Studybeesofficial

Where Minds Pollinate

Class 12

Mathematics

Chapter 5

Exercise 5.2

Handwritten Solution

Exercise 5.2

Concept. $\frac{ds}{dt} = v$ $\qquad$ $\frac{dv}{dt} = a$

$\int \frac{ds}{dt} dt = \int v dt$ $\qquad$ $\int \frac{dv}{dt} dt = \int a dt$

$\boxed{S = \int v dt}$ $\qquad$ $\boxed{V = \int a dt}$

Q1) Solution:- $S_0 = 129ft$; $V_0 = 87 ft/s$

(i) $a = -g \, ft/s^2$

$a = -32 \, ft/s^2$

$\because 1m = 39 inches$
$= 36 + 3$
$3f + 3 inches$
$1m = 3.3 ft$.
$= 9.8 \times 3.3$
$= \boxed{32 ft/s^2}$

As,

$V = \int a \, dt$

$= \int -32 dt$

$= -32 \int dt$

$V = -32t + C$ —— (1)

Initially at $t=0$, $v = 87$

$87 = -32(0) + C$

$\boxed{C = 87}$

So $\boxed{V = -32t + 87}$

As,

$S = \int V dt$

$= \int (-32t + 87) dt$

$S = -32\frac{t^2}{2} + 87t + C$

Initially at $t=0$ $S = 129$

$129 = -32(0) + 0 + C$; $\boxed{C = 129}$

So $S = -16t^2 + 87t + 129$ —— (1)

(ii) The projectile will hit the ground when $S=0$

$$-16t^2+87t+129=0$$

$$16t^2-87t-129=0$$

$a=16$, $b=-87$, $c=-129$.

$$t=\frac{87\pm5\sqrt{633}}{32}$$

$$t=\frac{87+5\sqrt{633}}{32} \quad ; \quad t=\frac{87-5\sqrt{633}}{32}$$

$\boxed{t=6.64\text{sec}}$ A $\qquad t=-1.21$ not possible

(IV) The projectile will be at max height if,

$$V=0$$

$$-32t+87=0$$

$$t=\frac{87}{32} \Rightarrow \boxed{t=2.71\text{ sec}}$$

(V) At $t=2.71$ projectile will be at max height so

put $t=2.71$ in (1)

$$S=-16(2.71)^2+87(2.71)$$

$$\boxed{S=247\text{ feet}}$$

(iii)

$$t=6.64-2.71$$

$$=3.93$$

6.64s → Time hitting the ground

2.71 → Time ~~before~~ at max height

As $V=-32t+87$

$V=+32t+87$ (Projectile moving downward so g will be +ve)

$$V=32(3.93)+87$$

$V=212.76$ ft/s Impact Velocity

Q6. $a = (3t+5)ms^{-2}$, $t=0$, $V = 2ms^{-1}$, $t = T$, $V = 6ms^{-1}$.

Sol:- As, $V = \int a\,dt$

$V = \int (3t+5)dt$

$V = \frac{3t^2}{2} + 5t + C$

At $t=0$, $V = 2ms^{-1}$

$2 = 0+0+C$

$\boxed{C = 2}$

So, $V = \frac{3t^2}{2} + 5t + 2$

At $t = T$, $V = 6ms^{-1}$

$6 = \frac{3T^2}{2} + 5T + 2$

$6 - 2 = \frac{3T^2}{2} + 5T$

$8 = 3T^2 + 10T$

$3T^2 + 10T - 8 = 0$

$a = 3$, $b = 10$, $c = -8$

$\boxed{T = 0.67}$, $T = -4$ not possible

Answer

Q2 Solution:- $S = t^3 - 9t^2 + 24t + 20$ in feet.

(i) $V = ?$ (ii) $a = ?$

$\frac{ds}{dt} = 3t^2 - 18t + 24$

$\boxed{V = 3t^2 - 18t + 24}$ — (i) (2)

$\frac{dv}{dt} = 6t - 18$

$\boxed{a = 6t - 18}$ — (ii)

(ii) $a = -6.5\ ft/s^2$

$a = 6t - 18$

$-6.5 + 18 = 6t$

$\boxed{t = \frac{11.5}{6}} \Rightarrow \boxed{t = 1.91}$ put in (1)

$S = (1.91)^3 - 9(1.91)^2 + 24(1.91) + 20$

$= 39.97\ ft$ ✓

put $t = 1.91$ in (2)

$V = 3(1.91)^2 - 18(1.91) + 24$

$V = 0.56\ ft/s.$

(iii) Displacement $= S(7) - S(1.5)$

$= (7)^3 - 9(7)^2 + 24(7) + 20 - [(1.5)^3 - 9(1.5)^2 + 24(1.5) + 20]$

$= 50.875\ m$

Distance $= S(7) + S(1.5)$

$= (7)^3 - 9(7)^2 + 24(7) + 20 + (1.5)^3 - 9(1.5)^2 + 24(1.5) + 20$

$= 129.125\ m$ Answer

IMP Exercise 5.2

Q3) Solution:- (i) $g = -9.8ms^{-2}$ (upward)

$1m \approx 3.28ft$

$g = -32 ft/s^2$

$a = g \quad v = \int g\, dt$

$v = -32\int dt$

$V = -32t + C$

Initial condition $V = 96$ $t = 0$

$96 = -32(0) + C$

$\boxed{C = 96}$

$V = -32t + 96$ ft/s — (1)

now

$S = \int V dt$

$S = \int(-32t + 96)dt$

$S = \frac{-32t^2}{2} + 96t + C$

$S = -16t^2 + 96t + C$

$S = 176ft;\ t = 0$

$176 = -16(0)^2 + 96(0) + C$

$\boxed{C = 176}$

$S = -16t^2 + 96t + 176 ft$ — (2)

Ans

(ii) when ball hit the grand

$S(t) = 0$.

$-16t^2 + 96t + 176 = 0$

$-16[t^2 - 6t - 11] = 0$

$t^2 - 6t - 11 = 0$

$a = 1 \quad b = -6 \quad c = -11$

$t = \frac{6 \pm \sqrt{36 - 4(1)(-11)}}{2(1)}$

$t = \frac{6 + \sqrt{36 + 44}}{2}$

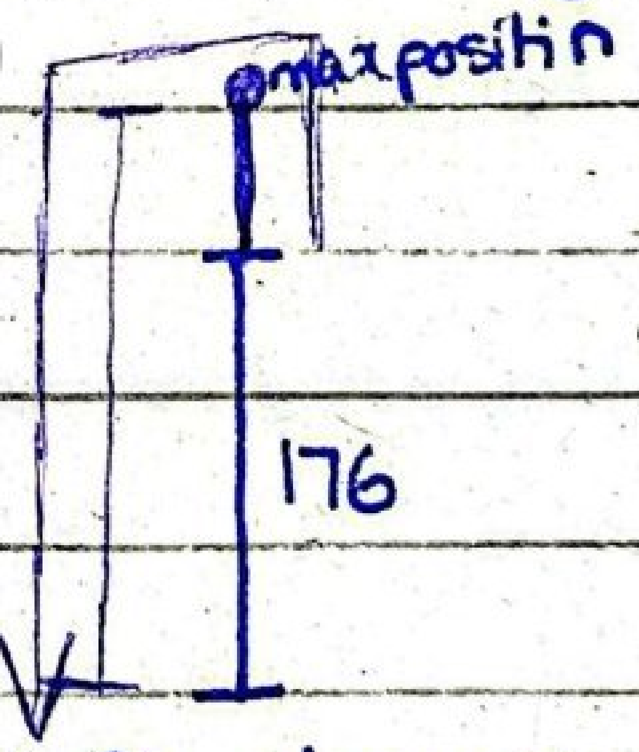
$\boxed{t = 7.47}$

$V(7.47) = -32(7.47) + 96$

$= -143.04 ft/s$

-ve shaw velocity downward direction

(iii)



put $V = 0$ at max position.

$-32t + 96 = 0$

$t = \frac{96}{32} \Rightarrow \boxed{t = 3s}$

$S(3) = -16(3)^2 + 96(3) + 176$

$= 320$ feet

upward distance

$= 320 - 176 = 144$

Total distance

$= 320 + 144$

$= 464 ft$ Answer

4) (i) $S = 2t^3 - 9t^2 + 12t - 4 \quad > 0$

$V = \frac{ds}{dt} = 6t^2 - 18t + 12$

when speed/velocity increase distance also increase.

$V = +ve$ $\quad V = -ve$ $\quad V = +ve$

$0 \quad 1 \quad 2 \quad +\infty$

put $V = 0 \Rightarrow 6t^2 - 18t + 12 = 0$

$6(t^2 - 3t + 2) = 0$

$t^2 - 3t + 2 = 0$

$t(t-2) - 1(t-2) = 0$

$\boxed{t = 1}$ $\boxed{t = 2}$

$[0,1), (1,2), (2,+\infty)$

check $[0,1)$

$V(0.5) = 6(0.5)^2 - 18(0.5) + 12$

$V(0.5) = 4.5 > 0$

check $[1,2)$

$V(1.5) = 6(1.5)^2 - 18(1.5) + 12$

$V(1.5) = -1.5 < 0$

check $(2,+\infty)$

$V(3) = 6(3)^2 - 18(3) + 12$

$= 12 > 0$

Hence $[0,1) \cup (2,+\infty)$

distance increasing.

(ii) when acceleration increase velocity will increase

$a = 12t - 18$

$a > 0$

$12t - 18 > 0$

so $V > 0$

$12t > 18$

$t > \frac{18}{12}$

$\boxed{t = \frac{3}{2}}$ Hence velocity will increase

(iii) when $a > 0$ and $V > 0$ then speed increase

or when $a < 0$ & $V < 0$ then also speed increase

$a > 0 \qquad V > 0$

$(1.5, +\infty) \qquad [0,1), (2,+\infty)$

$0 \quad 1 \quad 1.5 \quad 2 \quad +\infty$

$[0,1), (1,1.5), (1.5,2), (2,+\infty)$

check $[0,1)$

$V > 0$ & $a(0.5) = 12(0.5) - 18 = -12$

$a < 0$ speed decrease

check $(1,1.5)$

$V(1.1) = 6(1.1)^2 - 18(1.1) + 12$

$V(1.1) = -0.6 < 0$

$a(1.1) = 12(1.1) - 18$

$a(1.1) = -4.8 < 0$

speed will increase.

Check$(1.5, 2)$

$V(1.6) = 6(1.6)^2 - 18(1.6) + 12$

$= -1.4 < 0$

$a(1.6) = 12(1.6) - 18$

$= 37.2 > 0$ <u>speed decrease</u>

Check $(2, +\infty)$

$v > 0$

$a(3) = 36 - 18$ speed increase

$\boxed{a > 0}$

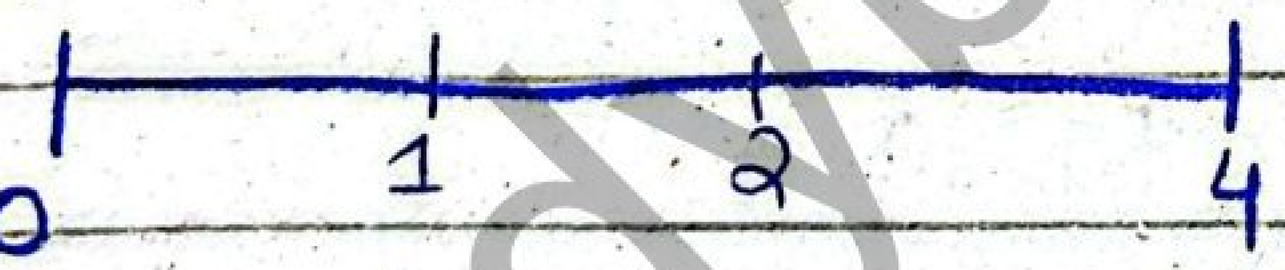
Hence speed will increase $(1, 1.5) \cup (2, +\infty)$

(IV) $V(1.5) = 6(1.5)^2 - 18(1.5) + 12$

$= -1.5$

speed $= |-1.5|$

speed $= 1.5$ Answer

(V) 

Solution:- $d = |S(1) - S(0)| + |S(2) - S(1)| + |S(4) - S(2)|$

Now $\boxed{S(0) = -4}$

$S(1) = 2 - 9 + 12 - 4$

$= \boxed{1}$

$S(2) = 2(2)^3 - 9(2)^2 + 12(2) - 4$

$\boxed{S(2) = 0}$

$S(4) = 2(4)^3 - 9(4)^2 + 12(4) - 4$

$\boxed{S(4) = 28}$

$d = |1 + 4| + |28 - 0| + |0 - 1|$

$\boxed{d = 34}$ <u>Answer</u>

Q5) Solution:- $S = 6t^2 - t^3$

(i) $S(t) = 6t^2 - t^3$

$v(t) = \frac{ds}{dt} = 12t - 3t^2$

$V(2) = 12(2) - 3(2)^2$

$\boxed{V(2) = 12ms^{-1}}$

$a(t) = \frac{dv}{dt} = 12 - 6t$

$a(2) = 12 - 6(2)$

$= \boxed{0ms^{-2}}$

(ii) $V=0$ (At rest)

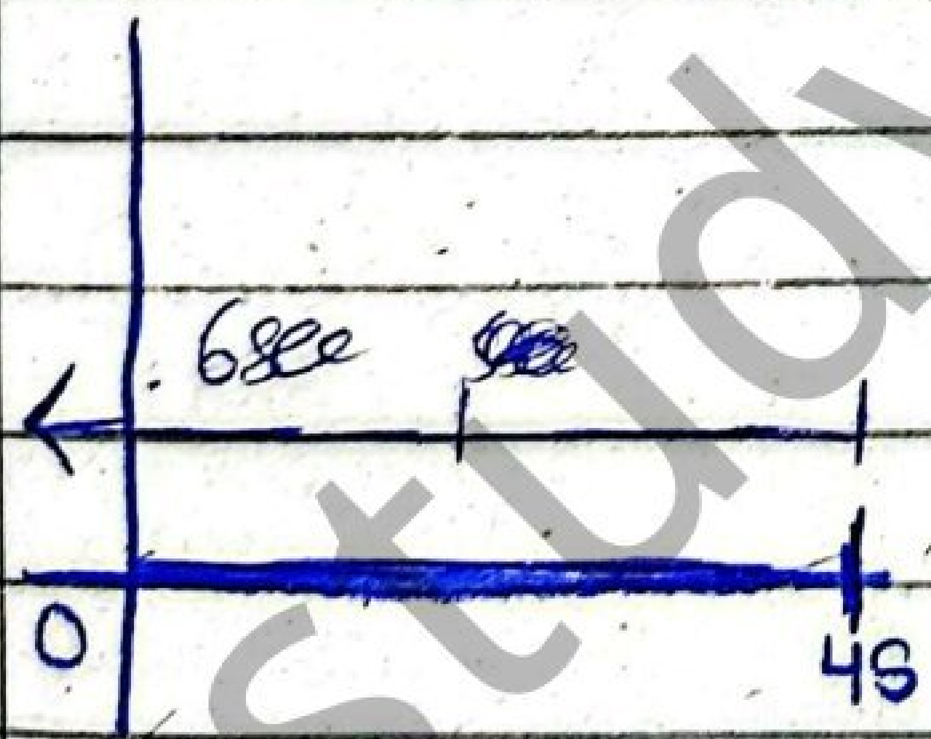
$12t - 3t^2 = 0$

$3t(4-t) = 0$

$3t = 0$ or $4 - t = 0$

$\boxed{t=0}$ $\boxed{t = 4 \text{ second}}$

initial point Answer



(iii) $V(5) = 12(5) - 3(5)^2$

$V(5) = -15ms^{-1}$

It shows that direction is -ve negative.

(iv) Put $V > 0$ Positive direction

$12t - 3t^2 > 0$

$3t(4-t) > 0$

$3t > 0$, $4 - t > 0$

$\boxed{t=0}$; $\boxed{4 > t}$

or $\boxed{t < 4}$

$(0, 4)$ Answer

(v) put $S(t) = 0$

$6t^2 - t^3 = 0$

$t^2(6-t) = 0$

$\boxed{t=0}$; $\boxed{6sec = t}$

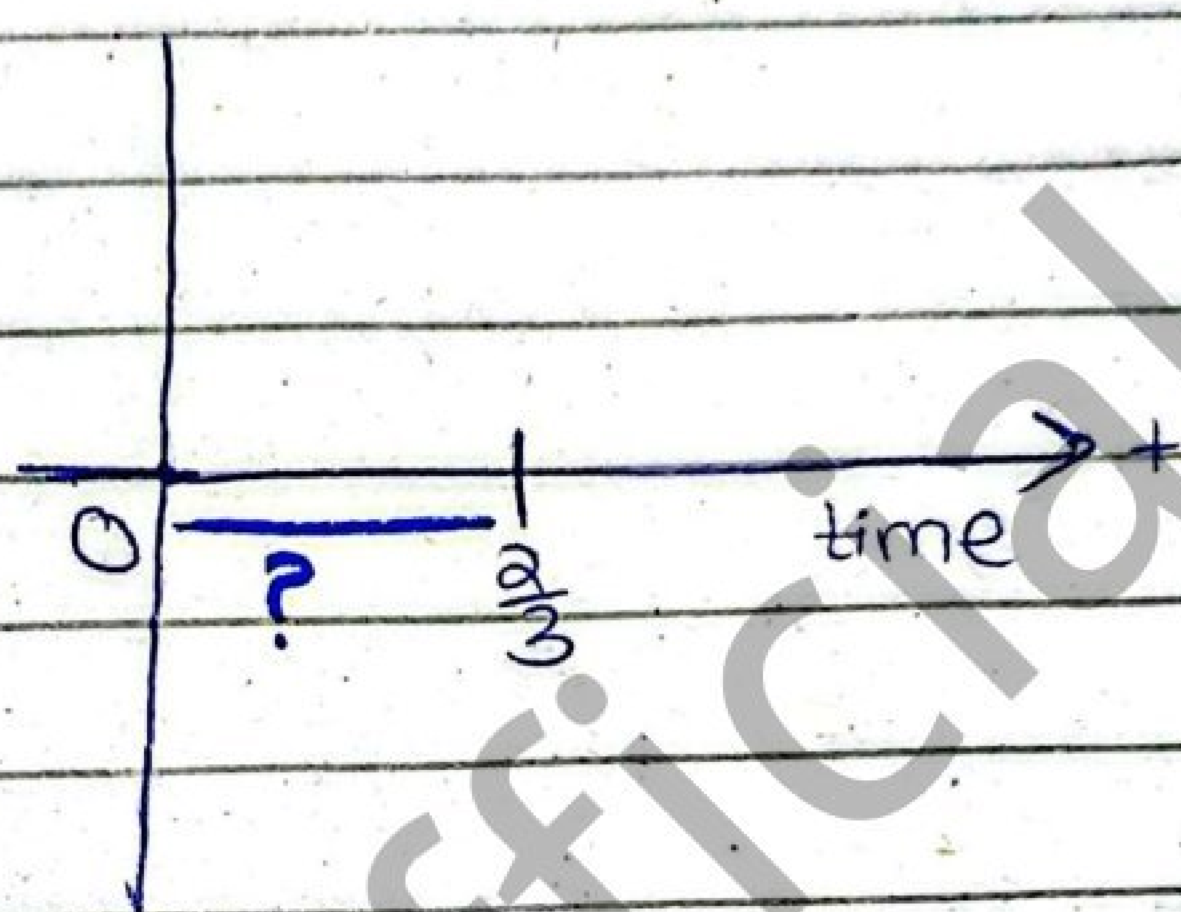
Answer.

Q7) Solution:- minimum velocity means acceleration is zero. $\boxed{a=0}$

Given $V = 3t^2 - 4t + 3$.

$a = \frac{dv}{dt} = 6t - 4$

$\boxed{a = 6t - 4}$



$0 = 6t - 4$

$6t = 4$

$\boxed{t = \frac{2}{3}\text{ sec}}$

$V = 3t^2 - 4t + 3$

$S = \int V dt$.

$S = \int (3t^2 - 4t + 3) dt$

$S = \frac{3t^3}{3} - \frac{4t^2}{2} + 3t + C$

$S(0) = 0$

$0 = (0)^3 - 2(0)^2 + 3(0) + C$

$\boxed{C = 0}$

$S(t) = t^3 - 2t^2 + 3t$

distance $= \left| S\left(\frac{2}{3}\right) - S(0) \right|$

$S\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right)$

$S\left(\frac{2}{3}\right) = \frac{38}{27}$

$S(0) = 0$

Distance $= \left| \frac{38}{27} - 0 \right| = \boxed{\frac{38}{27}\text{ m}}$ Answer.

(1) 8) Solution:- $V = \begin{cases} 10t - 2t^2 & , 0 \le t \le 6 \\ -\dfrac{432}{t^2} & , t > 6 \end{cases}$

Sol:- (i) for $t = 6s$.

$V = 10t - 2t^2$

now

$S = \int v\,dt$

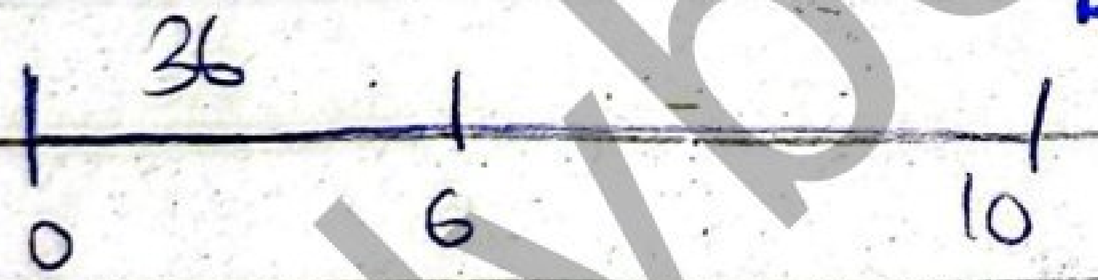
$S = \int (10t - 2t^2)\,dt$

$S = \int_0^6 (10t - 2t^2)\,dt$

$S(6) = \left| \dfrac{10t^2}{2} - \dfrac{2t^3}{3} \right|_0^6$

$S(6) = \left(5(6)^2 - \dfrac{2}{3}(6)^3\right) - (0)$

$\boxed{S(6) = 36m}$



(ii) $t = 10s$

$V = -\dfrac{432}{t^2}$

$S = \int v\,dt$

$S(10) = 36 + \int_6^{10} -432\,t^{-2}\,dt$

$S(10) = 36 + \left| \dfrac{-432\,t^{-1}}{-1} \right|_0^{10}$

$= 36 + \left[\dfrac{432}{10} - \dfrac{432}{6} \right]$

$\boxed{S(10) = 7.2m}$

$\beta = \tan^{-1}(-1)$

$\boxed{\beta = 135^0}$ Hence third line bisects them.

Exercise 5.2

Q9) $v = \begin{cases} 8t - \frac{3}{2}t^2 & 0 \le t \le 4 \\ 16 - 2t & t > 4 \end{cases}$

(i) Sol for $0 \le t \le 4$.

$V = 8t - \frac{3}{2}t^2$

$t = ?$

$\frac{dv}{dt} = 8 - \frac{3}{2}(2(t))$

$a = \frac{dv}{dt} = 8 - 3t$

$a = 0$

$0 = 8 - 3t$

$\boxed{t = \frac{8}{3} \text{ second}}$

greatest speed:-

$v(t) = 8t - \frac{3}{2}t^2$

$v\left(\frac{8}{3}\right) = 8\left(\frac{8}{3}\right) - \frac{3}{2}\left(\frac{8}{3}\right)^2$

$\boxed{v\left(\frac{8}{3}\right) = \frac{32}{3} \text{ ms}^{-1}}$

(ii) for $t=4$

$$V=8t-\frac{3}{2}t^2$$

$$V=\frac{ds}{dt}=8t-\frac{3}{2}t^2$$

$$s=\int V\,dt$$

$$s=\int_0^4 8t-\frac{3}{2}t^2$$

$$s=\left|\frac{8t^2}{2}-\frac{3}{2}\frac{t^3}{3}\right|_0^4$$

$$s=\left[4(4)^2-\frac{(4)^3}{2}\right]-[0]$$

$\boxed{s(4)=32m}$ Answer

Total distance =

$$|s(4)-s(0)|+|s(8)-s(4)|+|s(10)-s(8)|$$

$$d=32+|64-32|+|60-64|$$

$$d=32+32+4$$

$$\boxed{d=68m}$$

(iii) for $t>4$

$$V=16-2t$$

for rest $V=0$

$$0=16-2t$$

$$16=2t$$

$$\boxed{t=8sec}$$

(iv)

$V=16-2t$ for $t>4$

$$s=\int V\,dt$$

$$s=\int 16-2t\,dt$$

$$s(t)=16t-t^2+C$$

$$s(0)=0$$

$$0=16(0)-(0)^2+C$$

$$\boxed{C=0}$$

$$s(t)=16t-t^2$$

$$s(8)=16(8)-(8)^2$$

$$=128-64$$

$$\boxed{s(8)=64}$$

$$s(10)=16(10)-(10)^2$$

$$\boxed{s(10)=60m}$$

Total distance $=|s(4)$

Q10) Solution:- (i) $a = 4t - 8$. $v(0) = 6$

$\cancel{\frac{dv}{dt} = 4t - 8}$ $V = \int a\, dt$

$V = \int (4t-8)\,dt$

$\boxed{V = 2t^2 - 8 + C}$ — (1)

$V = 6$ $t = 0$

$6 = 2(0)^2 - 8 + C$

$\boxed{C = 6}$. put in (1)

$\boxed{V = 2t^2 - 8t + 6}$

(ii) when particle at rest velocity = 0

$0 = 2t^2 - 8t + 6$

$2(t^2 - 4t + 3) = 0$

$t^2 - 4t + 3 = 0$

$t^2 - 3t - t + 3 = 0$

$t(t-3) - 1(t-3) = 0$

$(t-1)(t-3) = 0$

$\boxed{t = 1}$, $\boxed{t = 3}$

$V = 2t^2 - 8t + 6$

$S = \int_1^3 2t^2 - 8t + 6$

$S = \left| \frac{2t^3}{3} - \frac{8t^2}{2} + 6t \right|_1^3$

$S = \left[\frac{2(3)^3}{3} - 4(3)^2 + 6(3) \right] - \left[\frac{2}{3} - 4 + 6 \right]$

$\boxed{S = +\frac{8}{3}\,m}$

or can do last Q same method

Q11) Solution:- $a = 4t - 9$; $\boxed{V(1) = -3}$

$V = \int a\,dt$

$V = \int 4t - 9$

$V = 2t^2 - 9t + C$ — (1)

$t = 1 \quad V = -3$

$-3 = 2(1)^2 - 9(1) + C$

$\boxed{C = 4}$ put in (1)

$\boxed{V = 2t^2 - 9t + 4}$

put $a = 0$

$4t - 9 = 0$

$\boxed{t = \frac{9}{4}}$

$V = 2\left(\frac{9}{4}\right)^2 - 9\left(\frac{9}{4}\right) + 4$

$\boxed{V = \frac{-49}{8}\ ms^{-1}}$

(ii) put $V = 0$

$2t^2 - 9t + 4 = 0$

$2t^2 - 8t - 1t + 4 = 0$

$(2t - 1)(t - 4) = 0$

$\boxed{t = \frac{1}{2}}$ $\boxed{t = 4}$ Answer

(iii)

$S = \int (2t^2 - 9t + 4)\,dt$

$S = \frac{2t^3}{3} - \frac{9t^2}{2} + 4t + C$

$t = 0 \quad S = 0$

$\boxed{C = 0}$

$\boxed{S(t) = \frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t}$

$S(0) = 0$

$S(0.5) = \frac{23}{24}$

$S(4) = \frac{-40}{3}$

$S(4.5) = -\frac{99}{8}$

$d = \left|S\left(\frac{1}{2}\right) - S(0)\right| + \left|S(4) - S\left(\frac{1}{2}\right)\right| + \left|S(4.5) - S(4)\right|$

$= d = \frac{23}{24} + \frac{343}{24} + \frac{23}{24} = \frac{389}{24}\ m$

IMP

Q12) (i) for $0 \le t \le 8$

Solution:- (i) $a = 4 - \frac{1}{2}t$

$V = \int a\,dt$

$V = \int \left(4 - \frac{1}{2}t\right)dt$

$V = 4t - \frac{1}{2(2)}t^2 + C$

$t = 0,\ v = 0$

$0 = 4(0) - \frac{1}{4}(0)^2 + C$

$\boxed{C = 0}$

$\boxed{v(t) = 4t - \frac{t^2}{4}}$

for $t > 8$

$a = 0$ (constant)

velocity constent

$V(8) = 4(8) - \frac{(8)^2}{4}$

$V(8) = 32 - \frac{64}{4}$

$\boxed{V(8) = 16}$

$$v(t) = \begin{cases} 4t - \frac{t^2}{4} & 0 \le t \le 8 \\ 16 & t > 8 \end{cases}$$

(ii) $a = 0 \therefore$ max speed

$0 = 4 - \frac{1}{2}t$

$\frac{1}{2}t = 4$ $\boxed{t = 8\text{sec}}$

(iii) for $0 \le t \le 8$

$V = 4t - \frac{t^2}{4}$

$S = \int V\,dt$

$S = \int \left(4t - \frac{t^2}{4}\right)dt$

$S = \frac{4t^2}{2} - \frac{1}{4}\frac{t^3}{3} + C$

$S = 2t^2 - \frac{t^3}{12} + C$

$t = 0,\ S = 0$

$\boxed{C = 0}$

$S = 2t^2 - \frac{t^3}{12}$

for $t > 8$

$V = 16$

$S = \int V\,dt$

$S = \int 16\,dt$

$S = 16t + C$

by Continuity

L.H.L = R.H.L = f(8)

$\lim_{x \to 8^-}\left(2t^2 - \frac{t^3}{12}\right) = \lim_{t \to 8^+}(16t + C) = f(8)$

$2(8)^2 - \frac{(8)^3}{12} = 16(8) + C$

$\boxed{C = -\frac{128}{3}} \Rightarrow \boxed{S = 16t - \frac{128}{3}}$

Displacement must be Continucus at t=8sec because jumps are physically not possible.

(iv) $S(t) = 16t - \frac{128}{3}$

$1000 = 16t - \frac{128}{3}$

$1000 + \frac{128}{3} = 16t$

$t = \frac{1000 + \frac{128}{3}}{16}$

$\boxed{t = 65.1667 \text{ sec}}$ A

Q13. (i) $v_f = v_i + at$

Proof (i) $a = \frac{\text{change in velocity}}{\text{change in time}}$

$a = \frac{v_f - v_i}{t - 0}$

$at = v_f - v_i$

$\boxed{v_f = v_i + at}$

proved

(ii) $S = v_i t + \frac{1}{2} at^2$

$\boxed{S = vt}$

$V_{avg} = \frac{v_i + v_f}{2}$ (v)

$(V_{avg})(t) = \left(\frac{v_i + v_f}{2}\right)t$ x by t on b/s

$S = \frac{v_i t}{2} + \frac{1}{2} v_f t$

$2S = v_i t + v_f t$

$2S = v_i t + (v_i + at)t$

$2S = v_i t + v_i t + at^2$

$\boxed{S = v_i t + \frac{1}{2} at^2}$ proved

from (2)

(iii) $V_{avg} \times t = \left(\frac{v_i + v_f}{2}\right)t$

$S = \left(\frac{v_i + v_f}{2}\right)t$

$2S = (v_f - v_i)t$

multiply a on b/s

$2aS = (v_f + v_i)(at)$

$2aS = (v_f + v_i)(v_f - v_i)$ using (i)

$\boxed{2aS = v_f^2 - v_i^2}$

proved.