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Where Minds Pollinate

Class 12

Mathematics

Chapter 3

Review Exercise 3

Handwritten Solution

Review Exercise # 3

i) $\int \frac{4x+2}{x^2+x+1}\,dx$

Solution: $2\int \frac{2x+1}{(x^2+x+1)}\,dx$

$= 2\ln|x^2+x+1| + c$ Answer

ii) $\int x(x^2+1)^4\,dx$

Solution: $\frac{1}{2}\int (x^2+1)^4\, 2x\,dx$

$= \frac{1}{2}\,\frac{(x^2+1)^5}{5} + c$

$= \frac{(x^2+1)^5}{10} + c$ Answer.

iii) $\int \cos^2 3x\,dx$

Solution: $\int \frac{1+\cos 2(3x)}{2}\,dx$

$= \frac{1}{2}\left[\int 1\,dx + \int \cos 2(3x)\,dx\right]$

$= \frac{1}{2}\left[x + \frac{\sin 2(3x)}{6}\right] + c$

$= \frac{1}{2}\left[x + \frac{2\sin 3x\cos 3x}{6}\right] + c$

$= \frac{1}{2}\left[x + \frac{\sin 3x\cos 3x}{3}\right] + c$

Answer

(iv) $\int \sin^{-1}x\,dx$

Sol: $\int 1\cdot \sin^{-1}x\,dx$

$= \sin^{-1}x \int 1\,dx - \int\left(\int dx \cdot \frac{d}{dx}(\sin^{-1}x)\right)dx$

$= x\sin^{-1}x - \int x \cdot \frac{1}{\sqrt{1-x^2}}\,dx$

$= x\sin^{-1}x + \frac{1}{2}\int (1-x^2)^{-1/2}\cdot 2x\,dx$

$= x\sin^{-1}x + \frac{1}{2}\,\frac{(1-x^2)^{1/2}}{1/2} + C$

$= x\sin^{-1}x + \sqrt{1-x^2} + C$ Answer

(iv) $\int \frac{x^2-29x+5}{(x-4)^2(x^2+3)}\,dx$ V.V.V IMPORTANT.

Sol: $\frac{x^2-29x+5}{(x-4)^2(x^2+3)} = \frac{A}{(x-4)} + \frac{B}{(x-4)^2} + \frac{Cx+D}{(x^2+3)}$ — (1)

$x^2-29x+5 = A(x-4)(x^2+3) + B(x^2+3) + (Cx+D)(x-4)^2$ — (2)

$x^2-29x+5 = A(x^3+3x-4x^2-12) + B(x^2+3) + Cx^3 - 8Cx^2 + 16Cx$
$+ Dx^2 - 8Dx + 16D$ — (3)

put $x-4=0 \Rightarrow \boxed{x=4}$, put in eq (2).

$16-116+5 = B(19)$

$\frac{-95}{19} = B$ $\boxed{B=-5}$.

comparing coefficients

x^3: $0 = A+C$

$\boxed{A=-C}$

$\underline{x^2}$ $1 = -4A + B - 8C + D$

$1 = -4(-C) - 5 - 8C + D$

$1 + 5 = -4C + D$

$\boxed{D = 4C + 6}$

$\underline{x}$ $-29 = 3A + 16C - 8D$

$-29 = 3(-C) + 16C - 8(4C + 6)$

$-29 = -3C + 16C - 32C - 48$

$\boxed{C = -1}$

$A = -(-1)$

$\boxed{A = 1}$

$D = 4(-1) + 6$

$\boxed{D = 2}$

$= \int \left(\frac{1}{x-4} - \frac{5}{(x-4)^2} + \frac{(-x+2)}{x^2+3} \right) dx$

$= \int \frac{1}{(x-4)} dx - 5\int (x-4)^{-2} dx - \frac{1}{2}\int \frac{2x}{x^2+3} dx + 2\int \frac{1}{x^2+3} dx$

$= \ln|x-4| - \frac{5(x-4)^{-1}}{-1} - \frac{1}{2}\ln|x^2+3| + 2\int \frac{1}{x^2+(\sqrt{3})^2} dx$

$= \ln|x-4| + \frac{5}{x-4} - \frac{1}{2}\ln|x^2+3| + 2\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + C$

$= \ln|x-4| + \frac{5}{x-4} - \frac{1}{2}\ln|x^2+3| + \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + C$

$= \ln|x-4| + \frac{5}{(x-4)} - \frac{1}{2}\ln|x^2+3| + \frac{2\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}}{3} + C$

(vi) $\int 2x \sin 3x \, dx$

Sol: $2\left[x\int \sin 3x\, dx - \int\left(\int \sin 3x\, dx \cdot \frac{d}{dx}(x)\right)dx\right]$

$= 2\left[x\cdot\left(\frac{-\cos 3x}{3}\right) - \int -\frac{\cos 3x}{3}\cdot(1)\, dx\right]$

$= -\frac{2}{3}x\cos 3x + \frac{2}{3}\int \cos 3x\, dx$

$= -\frac{2}{3}x\cos 3x + \frac{2}{3}\left(\frac{\sin 3x}{3}\right) + C$

$= -\frac{2}{3}x\cos 3x + \frac{2}{9}\sin 3x + C$ Answer.

(vii) $\int x^2 e^x dx$

Sol: $x^2\int e^x dx - \int\left(\int e^x dx \cdot \frac{d}{dx}(x^2)\right)dx$

$= x^2\cdot e^x - \int e^x \cdot 2x\, dx$

$= x^2 e^x - 2\int e^x \cdot x\, dx$

$= x^2 e^x - 2\left[x\int e^x dx - \int\left(\int e^x dx \cdot \frac{d}{dx}(x)\right)dx\right]$

$= x^2 e^x - 2\left[x e^x - \int e^x (1)\, dx\right]$

$= x^2 e^x - 2x e^x + 2\int e^x dx$

$= x^2 e^x - 2x e^x + 2e^x + C$

$= e^x(x^2 - 2x + 2) + C$

Answer.

(viii) $\int_0^{\pi/4} \sin 2x\,dx - 5\int_0^{\pi/4} \cos 4x\,dx$

Solution:- $-\left|\frac{\cos 2x}{2}\right|_0^{\pi/4} - 5\left|\frac{\sin 4x}{4}\right|_0^{\pi/4}$

$= -\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right) - \cos 2(0)\right]$

$= -\frac{5}{4}\left[\sin 4\left(\frac{\pi}{4}\right) - \sin 4(0)\right]$

$= -\frac{1}{2}\left[0-1\right] - \frac{5}{4}\left[0-0\right]$

$= \boxed{\frac{1}{2}}$ Answer.

(ix) $\int_1^4 \frac{\cos\sqrt{x}}{2\sqrt{x}}\,dx$

let $u=\sqrt{x}$ $x \to 1$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ $\Rightarrow \boxed{u=1}$

$x \to 4$

$du = \frac{1}{2\sqrt{x}}dx$ $\boxed{u=2}$.

$= \int_1^2 \cos u\,du$

$= \left|\sin u\right|_1^2$

$= \sin 2 - \sin 1$ Answer

Q3) $\int_0^1 \frac{x^2}{\sqrt{2x+1}}\,dx$

Sol. let $u = 2x+1 \Rightarrow \frac{du}{dx} = 2$

$du = 2dx$

$\boxed{\frac{du}{2} = dx}$

$u - 1 = 2x$

$x = \frac{u-1}{2}$

Taking sq root on b/s

$\left(\frac{u-1}{2}\right)^2 = x^2$

$\frac{(u-1)^2}{4} = x^2$

$\boxed{x^2 = \frac{u^2-2u+1}{4}}$

if $x \to 0$

then $u \to 1$.

if $x \to 1$

then $u \to 3$

$\Rightarrow \int_1^3 \frac{\frac{u^2-2u+1}{4}}{\sqrt{u}} \cdot \frac{du}{2}$

$= \frac{1}{4}\int_1^3 (u^2-2u+1)\,u^{-1/2} \cdot \frac{du}{2}$

$= \frac{1}{8}\int_1^3 \left(u^{2-1/2} - 2u^{1-1/2} + u^{-1/2}\right)du$

$= \frac{1}{8}\int_1^3 \left(u^{3/2} - 2u^{1/2} + u^{-1/2}\right)du$

$= \frac{1}{8}\left[\frac{u^{5/2}}{5/2} - \frac{2u^{3/2}}{3/2} + \frac{u^{1/2}}{1/2}\right]_1^3$

$= \frac{1}{8}\left[\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + 2u^{1/2}\right]_1^3$

$= \frac{2}{8}\left[u^{1/2}\left(\frac{u^2}{5} - \frac{2}{3}u + 1\right)\right]$

$= \frac{1}{4}\left[\sqrt{3}\left(\frac{9}{5} - \frac{2}{3}(3) + 1\right) - \sqrt{1}\left(\frac{1}{5} - \frac{2}{3} + 1\right)\right]$

$= \frac{1}{4}\left[\sqrt{3}\left(\frac{9}{5} - 1\right) - \frac{8}{15}\right]$

$= \frac{\sqrt{3}}{5} - \frac{2}{15}$

$= \frac{3\sqrt{3} - 2}{15}$

≈ 0.21 answer.

Q4) Sol: $v_i = 60ms^{-1}$, $g = -9.8ms^{-2}$

$t = ?$ $v_f = 0$

a) first equation of motion

$v_f = v_i + gt$

$0 = 60 + (-9.8)t$

$\boxed{t = 6.12s}$ Answer

(b) $S(t) = ?$ 2nd eq of motion

$S(t) = v_i t + \frac{1}{2}gt^2$

$S(t) = 60(6.12) + \frac{1}{2}(-9.8)(6.12)^2$

$S(t) = 367.2 - 183.52$

$\boxed{S(t) = 183.6m}$ Answer

Q5) Sol: $S(t) = \int v(t)dt$

$S(t) = \int \cos \pi t \, dt$

$S(t) = \frac{\sin \pi t}{\pi} + C$ — (1)

given Condition

$t = 0$, $S(0) = 4m$

from eq (1)

$S(0) = \frac{\sin \pi(0)}{\pi} + C$

$4 = 0 + C$

$\boxed{C = 4}$ put in eq (1)

$S(t) = \frac{\sin \pi t}{\pi} + 4$

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