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Where Minds Pollinate

Class 12
Mathematics

Chapter 4
Exercise 4.2
Handwritten Solution

Exercise # 4.2

Q1. $\frac{dy}{dx} = -\frac{1}{e^{3x}}$

By seperating variables:

Solution: $dy = -\frac{dx}{e^{3x}}$

Integrating both sides.

$\int 1 dy = -\int e^{-3x} \cdot dx$

$\int 1 \cdot dy = \frac{-1}{-3}\int e^{-3x} \cdot (-3) dx$

$\int 1 \cdot dy = \frac{1}{3}\int e^{-3x} (-3) dx$

$y = \frac{1}{3} e^{-3x} + C$ **Answer.**

Q2) $x \frac{dy}{dx} = 4y$

Solution:

$\frac{1}{y} dy = 4 dx \frac{1}{x}$

integrating b/s

$\int \frac{1}{y} dy = 4 \int \frac{1}{x} dx$

$\ln y = 4 \ln x + \ln C$ ∴ $\ln C$ because all equation is in $\ln$.

$= \ln x^4 + \ln C$

$\ln y = \ln x^4 C$ ∴ $\log m + \log n = \log(m \times n)$

Taking antilog on both sides

$$e^{\ln y} = e^{\ln x^4 c}$$

$$\boxed{y = cx^4}$$

Q3) $\frac{dy}{dx} = \frac{y^3}{x^2}$

Sol:- $\frac{1}{y^3} dy = dx \cdot \frac{1}{x^2}$

$y^{-3} dy = dx\, x^{-2}$

$\int y^{-3} dy = \int x^{-2} dx$

$\frac{y^{-2}}{-2} = \frac{x^{-1}}{-1} + C$

$\frac{1}{-2y^2} = \frac{1}{-x} + C$

$\frac{-1}{2y^2} + \frac{1}{x} - C = 0$

$$\boxed{\frac{1}{2y^2} - \frac{1}{x} + C = 0}$$

Q4) $\frac{dy}{dx} = e^{2x+3y}$

Sol:- $\frac{dy}{dx} = e^{2x} \cdot e^{3y}$

$\frac{dy}{e^{3y}} = e^{2x} \cdot dx$

$\int e^{-3y} \cdot dy = \int e^{2x} \cdot dx$

$\frac{1}{-3}\int e^{-3y}(-3) dy = \frac{1}{2}\int e^{2x} \cdot 2 dx$

$\frac{e^{-3y}}{-3} = \frac{1}{2} e^{2x}$

$\frac{1}{-3e^{3y}} - \frac{e^{2x}}{2} + C = 0$

$$\boxed{\frac{1}{3e^{3y}} + \frac{e^{2x}}{2} + C = 0}$$

Q5) $\frac{dy}{dx} = \frac{x^2y^2}{1+x}$ V.IMP

Sol:- $\int \frac{dy}{y^2} = \int \frac{x^2\,dx}{1+x}$

$$\begin{array}{r|l} & x - 1 \\ \hline 1+x & x^2 \\ & \pm x^2 \pm x \\ \hline & -x \\ & \mp x \mp 1 \\ \hline & 1 \end{array}$$

$\int y^{-2}dy = \int \left[(x-1) + \frac{1}{x+1}\right] dx$

$\frac{y^{-1}}{-1} = \int x\,dx - \int 1\,dx + \int \frac{1}{x+1}\,dx$

$\frac{-1}{y} = \frac{x^2}{2} - x + \ln|1+x| + C$

$\frac{1}{y} = \frac{-x^2}{2} + x - \ln(1+x) - C$

$\boxed{\frac{1}{y} + \frac{x^2}{2} - x + \ln(1+x) + C = 0}$ Answer

Q6) $2y(x+1)dy = x\,dx$

Sol:- $2y\,dy = \left(\frac{x}{x+1}\right)dx$

$$\begin{array}{r|l} & 1 \\ \hline x+1 & x \\ & \pm x \pm 1 \\ \hline & -1 \end{array}$$

$2\int y\,dy = \int \left(\frac{x}{x+1}\right)dx$

$2\int y\,dy = \int \left[1 - \frac{1}{x+1}\right]dx$

$\frac{2y^2}{2} = \int 1\,dx - \int \frac{1}{x+1}\,dx$

$y^2 = x - \ln(x+1) + C$ Answer

Q7) $\frac{dy}{dx} + y^2 \sin x = 0$

differential must be in multiply form not divide.

Solr $\frac{dy}{dx} = -y^2 \sin x$

$dy = -y^2 \sin x \, dx$

$\int \frac{dy}{y^2} = -\int \sin x \, dx$

$\int y^{-2} \, dy = -\int \sin x \, dx$

$\frac{y^{-1}}{-1} = -(-\cos x)$

$-\frac{1}{y} = \cos x + C$

$\boxed{\frac{1}{y} + \cos x + C = 0}$ Answer.

Q8) $(\sin x + \cos x)dx = \cot y \cos x \, dy$ IMP

Solution:- $\frac{(\sin x + \cos x)}{\cos x} dx = \cot y \, dy$

$\int \left(\frac{\sin x}{\cos x} + 1\right) dx = \int \cot y \, dy$

$-\int \frac{-\sin x}{\cos x} dx + \int 1 \, dx = \int \frac{\cos y}{\sin y} dy$

$-\ln(\cos x) + x = \ln(\sin y) + C$

$\ln((\cos x)^{-1}) + x = \ln(\sin y) + C$

$\ln\left(\frac{1}{\cos x}\right) + x = \ln(\sin y) + C$

$\ln(\sec x) + x = \ln(\sin y) + C$ Answer

Q9) $\frac{dy}{dx} = \cos x$; $y(0) = 1$

Sol:- $\int dy = \int \cos x \, dx$

$\boxed{y = \sin x + C}$ — (i)
General solution

put $x = 0 \Rightarrow y = 1$

$1 = \sin(0) + C$

$\boxed{C = 1}$ put in (1)

$\boxed{y = \sin x + 1}$ particular solution. Answer.

Q10) $2\frac{dy}{dx} = 4xe^{-x}$; $y(0) = 2$.

Sol:- $2dy = 4xe^{-x} dx$

$\int dy = 2\int xe^{-x} dx$

$y = 2\left[x\int e^{-x} dx - \int\left(\int e^{-x} dx \cdot \frac{d}{dx}(x)\right) dx\right]$

$y = 2\left[\frac{xe^{-x}}{-1} - \int \frac{e^{-x}}{-1} \cdot (1)\, dx\right]$

$y = 2\left[-xe^{-x} + \int e^{-x} dx\right.$

$\boxed{y = 2\left[-xe^{-x} - e^{-x}\right] + C}$

$x = 0$ $y = 2$

$2 = 2[0 - e^{0}] + C$

$\boxed{C = 4}$

$y = -2e^{-x}(x+1) + 4$

$\boxed{y = +2e^{-x}(1+x) - 4 = 0}$

Answer

Q11 $\frac{dy}{dx} + \left(\frac{1+x}{x}\right)y = 0$; $y(1) = 1$

Solution:- $\frac{dy}{dx} = -\frac{(1+x)}{x}y$

$\int \frac{dy}{y} = -\int \frac{(1+x)}{x}dx$

$\ln y = -\left[\int \left(\frac{1}{x} + 1\right)dx\right]$

$\ln y = -\ln x - x + C$

$\ln y + \ln x = -x + C$

$\boxed{\ln(xy) = -x + C}$

In particular solution

$x = 1$; $y = 1$

$\ln(1) = -1 + C$

$0 + 1 = C$

$\boxed{C = 1}$

$\ln(xy) = -x + 1$

$\boxed{\ln(xy) + x - 1 = 0}$

Particular Solution

Answer

Q12 $\frac{dy}{dx} + y\tan 2x = 0$; $y(0) = 2$.

Solution:- $dy = -y\tan 2x\, dx$

$\int \frac{dy}{y} = -\int \tan 2x\, dx$

$\ln y = +\int \frac{-\sin 2x}{\cos 2x}$

$\ln y = \frac{1}{2}\int \frac{-2\sin 2x}{\cos 2x}dx$

$\ln y = \frac{1}{2}\ln(\cos 2x) + C$

$\ln y = \ln\sqrt{(\cos 2x)} + C$

$\ln y = \ln\sqrt{\cos 2x} + C$

$e^{\ln y} = e^{\ln\sqrt{\cos 2x} + C}$

$\boxed{y = \sqrt{\cos 2x} \cdot C}$ General solution

$x = 0$; $y = 2$

$2 = \sqrt{\cos 2(0)} \cdot C$

$2 = \sqrt{1} \cdot C$

$\boxed{C = 2}$

$\boxed{y = 2\sqrt{\cos 2x}}$ Answer.

Q13) $\frac{dy}{dx} = y^2+4$; $y(0) = -2$.

Solution:- $dy = (y^2+4)\,dx$

$$\int \frac{dy}{y^2+2^2} = \int dx \qquad \because \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\boxed{\frac{1}{2}\tan^{-1}\frac{y}{2} = x + C}$$ General solution

$$\frac{1}{2}\tan^{-1}\left(\frac{-2}{2}\right) = 0 + C$$

$$\frac{1}{2}\tan^{-1}(-1) = C$$

$$\frac{1}{2}\left(-\frac{\pi}{4}\right) = C$$

$$\frac{1}{2}\left(-\frac{\pi}{4}+\pi\right) = C$$

$$\boxed{C = \frac{3\pi}{8}} \Rightarrow \boxed{\frac{1}{2}\tan^{-1}\frac{y}{2} = x + \frac{3\pi}{8}}$$ Answer.

Q14) $(1-x)dy + y^{-1}dx = 0$; $y(0) = 2$

Solution:- $(1-x)\,dy = -y^{-1}dx$

$$\int \frac{dy}{y^{-1}} = \int -\frac{(dx)}{(1-x)}$$

$$\int y\,dy = \int \frac{-1}{1-x}dx \Rightarrow \boxed{\frac{y^2}{2} = \ln(1-x) + C}$$

$x=0$, $y=2$

$\frac{(2)^2}{2} = \ln(1-0)+C$

$2 = \ln(1)+C$

$\boxed{C=2}$

$\frac{y^2}{2} = \ln(1-x)+2$ solution

Q15) $2(y-1)\,dy = (3x^2+4x+2)\,dx$; $y(0)=-1$

Solution:- $\int 2(y-1)\,dy = \int(3x^2+4x+2)\,dx$

$2\int(y-1)\,dy = \int(3x^2+4x+2)\,dx$

$2\left[\int y\,dy - \int 1\,dy\right] = 3\int x^2\,dx + 4\int x\,dx + 2\int dx$

$2\left[\frac{y^2}{2} - y\right] = \frac{3x^3}{3} + \frac{4x^2}{2} + 2x + C$

$\boxed{y^2-2y = x^3+2x^2+2x+C}$ General solution

$x=0$; $y=-1$

$(-1)^2 - 2(-1) = 0 + C$

$\boxed{C=3}$

$\boxed{y^2-2y = x^3+2x^2+2x+3}$ Answer