \frac{1}{3}\hat{k}$ — (2)

At $t=0$

$v = r' = \frac{2}{3}(1+0)^{1/2}\hat{i} - \frac{2}{3}(1-0)^{1/2}\hat{j} + \frac{1}{3}\hat{k}$

$= \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$

Diff w.r.t t

$a = \frac{d^2r}{dt^2} = \frac{2}{3}\left(\frac{1}{2}\right)(1+t)^{\frac{1}{2}-1} - \frac{2}{3}\left(\frac{1}{2}\right)(1-t)^{\frac{1}{2}-1}(-1)\hat{j} + 0\hat{k}$

$a = \frac{1}{3}(1+t)^{-1/2}\hat{i} + \frac{1}{3}(1-t)^{-1/2}\hat{j} + 0$

At $t=0$

$r = \frac{1}{3}(1+0)^{-1/2}\hat{i} + \frac{1}{3}(1-0)^{-1/2}\hat{j} + 0$

$= \frac{1}{3}\hat{i} + \frac{1}{3}\hat{j}$ Ans

Q2 (ii) $r(t) = \left(\frac{t}{\sqrt{2}}\right)i + \left(\frac{t}{\sqrt{2}} - 16t^2\right)\hat{j} - 2tk$

Sol: Diff w.r.t t

$\frac{dr}{dt} = \frac{1}{\sqrt{2}}\hat{i} + \left[\frac{1}{\sqrt{2}} - 16t + 2t\right]\hat{j} - 2\hat{k}$

$= \frac{1}{\sqrt{2}}\hat{i} + \left[\frac{1}{\sqrt{2}} - 32\cancel{t0}\right]\hat{j} - 2\hat{k}$

$=$ ~~[illegible]~~

At $t=0$

$$r' = \frac{1}{\sqrt{2}}\hat{i} + \left(\frac{1}{\sqrt{2}} - 32\times 0\right)\hat{j} - 2\hat{k}$$

$$= \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} - 2\hat{k}$$

Diff w.r.t t

$$r'' = \frac{d}{dt}\left[\frac{1}{\sqrt{2}}\hat{i} + \left(\frac{1}{\sqrt{2}} - 32t\right)\hat{j} - 2\hat{k}\right]$$

$$= 0\hat{i} + (0-32)\hat{j} - 2\hat{k}$$

$$= -32\hat{j} - 2\hat{k}$$ Answer.