Studybeesofficial

Where Minds Pollinate

Class 12
Mathematics

Chapter 3

Exercise 3.8

Handwritten Solution

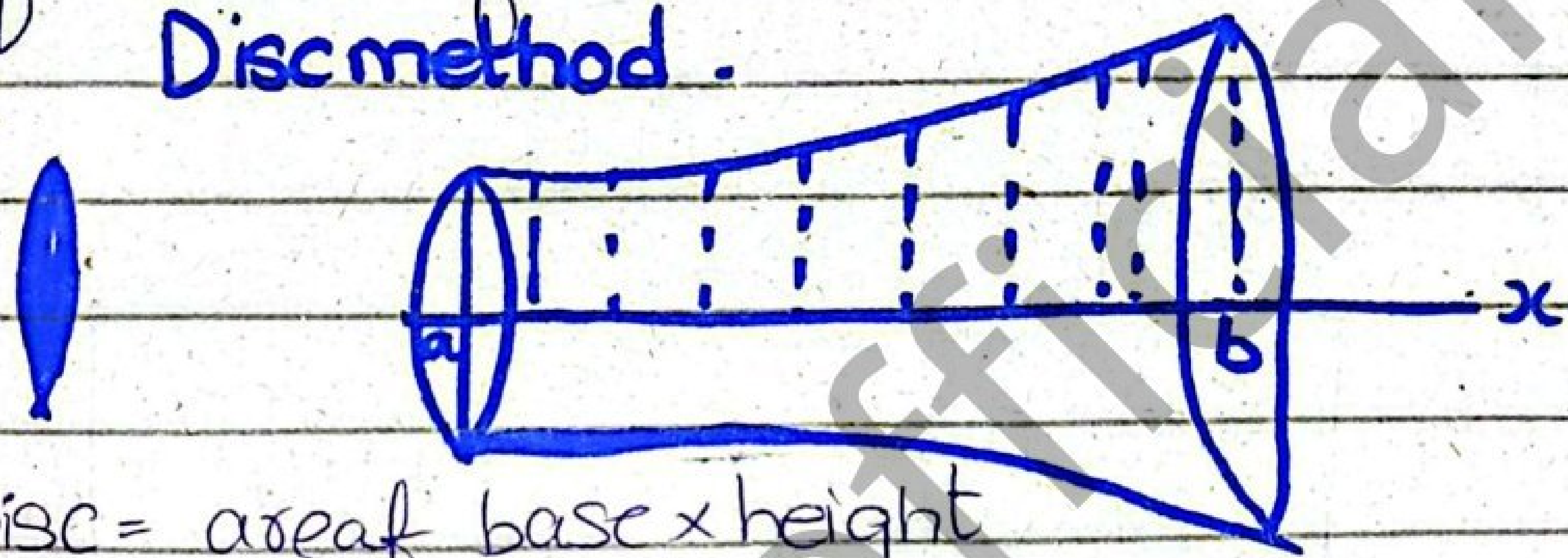
Exercise 3.8

Concepts:- 1) Area between a curve and the x-axis

2) Area b/w Curves Area $= \int_a^b [f(x) - g(x)]\,dx$; $f(x) > g(x)$

3) Volume of solids of revolution

→ **Disc method.**



Volume of Disc = area of base × height

Volume of disc $= \pi r^2 \times h$

volume of solid $= \sum_{i=1}^{n} \pi [f(x_i)]^2 \Delta x_i$

if $\Delta x_i \to 0$ then $V = \pi \int_a^b [f(x)]^2\,dx$

4) Applications: (i) Consumer and producer surpluses (ii) Work spring (iii) Rectilinear motion (iv) motion of spa…

Q1) Solution: $A = \int_a^b f(x)\,dx$

$A = \int_1^3 x^2\,dx$

$A = \left| \dfrac{x^3}{3} \right|_1^3 = \dfrac{1}{3}\left[(3)^3 - (1)^3\right]$

$\boxed{A = \dfrac{26}{3}}$ Ans

SLO

∴ if only one value of x is given suppose $x = 3$ then we have to put $y = 0$ in function e.g $y = x^2$ & take out x and then take limit

② Solution:- $A = \int_0^2 \sqrt{6x+4}\, dx$

$A = \frac{1}{6}\int_0^2 (6x+4)^{1/2} \cdot 6dx$

∴ for power rule derivative must be present.

$A = \frac{1}{6}\left| \frac{(6x+4)^{3/2}}{3/2} \right|_0^2$

$A = \frac{1}{6} \times \frac{2}{3}\left[(6(2)+4)^{3/2} - (6(0)+4)^{3/2}\right]$

$A = \frac{1}{9}\left[\{(16)^{1/2}\}^3 - [(4)^{1/2}]^3\right]$

$A = \frac{1}{9}\left[4^3 - 2^3\right]$

$\boxed{A = \frac{56}{9}}$

③ $y = \sqrt{4x}$ IMP

$y = \pm 2\sqrt{x} = f(x)$

$y^2 = 4x$

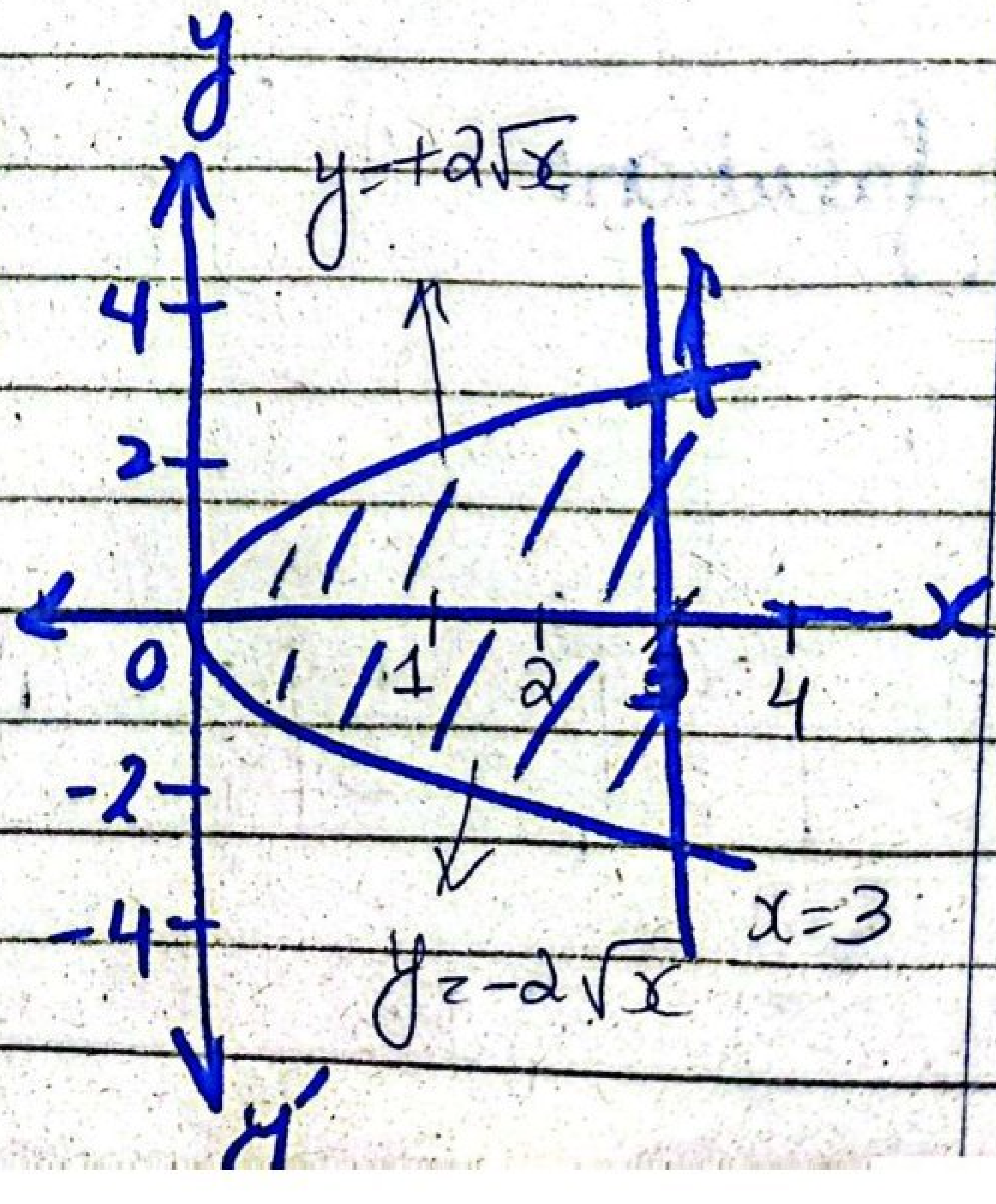
x	0	1	4	
y	0	±2	±4	

put $y = 0 \Rightarrow 0 = 4x$

$\boxed{x = 0}$

$A = \int_a^b \left(y_{upper} - y_{lower}\right) dx$

$A = \int_0^3 \left[2\sqrt{x} - (-2\sqrt{x})\right] dx$



$A = \int_0^3 (2\sqrt{x} + 2\sqrt{x})\,dx$

$= 4\int_0^3 \sqrt{x}\,dx$

$A = 4\left|\frac{x^{3/2}}{3/2}\right|_0^3 = 4\left(\frac{2}{3}\right)\left[(3)^{3/2} - (0)^{3/2}\right]$

$A = \frac{8}{3}(3)^{3/2}$

$\boxed{A = 8\sqrt{3}}$ Ans

Q 3) OR.

$y^2 = 4x \quad ; x = 3$

$y = \pm 2\sqrt{x}$

$y = y_1 + y_2$ where

$y_1 = 2\sqrt{x}$ & $y_2 = -2\sqrt{x}$

As $y_1 \geq 0$ at $[0, 3]$

$y_2 \leq 0$ at $[0, 3]$

Area $= \int_0^3 y\,dx$

$= \int_0^3 y_1\,dx - \int_0^3 y_2\,dx$

$= \int_0^3 2\sqrt{x}\,dx - \int_0^3 -2\sqrt{x}\,dx$

$2\int_0^3 \sqrt{x}\,dx + 2\int_0^3 \sqrt{x}\,dx$

$= 4\int_0^3 \sqrt{x}\,dx$

$= 4\left.\frac{x^{3/2}}{3/2}\right|_0^3$

$= \frac{8}{3}\left[(3)^{3/2} - 0^{3/2}\right]$

$= \boxed{8\sqrt{3}\ \text{sq units.}}$

Q4) (i) for $A = \int_1^3 \frac{1}{2}(0.2x^2 + x)\,dx$

$A = \int\left(\frac{1}{10}x^2 + \frac{1}{2}x\right)dx$

$A = \frac{1}{10}\int x^2\,dx + \frac{1}{2}\int x\,dx$

$= \frac{1}{10}\left|\frac{x^3}{3}\right| + \frac{1}{2}\left|\frac{x^2}{2}\right|$

$= \left[\frac{x^2}{30} + \frac{x^2}{4}\right]$

$A = \left[\frac{x^3}{30} + \frac{x^2}{4}\right]_1^3 = \left[\left\{\frac{(3)^3}{30} + \frac{(3)^2}{4}\right\} - \left\{\frac{1}{30} + \frac{1}{4}\right\}\right]$

$A = \frac{63}{20} - \frac{17}{60}$

$\boxed{A = \frac{43}{15}}$ Ans

(ii) $B = \left| \frac{x^3}{30} + \frac{x^2}{4} \right|_3^4$

$B = \left[\left\{ \frac{(4)^3}{30} + \frac{(4)^2}{4} \right\} - \left\{ \frac{(3)^3}{30} + \frac{(3)^2}{4} \right\} \right]$

$B = \frac{92}{15} - \frac{63}{20}$

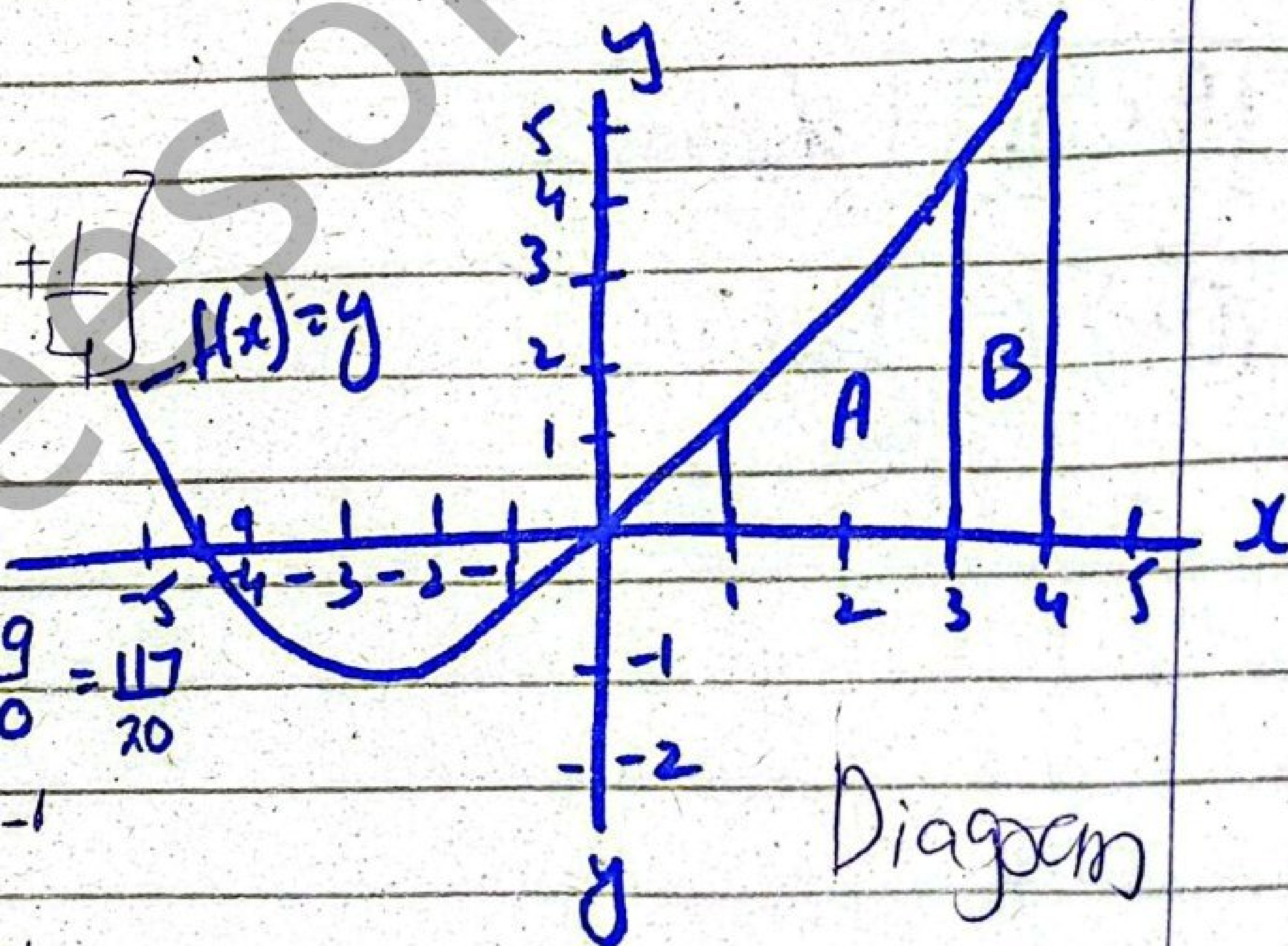
$\boxed{B = \frac{179}{60}}$ Answer

(iii) Area $= \left| \frac{x^3}{30} + \frac{x^2}{4} \right|_1^4$

$= \left[\frac{(4)^3}{30} + \frac{(4)^2}{4} \right] - \left[\frac{1}{30} + \frac{1}{4} \right]$

$= \frac{92}{15} - \frac{17}{60}$

$= \boxed{\frac{117}{20}}$ Ans or $\frac{43}{15} + \frac{179}{60} = \frac{117}{20}$



(iv) Area $= \left| \frac{x^3}{30} + \frac{x^2}{4} \right|_{-4}^{-1}$

$= \left[\frac{(-1)^3}{30} + \frac{(-1)^2}{4} \right] - \left[\frac{(-4)^3}{30} + \frac{(-4)^2}{4} \right]$

$= \frac{13}{60} - \frac{28}{15}$

$\boxed{\text{Area} = -\frac{33}{20}}$ A

neglecting -ve

Area $= \frac{33}{20}$ Ans

OR

$\frac{1}{2} \int_{-1}^{-4} (0.2x^2 + x)\,dx$ limit change.

$-\frac{1}{2} \int_{-4}^{-1} (0.2x^2 + x)\,dx$

solve

$= \frac{33}{20}$ Ans

Q5 (i) $y = 1+\cos x$; $[0, 3\pi]$

Sol $A = \int_a^b f(x)\,dx$

$A = \int_0^{3\pi} (1+\cos x)\,dx$

$A = \left| x + \sin x \right|_0^{3\pi}$

$= [(3\pi + \sin 3\pi) - (0 + \sin(0))]$

$= (3\pi + 0) - (0)$

$\boxed{A = 3\pi}$

(ii) $y = -1 + \sin x$; $\left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$

Sol $A = \int_{-3\pi/2}^{3\pi/2} (-1+\sin x)\,dx$

$A = \left| -x - \cos x \right|_{-3\pi/2}^{3\pi/2}$

$A = \left\{-\frac{3\pi}{2} - \cos\left(\frac{3\pi}{2}\right)\right\} - \left\{-\left(\frac{-3\pi}{2}\right) - \cos\left(\frac{3\pi}{2}\right)\right\}$

$A = -\frac{3\pi}{2} - \left[\frac{3\pi}{2}\right]$

$A = -\frac{6\pi}{2}$ $\boxed{A = -3\pi}$ -sign show that graph is below x axis

Q6) Solution: Point of intersection

$\boxed{y=x}$ (1) $\longrightarrow$ $\boxed{y=-2x}$ (2) comparing: $\therefore$ put (1) in (2).

Because 2 functions are given & single value of x

$x=-2x$

$x+2x=0$

$3x=0$ $[0,3]$

? $\boxed{x=0}$

$$A=\int_a^b(\text{upper curve}-\text{lower curve})dx$$

$$A=\int_0^3\left[x-(-2x)\right]dx$$

$$A=\int_0^3 3x\,dx$$

$$=3\left|\frac{x^2}{2}\right|_0^3$$

$$\frac{3}{2}\left[(3)^2-(0)^2\right]\Rightarrow\boxed{A=\frac{27}{2}}$$ square unit Answer.

Q7) Sol:- $y=x+6\rightarrow$ upper curve $[0,2]$

$y=x^2\longrightarrow$ lower curve

$$A=\int_0^2\left[(x+6)-(x^2)\right]dx$$

$$=\int_0^2(x+6-x^2)dx$$

$$=\left|\frac{x^2}{2}+6x-\frac{x^3}{3}\right|_0^2$$

$$\left(\frac{4}{2}+12-\frac{8}{3}\right)-(0)\Rightarrow\boxed{A=\frac{34}{3}}$$ Answer

x is given

Because only one value of x is given

Q8) put $y = 0$ $\therefore$ area ~~of~~ bounded by x-axis

$0 = x^3 + 1^3$

$0 = (x+1)(x^2 - x + 1)$

$x + 1 = 0$; $x^2 - x + 1 = 0$

$\boxed{x = -1}$ Imaginary (neglect).

$$A = \int_{-1}^{1} (x^3 + 1)\,dx = \left| \frac{x^4}{4} + x \right|_{-1}^{1}$$

$$A = \left(\frac{1}{4} + 1\right) - \left(\frac{1}{4} - 1\right)$$

$\boxed{A = 2}$ **Answer**

Q9) Sol $\boxed{x = y^2}$ $\longrightarrow$ $\boxed{x = y + 2}$ upper curve

lower curve

integrate with respect to y means ~~[illegible]~~ only y variable.

Point of Intersection

put eq(1) in (2)

$y^2 = y + 2$; $y^2 - y - 2 = 0$

$y^2 - 2y + y - 2 = 0$

$y(y-2) + 1(y-2) = 0$

$(y+1)(y-2) = 0$

$y + 1 = 0$ $y - 2 = 0$

$\boxed{y = -1}$, $y = 2$

$y \in [-1, 2]$

greater lower

$$A = \int_{-1}^{2} (y + 2 - y^2)\,dy = \left| \frac{y^2}{2} + 2y - \frac{y^3}{3} \right|_{-1}^{2}$$

$$A = \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$A = \frac{10}{3} + \frac{7}{6}$$

$A = \frac{9}{2}$ unit^2 **Answer**.

$f(x) = y^2$; $g(x) = x+2$

$-1 \to 1$	$-1 \to 1$
$0 \to 0$	$0 \to 2$
$1 \to 1$	$1 \to 3$
$2 \to 4$	$2 \to 4$

greater function.

Q10) Sol: $V = \pi \int_a^b [f(x)]^2 dx$

$\therefore$ Volume of solid about x-axis

$V = \pi \int_a^b [f(x)]^2 dx$

$\therefore$ volume of solid about y-axis

$V = \pi \int_a^b [f(y)]^2 dy$

$V = \pi \int_1^4 (\sqrt{x})^2 dx$

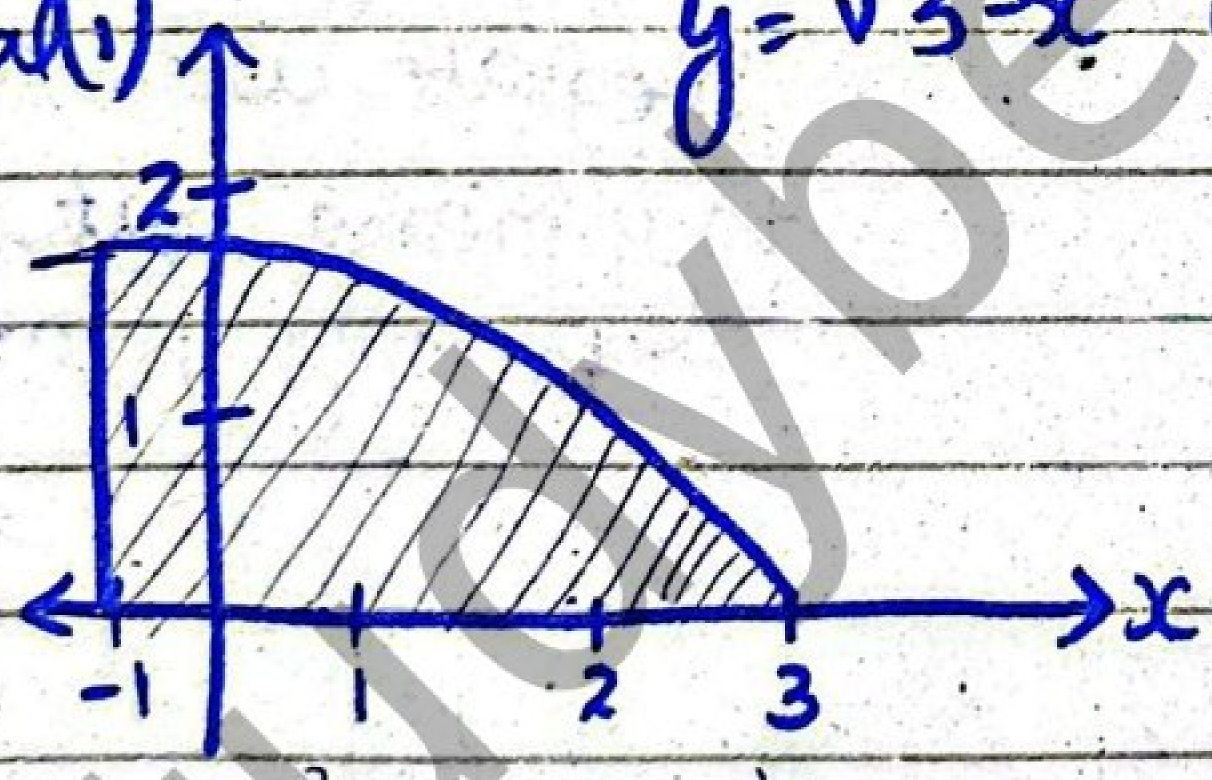
$= \pi \int_1^4 x\,dx$

$= \pi \left| \frac{x^2}{2} \right|_1^4$

$= \pi \left[\frac{(4)^2}{2} - \frac{1}{2} \right]$

$\boxed{V = \frac{15}{2} \pi \text{ unit}^3}$ Ans

Q11) Sol (i) $y = \sqrt{3-x}$ about x axis



Sol: $V = \pi \int_{-1}^{3} (\sqrt{3-x})^2 dx$

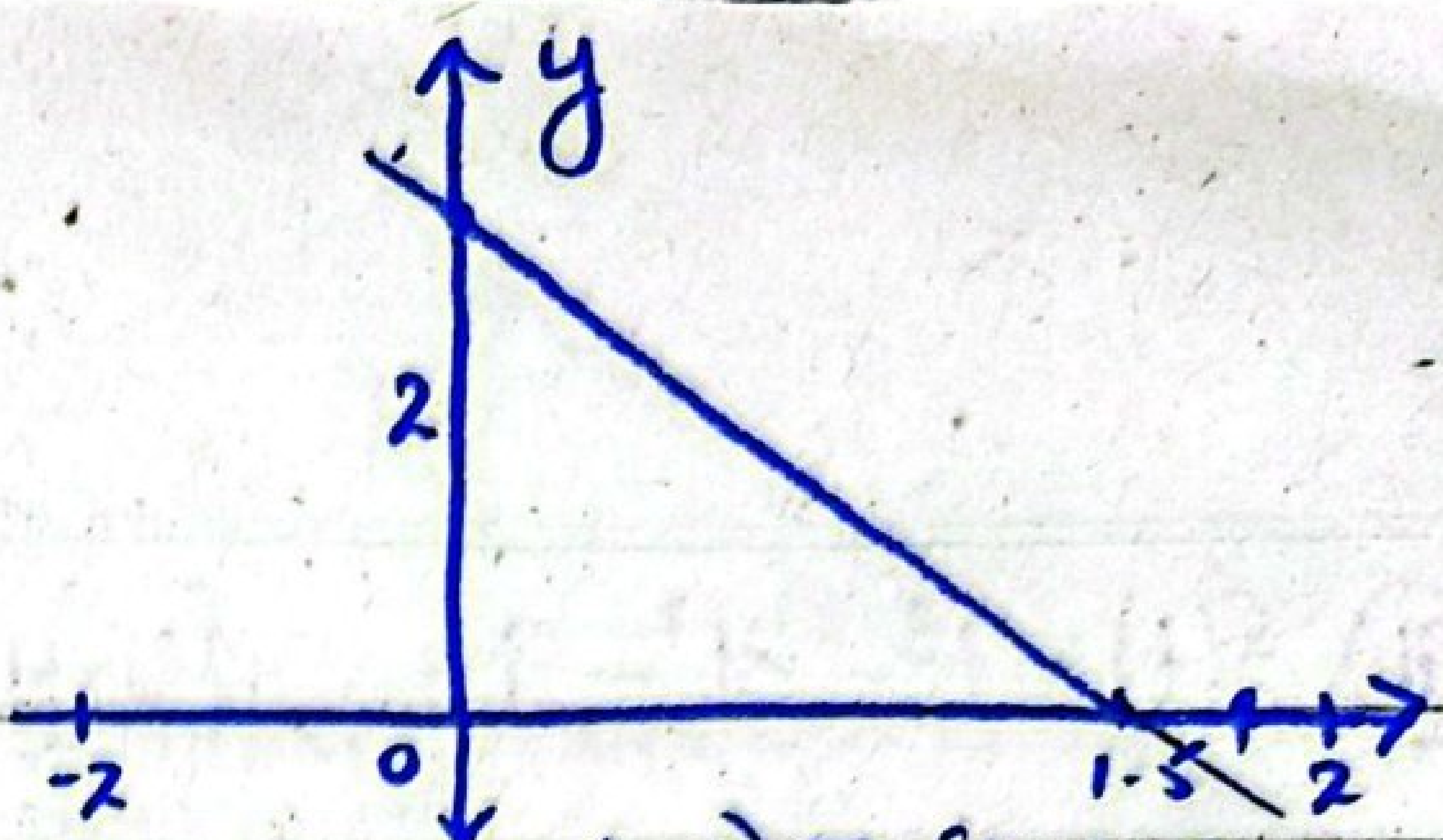
$V = \pi \int_{-1}^{3} (3-x) dx = \pi \left| 3x - \frac{x^2}{2} \right|_{-1}^{3}$

$V = \pi \left\{ \left(9 - \frac{9}{2}\right) - \left(-3 - \frac{1}{2}\right) \right\}$

$V = \pi \left[\frac{9}{2} - \left(-\frac{7}{2}\right) \right]$ $\boxed{V = 8\pi}$ unit^3

Answer

acceleration $= a(t) = \frac{dv}{dt}$



ii) $y = (3-2x)$ about y-axis $f(x)$

Sol: put $x=0$ (Because we dont know limit of y). To find why let $x=0$

$\boxed{y=3}$, $\boxed{y=0}$

$y = 3-2x$

$2x = 3-y$

$\boxed{x = \frac{3-y}{2}}$ $f(y)$

$V = \pi \int_0^3 [f(y)]^2 dy$

$V = \pi \int_0^3 \left(\frac{3-y}{2}\right)^2 dy$

$V = -\frac{\pi}{4} \int_0^3 (3-y)^2 (-1) dy$

$V = -\frac{\pi}{4} \left| \frac{(3-y)^3}{3} \right|_0^3$

$V = -\frac{\pi}{4} \left[0 - \frac{27}{3} \right]$

$V = \frac{27}{4(3)} \pi$

$\boxed{V = \frac{9\pi}{4} \text{ unit}^3}$

OR

$\pi \int_0^3 \frac{9+y^2-6y}{4} dy$

$\frac{\pi}{4} \int_0^3 (9+y^2-6y) dy \rightarrow \frac{9\pi}{4} \text{ unit}^3$

Q12 (i) $S(t) = t^2 - 2t$; $[0,5]$

Sol $V(t) = \frac{d}{dt} S(t)$

$V(t) = 2t - 2$ $\quad \therefore \frac{ds}{dt} = V(t)$

put $V(t) = 0$

$2t - 2 = 0$ $\quad \therefore S = \int v(t) dt$

$\boxed{t=1}$

0 —ve 1 +ve 5

$V(0.5) = 2(0.5) - 2$

$= -1$

$S(t) = -\int_0^1 v(t)dt + \int_1^5 v(t)dt$

$S(t) = -\int_0^1 (2t-2)dt + \int_1^5 (2t-2)dt$

$S(t) = -\left| t^2 - 2t \right|_0^1 + \left| t^2 - 2t \right|_1^5$

$S(t) = -[(1-2) - 0] + [(25-10) - (1-2)]$

$S(t) = 1 + 15 + 1$

$\boxed{S(t) = 17 \text{ cm}}$ Answer.

(ii) $S(t) = t^3 - 3t^2 - 9t$; $[0,4]$.

Sol: $v(t) = 3t^2 - 6t - 9$

put $v(t) = 0$

$0 = 3t^2 - 6t - 9$

$0 = 3(t^2 - 2t - 3)$

$0 = t(t-3) + 1(t-3)$

$0 = (t+1)(t-3)$

$\boxed{t = -1}$; $\boxed{t = 3}$

not possible

$-v(t)$ $\quad$ $+v(t)$

0 $\quad$ 3 $\quad$ 4

$$S(t) = \int_0^3 -v(t)\,dt + \int_3^4 v(t)\,dt$$

$$S(t) = -\int_0^3 (3t^2 - 6t - 9)\,dt + \int_3^4 (3t^2 - 6t - 9)\,dt$$

$$S(t) = -\left|t^3 - 3t^2 - 9t\right|_0^3 + \left|t^3 - 3t^2 - 9t\right|_3^4$$

$$S(t) = -\left[(27-27-27) - (0)\right] + \left[(64-48-36) - (-27)\right]$$

$S(t) = 27 - 20 + 27$

$S(t) = 7 + 27$

$\boxed{S(t) = 34\text{cm}}$ Answer.

$\therefore$ $v(t) = 3t^2 - 6t - 9$ at $[0,3]$

$= 3(1)^2 - 6(1) - 9$

$= -12 \leq 0$

$v(t) = 3(3.5)^2 - 6(3.5) - 9$

$= \boxed{6.75}$

(iii) $S(t) = 6\sin \pi t$; $[1,3]$

Sol: $\boxed{v(t) = 6\pi \cos \pi t}$

put $v(t) = 0$

$6\pi \cos \pi t = 0$

$\cos \pi t = \frac{0}{6\pi}$

$\pi t = \cos^{-1}(0)$

$\pi t = (2n+1)\frac{\pi}{2}$, $n \in Z$

$t = \frac{2n}{2} + \frac{1}{2} \Rightarrow \boxed{t = n + 0.5}$, $n \in Z$.

$t = n + 0.5 \quad , n \in \mathbb{Z}$

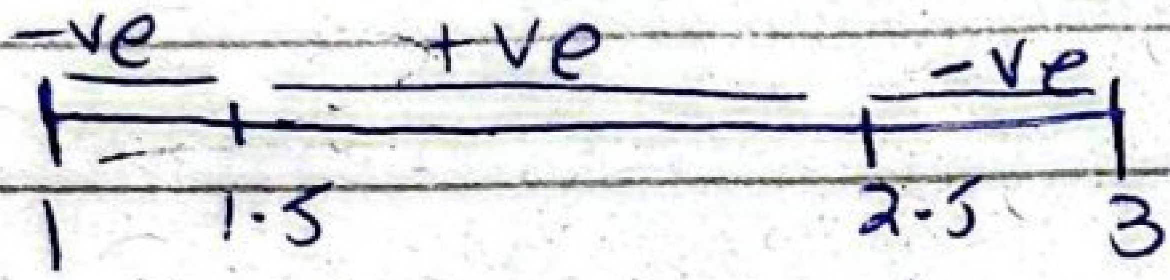
$n = 0, \pm 1, \pm 2, \ldots\ldots$

$\underline{n=0} \quad t = 0.5 \notin [1,3]$

$\underline{n=1} \quad t = 1.5 \in [1,3]$

$\underline{n=2} \quad t = 2.5 \in [1,3]$

$n = 3 \Rightarrow t = 3.5 \notin [1,3]$



$S(t) = \int_1^3 v(t)$

$$S(t) = \int_1^{1.5} -v(t)dt + \int_{1.5}^{2.5} v(t)dt + \int_{2.5}^{3} -v(t)dt$$

$$S(t) = -\int_1^{1.5} 6\pi \cos \pi t \, dt + \int_{1.5}^{2.5} 6\pi \cos \pi t \, dt - \int_{2.5}^{3} 6\pi \cos \pi t \, dt$$

$$S(t) = -\Big| 6 \sin \pi t \Big|_1^{1.5} + \Big| 6 \sin \pi t \Big|_{1.5}^{2.5} - \Big| 6 \sin \pi t \Big|_{2.5}^{3}$$

$$S(t) = -(-6-0) + (6-6) - (0-6)$$

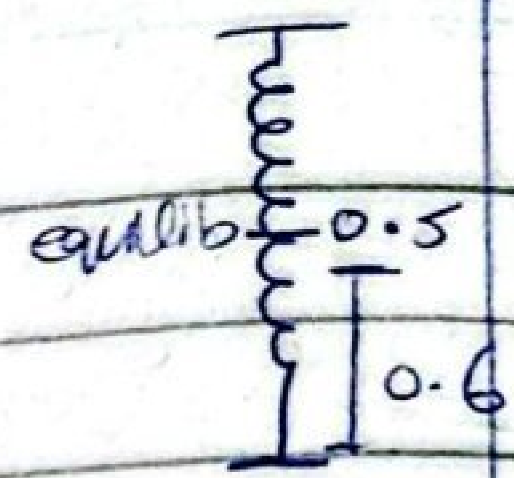
$$S(t) = +6+6$$

$\boxed{S(t) = 12cm}$ Answer

Q13) Sol $F \propto x$

$F = kx$

?



force $\propto$ displacement

$F = 50N, x = 0.5m \quad k = ?$

$F = kx$

$50N = k(0.5m)$

$k = 100 Nm^{-1}$

$\boxed{F = 100x}$

$w = \int_a^b F dx$

$W = \int_0^{0.6} 100x dx$

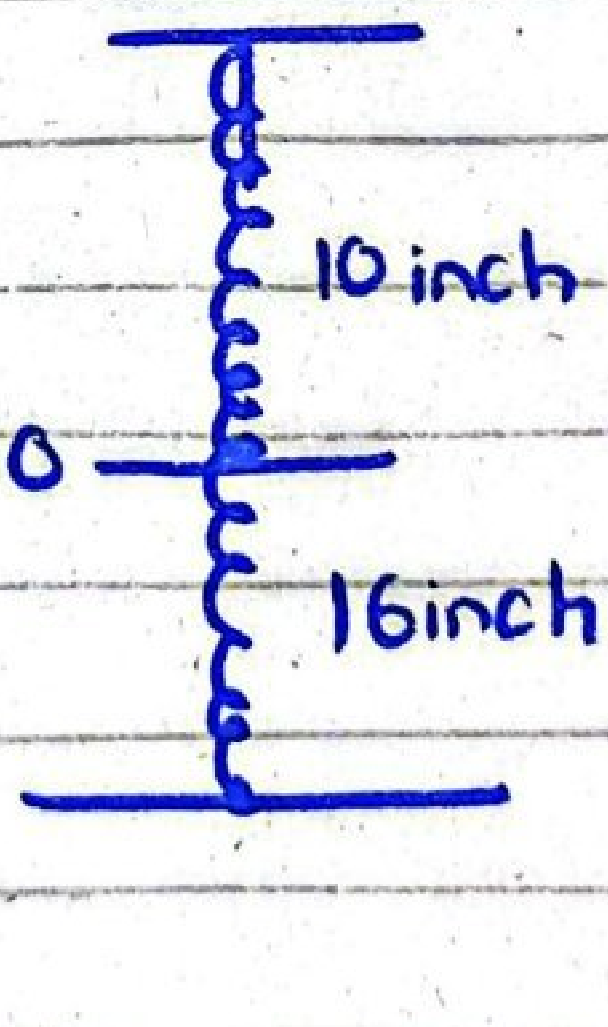
$W = \left| \frac{100x^2}{2} \right|_0^{0.6}$

$W = 50(0.6)^2 - 50(0)$

$\boxed{W = 18J}$ Answer.

Q14) Sol:- $F = 1.5x$

$W = \int_0^{16} (1.5x) dx$



$W = \left| \frac{1.5x^2}{2} \right|_0^{16}$

$W = \frac{1.5(16)^2}{2} - 0$

$\boxed{W = 192J}$ Answer

Q16) $f(t) = 200(t-5)^2$

Sol:- $T.R = \int_0^4 200(t-5)^2 dt$

$= 200\int_0^4 (t-5)^2 dt$

$= 200\left|\frac{(t-5)^3}{3}\right|_0^4$

$= \frac{200}{3}\left[(4-5)^3-(0-5)^3\right]$

$= \frac{200}{3}\left[-1+125\right]$

$\boxed{T.R = \$8266.66}$

Q17) $f(t) = 600\sqrt{1+3t}$

Sol $= \int_1^8 600\sqrt{1+3t}\, dt$

$= \frac{600}{3}\int_0^8 (1+3t)^{1/2} 3\,dt$

$= 200\left|\frac{(1+3t)^{3/2}}{3/2}\right|_0^8$

$= 200\times\frac{2}{3}\left[\left[1+3(8)\right]^{3/2}-\left[1\right]^{3/2}\right]$

$= \frac{400}{3}\left[(25)^{3/2}-1\right]$

$= \frac{400}{3}\left[124\right]$

$= \$16533.33$ Answer

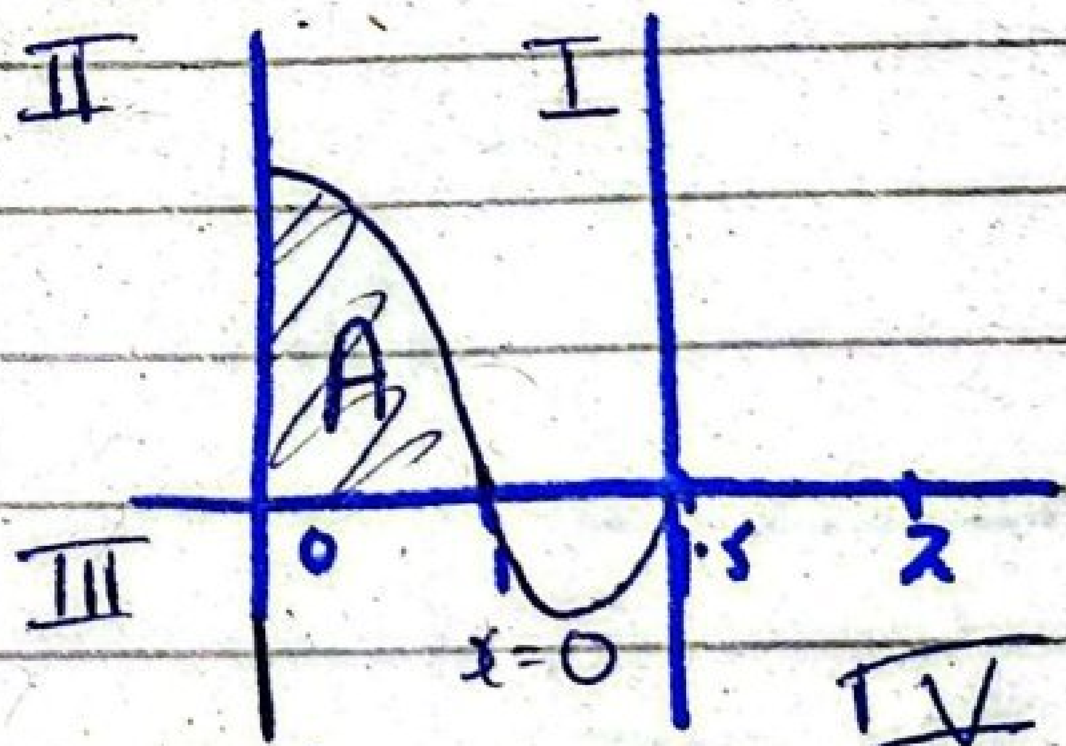
Q18) Sol:- put $y=0$

$0 = x^3-2x^2+1$

put $\boxed{x=1} \Rightarrow 0 = 1-2+1 = 0$

$x-1=0$

$$\begin{array}{r|l} & x^2-x-1 \\ \hline x-1 & x^3-2x^2+1 \\ & \underline{\pm x^3 \mp x^2} \\ & -x^2+1 \\ & \underline{\mp x^2 \pm x} \\ & -x+1 \\ & \underline{\mp x \pm 1} \\ & 0 \end{array}$$



$x = 1,\ -0.61,\ 1.61$

$x^2 - x - 1 = 0$

$a = 1, b = -1, c = -1$

$$x = \frac{1 \pm \sqrt{1+4}}{2}$$

$$x = \frac{1 + \sqrt{5}}{2}$$ $\boxed{x = \frac{1-\sqrt{5}}{2}}$ not included

$\boxed{x \approx 1.6}$ not included

let [0,1] check function is positive or negative

Hence given [0,1] as function is positive

and [1, 1.5] let $x = 1.2 \Rightarrow f(1.2) < 0$

it mean as function belong to **4th Quadrant**

$$A = \int_0^1 (x^3 - 2x^2 + 1)\, dx$$

$$= \left| \frac{x^4}{4} - \frac{2x^3}{3} + x \right|_0^1$$

$$A = \left(\frac{1}{4} - \frac{2}{3} + 1\right) - (0)$$

$$\boxed{A = \frac{7}{12} \text{ unit}^2}$$ A-1

Q15) ① $S(x) = 24$, $D(x) = 100-2x$

Sol:- • **For equilibrium quantity:-(x_e)**

$D(x) = S(x)$

$100-2x = 24$

$100-24 = 2x$

$\boxed{x_e = 38}$

equilibrium quantity.

• **Equilibrium Price P_e:-** Put $x = 38$ in $D(x)$

$D(38) = 100-2(38)$

$\boxed{P_e = 24}$

As

$C.S = \int_0^{x_e} (D(x) - P_e)\,dx$

$= \int_0^{38} (100-2x - 24)\,dx$

$= \int_0^{38} (76-2x)\,dx$

$= 76x - \frac{2x^2}{2} \Big|_0^{38}$

$= (76(38) - (38)^2) - 0$

$\boxed{C_S = 1444}$

As:- $PS = \int_0^{x_e} (P_e - S(x))\,dx$

$= \int_0^{38} (24-24)\,dx \Rightarrow \int_0^{38} 0\,dx$

As $\int 0\,dx = C$

So $\boxed{P.S = C}$

$PS = F(38) - f(0)$

$= C - C$

$= 0$

(ii) $S(x) = x^2 - 4$, $D(x) = -x+8$

Sol: $S(x) = D(x)$

$x^2 - 4 = -x + 8$

$x^2 + x - 12 = 0$

$x^2 + 4x - 3x - 12 = 0$

$x(x+4) - 3(x+4) = 0$

$(x-3)(x+4) = 0$

$\boxed{x=3}$; $\boxed{x=-4}$ not possible

$\boxed{P_0 = 5}$

$CS = \int_0^3 (-x+8-5)\,dx$

$CS = \int_0^3 (-x+3)\,dx$

$CS = \left| -\frac{x^2}{2} + 3x \right|_0^3$

$\boxed{CS = \frac{9}{2}}$

PS $= \int_0^3 (5 - x^2 + 4)\,dx$

$PS = \int_0^3 (9 - x^2)\,dx$

$PS = \left| 9x - \frac{x^3}{3} \right|_0^3$

$PS = \left(9(3) - \frac{(3)^3}{3}\right) - 0$

$\boxed{PS = 18}$ Answer

(iii) $S(x) = 2x^2 + 3x$; $D(x) = 36 - x^2$

Sol: $S(x) = D(x)$

$2x^2 + 3x = 36 - x^2$

$2x^2 + x^2 + 3x - 36 = 0$

$3x^2 + 3x - 36 = 0$

$3(x^2 + x - 12) = 0$

$x^2 + x - 12 = 0$

$x(x+4) - 3x - 12 = 0$

$(x-3)(x+4) = 0$

$\boxed{x=3}$ $\boxed{x=-4}$ not included

$x_0 = 3$

$P_0 = 36 - 9$

$P_0 = 27$

$CS = \int_0^3 (36 - x^2 - 27)\,dx$

$CS = \int_0^3 (9 - x^2)\,dx = \left| 9x - \frac{x^3}{3} \right|_0^3$

CS $= 27 - \frac{27}{3} = \boxed{18}$

PS $= \int_0^3 (27 - 2x^2 - 3x)\,dx$

$PS = \left| 27x - \frac{2x^3}{3} - \frac{3x^2}{2} \right|_0^3$

$PS = 27(3) - \frac{2(3)^3}{3} - \frac{3(3)^2}{2}$

$PS = 49.5$