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Where Minds Pollinate

Class 12
Mathematics

Chapter 3
Exercise 3.3
Handwritten Solution

Exercise 3.3

Concept: Some Useful Substitution:

(i) $\sqrt{a^2-x^2}$ put $\boxed{x = a\sin\theta}$

(ii) $\sqrt{x^2-a^2}$ put $\boxed{x = a\sec\theta}$

(iii) $\sqrt{a^2+x^2}$ put $\boxed{x = a\tan\theta}$

(iv) $\sqrt{x+a}$ or $\sqrt{x-a}$ put $\sqrt{x+a} = t$

(v) $\sqrt{2ax-x^2}$ put $x-a = a\sin\theta$

(vi) $\sqrt{2ax+x^2}$ put $x+a = a\sec\theta$

$\therefore \sin^2\theta + \cos^2\theta = 1$

$\therefore 1+\tan^2\theta = \sec^2\theta$

$\therefore 1+\cot^2\theta = \csc^2\theta$

Q1) $\int \frac{dx}{x^2+9}$

Sol $= \int \frac{dx}{x^2+3^2}$

$=$ put $x = 3\tan\theta$

$\frac{dx}{d\theta} = 3\sec^2\theta$

$dx = 3\sec^2\theta\, d\theta$

$= \int \frac{3\sec^2\theta\, d\theta}{9\tan^2\theta + 9}$

$= \int \frac{3\sec^2\theta\, d\theta}{9(\tan^2\theta+1)}$

$= \frac{1}{3}\int \frac{\sec^2\theta}{\sec^2\theta}\, d\theta$

$\therefore \frac{d}{dx}\left(\tan^{-1}\frac{x}{a}\right) = \frac{\frac{1}{a}\frac{d}{dx}(x)}{1+\left(\frac{x}{a}\right)^2}$

$= \frac{1}{a}\left[\frac{1}{\frac{a^2+x^2}{a^2}}\right]$

$= \frac{1}{a}\left[\frac{a^2}{a^2+x^2}\right]$

$\Rightarrow \frac{d}{dx}\left(\tan^{-1}\frac{x}{a}\right) = a\left[\frac{1}{a^2+x^2}\right]$

$\Rightarrow \frac{1}{a}\frac{d}{dx}\left(\tan^{-1}\frac{x}{a}\right) = \frac{1}{x^2+a^2}$

$\Rightarrow \int d\left[\frac{1}{a}\tan^{-1}\frac{x}{a}\right] = \int \frac{1}{x^2+a^2}\, dx$

$$\boxed{\int \frac{1}{x^2+a^2}\, dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c}$$

n

$= \frac{1}{3}\int d\theta$

$= \frac{1}{3}\theta + C$

now as $\therefore \frac{x}{3} = \tan\theta$

$\boxed{\theta = \tan^{-1}\left(\frac{x}{3}\right)}$

$\boxed{\frac{1}{3}\tan^{-1}\frac{x}{3} + C}$ Answer.

(2) $\int \frac{dx}{\sqrt{5-x^2}}$

Sol:- $\int \frac{dx}{\sqrt{(\sqrt{5})^2 - x^2}}$

put $\boxed{x = \sqrt{5}\sin\theta}$

$\frac{dx}{d\theta} = \sqrt{5}\cos\theta$

$\boxed{dx = \sqrt{5}\cos\theta\, d\theta}$

$\int \frac{\sqrt{5}\cos\theta\, d\theta}{\sqrt{(\sqrt{5})^2 - (\sqrt{5})^2\sin^2\theta}}$

$\int \frac{\sqrt{5}\cos\theta\, d\theta}{\sqrt{5(1-\sin^2\theta)}}$

$\int \frac{\cancel{\sqrt{5}}\cos\theta\, d\theta}{\cancel{\sqrt{5}}\sqrt{\cos^2\theta}}$

$\int \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}}\, d\phi$

$= \theta + C$

$= \sin^{-1}\frac{x}{\sqrt{5}} + C$ Answer

on last pages.

(3) $\int (2x+7)(x^2+7x+3)^{4/5}\, dx$ ★

Sol: $\frac{(x^2+7x+3)^{\frac{4}{5}+1}}{\frac{9}{5}} + C$

$\frac{5}{9}(x^2+7x+3)^{9/5} + C$

Answer.

(4) $\int \frac{x^2}{x^3+1}\, dx$ ★

Sol: $\frac{1}{3}\int \frac{3x^2}{x^3+1}\, dx$

$= \frac{1}{3}\ln(x^3+1) + C$

Q5) $\int \frac{dy}{y^2+8y+20}$ V.V IMP

$y^2+8y+20$

$y^2 + 4(2y) + 4^2 - 4^2 + 20$

$\therefore a^2+2ab+b^2$

$\boxed{(y+4)^2 + 4}$

now $\int \frac{dy}{(y+4)^2 + (2)^2}$

put $y+4 = 2\tan\theta$

$\frac{d}{d\theta}(y+4) = 2\frac{d}{d\theta}\tan\theta$

$\frac{dy}{d\theta} + 0 = 2\sec^2\theta$

$\boxed{dy = 2\sec^2\theta\, d\theta}$ $\Longrightarrow$ next page without substitution: $\int \frac{1}{2^2+(x+4)^2}dx = \frac{1}{2}\tan^{-1}\left(\frac{x+4}{2}\right)+C$.

$$= \int \frac{2\sec^2\theta \, d\theta}{(2\tan\theta)^2+4}$$

$$= \int \frac{2\sec^2\theta \, d\theta}{4(\tan^2\theta+1)}$$

$$= \frac{1}{2}\int \frac{\sec^2\theta \, d\theta}{\sec^2\theta}$$

$$= \frac{1}{2}\int d\theta$$

$$= \frac{1}{2}\theta + C \qquad \because \frac{y+4}{2} = \tan\theta \quad \therefore \theta = \tan^{-1}\left(\frac{y+4}{2}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{y+4}{2}\right)+C$$

Answer.

V.V.V.V.V.V.V. IMPORTANT Qs in Exercise 3.3.

Q6) $\int \frac{dx}{\sqrt{20-x^2-4x}}$

for $20-x^2-4x$

$20-(x^2+4x)$

$20-(x^2+2(x)(2)+2^2-2^2)$

$20-[(x+2)^2-4]$

$20-(x+2)^2+4$

$24-(x+2)^2$

$\boxed{(\sqrt{24})^2-(x+2)^2}$

$$\int \frac{dx}{\sqrt{(\sqrt{24})^2-(x+2)^2}}$$

$x+2 = \sqrt{24}\sin\theta$

$\frac{dx}{d\theta} = \sqrt{24}\cos\theta \, d\theta$

$\boxed{dx = \sqrt{24}\cos\theta \, d\theta}$

$$\int \frac{\sqrt{24}\cos\theta \, d\theta}{\sqrt{24-(\sqrt{24}\sin\theta)^2}}$$

$$= \int \frac{\sqrt{24}\cos\theta \, d\theta}{\sqrt{24-24\sin^2\theta}}$$

$$= \int \frac{\sqrt{24}\cos\theta \, d\theta}{\sqrt{24(1-\sin^2\theta)}}$$

$$\int \frac{\sqrt{24}\cos\theta \, d\theta}{\sqrt{24}(\sqrt{\cos^2\theta})}$$

$$\int \frac{\cos\theta}{\cos\theta}d\theta = \int d\theta = \theta + C$$

Direct

$$\int \frac{1 \, dx}{\sqrt{(\sqrt{24})^2-(x+2)^2}}$$

$$\sin^{-1}\left(\frac{x+2}{\sqrt{24}}\right)+C$$

now

$\frac{x+2}{\sqrt{24}} = \sin\theta \Rightarrow \theta = \sin^{-1}\frac{x+2}{\sqrt{24}}$

$\theta = \boxed{\sin^{-1}\frac{x+2}{\sqrt{24}}+C}$ Answer

⑦ $\int \frac{x\,dx}{(4x^2+1)^3}$

$\therefore \frac{d}{dx}(4x^2+1)$

$= 4(2x) = 8x$

$$\frac{1}{8}\int \frac{8x\,dx}{(4x^2+1)^3}$$

$$= \frac{1}{8}\int (4x^2+1)^{-3}(8x)\,dx$$

$$= \frac{1}{8}\frac{(4x^2+1)^{-2}}{-2}+C$$

$$\frac{-1}{16(4x^2+1)^2}+C$$

(8) $\int x^4\sqrt{3x^5-5}\;dx$

Sol $\int (3x^5-5)^{1/2}(x^4)\,dx$

$\frac{d}{dx}(3x^5-5) = 15x^4$

$= \frac{1}{15}\int(3x^5-5)^{1/2}(15x^4)\,dx$

$= \frac{1}{15}\,\frac{(3x^5-5)^{3/2}}{3/2}+C$

$= \frac{1}{15}\times\frac{2}{3}(3x^5-5)^{3/2}+C$

$= \boxed{\frac{2}{45}(3x^5-5)^{3/2}+C}$

Answer.

(9) $\int\frac{2ax+b}{ax^2+bx+c}\,dx$

Sol $\ln|ax^2+bx+c|+C$

Answer

(10) $\int\frac{dx}{(1-3x)^2}$

Sol $\int(1-3x)^{-2}\,dx$

$= -\frac{1}{3}\int(1-3x)^{-2}(-3)\,dx$

$= -\frac{1}{3}\,\frac{(1-3x)^{-1}}{-1}$

$= \frac{-1}{3(1-3x)}+C$ Answer

(11) $\int\frac{z^3}{1+z^4}\,dz$

$\frac{1}{4}\int\frac{4z^3}{(1+z^4)}\,dz$

$\frac{1}{4}\ln|1+z^4|+C$ Answer.

(12) $\int\frac{\cot^{-1}x}{1+x^2}\,dx$

$= -\int[\cot^{-1}x]\cdot\left(\frac{-1}{1+x^2}\right)dx$

$= -\frac{(\cot^{-1}x)^2}{2}+C$

Answer.

(7) $\int \frac{x\,dx}{(4x^2+1)^3}$ By substitution.

for let $t = 4x^2+1$ (without power)

$1 \cdot dt = 8x\,dx$

$\boxed{x\,dx = \frac{1}{8}dt}$

$\int \frac{\frac{1}{8}dt}{t^3}$

$= \frac{1}{8}\int \frac{dt}{t^3}$

$= \frac{1}{8}\int t^{-3}dt$

$= \frac{1}{8}\left(\frac{t^{-2}}{-2}\right) + C$

$= \frac{-1}{16t^2} + C$

$= \boxed{\frac{-1}{16(4x^2+1)^2} + C}$ Ans

(Q8) $\int x^4\sqrt{3x^5-5}\,dx$

let $t = 3x^5-5$

$1 \cdot dt = 15x^4dx$

$\frac{1}{15}dt = x^4 \cdot dx$

So $\int \sqrt{3x^5-5} \cdot x^4dx$

$\int \sqrt{t} \cdot \frac{1}{15}dt$

$= \frac{1}{15}\int t^{1/2}\,dt$

$\frac{1}{15}\frac{t^{3/2}}{3/2}$

$= \frac{2t^{3/2}}{45} + C$

$= \boxed{\frac{2(3x^5-5)^{3/2}}{45} + C}$ Answer

(Q9) $\int \frac{2ax+b}{ax^2+bx+c}dx$

let $t = ax^2+bx+c$

$dt = (2ax+b)dx$

$= \int \frac{1}{t}dt$

$= \ln t + c$

$= \ln(ax^2+bx+c) + C$ Ans

Q12 $\int \frac{\cot^{-1}x}{1+x^2}dx$

let $t = \cot^{-1}x$

$1 \cdot dt = \frac{-1}{1+x^2}dx$

$-1\,dt = \frac{1}{1+x^2}dx$

$\int t\,(-1)dt$

$= -\frac{t^2}{2} + C$

$= -\frac{(\cot^{-1}x)^2}{2} + C$ Ans

$(1-3x)=t$; $(1+3x^4)=t$

Q10) $\int \frac{dx}{(1-3x)^2}$

$t = 1-3x$

$dt = -3dx$

$$\boxed{dx = -\frac{dt}{3}}$$

$\int \frac{1}{t^2}\left(\frac{-dt}{3}\right)$

$= -\frac{1}{3}\int t^{-2}.dt$

$= -\frac{1}{3}\frac{t^{-1}}{-1} + C$

$= +\frac{1}{3t} + C$

$= \frac{1}{3(1-3x)} + C$ Ans

Q11) $\int \frac{x^3}{1+x^4}dx$

Sol:- let $t = 1+x^4$

$dt = 4x^3 dx$

$$\boxed{\frac{dt}{4} = x^3 dx}$$

$\int \frac{1}{t}\left(\frac{dt}{4}\right)$

$\frac{1}{4}\int \frac{1}{t}dt$

$\frac{1}{4}\ln t + C.$

$$\boxed{\frac{\ln(1+x^4)}{4} + C}$$

Exercise 3.3.

Q3 $\int (2x+7)(x^2+7x+3)^{4/5}dx$

Sol: **By Substitution method.**

let $t = x^2+7x+3$

$\therefore t = x^2+7x+3$

Diff b/s w.r.t x.

$\frac{dt}{dx} = \frac{d}{dx}(x^2+7x+3)$

$\boxed{dt = (2x+7)dx}$.

$1.dt = (2x+7)dx$

$= \int (x^2+7x+3)^{4/5}.(2x+7)dx$

$= \int t^{4/5}.dt$

$= \frac{t^{4/5+1}}{\frac{4}{5}+1} + C$

$= \frac{t^{9/5}}{9/5} + C$

But $t = x^2+7x+3$

$= \frac{5}{9}(x^2+7x+3)^{9/5} + C$ **Answer.**

Q4) $\int \frac{x^2}{x^3+1}dx$

Substitution method.

let $t = x^3+1$

$\boxed{1.dt = 3x^2.dx}$

$\times$ & $\div$ by 3

$\boxed{\frac{1}{3}dt = x^2dx}$

now $\int \frac{\frac{1}{3}dt}{t}$

$= \frac{1}{3}\int \frac{1}{t}dt$

$= \frac{1}{3}\ln t + C$

$= \boxed{\frac{1}{3}\ln(x^3+1)+C}$ Answer