Studybeesofficial

Where Minds Pollinate

Class 12
Mathematics

Chapter 4

Exercise 4.4

Handwritten Solution

Exercise 4.4

(1)

Sol:

$\frac{dp}{dt} \propto P$

$\frac{dp}{dt} = KP$

$\frac{1}{P} dP = K dt$

$\int \frac{1}{P} dp = K \int dt$

$\ln p = Kt + C$ —— ①

At $t=0$, $P=20$

$\ln 20 = K(0) + C$

$C = \ln 20$ put in ①

$\ln P = Kt + \ln 20$ —— ②

After 4 years

$t = 4$, $P = 25$ in ②

$\ln 25 = K(4) + \ln 20$

$\ln 25 - \ln 20 = 4K$

$\ln\left(\frac{25}{20}\right) = 4K$

$\frac{0.2231}{4} = K$

$K = 0.0557$ put in ②

$\ln P = 0.0557t + \ln 20$

After 12 years

put $t = 12$

$\ln P = 0.0557(12) + \ln 20$

$\ln P = 3.6641$

$e^{\ln P} = e^{3.6641}$

$P = e^{3.6641}$

$\boxed{P = 39}$ million

(3)

Solution:- $\frac{dp}{dt} \propto P$

$\frac{dp}{dt} = KP$

$\int \frac{1}{P} dp = K \int 1\, dt$

$\ln p = Kt + C$ —— ①

Initially at $t=0$, $P=20$

$\ln 20 = K(0) + C$

$\boxed{C = \ln 20}$

After 2 days

$t = 2$, $P = 4 \times 20 = 80$ put

$\ln 80 = K(2) + \ln 20$ —— ②

$\ln\left(\frac{80}{20}\right) = 2K$

$\frac{\ln 4}{2} = K \Rightarrow \boxed{K = 0.69314}$

put in ②

$\ln P = 0.6931t + \ln 20$ —— ③

After 10 years

put $t = 10$

$\ln P = 0.6931(10) + \ln 20$

$\ln P = 9.926$

$P = e^{9.926} \Rightarrow \boxed{P = 20470}$

(2)

Solution:- $\frac{dT}{dt} \propto (T - T_r)$ Temp Time

$\frac{dT}{dt} = k(T - T_r)$

$dT \cdot \frac{1}{T - T_r} = k\,dt$

$\frac{1}{T-20} dT = k\,dt$

$\int \frac{1}{T-20} dT = k \int 1\,dt$

$\ln(T-20) = kt + C$ —— (1)

Initially at $t=0$, $T = 150°C$.

$\ln(150-20) = k(0) + C$

$C = \ln 130$

So in eq (1)

$\ln(T-20) = kt + \ln 130$ —— (2)

After 4 minuites.

$t = 4$, $T = 90°C$

$\ln(90-20) = k(4) + \ln 130$

$\ln(70) - \ln(130) = 4k$

$\ln \frac{70}{130} = 4k$

$k = \frac{-0.6190}{4}$

$\boxed{k = -0.1547}$

So $\ln(T-20) = -0.1547t + \ln 130$.

At 40°C

$T = 40$.

$\ln(40-20) = -0.1547t + \ln 130$

$\ln 2 = -0.1547t + \ln 130$

$0.1547t = \ln 130 - \ln 2$

$t = \frac{1.8718}{0.1547}$

$\boxed{t = 12.1}$

So After 12 minuites the temp will be 40°C.

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Solution:- $\frac{dT}{dt} \propto (T - T_0)$

$\frac{dT}{dt} = k(T - T_0)$

$\int \frac{1}{T - T_0} dT = k\int 1\, dt \Rightarrow \int \frac{1}{T-30} dT = k\int dt$

$\ln(T-30) = kt + C$ —— ① $\Rightarrow T - 30 = e^{kt+C}$

$T - 30 = e^{kt} \cdot e^{C}$

$T - 30 = e^{kt} \cdot C$ —— ①

Initially at $t = 0$, $T = 80F°$

$80 - 30 = e^{k(0)} \cdot C$

$\boxed{+50 = C}$ put in ①

$\boxed{T - 30 = 50e^{kt}}$

$t = 1$ $T = 40$

$40 - 30 = 50e^{k(1)}$

$e^{k} = \frac{10}{50}$

$\ln e^{k} = \ln\left(\frac{1}{5}\right)$

$k = \ln 1 - \ln 5$

$\boxed{k = -\ln 5}$

$T - 30 = 50^{(-\ln 5)t}$

$70 = T$ $t = ?$

$70 - 30 = 50e^{(-\ln 5)t}$

$\frac{4}{5} = e^{\ln 5^{-t}}$

$\frac{4}{5} = (5)^{-t}$

$\frac{4}{5} = \left(\frac{1}{5}\right)^{t}$

$\ln\frac{4}{5} = t\ln\left(\frac{1}{5}\right)$

$\frac{\ln 4/5}{\ln(1/5)} = t$

$\boxed{t = 0.14 \text{min}}$ Answer.

amount of substance

amount present

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Solution: $\frac{dA}{dt} \propto A$, $A(t)$ is present amount of radioactive substance.

$\frac{dA}{dt} \propto kA$.

$\int \frac{dA}{A} = k\int dt$

$\ln_e A = kt + c$
$\quad e \quad\;\; e$

$A = e^{kt} \cdot e^{c}$

$\boxed{A = e^{kt} \cdot C}$ ~~e^{c}~~

At $t=0$ $A=50$

$50 = Ce^{k(0)}$

$\boxed{C = 50}$

$A = 50e^{kt}$ —(2)

At $t=1000$, $A = \frac{50}{2} = 25$

$25 = 50e^{1000k}$

$\frac{25}{50} = e^{1000k}$

$\ln\left(\frac{1}{2}\right) = \ln e^{1000k}$

$\ln 1 - \ln 2 = 1000k$

$-\frac{\ln 2}{1000} = k$ put in (2)

$\boxed{A = 50e^{-\frac{\ln 2}{1000}t}}$

$A = ?$ $t = 800$ years

$A = 50e^{-\frac{\ln 2}{1000} \times 800}$

$A = 50e^{-(\ln 2)\frac{8}{10}}$

$A = 50e^{-4/5 \ln 2}$

$A = 50e^{\ln(2)^{-4/5}}$

$A = 50(2)^{-4/5}$

$A = 50\left(\frac{1}{2}\right)^{4/5}$

$\boxed{A = 28.7\text{ gram}}$ Ans

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$\frac{dv}{dt} = -g$

$dv = -g\,dt$

$\boxed{g = 9.8ms^{-1}}$

$\int dv = -9.8 \int dt$

$V = -9.8t + C$

Initial Condition

$t = 0,\ V = 40ms^{-1}$

$40 = -9.8(0) + C$

$\boxed{C = 40}$

$\boxed{V = -9.8t + 40}$ — (1)

$V = -9.8(1) + 40$

$\boxed{V = 30.2ms^{-1}}$ Answer

S_{max} height = ?

$\frac{ds}{dt} = V$

$\frac{ds}{dt} = -9.8t + 40$

$\int ds = \int(-9.8t + 40)\,dt$

$S = -9.8\frac{t^2}{2} + 40t + C$

$\boxed{S = -4.9t^2 + 40t + C}$ — (2)

I.C $t = 0$ (initially) $S = 0$ (Initially)

$0 = -4.9(0) + 40(0) + C$

$\boxed{C = 0}$

$\boxed{S = -4.9t^2 + 40t}$ — (★)

max height = velocity is zero

from (1)

$0 = -9.8t + 40$

$9.8t = 40$

$t = \frac{40}{9.8}$

$\boxed{t = 4.08}$ put in (★)

$S = -4.9(4)^2 + 40(4)$

$\boxed{S = 81.6}$

Answer.