Studybeesofficial

Where Minds Pollinate

Class 12
Mathematics

Chapter 3

Exercise 3.7

Handwritten Solution

Excercise 3.7

Definite Integral

$$A = \int_{x=a}^{x=b} f(x)dx = F(b) - F(a)$$] fundamental Theorm of Calculus.

Properties:

(i) $\int_a^a f(x)dx = 0$

(ii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(iii) If 'c' is the point between 'a' and 'b' then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Q1 (1) $\int_{-1}^{2} (2x+3)dx$

Sol:- $2\int_{-1}^{2} x dx + 3\int_{-1}^{2} dx$

$= 2\left|\frac{x^2}{2}\right|_{-1}^{2} + 3\left|x\right|_{-1}^{2}$

$= (2)^2 - (-1)^2 + 3(2) - (-1)$

$= \boxed{A = 12}$ Answer

SLO

$$\int_x^e x(\ln x)\,dx$$

② $\int_{-4}^{12} \sqrt{y+4}\,dy$

Sol:- $\int_{-4}^{12} (y+4)^{1/2}\,dy$

$= \left[\dfrac{|y+4|^{3/2}}{3/2}\right]_{-4}^{12}$

$= \dfrac{2}{3}\left[(12+4)^{3/2} - (-4+4)^{3/2}\right]$

$= \dfrac{2}{3}\left[(16)^{3/2}\right]$

$= \dfrac{2}{3}(4^3) = \boxed{\dfrac{128}{3}}$

③ $\int_0^{1/2} (2x+1)^{1/3}\,dx$

$\dfrac{1}{2}\int_0^{1/2} (2x+1)^{-1/3}\, 2dx$

$= \dfrac{1}{2}\left|\dfrac{(2x+1)^{2/3}}{2/3}\right|_0^{1/2}$

$= \dfrac{1}{2}\left(\dfrac{3}{2}\right)\left[\left(2\times\dfrac{1}{2}+1\right)^{2/3} - (0+1)^{2/3}\right]$

$= \dfrac{3}{4}\left[(2)^{2/3} - 1\right]$

$= \dfrac{3}{4}\left[\sqrt[3]{4} - 1\right] = A$

④ $\int_0^3 (6x^2-4x+5)\,dx$

Sol:- $6\int_0^3 x^2\,dx - 4\int_0^3 x\,dx + 5\int_0^3 dx$

$= 6\left|\frac{x^3}{3}\right|_0^3 - 4\left|\frac{x^2}{2}\right|_0^3 + 5\left|x\right|_0^3$

$= 2\left[(3)^3-(0)^3\right] - 2\left[(3)^2-(0)^2\right] + 5\left[3-0\right]$

$= 2[27] - 2[9] + 15$

$\boxed{A = 51}$ Ans

⑤ $\int_{-2}^{1} (12x^5-36)\,dx$

Sol:- $12\int_{-2}^{1} x^5\,dx - 36\int_{-2}^{1} dx$

$= 12\left|\frac{x^6}{6}\right|_{-2}^{1} - 36\left|x\right|_{-2}^{1}$

$= 2\left[1-(-2)^6\right] - 36\left[1-(-2)\right]$

$\boxed{A = -234}$ Ans

Q6) $\int_{-\pi/3}^{\pi/4} \cos\theta\, d\theta$

$= \left|\sin\theta\right|_{-\pi/3}^{\pi/4}$

$= \sin\left(\frac{\pi}{4}\right) - \sin\left(-\frac{\pi}{3}\right)$

$= \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2}\right)$

$= \boxed{\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}}$ Answer.

(7) $\int_0^{\pi/4} \sec^2 2\theta \, d\theta$

Sol:- $\left| \frac{\tan 2\theta}{2} \right|_0^{\pi/4}$

$= \frac{1}{2}\left[\tan 2\left(\frac{\pi}{4}\right) - \tan 2(0)\right]$

$= \frac{1}{2}\left[\tan \frac{\pi}{2} - \tan(0)\right]$

$= \frac{1}{2}[\infty]$

$\boxed{A = \infty}$ Improper Integral

(8) $\int_2^4 \frac{x^2+8}{x^2} dx$

$= \int_2^4 \left(\frac{x^2}{x^2} + \frac{8}{x^2}\right) dx$

$= \int_2^4 1 dx + 8\int_2^4 x^{-2} dx$

$= \left| x \right|_2^4 + 8 \left| \frac{x^{-1}}{-1} \right|_2^4$

$= (4-2) - 8 \left| \frac{1}{x} \right|_2^4$

$= 2 - 8\left(\frac{1}{4} - \frac{1}{2}\right)$

$= 2+2 = \boxed{4}$ ~~Answer~~

9) $\int_{-1/2}^{3/2} (x - \cos\pi x)\,dx$

Sol: $\int_{-1/2}^{3/2} x\,dx - \int_{-1/2}^{3/2} \cos\pi x\,dx$

$= \left| \frac{x^2}{2} \right|_{-1/2}^{3/2} - \left| \frac{\sin\pi x}{\pi} \right|_{-1/2}^{3/2}$

$= \frac{1}{2}\left[\left(\frac{3}{2}\right)^2 - \left(\frac{-1}{2}\right)^2\right] - \frac{1}{\pi}\left[\sin\pi\left(\frac{3}{2}\right) - \sin\pi\left(\frac{-1}{2}\right)\right]$

$= \frac{1}{2}\left[\frac{9}{4} - \frac{1}{4}\right] - \frac{1}{\pi}\left[-1+1\right]$

$\Rightarrow \boxed{A = 1}$

10) $\int_1^4 \frac{\cos\sqrt{x}}{2\sqrt{x}}\,dx = \int \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}\,dx$

let $t = \sqrt{x}$, $\frac{dt}{dx} = \frac{1}{2} x^{-1/2}$

$dt = \frac{1}{2\sqrt{x}}\,dx$ if $x = 1 \Rightarrow \boxed{t = 1}$

if $x = 4 \Rightarrow \boxed{t = 2}$

$$= \int_1^2 \cos t\,dt$$

$$= \left| \sin t \right|_1^2$$

$\boxed{A = \sin 2 - \sin 1}$ Answer

(11) $$\int_{\pi/6}^{\pi/3} \underset{f(x)}{\sin x}\,\underset{f'(x)}{\cos x}\,dx$$

$$= \left| \frac{\sin^2 x}{2} \right|_{\pi/6}^{\pi/3}$$

$$= \frac{1}{2}\left[\left(\sin\frac{\pi}{3}\right)^2 - \left(\sin\frac{\pi}{6}\right)^2\right]$$

$$= \frac{1}{2}\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right]$$

$$= \frac{1}{2}\left[\frac{2}{4}\right]$$

$\boxed{A = \frac{1}{4}}$ Ans

(12) $$\int_{\pi/6}^{\pi/2} \frac{1+\cos\theta}{(\theta+\sin\theta)^2}\,d\theta$$

$$\int_{\pi/6}^{\pi/3} (\theta+\sin\theta)^{-2}(1+\cos\theta)\,d\theta$$

$$= -\left|(\theta + \sin\theta)^{-1}\right|_{\pi/6}^{\pi/2}$$

$$= -\left[\left(\frac{\pi}{2} + \sin\frac{\pi}{2}\right)^{-1} - \left(\frac{\pi}{6} + \sin\frac{\pi}{6}\right)^{-1}\right]$$

$$= \left(\frac{\pi}{6} + \frac{1}{2}\right)^{-1} - \left(\frac{\pi}{2} + 1\right)^{-1}$$

$$= \frac{6}{\pi+3} - \frac{2}{\pi+2}$$

$$= 0.9769 - 0.3889$$

$$= \boxed{A = 0.588}$$ Answer.

(13) $$\int_{-\pi/4}^{\pi/4} (\sec x + \tan x)^2 \, dx$$

Sol:- $$= \int_{-\pi/4}^{\pi/4} (\sec^2 x + \tan^2 x + 2\sec x \tan x) \, dx$$

$$= \int_{-\pi/4}^{\pi/4} (\sec^2 x + \sec^2 x - 1 + 2\sec x \tan x) \, dx$$ $\because 1 + \tan^2 x = \sec^2 x$

$$= 2\int_{-\pi/4}^{\pi/4} \sec^2 x \, dx - \int_{-\pi/4}^{\pi/4} 1 \cdot dx + 2\int_{-\pi/4}^{\pi/4} \sec x \tan x \, dx$$

$$= 2\left|\tan x\right|_{-\pi/4}^{\pi/4} - \left|x\right|_{-\pi/4}^{\pi/4} + 2\left|\sec x\right|_{-\pi/4}^{\pi/4}$$

$$= 2\left[\tan\frac{\pi}{4} - \tan\left(-\frac{\pi}{4}\right)\right] - \left[\frac{\pi}{4} + \frac{\pi}{4}\right] + 2\left[\frac{1}{\cos\frac{\pi}{4}} - \frac{1}{\cos\left(-\frac{\pi}{4}\right)}\right]$$

$$= 2[1+1] - \left(\frac{2\pi}{4}\right) + 2\left[\frac{1}{\cos\frac{\pi}{4}} - \frac{1}{\cos\left(\frac{\pi}{4}\right)}\right] \Rightarrow 4 - \frac{\pi}{2}$$ Answer

(14) $\int_{-\pi/2}^{\pi/2} \cos^2 x \, dx$

Sol $\int_{-\pi/2}^{\pi/2} \frac{1+\cos 2x}{2} dx$

$= \frac{1}{2}\left[\int_{-\pi/2}^{\pi/2} 1 dx + \int_{-\pi/2}^{\pi/2} \cos 2x dx\right]$

$= \frac{1}{2}\left[|x|_{-\pi/2}^{\pi/2} + \left|\frac{\sin 2x}{2}\right|_{-\pi/2}^{\pi/2}\right]$

$= \frac{1}{2}\left[\left(\frac{\pi}{2}+\frac{\pi}{2}\right) + \frac{1}{2}\left(\sin 2\left(\frac{\pi}{2}\right) - \sin 2\left(\frac{-\pi}{2}\right)\right)\right]$

$\Rightarrow \frac{1}{2}\left\{\frac{2\pi}{2} + \frac{1}{2}(0+0)\right\}$

$\boxed{A = \frac{\pi}{2}}$

(15) $\int_1^3 \ln x \, dx$

Sol $= \ln x \int dx - \int\left(\int 1 \cdot dx \cdot \frac{d}{dx}(\ln x)\right) dx$

$= x\ln x - \int\left(x \cdot \frac{1}{x}\right) dx$

$\Rightarrow x\ln x - \int dx$

$\boxed{x\ln x - x}$

$= |x\ln x - x|_1^3$

$= (3\ln 3 - 3) - (1\ln 1 - 1) \Rightarrow 3\ln 3 - 3 + 1$

$\boxed{A = 3\ln 3 - 2}$ Ans

(16) $\int_2^4 \left(e^{x/2} - e^{x/4}\right)dx$

Sol:- $\int_2^4 e^{x/2}\,dx - \int_2^4 e^{x/4}\,dx$ OR $\Rightarrow$ $2\int_2^4 e^{x/2}\cdot\frac{1}{2}\,dx - 4\int e^{x/4}\cdot\frac{1}{4}\,dx$

$= \left|\frac{e^{x/2}}{1/2}\right|_2^4 - \left|\frac{e^{x/4}}{1/4}\right|_2^4$

$= 2\left|e^{4/2} - e^{2/2}\right| - 4\left|e^{4/4} - e^{2/4}\right|$

$= 2\left|e^2 - e\right| - 4\left|e - e^{1/2}\right|$

$= 2e^2 - 2e - 4e + 4e^{1/2}$

$\boxed{2e^2 - 6e + 4e^{1/2}}$ Answer.

(17) $\int_0^{\pi/4} \frac{1}{1-\sin x}\,dx$

$= \int_0^{\pi/4} \left(\frac{1}{1-\sin x} \times \frac{1+\sin x}{1+\sin x}\right)dx$

$= \int_0^{\pi/4} \frac{1+\sin x}{1-\sin^2 x}\,dx = \int_0^{\pi/4} \frac{1+\sin x}{\cos^2 x}\,dx$

$= \int_0^{\pi/4} \frac{1}{\cos^2 x} + \int_0^{\pi/4} \frac{\sin x}{\cos x \cos x}\,dx$ or $\int_0^{\pi/4} \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}$

$= \int_0^{\pi/4} \sec^2 x\,dx + \int_0^{\pi/4} \tan x \sec x\,dx$

$$= \left| \tan x \right|_0^{\pi/4} + \left| \sec x \right|_0^{\pi/4}$$

$$= \tan\frac{\pi}{4} - \tan(0) + \frac{1}{\cos(\pi/4)} - \frac{1}{\cos(0)}$$

$$\boxed{A = \sqrt{2}}$$ Answer

(18) $$\int_0^{\pi/4} \tan^{-1} y \, dy$$

$$= \tan^{-1} y \int 1 dy - \int \left(\int dy \cdot \frac{d}{dx}(\tan^{-1} y) \right) dy$$

$$= y \tan^{-1} y - \int \left(y \cdot \frac{1}{1+y^2} \right) dy$$

$$= y \tan^{-1} y - \frac{1}{2} \int \frac{2 \cdot y}{(1+y^2)} dx$$

$$= \left| y \tan^{-1} y - \frac{1}{2} \ln|1+y^2| \right|_0^{\pi/4}$$

$$= \left(\frac{\pi}{4} \tan^{-1} \frac{\pi}{4} - \frac{1}{2} \ln\left|1+\left(\frac{\pi}{4}\right)^2\right| \right) - \left(0 \tan^{-1}(0) - \frac{1}{2} \ln(1+0) \right)$$

$$= \frac{\pi}{4}\left(\frac{1}{1}\right) - \frac{1}{2} \ln\left|1+\frac{\pi^2}{16}\right|$$

$$= \frac{\pi}{4} - \frac{1}{2} \ln\left|\frac{16+\pi^2}{16}\right|$$ Answer

(19) $\int_0^{\pi/2} \frac{\sin x}{(2+\cos x)(5+\cos x)} dx$

let $\cos x = t$

$-\sin x = \frac{dt}{dx}$

$\boxed{-dt = \sin x\, dx}$

if $x=0$ then $t=1$

if $x=\frac{\pi}{2}$ then $t=0$

$\int_1^0 \frac{-dt}{(2+t)(5+t)} = -\int_1^0 \frac{dt}{(2+t)(5+t)}$

$= \int_0^1 \frac{dt}{(2+t)(5+t)}$ — (*)

Consider $\frac{1}{(2+t)(5+t)} = \frac{A}{2+t} + \frac{B}{5+t}$ — (1)

$1 = A(5+t) + B(2+t)$ — (2)

put $2+t=0 \Rightarrow \boxed{t=-2}$ in (2)

$1 = A(5-2) + B(2-2)$

$\boxed{A = \frac{1}{3}}$

put $5+t=0 \Rightarrow \boxed{t=-5}$ in (2).

$1 = A(5-5) + B(2-5)$

$1 = B(-3)$

$\boxed{B = -\frac{1}{3}}$

$\frac{1}{(2+t)(5+t)} = \frac{1}{3(2+t)} - \frac{1}{3(5+t)}$ put in (*).

$= \int_0^1 \left(\frac{1}{3(2+t)} - \frac{1}{3(5+t)} \right) dt$

$= \frac{1}{3} \left| \ln(2+t) \right|_0^1 - \frac{1}{3} \left| \ln(5+t) \right|_0^1$

$= \frac{1}{3}\left[\ln 3 - \ln 2\right] - \frac{1}{3}\left[\ln 6 - \ln 5\right]$

$= \frac{1}{3}\left[\ln 3 - \ln 2 - \ln 6 + \ln 5\right]$

$= \frac{1}{3}\left[\ln 3 - \ln 2 - \ln(3 \times 2) + \ln 5\right]$

$= \frac{1}{3}\left[\ln 3 - \ln 2 - \ln 3 - \ln 2 + \ln 5\right]$

(20) $\int_2^5 \frac{1}{x(x+1)} dx \Rightarrow \frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$ —(1)

$1 = A(x+1) + B(x)$ —(2) put $\boxed{x=0}$ in eq (2)

$1 = A(0+1) + B(0) \Rightarrow A = 1$ put $x+1 = 0$

$\boxed{x = -1}$ put in (2)

$1 = A(-1+1) + B(-1) \Rightarrow \boxed{B = -1}$ put values A & B in eq (1)

$\int_2^5 \left(\frac{1}{x} - \frac{1}{x+1} \right) dx$

$= \int_2^5 \frac{1}{x} dx - \int_2^5 \frac{1}{x+1} dx$

$= \left|\ln x\right|_2^5 - \ln|x+1|\Big|_2^5$

$= (\ln 5 - \ln 2) - (\ln 6 - \ln 3)$

$= \ln 5 - \ln 2 - \ln(3\times 2) + \ln 3$

$= \ln 5 - \ln 2 - (\ln 3 + \ln 2) + \ln 3$

$= \ln 5 - \ln 2 - (\ln 3 + \ln 2) + \ln 3$

$= \ln 5 - \ln 2 - \ln 3 - \ln 2 + \ln 3$

$= \ln 5 - 2\ln 2$

$= \ln 5 - \ln 2^2$

$= \ln 5 - \ln 4$

$\boxed{= \ln \frac{5}{4}}$ Ans

Ex 2.7

Q16 Sol: $\int_2^4 e^{x/2}\,dx - \int_2^4 e^{x/4}\,dx$

$= 2\int e^{x/2}\cdot\frac{1}{2}\,dx - 4\int_2^4 e^{x/4}\cdot\frac{1}{4}\,dx$

$= 2e^{x/2} - 4e^{x/4}\Big|_2^4$

$= \left(2e^{4/2} - 4e^{4/4}\right) - \left(2e^{2/2} - 4e^{2/4}\right)$

$= (2e^2 - 4e) - (2e^1 - 4e^{1/2})$

$= 2e^2 - 6e + 4e^{1/2}$ Answer