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Where Minds Pollinate

Class 12

Mathematics

Chapter 4

Exercise 4.1

Handwritten Solution

Exercise 4.1

Diffrential equations:-

Concept: $3\frac{d^2y}{dx^2} - r\left(\frac{dy}{dx}\right)^5 = 5$

order = 2

Degree = 1

$\therefore \left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right) + y = x+1$

order, degree, Dependent variable, Independent variable

• dependent variable should not multiply with derivative or trancadential function.

Q1) (i) $(1-x)y'' - 4xy' + 5y = \cos x$

Sol:- order = 2
degree = 1

(ii) $yy' + 2y = 1 + x^2$
order = 1
degree = 1

(iii) $(y'')^3 - 3y' + 2y = x$
order = 2
degree = 3

(iv) $(y')^2 - yy'' + 2 = 0$
order = 2
degree = 1

(v) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$
order = 2
degree = 1

vi) $\frac{dy}{dx} = \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^2}$

Sol:- $\left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{d^2y}{dx^2}\right)^2$

order 2
degree = 2

vii) $x^3\frac{d^4y}{dx^4} - x^2\frac{d^2y}{dx^2} + 4\left(\frac{dy}{dx}\right)^5 + 4xy - 3y = 0$

Sol: order = 4
degree 1

$Q_2)$ (ii) $y = \cos(x+b)$

Sol Diff w.r.t x

$$\frac{dy}{dx} = -\sin(x+b) \cdot \frac{d}{dx}(x+b)$$

$$\frac{dy}{dx} = -\sin(x+b)(1+0)$$

$$\frac{dy}{dx} = -\sin(x+b)$$

Again diff w.r.t x.

$$\frac{d^2y}{dx^2} = -\cos(x+b) \cdot (1+0)$$

$$\frac{d^2y}{dx^2} = -y$$

$$\boxed{\frac{d^2y}{dx^2} + y = 0}$$

(i) $y = ae^{x} + be^{-x} + c$

Sol:- Diff w.r.t x

$$\frac{dy}{dx} = ae^{x} \cdot 1 + be^{-x} \cdot (-1) + 0$$

$$\frac{dy}{dx} = ae^{x} - be^{-x}$$

$$\frac{d^2y}{dx^2} = ae^{x} - be^{-x}(-1)$$

$$\frac{d^2y}{dx^2} = ae^{x} + be^{-x}$$

$$\frac{d^2y}{dx^2} = y - c.$$

Again diff w.r.t x

$$\frac{d^3y}{dx^3} = \frac{dy}{dx} - 0$$

$$\frac{d^3y}{dx^3} - \frac{dy}{dx} = 0$$

(iii) $y = mx + c$

Sol $\frac{dy}{dx} = m$

$\frac{d^2y}{dx^2} = 0$ Answer

$\boxed{\frac{d^3y}{dx^3} = 0}$

(iv) $y = bx^2 + 2ax$

Sol:- $\frac{dy}{dx} = 2bx + 2a$

$\frac{d^2y}{dx^2} = 2b + 0$

$\boxed{\frac{d^3y}{dx^3} = 0}$ Answer.

Q3) (i) $2y' + y = 0$ — (1) ; $y = e^{-\frac{x}{2}}$

Solution:- Diff w.r.t x

$\frac{dy}{dx} = e^{-x/2} \frac{d}{dx}\left(\frac{-x}{2}\right)$

$y' = e^{-x/2} \cdot \frac{-1}{2}$

$\boxed{y' = \frac{-1}{2} e^{-x/2}}$ So put in (1)

$2\left(\frac{-1}{2} e^{-x/2}\right) + e^{-x/2} = 0$

$-\cancel{e^{-x/2}} + \cancel{e^{-x/2}} = 0$

$0 = 0$ True.

Hence $y = e^{-x/2}$ is the solution of given DE (differential eq).

(iv) $y' + y = \sin x$ (1) ; $y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + 10e^{-x}$

Sol:- Diff w.r.t x

$$\frac{dy}{dx} = \frac{1}{2}\cos x + \frac{1}{2}\sin x - 10e^{-x}$$

putting y & y' in (1)

$$y' + y = \sin x$$

$$\cancel{\frac{1}{2}\cos x} + \frac{1}{2}\sin x - \cancel{10e^{-x}} + \frac{1}{2}\sin x - \cancel{\frac{1}{2}\cos x} + \cancel{10e^{-x}} = \sin x$$

$$\sin x\left(\frac{1}{2} + \frac{1}{2}\right) = \sin x$$

$\sin x = \sin x$ Hence $y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + 10e^{-x}$ is solution of given D.E.

(ii) $\frac{dy}{dx} - 2y = e^{3x}$; $y = e^{3x} + 10e^{2x}$

Solution: $\frac{dy}{dx} = 3e^{3x} + 20e^{2x}$

$$[3e^{3x} + 20e^{2x}] - 2[e^{3x} + 10e^{2x}] = e^{3x}$$

$$3e^{3x} + \cancel{20e^{2x}} - 2e^{3x} - \cancel{20e^{2x}} = e^{3x}$$

$e^{3x} = e^{3x}$ Proved!

(iii) $y' = 25 + y^2$; $y = 5\tan 5x$

Sol $\frac{dy}{dx} = 5\sec^2 5x \cdot 5$

$$\frac{dy}{dx} = 25\sec^2 5x$$

$$25\sec^2(5x) = 25 + (5\tan 5x)^2$$

$$25\sec^2(5x) = 25 + 25\tan^2(5x)$$

$$= 25\{1+\tan^2(5x)\}$$

$$25\sec^2(5x) = 25\sec^2(5x) \quad \text{proved}$$

(v) $x^3dy - 2dx = 0 \quad ; \quad y = \frac{-1}{x^2} + 6$

Sol:- $y = -x^{-2} + 6$

$$\frac{dy}{dx} = 2x^{-3} + 0$$

$$\boxed{\frac{dy}{dx} = \frac{2}{x^3}}$$

$$x^3dy = 2dx$$

$$\boxed{x^3\frac{dy}{dx} = 2}$$

$$x^3\left[\frac{2}{x^3}\right] = 2$$

$2 = 2$ Proved

(vi) $y' - \frac{1}{x}y = 1 \quad ; \quad y = x\ln x \quad , x > 0$

Sol $\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x(1)$

$$y' = \boxed{\frac{dy}{dx} = 1 + \ln x}$$

$$(1+\ln x) - \frac{1}{x}(x\ln x) = 1$$

$1 + \ln x - \ln x = 1$

$1 = 1$ proved

Q4)

Sol:- $dy - \sin x dx = 0$

$dy = \sin x dx$

$\frac{dy}{dx} = \sin x$

Order $= 1$

Degree $= 1$ Ans

Q5) Sol:-

$\frac{dy}{dx} = -a\sin x + b\cos x$

$\boxed{\frac{d^2y}{dx^2} = -a\cos x - b\sin x}$

$\frac{d^2y}{dx^2} + y = 0$

$-a\cos x - b\sin x + a\cos x + b\sin x = 0$

$0 = 0$ proved

Q6) Sol:- $f(x) = y_1 = x^2$

$f(x) = y = x^2$

$y' = 2x$

$y'' = 2$

$x^2y'' - 4xy' + 6y = 0$

$x^2(2) - 4x(2x) + 6(x^2) = 0$

$2x^2 - 8x^2 + 6x^2 = 0$

$-6x^2 + 6x^2 = 0$

$0 = 0$

$y_1 = x^2$ is solution

Now check

$y_2 = x^3$

$y = x^3$

$y' = 3x^2$ $\quad y'' = 6x$

$x^2y'' - 4xy' + 6y = 0$

$x^2(6x) - 4x(3x^2) + 6(x^3) = 0$

$6x^3 - 12x^3 + 6x^3 = 0$

$0 = 0$

$y_2 = x^3$ is a solution Answer.

(1) $y_1 = x^2$

$c_1y_1 = c_1x^2$ let $y = c_1x^2$

$\boxed{y = c_1x^2}$

$y' = c_1(2x)$

$y'' = 2c_1$

$x^2y'' - 4xy' + 6y = 0$

$x^2(2c_1) - 4x(2c_1x) + 6(c_1x^2) = 0$

$2c_1x^2 - 8c_1x^2 + 6c_1x^2 = 0$

$0 = 0$

c_1y_1 is a solution of given D.E

$y_2 = x^3$

$c_2y_2 = c_2x^3$ let $y = c_2y_2$

$\boxed{y = c_2x^3}$

$y' = 3c_2x^2$

$y'' = 6c_2x$

$x^2y'' - 4xy' + 6y = 0$

$x^2(6C_2x) - 4x(3C_2x^2) + 6(C_2x^3) = 0$

$6C_2x^3 - 12C_2x^3 + 6C_2x^3 = 0$

$0 = 0$

C_2y_2 is also a solution of given D.E

(ii) $y_1 + y_2 = x^2 + x^3$

$f(x) = x^2 + x^3$

$\boxed{y = x^2 + x^3}$

$y' = 2x + 3x^2$

$y'' = 2 + 6x$

$x^2y'' - 4xy' + 6y = 0$

$x^2(2+6x) - 4x(2x+3x^2) + 6(x^2+x^3) = 0$

$2x^2 + 6x^3 - 8x^2 - 12x^3 + 6x^2 + 6x^3 = 0$

$\cancel{-6x^2} + \cancel{6x^2} \cancel{- 6x^3} + \cancel{6x^3} = 0$

$0 = 0$

Hence $(y_1 + y_2)$ is also a solution of given DE.