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Where Minds Pollinate

Class 12
Mathematics

Chapter 3

Exercise 3.5

Handwritten Solution

Exercise 3.5

Concept: Proper Partial fraction

In rational fraction $\frac{P(x)}{Q(x)}$ if degree of $Q(x)$ greater than $P(x)$. Other wise Improper Partial fraction. If equal then also improper.

∴ Polynomial: Power of x should be +ve & not be in points

Case 1: when $Q(x)$ has only non-repeated linear factors

Case 2: when $Q(x)$ has repeated linear factors

Case 3: when $Q(x)$ contains non repeated Irreducable quadratic factors

Case 4: when $Q(x)$ contains repeated irreducable quadratic factors

Q1 $\int \frac{3x+7}{(x+2)(x+3)} dx$

Sol:- Proper partial fraction

Case 1

$$\frac{3x+7}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$\frac{3x+7}{\cancel{(x+2)(x+3)}} = \frac{A(x+3) + B(x+2)}{\cancel{(x+2)(x+3)}}$$

$$3x+7 = A(x+3) + B(x+2) \quad —(2)$$

put $x = -2$ in (2)

$$3(-2)+7 = A(-2+3) + B(-2+2)$$

$\boxed{A=1}$

put $x=-3$ put in 2

$3(-3)+7 = A(-3+3)+B(-3+2)$

$-9+7 = A(0)+B(-1)$

$+2 = B(+1)$

$\boxed{B=2}$

put A & B in eq 1

$$\boxed{\frac{3x+7}{(x+2)(x+3)} = \frac{1}{x+2}+\frac{2}{x+3}}$$

$$= \int\left(\frac{1}{x+2}+\frac{2}{x+3}\right)dx$$

$$= \int\frac{1}{x+2}dx+2\int\frac{1}{x+3}dx$$

$$= \ln|x+2|+2\ln|x+3|+C \text{ Ans}$$

Q2) $\int\frac{4x+9}{x^2+x-12}dx$

Proper fraction

for x^2+x-12

$x^2+4x-3x-12$

$x(x+4)-3(x+4)$

$(x-3)(x+4)$

$$\int\frac{4x+9}{(x-3)(x+4)}dx$$

$$\frac{4x+9}{(x-3)(x+4)} = \frac{A}{x-3} + \frac{B}{x+4} \quad (1)$$

$$\frac{4x+9}{(x-3)(x+4)} = \frac{A(x+4)+B(x-3)}{(x-3)(x+4)}$$

$4x+9 = A(x+4)+B(x-3)$ — (2)

put $x=3$ in (2)

$4(3)+9 = A(3+4)+B(3-3)$

$\boxed{A=3}$

put $x=-4$ put in (2)

$4(-4)+9 = A(-4+4)+B(-4-3)$

$\boxed{B=+1}$

put values of A & B in (1)

$$\boxed{\frac{4x+9}{(x-3)(x+4)} = \frac{3}{x-3} + \frac{1}{x+4}}$$

$$\int\left(\frac{3}{x-3} + \frac{1}{x+4}\right)dx$$

$$3\int\frac{1}{x-3}dx + \int\frac{1}{x+4}dx$$

$3\ln|x-3| + \ln|x+4| + C$

Q3) $\int \frac{21-8x}{x^2+x-6}dx$

x^2+x-6

$x^2+3x-2x-6$

$x(x+3)-2(x+3)$

$(x-2)(x+3)$

$$\int \frac{21-8x}{(x-2)(x+3)}\,dx$$

$$\frac{21-8x}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} \quad \text{--- (1)}$$

$$21-8x = A(x+3) + B(x-2) \quad \text{--- (2)}$$

put $\boxed{x=2}$ in eq (2)

$$21-8(2) = A(2+3) + B(2-2)$$

$$\boxed{A=1}$$

put $x=-3$ in (2)

$$21-8(-3) = A(-3+3) + B(-3-2)$$

$$\boxed{B=-9}$$

put A & B in eq (1)

$$\boxed{\frac{21-8x}{(x-2)(x+3)} = \frac{1}{x-2} - \frac{9}{x+3}}$$

$$= \int \left(\frac{1}{x-2} - \frac{9}{x+3}\right) dx$$

$$= \int \frac{1}{x-2}\,dx - 9\int \frac{1}{x+3}\,dx$$

$$= \ln|x-2| - 9\ln|x+3| + C$$

Q4) $\int \frac{3x-7}{(x+2)^2}\,dx$

for Proper fraction: Case 2.

for $\frac{3x-7}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$ --- (1)

$3x+7 = A(x+2)+B \longrightarrow (2)$

put $x = -2$ in eq (2)

$3(-2)+7 = A(-2+2)+B$

$\boxed{B=1}$

Comparing coefficient of x

$\boxed{A=3}$

putting values of A & B in (1)

$$\boxed{\frac{3x+7}{(x+2)^2} = \frac{3}{(x+2)} + \frac{1}{(x+2)^2}}$$

$$= \int \left(\frac{3}{x+2} + \frac{1}{(x+2)^2} \right) dx$$

$$= 3\int \frac{1}{x+2}\,dx + \int \frac{1}{(x+2)^2}\,dx$$

$$= 3\ln|x+2| + \int (x+2)^{-2}\,dx$$

$$= 3\ln|x+2| - \frac{1}{(x+2)} + C$$

Answer

Q5) $$\int \frac{5x^2-5x+2}{(x+1)(x-1)^2}\,dx$$

Sol $$\frac{5x^2-5x+2}{(x+1)(x-1)^2} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \quad (1)$$

$$5x^2-5x+2 = A(x-1)^2 + B(x-1)(x+1) + C(x+1) \quad (2)$$

put $x = -1$.

$$5(-1)^2-5(-1)+2 = A(-1-1)^2 + B(-1-1)(-1+1) + C(-1+1)$$

$\boxed{A=3}$

put $x = 1$ in eq (2)

$5-5+2 = A(1-1) + B(1+1) + C(1+1)$

$\boxed{C=1}$

from eq (2).

comparing coefficients of x^2

$5 = A+B$

$5-3 = B \Rightarrow \boxed{B=2}$

put values of A, B, & C in eq (1)

$$\boxed{\frac{5x^2-5x+2}{(x+1)(x-1)^2} = \frac{3}{x+1} + \frac{2}{x-1} + \frac{1}{(x-1)^2}}$$

$$= \int \frac{3}{x+1} + \frac{2}{x-1} + \frac{1}{(x-1)^2} dx$$

$$= 3\int \frac{1}{x+1} dx + 2\int \frac{1}{x-1} dx + \int (x-1)^{-2} dx$$

$$= 3\ln|x+1| + 2\ln|x-1| + \frac{(x-1)^{-1}}{-1} + C$$

$$= \boxed{3\ln|x+1| + 2\ln|x-1| - \frac{1}{x-1} + C}$$ Answer.

Q6) $\int \frac{9x^2+3x+29 dx}{(x+1)(x^2+4)} dx$

for Proper fraction; Case 3

$$\frac{9x^2+3x+29}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4}$$ quadratic factor — (1)

$$9x^2+3x+29 = A(x^2+4) + (Bx+C)(x+1) \quad (2)$$

put $x = -1$ in eq (2).

$9(-1)^2+3(-1)+29 = A((-1)^2+4)+B(-1)+C(-1+1)$

$\boxed{A=7}$

comparing coefficient of x^2

$9 = A+B$

$9 = 7+B \Rightarrow \boxed{B=2}$.

comparing coefficient of x

$3 = B+C$

$3 = 2+C \Rightarrow \boxed{C=1}$

put A, B, and C in eq (1).

$$\boxed{\frac{9x^2+3x+29}{(x+1)(x^2+4)} = \frac{7}{x+1} + \frac{2x+1}{x^2+4}}$$

$$= \int \left(\frac{7}{x+1} + \frac{2x-1}{x^2+4}\right) dx$$

$$= 7\int \frac{1}{x+1}\,dx + \int \frac{2x+1}{x^2+4}\,dx$$

$$= 7\ln|x+1| + \int \frac{2x}{x^2+4}\,dx + \int \frac{1\,dx}{x^2+2^2}$$

$$= 7\ln|x+1| + \ln|x^2+4| + \frac{1}{2}\tan^{-1}\frac{x}{2} + c \quad \text{Answer.}$$

Q8 $\int \frac{7x^2+7x+4}{(2x+1)(x^2+x+1)}\,dx$

for Proper fraction: Case 3.

$$\frac{7x^2+7x+4}{(2x+1)(x^2+x+1)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+x+1} \quad (1)$$

$$= \cancel{(2x+1)(x^2+x+1)} \quad 7x^2+7x+4 = A(x^2+x+1)+(Bx+C)(2x+1) \quad (2)$$

put $2x+1=0$ $\boxed{x = -\frac{1}{2}}$ put in (2)

$$7\left(-\frac{1}{2}\right)^2 + 7\left(-\frac{1}{2}\right)+4 = A\left(\left(-\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)+1\right) + \left(B\left(-\frac{1}{2}\right)+C\right)\left(2\left(-\frac{1}{2}\right)+1\right)$$

$$\frac{7}{4} - \frac{7}{2} + 4 = A\left(\frac{1}{4}-\frac{1}{2}+1\right)$$

$$\frac{7-14+16}{4} = A\left(\frac{1}{4}+\frac{1}{2}\right)$$

$$\frac{9}{4} = A\left(\frac{3}{4}\right)$$

$$\boxed{A=3}$$

from eq (2) $7x^2+7x+4 = Ax^2+Ax+A+2Bx^2+2Bx+2Cx+C$

comparing coefficient of x^2

$$7 = A+2B$$

$$7 = 3+2B$$

$$\boxed{B=2}$$

comparing coefficient of x

$$7 = A+B+2C$$

$$7 = 3+2+2C$$

$$\boxed{C=1}$$

put A, B & C in eq (1)

$$\int\left(\frac{3}{2x+1}+\frac{2x+1}{x^2+x+1}\right)dx$$

$$= \frac{3}{2}\int\frac{1.2}{2x+1}dx + \int\frac{2x+1}{x^2+x+1}dx$$

$$= \frac{3}{2}\ln|2x+1| + \ln|x^2+x+1| + C$$ Answer.

Q8) $\int \frac{x^3+4x^2+9x+14}{x^2+4x+3}\,dx$

for Improper fraction:

$$\begin{array}{r} x \\ x^2+4x+3 \,\overline{\big)\, x^3+4x^2+9x+14} \\ \pm x^3 \pm 4x^2 \pm 3x \\ \hline 6x+14 \end{array}$$

$\int \left(x + \frac{6x+14}{x^2+4x+3} \right) dx$

$= \int x\,dx + \int \frac{6x+14}{x^2+4x+3}\,dx$] proper frac

x^2+4x+3

$x(x+3)+1(x+3)$

$(x+1)(x+3)$

$= \int x\,dx + \int \frac{6x+14}{(x+1)(x+3)}\,dx$

$= \frac{x^2}{2} + \int \frac{6x+14}{(x+1)(x+3)}\,dx$

$\frac{6x+14}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$ — (1)

$6x+14 = A(x+3) + B(x+1)$ — (2)

put $\boxed{x=-1}$ put in (2).

$6(-1)+14 = A(-1+3) + B(-1+1)$

$\boxed{A=4}$

put $x+3=0 \Rightarrow \boxed{x=-3}$

$6(-3)+14 = A(-3+3) + B(-3+1)$

$\boxed{B=2}$

put values of A & B in eq (1)

$\frac{6x+14}{(x+1)(x+3)} = \frac{4}{x+1} + \frac{2}{x+3}$

$= \frac{x^2}{2} + \int \left(\frac{4}{x+1} + \frac{2}{x+3} \right) dx$

$= \frac{x^2}{2} + 4\int \frac{1}{x+1}\,dx + 2\int \frac{1}{x+3}\,dx$

$= \frac{x^2}{2} + 4\ln|x+1| + 2\ln|x+3| + c$

Answer

9) $\int \frac{1}{x^2-9}\,dx$

fait Proper fraction

$= \int \frac{1}{x^2-3^2}\,dx$

$= \int \frac{1}{(x-3)(x+3)}\,dx$

$= \frac{1}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$ — (1)

$1 = A(x+3) + B(x-3)$ — (2)

put $\boxed{x=3}$ put (2)

$1 = A(3+3) + B(3-3)$

$\boxed{A = \frac{1}{6}}$

put $\boxed{x=-3}$ in (2)

$1 = A(-3+3) + B(-3-3)$

$1 = B(-6)$

$\boxed{B = \frac{-1}{6}}$

put values of A & B in eq (1)

$\frac{1}{(x-3)(x+3)} = \frac{1}{6(x-3)} - \frac{1}{6(x+3)}$

$= \int \left[\frac{1}{6(x-3)} - \frac{1}{6(x+3)}\right] dx$

$= \frac{1}{6}\left[\int \frac{1}{x-3}\,dx - \int \frac{1}{x+3}\,dx\right]$

$\frac{1}{6}\left[\ln|x-3| - \ln|x+3|\right] + C$

$= \frac{1}{6}\left[\ln\frac{|x-3|}{|x+3|}\right] + C$

Q10) $\int \frac{1}{x^3+2x^2+x}\, dx$

Sol, $\int \frac{1}{x(x^2+2x+1)}\, dx$

x^2+2x+1

$x^2+1x+1x+1$

$x(x+1)+1(x+1)$

$(x+1)(x+1)$

$\Rightarrow \int \frac{1}{x(x+1)^2}\, dx$

$\frac{1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ — (1)

$1 = A(x+1)^2 + B(x)(x+1) + C(x)$ — (2)

put $x=0$ in eq (2)

$1 = A(0+1)^2 + B(0)(0+1) + C(0)$

$\boxed{A=1}$

put $\boxed{x=-1}$ put in eq (2)

$1 = A(-1+1)^2 + B(-1)(-1+1) + C(-1)$

$\boxed{C=-1}$

from eq (2)

$1 = A(x^2+2x+1) + B(x^2+x) + C(x)$

comparing coefficients of x^2

$0 = A+B$

$0 = 1+B$

$\boxed{B=-1}$

put A, B, C in eq (1) $\Rightarrow$ $\boxed{\frac{1}{x(x+1)^2} = \frac{1}{x} - \frac{1}{x+1} - \frac{1}{(x+1)^2}}$

$\Rightarrow \int \frac{1}{x}\, dx - \int \frac{1}{x-1}\, dx - \int \frac{1}{(x+1)^2}\, dx$

$= \ln|x| - \ln|x-1| - \int (x+1)^{-2}\, dx$

$= \ln|x| - \ln|x+1| - \frac{(x+1)^{-1}}{-1} + C$

$= \ln|x| - \ln(x+1) + \frac{1}{x+1} + C$

☆ ☆ IMPORTANT

Q11) $\int \frac{e^x}{(e^x+1)^2(e^x-2)} dx$

Sol let $e^x = y$

$\boxed{dy = e^x dx}$

$= \int \frac{dy}{(y+1)^2(y-2)}$

$\Rightarrow \frac{1}{(y-2)(y+1)^2} = \frac{A}{y-2} + \frac{B}{y+1} + \frac{C}{(y+1)^2}$ —(1)

$\cancel{(y-2)(y+1)^2} =$

$1 = A(y+1)^2 + B(y+1)(y-2) + C(y-2)$ —(2)

put $y-2=0$ $\boxed{y=2}$ in eq (2).

$1 = A(2+1)^2 + B(2+1)(2-2) + C(2-2)$

$\boxed{A = \frac{1}{9}}$

put $y=-1$ in eq (2).

$1 = A(-1+1) + B(-1+1)(-1-2) + C(-1-2)$

$\boxed{C = -\frac{1}{3}}$.

from eq (2)

$1 = A(y^2+2y+1) + B(y^2-2y+y-2) + C(y-2)$

comparing coefficient of y^2

$0 = A+B$

$0 = \frac{1}{9} + B$ $\boxed{B = -\frac{1}{9}}$

Put values of A, B & C in eq (1).

$$= \int \frac{1}{9(y-2)}\,dy - \int \frac{1}{9(y+1)}\,dy - \int \frac{1}{3(y+1)^2}\,dy.$$

$$= \frac{1}{9}\int \frac{1}{y-2}\,dy - \frac{1}{9}\int \frac{1}{y+1}\,dy - \frac{1}{3}\int \frac{1}{(y+1)^2}\,dy$$

$$= \frac{1}{9}\ln|y-2| - \frac{1}{9}\ln|y+1| - \frac{1}{3}\int (y+1)^{-2}\,dy$$

$$= +\frac{1}{9}\ln|y-2| - \frac{1}{9}\ln|y+1| - \frac{1}{3}\cdot\frac{(y+1)^{-1}}{-1} + C$$

$$= \frac{1}{9}\left[\ln|y-2| - \ln|y+1|\right] + \frac{1}{3(y+1)} + C$$

$$= \frac{1}{9}\ln\left|\frac{y-2}{y+1}\right| + \frac{1}{3(y+1)} + C$$

$$= \frac{1}{9}\ln\left|\frac{e^x-2}{e^x+1}\right| + \frac{1}{3(e^x+1)} + C$$

Q12) $\int \frac{x}{(x+1)^2(x^2+1)}\,dx$ IMP

Sol $\frac{x}{(x+1)^2(x^2+1)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{Cx+D}{(x^2+1)}$ —(1)

$x = A(x+1)(x^2+1) + B(x^2+1) + Cx+D\,(x+1)^2$ —(2)

put $x = -1$ in (2)

$-1 = A(-1+1)((-1)^2+1) + B((-1)^2+1) + (C(-1)+D)(-1+1)^2$

$$\boxed{B = -\frac{1}{2}}$$

from eq (2)

$x = A(x^3+x+x^2+1) + B(x^2+1) + (Cx+D)(x^2+2x+1)$

$x = A(x^3+x^2+x+1) + B(x^2+1) + Cx^3+2Cx^2+Cx+Dx^2+2Dx+D$

Comparing coefficients

x^3; $0 = A + C \Rightarrow \boxed{A = -C}$

x^2; $0 = A + B + 2C + D$

$0 = A - \frac{1}{2} + 2C + D$

$\frac{1}{2} = -C + 2C + D$

$\boxed{C + D = \frac{1}{2}}$

x; $1 = (A + C) + 2D$

$1 = 0 + 2D$

$\boxed{D = \frac{1}{2}}$

$\frac{1}{2} = C + \frac{1}{2}$

$\cancel{A = 0}$

$A = -0$

$\boxed{A = 0}$

$\boxed{C = 0}$

put A, B, C, D in (1).

$$\frac{x}{(x+1)^2(x^2+1)} = \frac{0}{x+1} - \frac{1}{2(x+1)^2} + \frac{0x + \frac{1}{2}}{x^2+1}$$

$$\boxed{\frac{x}{(x+1)^2(x^2+1)} = \frac{-1}{2(x+1)^2} + \frac{1}{2(x^2+1)}}$$

$$= \frac{-1}{2}\int \frac{1}{(x+1)^2}dx + \frac{1}{2}\int \frac{1}{x^2+(1)^2}dx$$

$$= -\frac{1}{2}\int (x+1)^{-2}\,dx + \frac{1}{2}\tan^{-1}x$$

$$= -\frac{1}{2}\frac{(x+1)^{-1}}{-1} + \frac{1}{2}\tan^{-1}x + C$$

$$= \frac{1}{2}\left[\frac{1}{(x+1)} + \tan^{-1}x\right] + C$$

Answer