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Where Minds Pollinate

Class 12
Mathematics

Chapter 4
Exercise 4.3
Handwritten Solution

Exercise 4.3

Homogenous eqⁿ non seperatable differential.

Q1) $f(x,y) = 6xy^3 - x^2y^2$

Solution: Replace x by tx, y by ty.

$f(tx, ty) = 6(tx)(ty)^3 - (tx)^2(ty)^2$

$= 6tx(t^3y^3) - t^2x^2t^2y^2$

$= 6t^4xy^3 - t^4x^2y^2$

$= t^4[6xy^3 - x^2y^2]$

$f(tx, ty) = t^4 f(x,y)$

Hence $f(x,y)$ is homogenous function of degree 4.

∴ If degree same in equation then homogenous equation.

e.g. $6xy^3 - x^2y^2$ (4) 4 degree = 4.

Q2) $f(x,y) = x^2 - y$

Solution: $f(tx,ty) = (tx)^2 - ty$

$= t^2x^2 - ty$

$= t(tx - t)$

$\neq t^n f(x,y)$ non homogenous (no degree).

Q4) $(x-y)\,dx + x\,dy = 0$

Solution: $x\,dy = -(x-y)\,dx$

$\frac{dy}{dx} = \frac{-(x-y)}{x}$ —— (1)

let $y = vx$

y dependent variable, v dependent variable, x Independent variable.

Diff w.r.t x

$$\frac{dy}{dx} = u\frac{d}{dx}(x) + x\frac{d}{dx}(u)$$

$$\boxed{\frac{dy}{dx} = u + x\frac{du}{dx}}$$

$$u + x\frac{du}{dx} = -\frac{(x - ux)}{x}$$

$$= -\frac{\cancel{x}(1-u)}{\cancel{x}}$$

$$x\frac{du}{dx} = -1 + \cancel{u} - \cancel{u}$$

$$x\,du = -1\,dx$$

$$\int 1\,du = -\int \frac{1}{x}\,dx$$

$$u = -\ln x + C$$

But $y = ux \Rightarrow u = \frac{y}{x}$

$\boxed{\frac{y}{x} = -\ln x + C}$ Answer.

Q3) $f(x,y) = \frac{2y^3}{x^2y} - 7$

Solution:- let $x = tx$ $y = ty$

$$f(tx,ty) = \frac{2(ty)^3}{(tx)^2(ty)} - 7$$

degree = 0

$$= \frac{2\cancel{t^3}y^3}{\cancel{t^3}x^2y} \Rightarrow \frac{2y^3}{x^2y} - 7 \quad \text{so } t^0\left(\frac{2y^3}{x^2y} - 7\right) \text{ is homogenous}$$

Imp

Q5) $y\,dx - (y-x)\,dy = 0$

sol $y\,dx = (y-x)\,dy$

$$\boxed{\frac{dy}{dx} = \frac{y}{y-x}}$$

put $y = vx$

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

$$v + x\frac{dv}{dx} = \frac{vx}{vx - x}$$

$$v + x\frac{dv}{dx} = \frac{vx}{x(v-1)}$$

$$x\frac{dv}{dx} = \frac{v}{v-1} - v$$

$$x\frac{dv}{dx} = \frac{v - v^2 + v}{v-1}$$

$$x\frac{dv}{dx} = \frac{2v - v^2}{v-1}$$

$$x\,dv = \frac{2v - v^2}{v-1}\,dx$$

$$\int \frac{(v-1)}{(2v-v^2)}\,dv = \int \frac{1}{x}\,dx$$

$$\frac{1}{2}\int \frac{2(-1+v)}{2v-v^2}\,dv = \int \frac{1}{x}\,dx$$

$$-\frac{1}{2}\int \frac{+2-2v}{2v-v^2}\,dv = \int \frac{1}{x}\,dx$$

$$-\frac{1}{2}\ln(2v-v^2) = \ln x + \ln c$$

$$\ln(2v-v^2)^{-1/2} = \ln xc$$

$$e^{\ln(2v-v^2)^{-1/2}} = e^{\ln xc}$$

$$(2v-v^2)^{-1/2} = xc$$

$$\left(2\frac{y}{x} - \frac{y^2}{x^2}\right)^{-1/2} = xc$$

$$\left(\frac{2xy - y^2}{x^2}\right)^{-1/2} = xc$$

$$\sqrt{\frac{x^2}{y(2x-y)}} = xc$$

$$\frac{\cancel{x}}{\sqrt{y(2x-y)}} = \cancel{x}c$$

$c = \frac{1}{c}$

$$\therefore \frac{1}{c} = \sqrt{y(2x-y)}$$

$$c = \sqrt{y(2x-y)}$$

Answer

Q6) $\frac{dy}{dx} = \frac{y-x}{x+y}$

Sol:- put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{vx - x}{x + vx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(v-1)}{\cancel{x}(1+v)} - v$$

$$x\frac{dv}{dx} = \frac{v-1-v(1+v)}{(1+v)}$$

$$x\frac{dv}{dx} = \frac{\cancel{v}-1-\cancel{v}-v^2}{(v+1)}$$

$$x\frac{dv}{dx} = -\frac{(v^2+1)}{v+1}$$

$$x\,dv = -\frac{(v^2+1)}{v+1}dx$$

$$\int \frac{v+1}{v^2+1}\,dv = -\int \frac{1}{x}\,dx$$

$$\frac{1}{2}\int \frac{2v}{v^2+1}\,dv + \int \frac{1}{v^2+1^2}\,dv = -\ln x + C$$

$$\frac{1}{2}\ln(v^2+1) + \tan^{-1} v = -\ln x + C$$

$$\frac{1}{2}\ln\left(\frac{y^2}{x^2}+1\right) + \tan^{-1}\left(\frac{y}{x}\right) = -\ln x + C$$

$$\boxed{\frac{1}{2}\ln\left(\frac{y^2+x^2}{x^2}\right) + \tan^{-1}\frac{y}{x} + \ln x = C}$$

Q7) $\frac{dy}{dx} = \frac{y^2+yx}{x^2}$

Sol: $y = vx$

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

$$v + x\frac{dv}{dx} = \frac{v^2x^2 + vx\cdot x}{x^2}$$

$$x\frac{dv}{dx} = \frac{\cancel{x^2}(v^2+v)}{\cancel{x^2}} - v$$

$$x\cdot\frac{dv}{dx} = v^2 + \cancel{v} - \cancel{v}$$

$$x\,dv = v^2\,dx$$

$$\int \frac{dv}{v^2} = \int \frac{dx}{x}$$

$$\int v^{-2}\,dv = \int \frac{1}{x}\,dx$$

$$\frac{v^{-2+1}}{-1} = \ln x + C$$

$$-\frac{1}{v} = \ln x + C$$

$$-\frac{1}{\frac{y}{x}} = \ln x + C$$

$$\boxed{-\frac{x}{y} = \ln x + C}$$

Answer.

(Q8) $\frac{dy}{dx} = \frac{y}{x+\sqrt{xy}}$

Sol:- put $y = vx$

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

$$v + x\frac{dv}{dx} = \frac{vx}{x+\sqrt{v.x.x}}$$

$$x\frac{dv}{dx} = \frac{vx}{x+\sqrt{x^2 v}} - v$$

$$x\frac{dv}{dx} = \frac{vx}{x+x\sqrt{v}} - v$$

$$x\frac{dv}{dx} = \frac{vx}{x(1+\sqrt{v})} - v$$

$$= \frac{v - v(1+\sqrt{v})}{1+\sqrt{v}}$$

$$x\,dv = \frac{v - v - v^{3/2}}{1+\sqrt{v}}dx$$

$$\int \frac{1+v^{1/2}}{v^{3/2}}dv = -\int \frac{1}{x}dx$$

$$\int\left(\frac{1}{v^{3/2}} + \frac{v^{1/2}}{v^{3/2}}\right)dv = -\ln x + C$$

$$\int v^{-3/2}dv + \int v^{-1}dv = -\ln x + C$$

$$\frac{v^{-1/2}}{-1/2} + \int \frac{1}{v}dv = -\ln x + C$$

$$\frac{-2}{\sqrt{v}} + \ln v + \ln x = C$$

$$\frac{-2}{\sqrt{\frac{y}{x}}} + \ln\frac{y}{x} + \ln x = C$$

$$\boxed{-2\frac{\sqrt{x}}{\sqrt{y}} + \ln y - \cancel{\ln x} + \cancel{\ln x} = C}$$

Answer.

(Q9) $\frac{dy}{dx} = \frac{3x^3+y^3}{xy^2}$

Sol:- put $y = vx$

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

$$v + x\frac{dv}{dx} = \frac{3x^3+v^3x^3}{xv^2x^2}$$

$$x\frac{dv}{dx} = \frac{x^3(3+v^3)}{x^3v^2} - v$$

$$x\frac{dv}{dx} = \frac{3+v^3-v^3}{v^2}$$

$$x\,dv = \frac{3}{v^2}dx$$

$$\int v^2 dv = \int \frac{3}{x}dx$$

$$\frac{v^3}{3} = 3\int \frac{1}{x}dx$$

$$\frac{v^3}{3} = 3\ln x + C$$

$$v^3 = 9\ln x + 3C$$

$$\boxed{\frac{y^3}{x^3} = 9\ln x + C}$$

Ans

Q10) $\frac{dy}{dx} = \frac{x+3y}{3x+y}$

Sol: put $y = ux$

$\boxed{\frac{dy}{dx} = u + x\frac{du}{dx}}$

$u + x\frac{du}{dx} = \frac{x+3ux}{3x+ux}$

$x\frac{du}{dx} = \frac{\cancel{x}(1+3u)}{\cancel{x}(3+u)} - u$

$x\frac{du}{dx} = \frac{1+3u-u(3+u)}{3+u}$

$x\frac{du}{dx} = \frac{1+\cancel{3u}-\cancel{3u}-u^2}{3+u}$

$x\,du = \frac{1-u^2}{3+u}\,dx$

$\int\frac{3+u}{1-u^2}\,du = \int\frac{1}{x}\,dx$

$\left(\frac{3+u}{1-u^2}\right) = \frac{3+u}{(1-u)(1+u)} = \frac{A}{1-u} + \frac{B}{1+u}$ — (1)

$(1-u)(1+u)$ multiply by eq(1) on b/s

$3+u = A(1+u) + B(1-u)$ — (2)

put $1-u=0$ $\boxed{u=1}$ put in (2)

$4 = A(2) \Rightarrow \boxed{A=2}$ put $1+u=0$ $\boxed{u=-1}$

$2 = B(2) \Rightarrow \boxed{B=1}$

$\int\left(\frac{2}{1-u} + \frac{1}{1+u}\right)du = \int\frac{1}{x}\,dx$

$\int\frac{2}{1-u}\,du + \int\frac{1}{1+u}\,du = \int\frac{1}{x}\,dx$

$-2\int\frac{-1}{1-u}\,du + \ln(1+u) = \ln x + \ln C$

$-2\ln(1-u) + \ln(1+u) = \ln x + \ln C$

$-\ln\left(1-\frac{y}{x}\right)^2 + \ln\left(1+\frac{y}{x}\right) = \ln xC$

$\ln\left(\frac{x+y}{x}\right) - \ln\left(\frac{x-y}{x}\right)^2 = \ln xC$

$\ln(x+y) - \ln x - \ln(x-y)^2 + \ln x^2 = \ln x$

$\ln(x+y) - \ln x - \ln(x-y)^2 + 2\ln x = \ln x$

$\ln(x+y) + \ln x - \ln(x-y)^2 = \ln xC$

$e^{\ln\frac{(x+y)x}{(x-y)^2}} = e^{\ln xC}$

$\boxed{\frac{x(x+y)}{(x-y)^2} = xC}$ Ans

Q11) $\left[y + x\cot\left(\frac{y}{x}\right)\right]dx - x\,dy = 0$

Sol: $x\,dy = \left[y + x\cot\left(\frac{y}{x}\right)\right]dx$

$$\frac{dy}{dx} = \frac{\left[y + x\cot\left(\frac{y}{x}\right)\right]}{x}$$

put $y = vx$

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

$$v + x\frac{dv}{dx} = \frac{vx + x\cot\left(\frac{vx}{x}\right)}{x}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(v + \cot v)}{\cancel{x}} - v$$

$$x\frac{dv}{dx} = \cancel{v} + \cot v - \cancel{v}$$

$$x\,dv = \cot v\,dx$$

$$\frac{dv}{\cot v} = \frac{1}{x}dx$$

$$\frac{1}{\frac{\cos v}{\sin v}}dv = \frac{1}{x}dx$$

$$-\int \frac{-\sin v}{\cos v}dv = \int \frac{1}{x}dx$$

$$-\ln(\cos v) = \ln x + \ln c$$

$$\ln(\cos v)^{-1} = \ln xc$$

$$\ln \frac{1}{\cos v} = \ln xc$$

$$e^{\ln \sec v} = e^{\ln xc} \Rightarrow \boxed{\sec\left(\frac{y}{x}\right) = xc}$$ Ans

(Q14) $2x^2\frac{dy}{dx} = 3xy + y^2$; $y(1) = 2$.

Solt put $y = ux$

$$\boxed{\frac{dy}{dx} = u + x\frac{du}{dx}}$$

$$u + x\frac{du}{dx} = \frac{3x \cdot ux + u^2x^2}{2x^2}$$

$$x\frac{du}{dx} = \frac{x^2(3u+u^2)}{2x^2} - u$$

$$x\frac{du}{dx} = \frac{3u + u^2 - 2u}{2}$$

$$x\,du = \frac{u^2+u}{2}\,dx$$

$$\int \frac{2}{u^2+u}\,du = \int \frac{1}{x}\,dx$$

$$2\int \frac{1}{u(u+1)}\,du = \int \frac{1}{x}\,dx$$

$$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1} \quad (1)$$

$u(u+1)$ x on b/s (1)

$$1 = A(u+1) + Bu \quad (2)$$

$\boxed{u=0}$ put in (2)

$$1 = A(1) + 0$$

$$\boxed{A = 1}$$

$u + 1 = 0 \Rightarrow \boxed{u = -1}$

$A(0) + B(-1) = 1 \Rightarrow \boxed{B = -1}$

$$\frac{1}{u(u+1)} = \frac{1}{u} - \frac{1}{u+1}$$

$$2\left[\int \frac{1}{u}\,du - \int \frac{1}{u+1}\,dx\right] = \int \frac{1}{x}\,dx$$

$$2[\ln u - \ln(u+1)] = \ln x + \ln C$$

$$2\ln\left(\frac{u}{u+1}\right) = \ln xC$$

$$e^{\ln\left(\frac{u}{u+1}\right)^2} = e^{\ln xC}$$

$$\left(\frac{u}{u+1}\right)^2 = xC$$

$$\left(\frac{\frac{y}{x}}{\frac{y}{x}+1}\right)^2 = xC$$

$$\left(\frac{y}{x+y}\right)^2 = xC \quad , \quad x = 1 \;\; y = -2$$

$$\left(\frac{-2}{1-2}\right)^2 = (1)C$$

$$\boxed{C = 4}$$

$$\boxed{\left(\frac{y}{x+y}\right)^2 = 4x}$$

Q 15) $(x+ye^{y/x})dx - xe^{y/x}dy = 0$; $y(1)=0$

Sol $(x+ye^{y/x})dx = xe^{y/x}dy$

$$\boxed{\frac{(x+ye^{y/x})}{xe^{y/x}} = \frac{dy}{dx}}$$

put $y = vx$

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

$$v + x\frac{dv}{dx} = \frac{x+vxe^{\frac{vx}{x}}}{xe^{\frac{vx}{x}}}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(1+ve^{v})}{\cancel{x}e^{v}} - v$$

$$x\frac{dv}{dx} = \frac{1+ve^{v}-ve^{v}}{e^{v}}$$

$$x\,dv = \frac{1}{e^{v}}dx$$

$$\int e^{v}dv = \int \frac{1}{x}dv$$

$$e^{v} = \ln x + c$$

$$\boxed{e^{y/x} = \ln x + c} \quad G.S$$

$x=1$ $y=0$

$$e^{0} = \ln 1 + c$$

$$\boxed{c = 1}$$

$$e^{y/x} = \ln x + 1 \quad A$$

Q13) $(x^2+2y^2)dx = xy\,dy$; $y(1)=1$.

Solution:- $\dfrac{x^2+2y^2}{xy} = \dfrac{dy}{dx}$ —— ①

let $y = ux$

$\dfrac{dy}{dx} = u + x\dfrac{du}{dx}$

$u + x\dfrac{du}{dx} = \dfrac{x^2+2y^2}{xy}$

$u + x\dfrac{du}{dx} = \dfrac{x^2+2u^2x^2}{x^2u}$

$u + x\dfrac{du}{dx} = \dfrac{x^2(1+2u^2)}{x^2 u}$

$u + x\dfrac{du}{dx} = \dfrac{1+2u^2}{u}$

$x\dfrac{du}{dx} = \dfrac{1+2u^2}{u} - u$

$x\dfrac{du}{dx} = \dfrac{1+2u^2-u^2}{u}$

$x\dfrac{du}{dx} = \dfrac{1+u^2}{u}$

$\dfrac{1}{x}dx = \dfrac{u}{1+u^2}\,du$

$\int \dfrac{1}{x}\,dx = \dfrac{1}{2}\int \dfrac{u}{1+u^2}\,du$

$\ln x + c = \dfrac{1}{2}\ln(1+u^2)$

$e^{\ln xc} = e^{\ln(1+u^2)^{1/2}}$

$xc = (1+u^2)^{1/2}$

$xc = \left(1+\dfrac{y^2}{x^2}\right)^{1/2}$ —— ②

$x=1 \quad y=1$

$(1)c = \left(1+\dfrac{1^2}{1^2}\right)^{1/2}$

$\boxed{c = \sqrt{2}}$

$\sqrt{2}\,x = \left(\dfrac{y^2+x^2}{x^2}\right)^{1/2}$ Answer

OR

$\dfrac{\sqrt{y^2+x^2}}{\sqrt{x^2}} = \sqrt{2}\,x$

$\sqrt{2}\,x = \dfrac{\sqrt{x^2+y^2}}{x}$

$\sqrt{2}\,x^2 = \sqrt{x^2+y^2}$

Answer

Q12) $xy^2\frac{dy}{dx} = y^3 - x^3$; $y(1) = 2$.

Sol: put $y = vx$

$$\frac{dy}{dx} = \frac{y^3 - x^3}{xy^2}$$

$$v + x\frac{dv}{dx} = \frac{v x^3 - x^3}{x^3 v^2}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{x^3}(v-1)}{\cancel{x^3}v^2}$$

$$x\frac{dv}{dx} = \frac{v^3 - 1 - v^3}{v^2}$$

$$x\frac{dv}{dx} = \frac{-1}{v^2}$$

$$dx \cdot \frac{-1}{x} = v^2 dv$$

$$-\int \frac{1}{x}dx = \int v^2 dv$$

$$-\ln x + C = \frac{v^3}{3}$$

$$\boxed{-\ln x + C = \frac{y^3}{3x^3}} \text{ --- } (1)$$

$x = 1$ $y = 2$

$$\frac{8}{3} = -\ln 1 + C$$

$\boxed{C = \frac{8}{3}}$ put in (1)

$$\frac{y^3}{3x^2} + \ln x = \frac{8}{3} \text{ Ans} \quad \text{or} \quad \boxed{\frac{y^3}{x^2} + 3\ln x = 8}$$