Studybeesofficial

Where Minds Pollinate

Class 12
Mathematics

Chapter 3

Exercise 3.4

Handwritten Solution

Exercise 3.4

Calculus corner

Concept: $\int uv\,dx = u\int v\,dx - \int\left(\int v\,dx \cdot \frac{d}{dx}(u)\right)dx$

ILATE

I → Inverse function] no direct integration.
L → log function.]
A → Algebric function. e.g $x^2, x^3, (x+2)^5$ $x^0 = 1$ also
T → Trignometric function.] choice can take u.
E → Exponential function.]

Q1) $\int \ln x\,dx$

Sol = $\int 1 \cdot \ln x\,dx$ Integrating by parts

let $u = \ln x$; $v = 1$

$\int \ln x\,dx = \ln x \int 1\,dx - \int\left(\int 1\,dx \cdot \frac{d}{dx}(\ln x)\right)dx$

$\int \ln x\,dx = \ln x \cdot x - \int\left(x \cdot \frac{1}{x}\right)dx$

$\int \ln x\,dx = x\ln x - \int 1\,dx$

$\boxed{\int \ln x\,dx = x\ln x - x + C}$ Answer.

(Q2) $\int (\ln x)^2 dx$

Sol: $\int 1 \cdot (\ln x)^2 dx$

$u = (\ln x)^2 ; \; v = \underline{1}$

$$\int (\ln x)^2 dx = (\ln x)^2 \int 1 \cdot dx - \int \left(\int 1 \cdot dx \cdot \frac{d}{dx} (\ln x)^2 \right) dx$$

$$= (\ln x)^2 x - \int \left(\cancel{x} \cdot 2(\ln x) \left(\frac{1}{\cancel{x}} \right) \right) dx$$

$$= x(\ln x)^2 - 2 \int \ln x \, dx$$

$$= x(\ln x)^2 - 2 \left[\ln x \cdot \int 1 \cdot dx - \int \left(\int 1 \cdot dx \, \frac{d}{dx} (\ln x) \right) dx \right]$$

$$= x(\ln x)^2 - 2 \left[\ln x \cdot (x) - \int \left(\cancel{x} \cdot \frac{1}{\cancel{x}} \right) dx \right]$$

$$= x(\ln x)^2 - 2 \left[x \ln x - \int 1 \cdot dx \right]$$

Answer.

$$= \boxed{x(\ln x)^2 - 2x \ln x + 2x + C}$$

(3) $\int 1 \cdot \sin(\ln x) dx = I$

Sol: $U = \sin(\ln x), \; V = \underline{1}$.

$$\int \sin(\ln x) dx = \sin(\ln x) \int 1 \cdot dx - \int \left[\int 1 \cdot dx \cdot \frac{d}{dx} (\sin(\ln x)) \right] dx$$

$$= x \cdot \sin(\ln x) - \int \left(\cancel{x} \cdot \cos(\ln x) \cdot \frac{1}{\cancel{x}} \right) dx$$

$$= x \cdot \sin(\ln x) - \int 1 \cdot \cos(\ln x) dx$$

$u = \cos(\ln x)$; $v = 1$

$= x\sin(\ln x) - \left[\cos(\ln x)\cdot\int 1\cdot dx - \int\left[\int 1\cdot dx \frac{d}{dx}(\cos(\ln x))\right]dx\right]$

$= x\sin(\ln x) - \left[x\cos(\ln x) - \int\left[-x\cdot\sin(\ln x)\cdot\frac{1}{\cancel{x}}\right]dx\right]$

$I = x\sin(\ln x) - x\cos(\ln x) - \int\sin(\ln x)dx$

$I = x\sin(\ln x) - x\cos(\ln x) - I$

$I + I = x\sin(\ln x) - x\cos(\ln x) + C$

$2I = x\sin(\ln x) - x\cos(\ln x) + C$

$I = \frac{1}{2}x\sin(\ln x) - \frac{1}{2}x\cos(\ln x) + C$

$\int\sin(\ln x)dx = \frac{1}{2}x\sin(\ln x) - \frac{1}{2}x\cos(\ln x) + C$ Ans

Q4) $\int x^3 \ln x \, dx$

Sol:- $u = \ln x$, $v = x^3$

$= \ln x\int x^3 dx - \int\left(\int x^3 dx\cdot\frac{d}{dx}(\ln x)\right)dx$

$= \ln x\cdot\frac{x^4}{4} - \int\left(\frac{x^4}{4}\cdot\frac{1}{x}\right)dx$

$= \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3 dx$

$= \frac{x^4}{4}\ln x - \frac{1}{4}\frac{x^4}{4} + C$

$= \int x^3 \ln x\, dx = \frac{x^4}{4}\ln x - \frac{x^4}{16} + C$

Answer.

Q5) $\int y \sin 2y \, dy$

Sol: $u = y$, $v = \sin 2y$

$= y\int \sin 2y\,dy - \int\left(\int \sin 2y\,dy \cdot \frac{d}{dy}(y)\right)dy$

$= y\frac{1}{2}\int \sin 2y \cdot 2dy - \int\left(\frac{1}{2}\sin 2y \cdot 2dy\right)(1)dy$

$= \frac{y}{2}(-\cos 2y) - \frac{1}{2}\int -\cos 2y \cdot dy$

$= -\frac{y}{2}\cos 2y + \frac{1}{2}\int \cos 2y\,dy$

$= -\frac{y}{2}\cos 2y + \frac{1}{2}\left[\frac{\sin 2y}{2}\right] + C$

$= \boxed{\int y \sin 2y\,dy = -\frac{y}{2}\cos 2y + \frac{1}{4}\sin 2y + C}$ Answer.

Q6) $\int e^x \cos x\,dx = I$

Soln $u = \cos x$, $v = e^x$

$I = \cos x\int e^x dx - \int\left(\int e^x dx \cdot \frac{d}{dx}(\cos x)\right)dx$

$I = \cos x \cdot e^x - \int e^x(-\sin x)dx$

$I = e^x \cos x + \int e^x \sin x\,dx$

$I = e^x \cos x + \sin x\int e^x dx - \int\left(\int e^x dx \cdot \frac{d}{dx}(\sin x)\right]dx$

$I = e^x \cos x + \sin x \cdot e^x \int e^x \cdot \cos x dx$

$I = e^x \cos x + e^x \sin x - I$

$I + I = e^x [\cos x + \sin x]$

$2I = e^x [\cos x + \sin x]$

$I = \frac{1}{2} e^x [\cos x + \sin x] + C$ Ans

Q7) $\int x \sec^{-1} x dx$

$u = \sec^{-1} x \; ; \; v = x$

$= \sec^{-1} x \int x dx - \int \left(\int x dx \cdot \frac{d}{dx}(\sec^{-1} x) \right) dx$

$= \frac{x^2}{2} \sec^{-1} x - \int \left(\frac{x^2}{2} \cdot \frac{1}{x\sqrt{x^2-1}} \right) dx$

$= \frac{x^2}{2} \sec^{-1} x - \frac{1}{2 \cdot 2} \int (x^2-1)^{-1/2} \cdot 2x dx$

$= \frac{x^2}{2} \sec^{-1} x - \frac{1}{4} \left[\frac{(x^2-1)^{1/2}}{1/2} \right] + C$

$= \frac{x^2}{2} \sec^{-1} x - \frac{1}{4} \left(\frac{2}{1} \right) \sqrt{x^2-1} + C$

$= \boxed{\frac{x^2}{2} \sec^{-1} x - \frac{1}{2} \sqrt{x^2-1} + C}$ Ans

Q8) $\int \ln(2x+3) dx$

Sol: $= \int 1 \cdot \ln(2x+3) dx$

$u = \ln(2x+3), \; v = 1$

$= \ln(2x+3) \int 1 \cdot dx - \int \left(\int 1 \cdot dx \cdot \frac{d}{dx}(\ln(2x+3)) \right) dx$

$= x\ln(2x+3) - \int\left(x \cdot \frac{1}{2x+3} \cdot 2\right) dx$

$= x\ln(2x+3) - \int \frac{(2x+3)-3}{2x+3} dx$

$= x\ln(2x+3) - \int\left(\frac{2x+3}{2x+3} - \frac{3}{2x+3}\right) dx$

$= x\ln(2x+3) - \int 1 \cdot dx + \frac{3}{2}\int (1)\frac{2}{2x+3}\, dx$

$= x\ln(2x+3) - x + \frac{3}{2}\ln(2x+3) + C$ Ans

Second way

$$\begin{array}{r} 1 \\ 2x-3 \overline{) \; 2x \quad} \\ \underline{\pm 2x \pm 3} \\ -3 \end{array}$$

$\left(1 - \frac{3}{2x+3}\right)$

Q9) $\int x^2 e^x dx$

$u = x^2, \; v = e^x$

$= x^2 \int e^x dx - \int\left(\int e^x dx \cdot \frac{d}{dx}(x^2)\right) dx$

$= x^2 e^x - \int e^x \cdot (2x) dx$

$= x^2 e^x - 2\left[\int x e^x dx\right]$

$= x^2 e^x - 2\left[x\int e^x dx - \int\left[\int e^x dx \cdot \frac{d}{dx}(x)\right]\right] dx$

$= x^2 e^x - 2\left[x e^x - \int e^x (1) dx\right]$

$= x^2 e^x - 2\left[x e^x - e^x\right] + C$

$= x^2 e^x - 2x e^x + 2e^x + C$ Answer

Q10) $\int x \cos x \, dx$

$u = x, \; v = \cos x$

$= x\int \cos x \, dx - \int\left(\int \cos x \, dx \cdot \frac{d}{dx}(x)\right) dx$

$= x(\sin x) - \int \sin x \cdot dx$

$= x \sin x - (-\cos x) + C$

$= \boxed{x \sin x + \cos x + C}$ Ans

Q11) $\int 1 \cos^{-1} x \, dx$

$u = \cos^{-1} x, \; v = 1$

$= \cos^{-1} x \int 1 \cdot dx - \int \left(\int 1 \cdot dx \cdot \frac{d}{dx} (\cos^{-1} x) \right) dx$

$= x \cos^{-1} x - \int x \cdot \left(\frac{-1}{\sqrt{1-x^2}} \right) dx$

$= x \cos^{-1} x + \frac{1}{-2} \int (1-x^2)^{-1/2} (-2x) \, dx$

$= x \cos^{-1} x - \frac{1}{2} \left[\frac{(1-x^2)^{1/2}}{1/2} \right] + C$

$= \boxed{x \cos^{-1} x - \left(\sqrt{1-x^2} \right) + C}$ Answer

Q12 $\int \tan^{-1} x \cdot 1 dx$

for $u = \tan^{-1} x, \; v = 1$

$\tan^{-1} x \int 1 \cdot dx - \int \left(\int 1 dx \cdot \frac{d}{dx} (\tan^{-1} x) \right) dx$

$= x \tan^{-1} x - \int x \cdot \frac{1}{(1+x^2)} \, dx$

$= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{(1+x^2)} \, dx$

$= \boxed{x \tan^{-1} x - \frac{1}{2} \ln |1+x^2| + C}$

Ans

(13) $\int x \sec^2 x \, dx$

Sol $u = x, \; v = \sec^2 x$

$= x\int \sec^2 x \, dx - \int\left(\int \sec^2 x \, dx \cdot \frac{d}{dx}(x)\right)dx$

$= x\tan x - \int \tan x \, dx$

$= x\tan x + \int \frac{-\sin x}{(\cos x)} \, dx$

$= x\tan x + \ln|\cos x| + C$ Ans

(14) $\int x^2 \sin^{-1} x \, dx$ v. IMPORTANT

$u = \sin^{-1} x, \; v = x^2$

$= \sin^{-1} x \int x^2 \, dx - \int\left(\int x^2 \, dx \cdot \frac{d}{dx}(\sin^{-1} x)\right)dx$

$= \frac{x^3}{3}\sin^{-1} x - \int\left(\frac{x^3}{3} \cdot \frac{1}{\sqrt{1-x^2}}\right)dx$

$= \frac{x^3}{3}\sin^{-1} x - \frac{1}{3}\underbrace{\boxed{\int \frac{x^3}{\sqrt{1-x^2}} \, dx}}_{I}$

$I = \int \frac{x^3}{\sqrt{1-x^2}} \, dx$ ∴

let $1 - x^2 = y \Rightarrow 0 - 2x = \frac{dy}{dx}$

$\boxed{-2x\,dx = dy}$ $\boxed{x \cdot dx = \frac{-1}{2} dy}$

$\boxed{1 - y = x^2}$

$$= \int \frac{x^2 \cdot x\,dx}{\sqrt{1-x^2}}$$

$$= \int \frac{(1-y)\left(-\frac{1}{2}\right)dy}{\sqrt{y}}$$

$$= \frac{-1}{2}\int \frac{1-y}{\sqrt{y}}\,dy$$

$$= \frac{-1}{2}\left[\int\left(\frac{1}{\sqrt{y}} - \frac{y}{\sqrt{y}}\right)dy\right]$$

$$= -\frac{1}{2}\left[\int y^{-1/2}dy - \int y^{1/2}dy\right]$$

$$= -\frac{1}{2}\left[\frac{y^{1/2}}{1/2} - \frac{y^{3/2}}{3/2}\right] + C$$

$$= -\frac{1}{2}\left[2y^{1/2} - \frac{2}{3}y^{3/2}\right] + C$$

$$= \frac{-1}{2}(2y^{1/2}) + \frac{1}{2}\left(\frac{2}{3}\right)y^{3/2} + C$$

$$I = -y^{1/2} + \frac{1}{3}y^{3/2} + C$$

$$\boxed{I = -\sqrt{1-x^2} + \frac{1}{3}(1-x^2)^{3/2} + C}$$

$$= \frac{x^3}{3}\sin^{-1} - \frac{1}{3}\left[-\sqrt{1-x^2} + \frac{1}{3}(1-x^2)^{3/2}\right] +$$

$$= \frac{x^3}{3}\sin^{-1}x + \frac{\sqrt{1-x^2}}{3} - \frac{1}{9}(1-x^2)^{3/2} + C$$

A

Q15) $\int 1 \cdot \ln\left(x+\sqrt{1+x^2}\right)dx$

Sol $u = \ln\left(x+\sqrt{1+x^2}\right)$, $v = 1$.

$$= \ln\left(x+\sqrt{1+x^2}\right)\int 1 \cdot dx - \int\left[\int 1 \cdot dx \cdot \frac{d}{dx}\left(\ln\left(x+\sqrt{1+x^2}\right)\right)\right]dx$$

$$\frac{d}{dx}\left(\ln\left(x+\sqrt{1+x^2}\right)\right) = \frac{1}{x+\sqrt{1+x^2}} \times \frac{d}{dx}\left(x+\sqrt{1+x^2}\right)$$

$$= \frac{1}{x+\sqrt{1+x^2}} \times \left(1 + \frac{1}{2}(1+x^2)^{-1/2}(2x)\right)$$

$$= \frac{1}{x+\sqrt{1+x^2}}\left(1 + \frac{x}{\sqrt{1+x^2}}\right)$$

$$= \frac{1}{\left(x+\sqrt{1+x^2}\right)}\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right)$$

$$= \boxed{\int \frac{1}{\sqrt{1+x^2}}}$$

$$= \ln\left(x+\sqrt{1+x^2}\right) \cdot x - \int \left(x \cdot \frac{1}{\sqrt{1+x^2}}\right) dx$$

$$= x\ln\left(x+\sqrt{1+x^2}\right) - \frac{1}{2}\int (1+x^2)^{-1/2} \cdot 2x\, dx$$

$$= x\ln\left(x+\sqrt{1+x^2}\right) - \frac{1}{2}\left[\frac{(1+x^2)^{1/2}}{1/2}\right] + C$$

$$= x\ln\left(x+\sqrt{1+x^2}\right) - \sqrt{1+x^2} + C$$ Answer

Q16) $\int x^3 e^{x^2} dx$

put $x^2 = y$

$2x = \frac{dy}{dx}$

$$\boxed{x\,dx = \frac{dy}{2}}$$

$$= \int x^2 \cdot e^{x^2} \cdot x\, dx$$

$$= \int y \cdot e^y \frac{dy}{2}$$

$$= \frac{1}{2}\int y e^y dy$$

$u = y, \; v = e^y$

$$= \frac{1}{2}\left[y\int e^y dy - \int\left(\int e^y dy \cdot \frac{d}{dy}y\right) dy\right]$$

$$= \frac{1}{2}\left[y e^y - \int e^y dy\right] \Rightarrow \frac{1}{2}\left[y e^y - e^y\right] + C$$

$$\Rightarrow \frac{1}{2}\left[x^2 e^{x^2} - e^{x^2}\right] + C$$ Ans

(18) $\int \frac{\ln x}{\sqrt{x}} dx$

Sol $\int \underset{v}{x^{-1/2}} \cdot \underset{u}{\ln x}\, dx$

$= \ln x \int x^{-1/2} dx - \int\left[\int x^{-1/2} dx \cdot \frac{d}{dx}(\ln x)\right] dx$

$= \ln x \left(\frac{x^{1/2}}{1/2}\right) - \int \frac{x^{1/2}}{1/2} \cdot \left(\frac{1}{x}\right) dx$

$= 2\sqrt{x}\, \ln x - 2\int x^{1/2} x^{-1} \cdot dx$

$= 2\sqrt{x}\, \ln x - 2\int x^{-1/2} \cdot dx$

$= 2\sqrt{x}\, \ln x - 2\left[\frac{x^{1/2}}{1/2}\right] + c$

$= 2\sqrt{x} \ln x - 2(2\sqrt{x}) + c$

$= 2\sqrt{x} \ln x - 4\sqrt{x} + C$

$= \boxed{2\sqrt{x}\,[\ln x - 2] + C}$

(17) $\int x^2 \sin x\, dx$

$x^2 \int \sin x\, dx - \int\left(\sin x\, dx \cdot \frac{d}{dx}(x^2)\right) dx$

$= x^2(-\cos x) - \int(-\cos x)(2x) dx$

$= -x^2 \cos x + 2\int x \cos x\, dx$

$= -x^2 \cos x + 2\left[x \int \cos x\, dx - \int\left(\int \cos x\, dx \frac{d}{dx}(x)\right) dx\right.$

$= -x^2 \cos x + 2\left[x \sin x - \int \sin x\, dx\right]$

$= -x^2 \cos x + 2\left[x \sin x - (-\cos x) + C\right]$

$= -x^2 \cos x + 2x \sin x + 2\cos x + C$