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Where Minds Pollinate

Class 12
Mathematics

Chapter 3

Exercise 3.6

Handwritten Solution

Exercise 3.6 V.V.IMP

- **Indefinate Integral, or anti derivative**

Result → function. $\boxed{f(x) = x^2 + x}$

How?

let $\int (2x+1)\,dx = ?$

$\frac{2 \cdot x^2}{2} + x$

$\boxed{= x^2 + x}$ → $f(x)$.

- **Definite Integral :-**

Result → Number (defined)

$$A = \int_{x=a}^{x=b} f(x)\,dx$$

Properties:

(i) $\int_a^a f(x)\,dx = 0$ e.g $\int_a^a \sin x\,dx = 0$

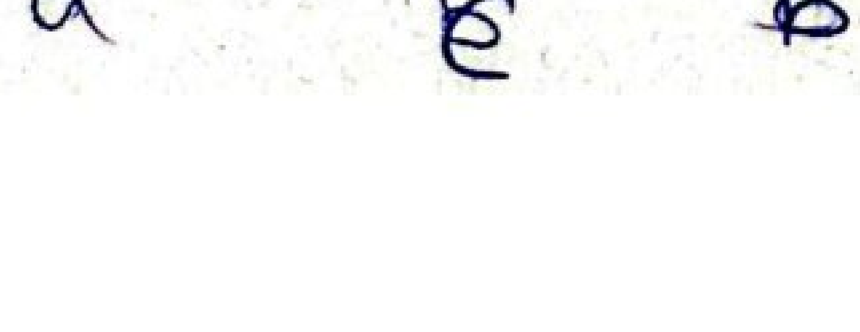
$= -\cos x \Big|_a^a$ $-(\cos a + \cos a)$ $= \boxed{0}$

(ii) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ ∴ limit change

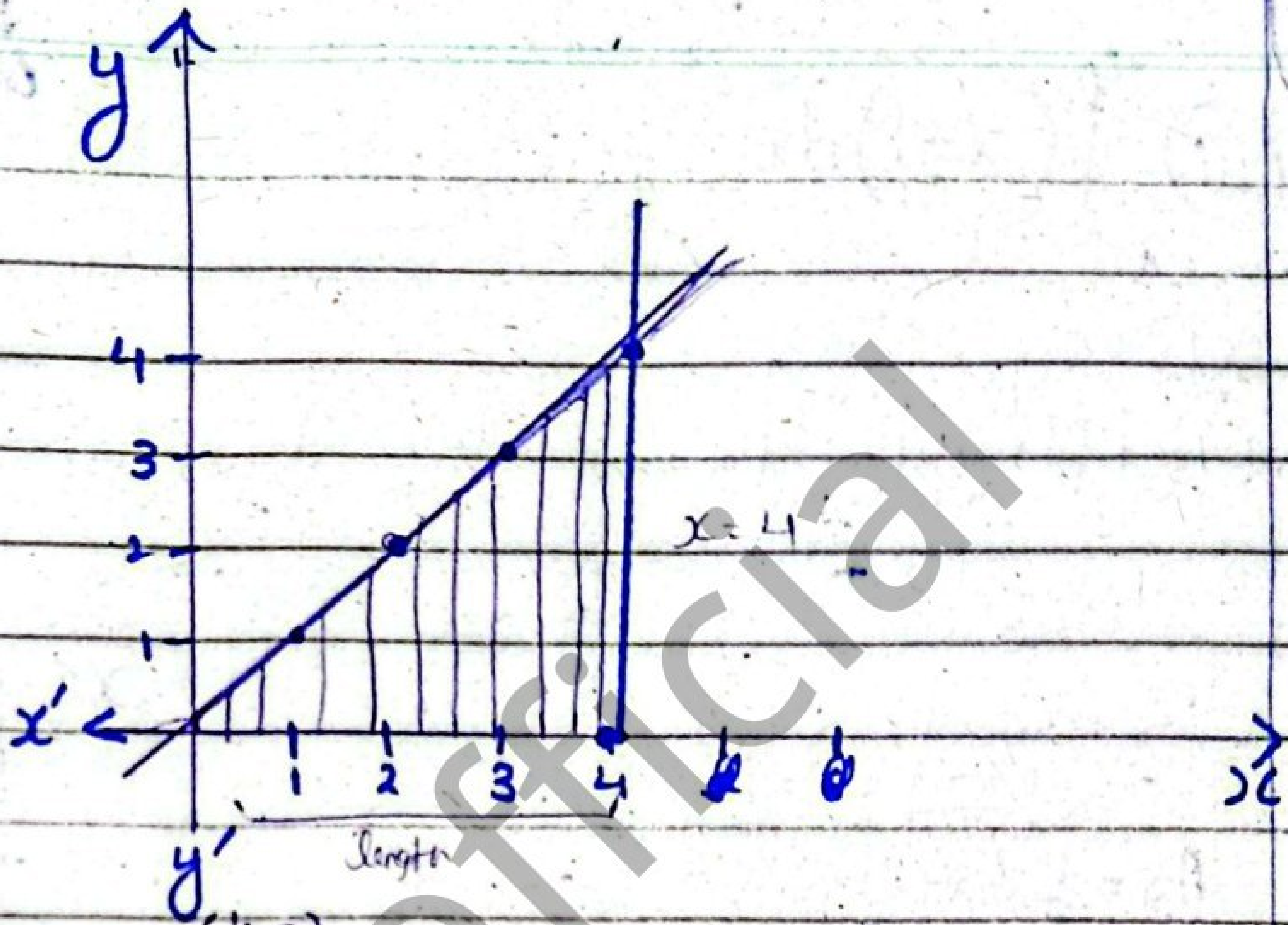
(iii) If c is the point between a and b then

$$\int_a^b f(x)\,dx = +\int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

E.g $\int_2^5 x\,dx = \int_2^3 x\,dx + \int_3^5 x\,dx$



Q1 (i) $\int_0^4 x\,dx$ $x=0$, $x=4$



Sol: $f(x)=x$

$\boxed{y=f(x)}$

x	0	1	2	3
y	0	1	2	3

Area of triangle $= \frac{1}{2}$ length $\times$ width or heigth.

$A = \frac{1}{2}(4)(4)$

$A = 8\,\text{unit}^2$ Answer

Trick: Area under curve from a to b

$\int_0^4 x\,dx = \left|\frac{x^2}{2}\right|_0^4$

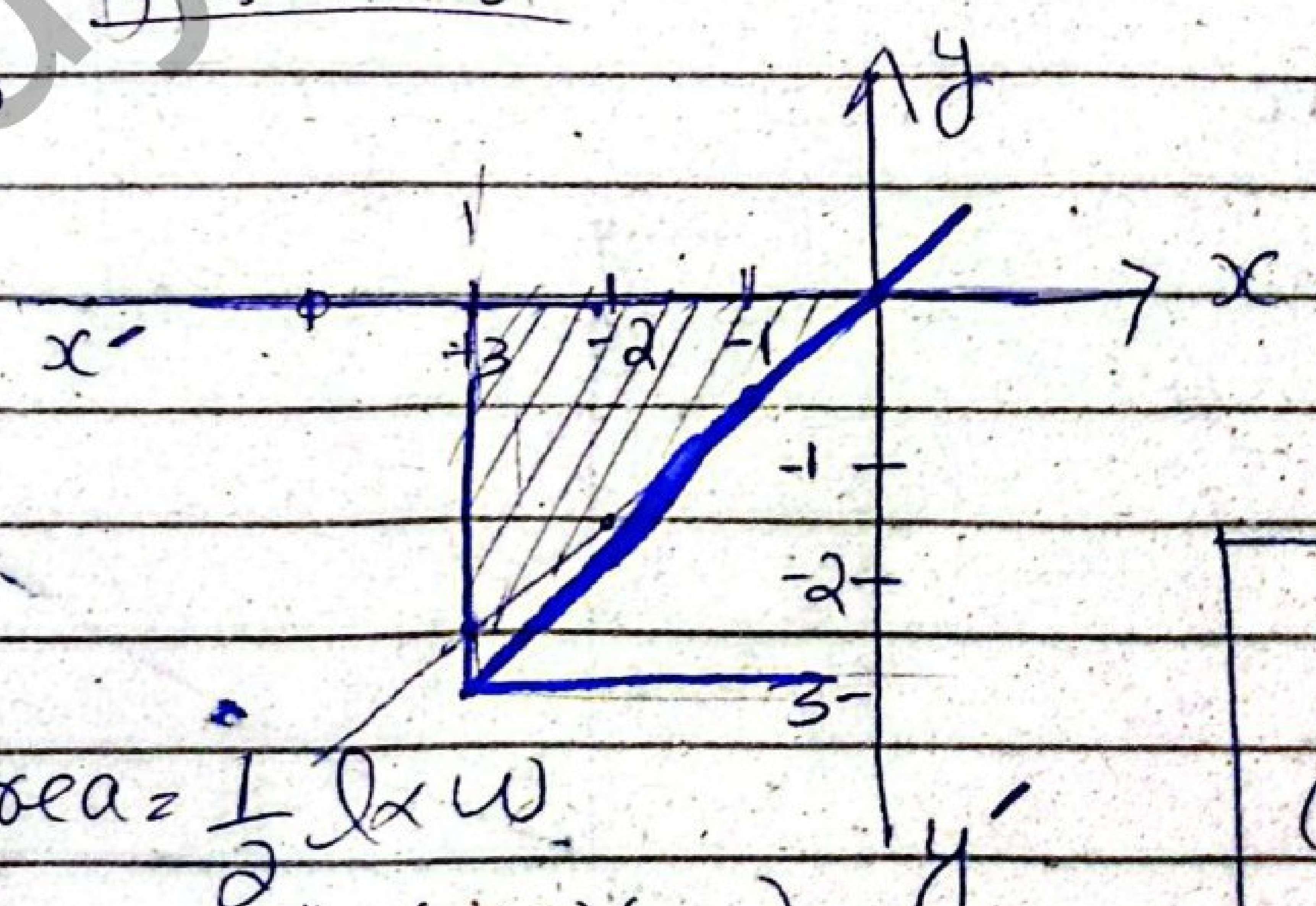
$\frac{(4)^2}{2} - \frac{0^2}{2} = \boxed{8}$

(ii) $\int_{-3}^{0} x\,dx$

x	-3	-2	-1	0
y	-3	-1	-1	0

Sol:- $x=-3$ $x=0$

$\therefore y = f(x) = x$



$\left|\frac{x^2}{2}\right|_{-3}^{0}$

$\frac{(0)^2}{2} - \frac{(+3)^2}{2}$

$= 4.5$

$\therefore$ length is always +ve

Area $= \frac{1}{2} l \times w$

$= \frac{1}{2}(+3)(+3)$

$= \frac{9}{2} = -4.5$ -ve sign shows shaded region is below x-axis

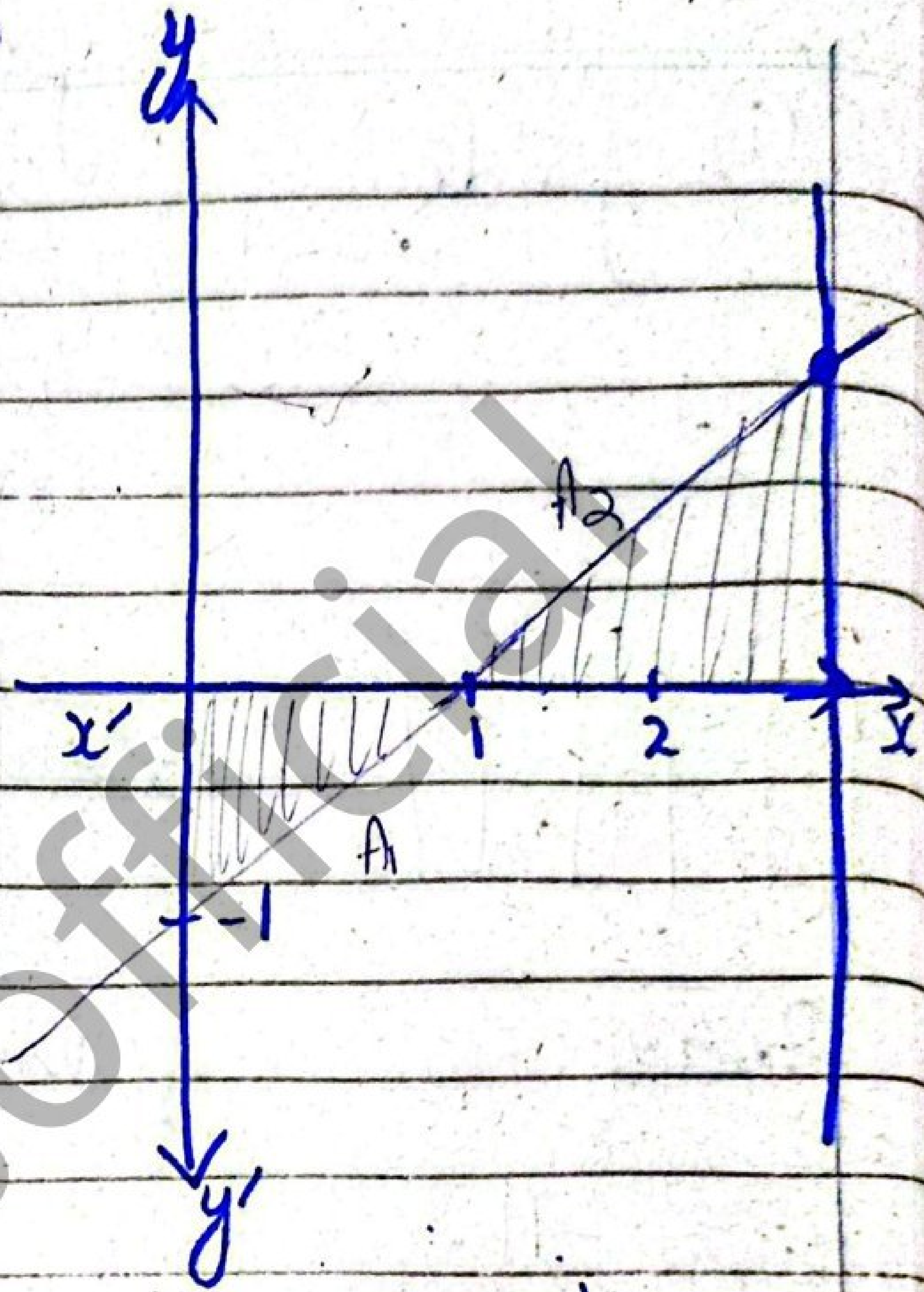
(iii) $\int_0^2 (x-1)\,dx$

$x-1=0$

$\boxed{x=1}$

$x=0 \quad x=2$

x	0	1	2
y	-1	0	1



$A_1 = \frac{1}{2}(1)\times(-1)$

$\boxed{A_1 = -\frac{1}{2}}$

$A_2 = \frac{1}{2}\, l \times w$

$A_2 = \frac{1}{2}(1)(1)$

$\boxed{A_2 = \frac{1}{2}}$

Total Area $= A_1 + A_2$

$= -\frac{1}{2} + \frac{1}{2}$

$\boxed{A = 0}$

Short

$\left|\frac{x^2}{2} - x\right|_0^2$

$\left[\frac{(2)^2}{2} - 2\right] - \left[\frac{0^2}{2} - 0\right]$

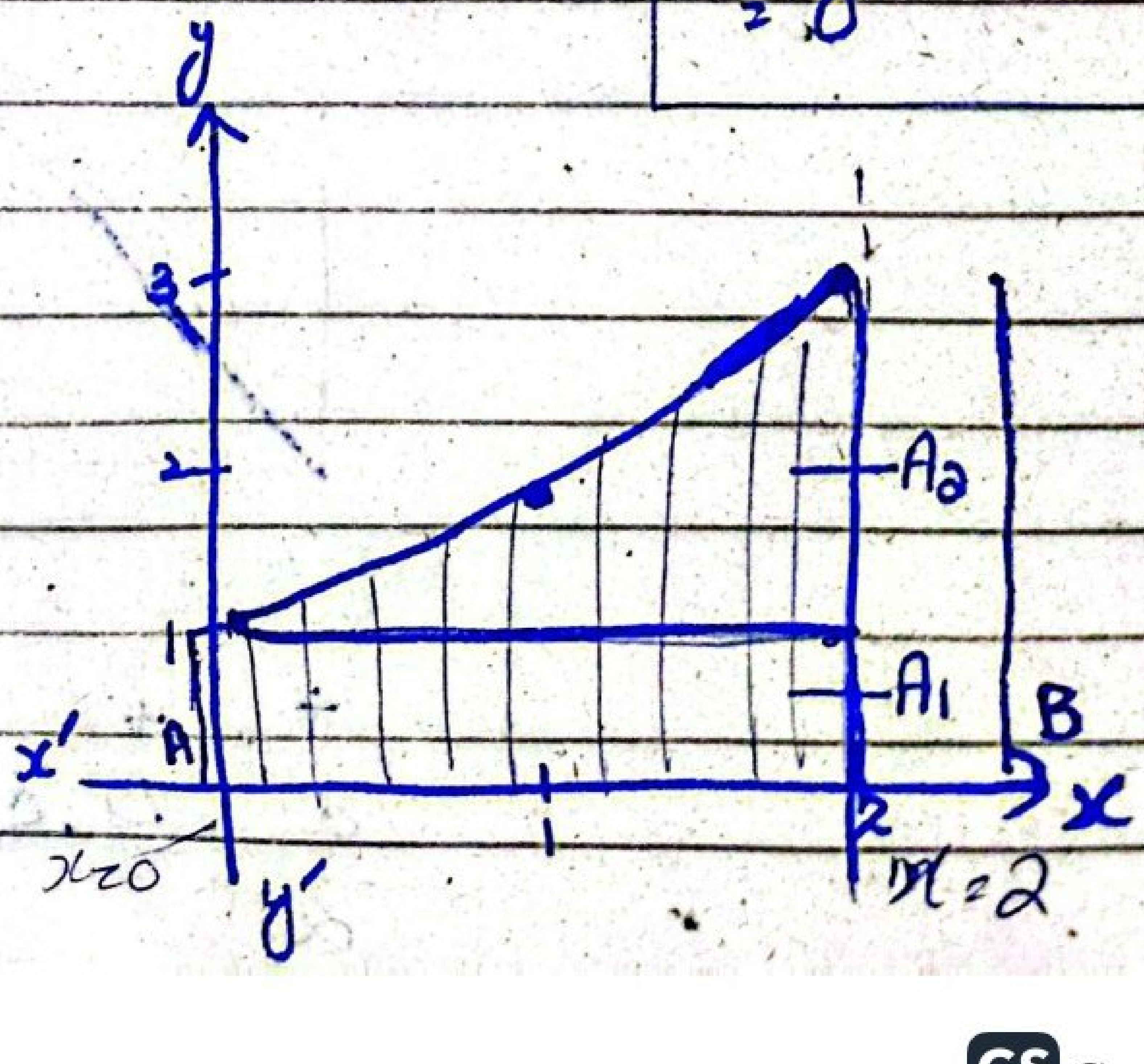
$= 0$

(IV) $\int_0^2 (x+1)\,dx$

so $y = x+1$

$\boxed{x=0}$ $\boxed{x=2}$

x	0	1	2
y	1	2	3



Rectangle

$A_1 = l \times w$

$A_1 = 2 \times 1$

$\boxed{A_1 = 2 unit^2}$

triangle

$A_2 = \frac{1}{2} l \times w$

$= \frac{1}{2}(2)(2)$

$\boxed{A_2 = 2 unit^2}$

Total Area

$A = A_1 + A_2$

$A = 2 + 2$

$\boxed{A = 4 unit^2}$ Answer

Another way $\boxed{\text{Area of Trapezium.}}$

$A = \frac{1}{2}\begin{pmatrix}\text{Sum of} \\ \text{parallel} \\ \text{sides}\end{pmatrix}(\text{height})$

$A = \frac{1}{2}(A+B)h$

$A = \frac{1}{2}(1+3)(2)$

$\boxed{A = 4 unit^2}$

∴ Verification MCQs

$$\int_0^2 (x+1)\,dx$$

$$= \left.\frac{x^2}{2} + x\right|_0^2$$

$$= \frac{(2)^2}{2} + 2 - (0+0)$$

$$= \frac{4}{2} + 2$$

$$= \boxed{4} u$$

(v) $\int_{-3}^{3} 2\,dx$

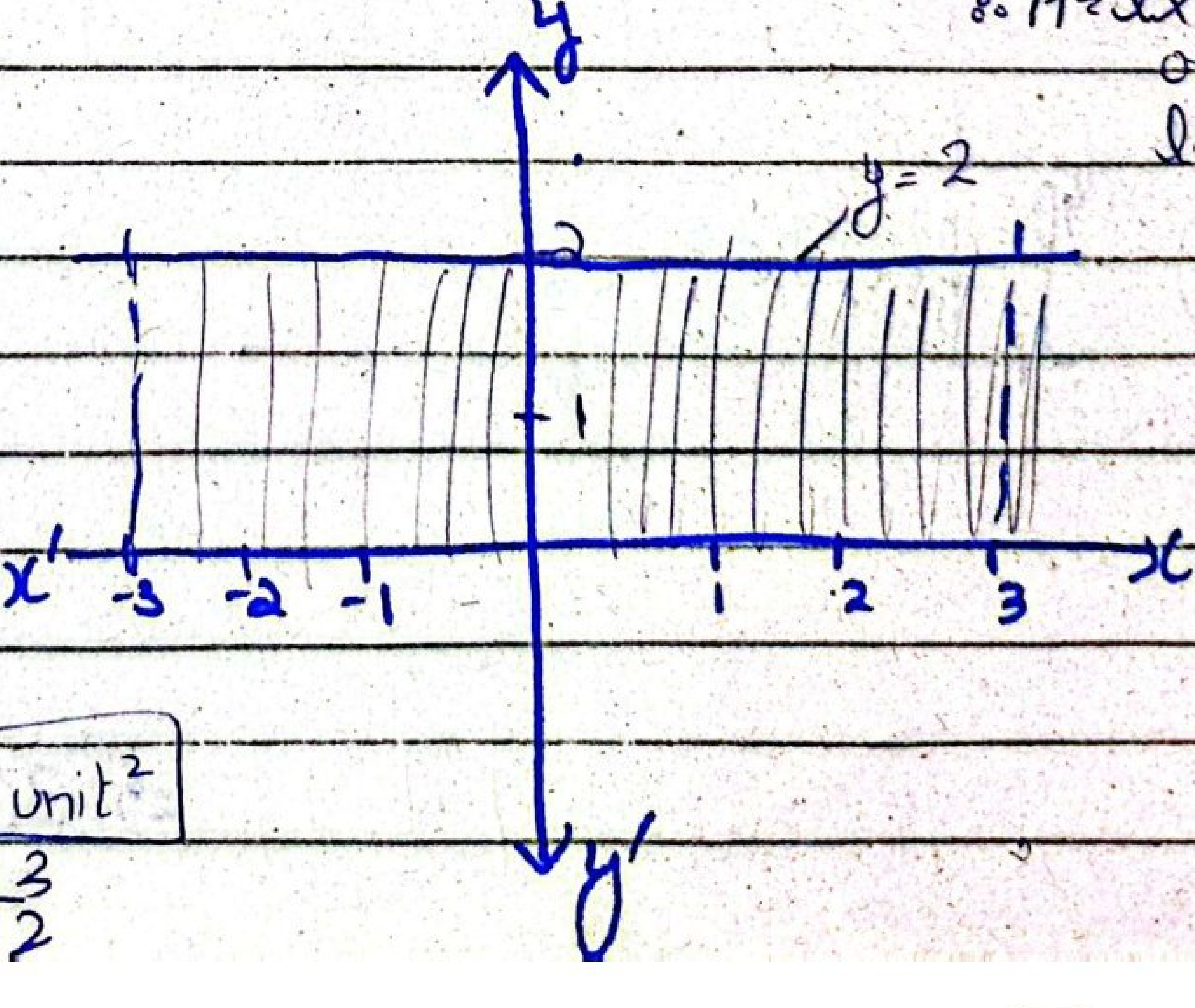
$y = 2$

$x = -3$, $x = 3$

$A = l \times B$

$= 6 \times 2 = \boxed{12 unit^2}$

So $A = l \times B$ or $l \times H$ same

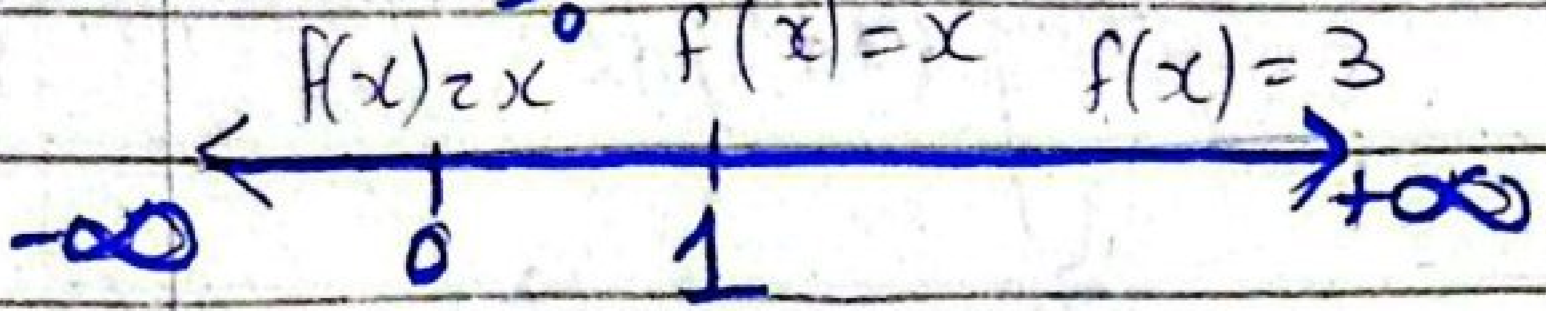


x	-3	-2	-1	0	1	2	3
y	2	2	2	2	2	2	2

CamScanner

$$\begin{Bmatrix} x & ; x \le 1 \\ 3 & : x > 1 \end{Bmatrix}$$

Q2 (i) $\int_0^1 f(x)\,dx$



Sol $\int_0^1 f(x)\,dx = \int_0^1 x\,dx = \left|\frac{x^2}{2}\right|_0^1$

$= \underbrace{\left[\frac{(1)^2}{2} - \frac{(0)^2}{2}\right]}_{\text{upperlimit - lowerlimit}}$

$\int_0^1 f(x)\,dx = \frac{1}{2}$ Answer

(ii) $\int_{-1}^{1} f(x)\,dx$

Sol:- $\int_{-1}^{1} x\,dx$

$= \left|\frac{x^2}{2}\right|_{-1}^{1} \Rightarrow \frac{(1)^2}{2} - \frac{(-1)^2}{2} \Rightarrow \frac{1}{2} - \frac{1}{2} = 0$ Answer.

(iii) $\int_1^4 f(x)\,dx$

$= \int_1^4 3\,dx$

$= 3\int_1^4 dx$

$= 3|x|_1^4 \Rightarrow 3[4-1] \Rightarrow 3(3) = \boxed{9}$ Answer.

(iv) $\int_{-1}^{2} f(x)\,dx$

$= \int_{-1}^{2} f(x)\,dx = \int_{-1}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$

$= 0 + \int_{1}^{2} 3\,dx$

$= 3|x|_{1}^{2}$

$= 3(2-1)$

$= 3(1) = \boxed{3}$ Ans

Q3. (i) $\int_{a}^{b} f(x)\,dx$

Sol 2 Answer

(ii) $\int_{b}^{c} f(x)\,dx$

$=$ 3 Answer

(iii) $\int_{c}^{d} f(x)\,dx$

$=$ 3.5 Answer.

(iv) $\int_{a}^{c} f(x)\,dx$.

$\int_{a}^{b} f(x)\,dx + \int_{b}^{c} f(x)\,dx$

$= 2+3$

$=$ 5 Answer

(v) $\int_{b}^{d} f(x)\,dx$

Sol:- $\int_{b}^{c} f(x)\,dx + \int_{c}^{d} f(x)\,dx$

$= 3+3.5 = \boxed{6.5}$ Ans

(vi) $\int_{a}^{d} f(x)\,dx$

Sol:- $\int_{a}^{b} f(x)\,dx + \int_{b}^{c} f(x)\,dx + \int_{c}^{d} f(x)\,dx$

$= 2+3+3.5$

$= \boxed{8.5}$ Ans

Q4) find: $\int_1^5 [3f(x) - 2g(x)]dx$ if $\int_1^5 f(x)\,dx = 4$

and $\int_1^5 g(x)dx = 5$

Sol: $3\int_1^5 f(x)dx - 2\int_1^5 g(x)dx$

$= 3(4) - 2(5)$

$= 12 - 10$

$= \boxed{2}$ Answer

Q5. $\int_1^4 f(x)dx$ if $\int_1^2 f(x)dx = 1$ & $\int_2^4 f(x)dx = 2$

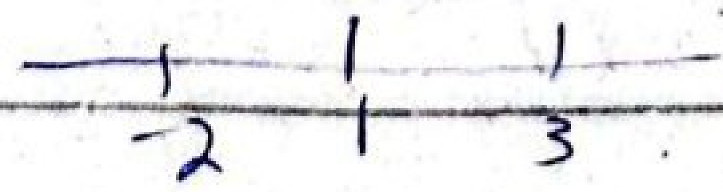
Sol:- $\int_1^4 f(x)dx = \int_1^2 f(x)dx + \int_2^4 f(x)dx$

$\int_1^4 f(x)dx = 1 + 2$

$= 3$ Ans

Q6) find: $\int_3^{-2} f(x)dx$ if $\int_{-2}^1 f(x)dx = 1$ & $\int_1^3 f(x)dx = -5$

Sol



$= -\int_{-2}^3 f(x)$

$= -\left[\int_{-2}^1 f(x)dx + \int_1^3 f(x)dx\right]$

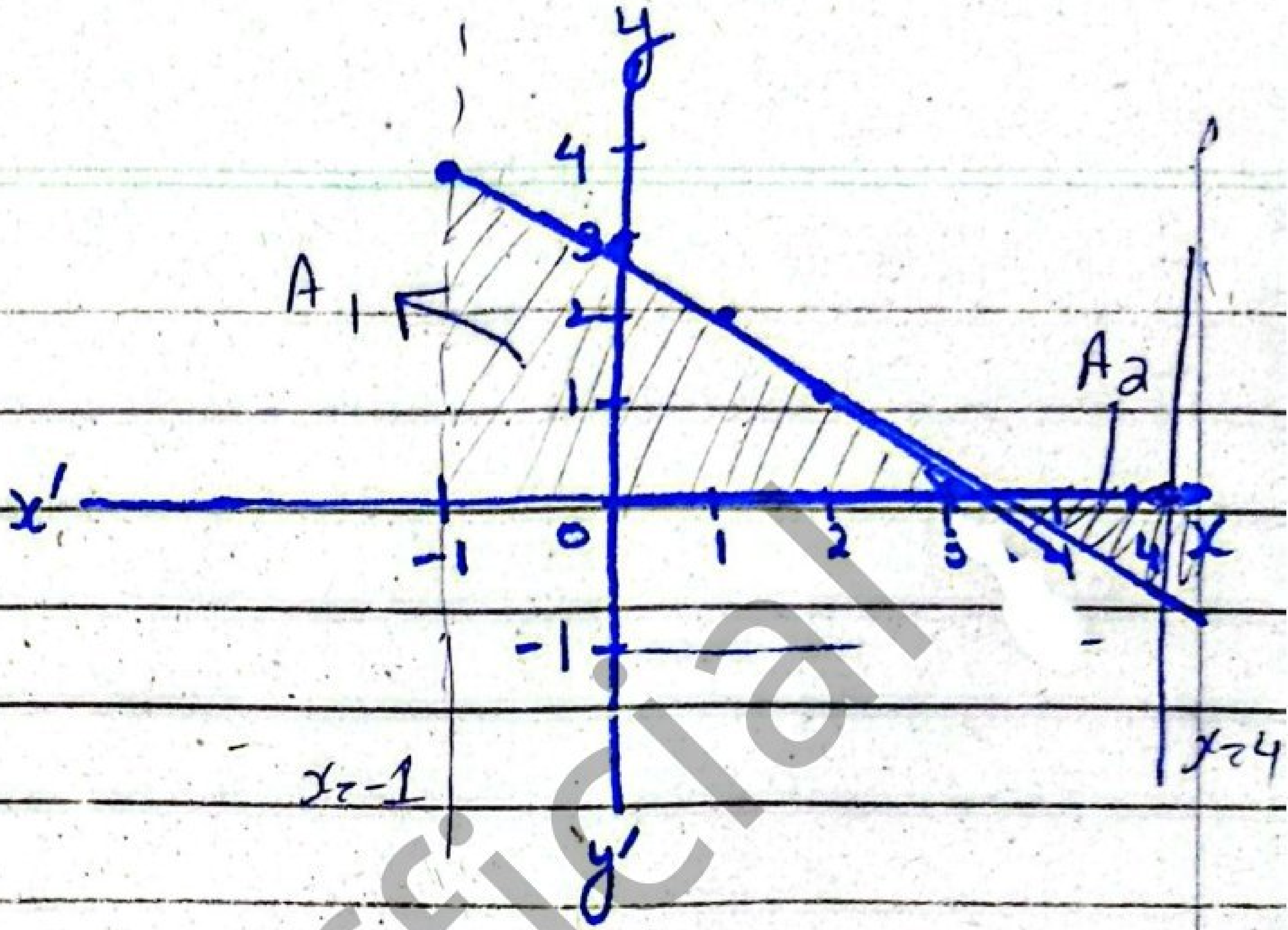
$= -[1 + (-5)]$

$= \boxed{4}$ Ans

$\therefore$ big limit should be above.

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

Q7 (i) $\int_{-1}^{4}(3-x)\,dx$



Sol:- $x=-1 \quad x=4$

$f(x)=y=3-x$

x	-1	0	1	2	3	4
y	4	3	2	1	0	-1

$A_1=\frac{1}{2}l\times w$

$=\frac{1}{2}(4)(4)=8.$

$A_2=\frac{1}{2}l\times w=\frac{1}{2}(1)(1)=-\frac{1}{2}$ (below-ve axis

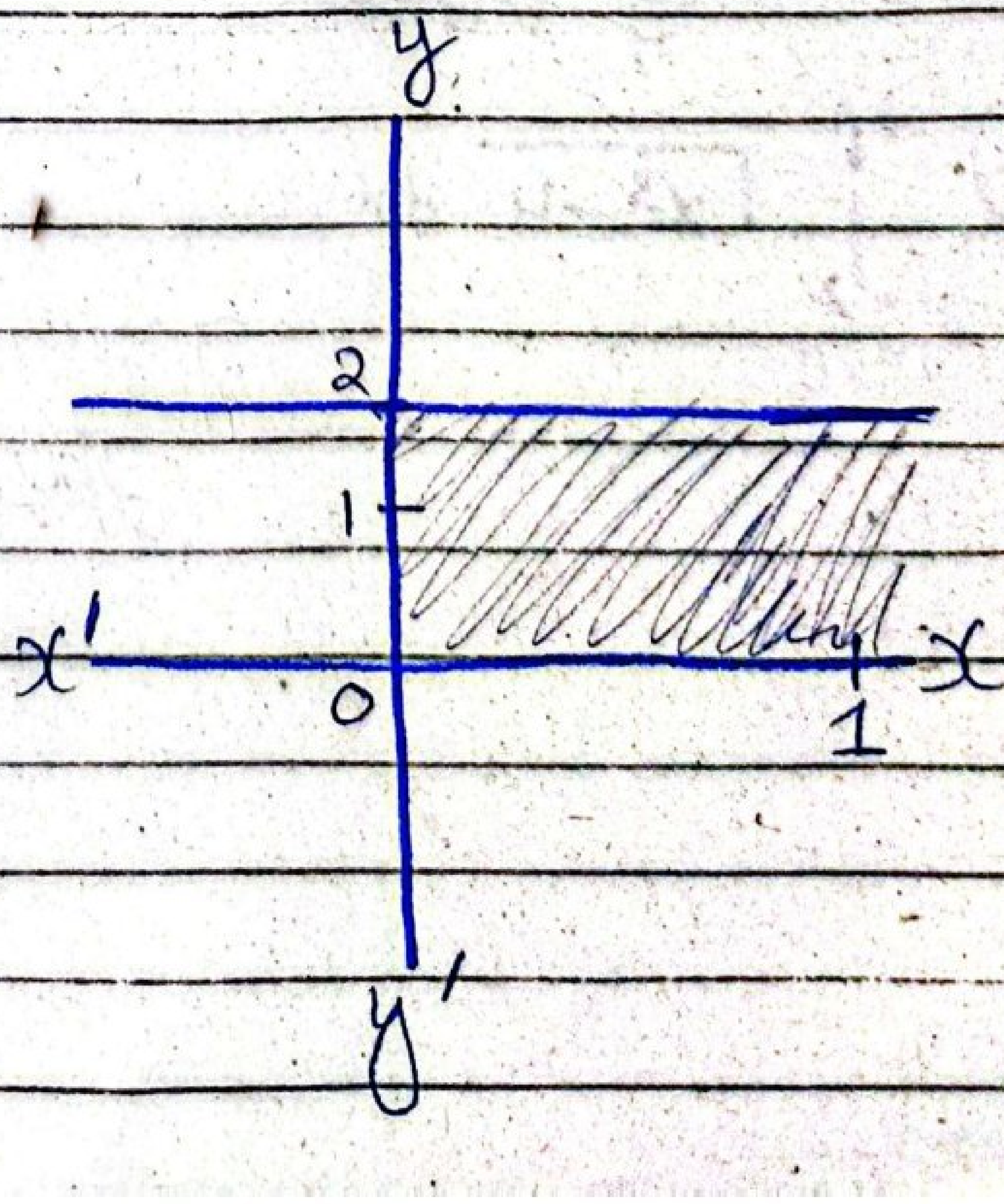
Total area

$=A_1+A_2$

$=8-\frac{1}{2}$

$=\boxed{\frac{15}{2}}$ Answer

(ii) $\int_0^1\left[2+\sqrt{1-x^2}\right]dx$



Sol $\underbrace{\int_0^1 2\,dx}_{A_1}+\underbrace{\int_0^1\sqrt{1-x^2}\,dx}_{A_2}$

$A_1=\int_0^1 2\,dx$

$y=2=f(x)$

$A_1=l\times w$

$=1\times 2$

$\boxed{A_1=2}$

+92 3193683107

$f(x) = \sqrt{1-x^2}$

$(y)^2 = (\sqrt{1-x^2})^2$

$y^2 = 1-x^2$

$\boxed{x^2+y^2=1}$ eq of circle with radius 1

(0,0) origion

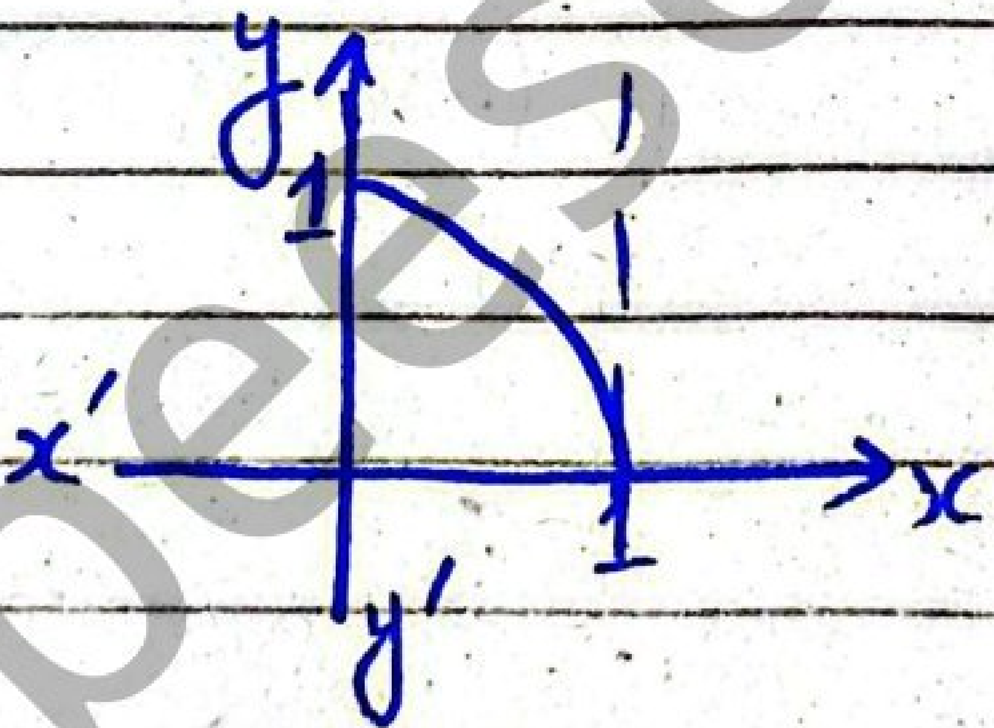
radius r

$x^2+y^2 = r^2$

Area of circle

$= \pi r^2$

$\boxed{A_2 = \frac{\pi}{4}}$



$\int_{-1}^{4} 3dx - \int_{-1}^{4} x dx$

$= 3|x|_{-1}^{4} - \left|\frac{x^2}{2}\right|_{-1}^{4}$

$= 3(4+1) - \left(\frac{16}{2} - \frac{1}{2}\right)$

$= 15 - \frac{15}{2}$ $\boxed{\frac{15}{2}}$

Total area = $A_1 + A_2$

$A = 2 + \frac{\pi}{4}$ Answer

(iii) $\int_{2}^{2} \sqrt{x^3-4}\, dx$

So $\boxed{\int_{2}^{2} \sqrt{x^3-4}\, dx = 0}$ Answer

$\therefore \int_{a}^{a} f(x)\, dx = 0$