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Where Minds Pollinate

Class 12

Mathematics

Chapter 3

Exercise 3.2

Handwritten Solution

Exercise # 3.2

Concepts:-

Formulas:

- Formula for derivative:

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

① $\int \sin kx \, dx = \frac{-\cos kx}{\frac{d}{dx}(kx)} + c$ — ①

② $\int \cos x \, dx = \frac{\sin x}{\frac{d}{dx}(x)} + c$ — ②

③ $\int \sec^2 x \, dx = \frac{\tan x}{\frac{d}{dx}(x)} + c$ — ③

④ $\int \sec x \tan x \, dx = \frac{\sec x}{\frac{d}{dx}(x)} + c$ — ④

⑤ $\int \operatorname{cosec}^2 x \, dx = -\frac{\cot x}{\frac{d}{dx}(x)} + c$ — ⑤

⑥ $\int \operatorname{cosec} x \cot x \, dx = \frac{-\operatorname{cosec} x}{\frac{d}{dx}(x)} + c$ — ⑥

(7) $\int \sec x \, dx = \ln|\sec x + \tan x| + C$ — (7)

(8) $\int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x - \cot x| + C$ — (8)

(9) $\int \tan x \, dx = -\ln(\cos x) = \ln(\sec x)$ — (9)

(10) $\int \cot x \, dx = \ln(\sin x) + C$ — (10)

(11) $\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$ — (11)

(12) $\int \cos^2 x \, dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$ — (12).

$\therefore \sin^2 x = \frac{1-\cos 2x}{2}$

$\therefore \cos^2 x = \frac{1+\cos 2x}{2}$

- formula no (7). $\int \sec x \, dx = \ln|\sec x + \tan x| + C$.

Proof:- L.H.S $= \int \sec x \, dx$

$= \int (\sec x) \times \frac{(\sec x + \tan x)}{(\sec x + \tan x)}$

$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}$

$= \ln(\sec x + \tan x) + C =$ RH.S.

- formula no (8) $\int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x - \cot x| + C$

Proof:- L.H.S $= \int \operatorname{cosec} x \, dx$

$= \int \frac{\operatorname{cosec} x (\operatorname{cosec} x - \cot x)}{\operatorname{cosec} x - \cot x} dx$

$= \int \frac{\operatorname{cosec}^2 x - \operatorname{cosec} x \cot x}{\operatorname{cosec} x - \cot x} dx$

$= \ln(\operatorname{cosec} x - \cot x) + C =$ **RHS**.

- $\int \tan x\, dx = \ln(\cos x) = \ln(\sec x)$

Proof:- L.H.S: $\int \tan x\, dx$

$= -\int \frac{-\sin x}{\cos x} dx$

$= -\ln|\cos x| + C$

$= \ln(\cos x)^{-1} + C$

$= \ln\left(\frac{1}{\cos x}\right) + C$

$= \ln|\sec x| + C$ RHS

note:

no direct integration formula for $\sin^2 x, \cos^2 x$ $\tan^2 x, \cot^2 x$

formulano 10: $\int \cot x\, dx = \ln(\sin x) + C$

LHS $\int \frac{\cos x\, dx}{\sin x}$

$= \ln|\sin x|$ proved

formula no 11: $\int \sin^2 x\, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$.

$\int \sin^2 x\, dx = \int \frac{1 - \cos 2x}{2} dx$

$\frac{1}{2}\left[\int 1\, dx - \int \cos 2x\, dx\right]$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{\frac{d}{dx}(2)}\right] + C$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + C$

$= \boxed{\frac{1}{2}x - \frac{\sin 2x}{4} + C}$ Ans

formula no 12

$\int \cos^2 x\, dx = \int \frac{1 + \cos 2x}{2} dx$

$\frac{1}{2}\left[\int 1\, dx + \int \cos 2x\, dx\right]$

$\frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + C$

$\frac{x}{2} + \frac{\sin 2x}{4} + C$ Answer.

Q1) $\int(\sin\pi x - 3\sin 3x)\,dx$

Solution: $-\dfrac{\cos\pi x}{\pi} + \cancel{3}\dfrac{\cos 3x}{\cancel{3}} + C$

$= \boxed{-\dfrac{1}{\pi}\cos\pi x + \cos 3x + C}$ Answer

Rechecking: $\dfrac{d}{dx}\left[-\dfrac{1}{\pi}\cos\pi x + \cos 3x + C\right]$

$= -\dfrac{1}{\pi}\dfrac{d}{dx}(\cos\pi x) + \dfrac{d}{dx}(\cos 3x) + \dfrac{d}{dx}(C)$

$= -\dfrac{1}{\pi}\boxed{-\cancel{\pi}\sin\pi x} + (-3\sin 3x) + 0$

$\boxed{\sin\pi x - 3\sin 3x}$ Verified.

Q2) $\int -\sec^2\left(\dfrac{3y}{2}\right)dy$

Sol: $-\int \sec^2\left(\dfrac{3y}{2}\right)dy$

$= -\dfrac{\tan\frac{3y}{2}}{\frac{d}{dy}\left(\frac{3y}{2}\right)} + C$

$= \boxed{-\dfrac{2}{3}\tan\dfrac{3y}{2} + C}$

Recheck

$\dfrac{d}{dx}\left(-\dfrac{2}{3}\tan\dfrac{3y}{2} + C\right)$

$-\dfrac{2}{3}\dfrac{d}{dy}\left[\tan\dfrac{3y}{2}\right] + \dfrac{d}{dy}(C)$

$-\dfrac{2}{3}\left[\sec^2\left(\dfrac{3y}{2}\right)\cdot\dfrac{d}{dy}\left(\dfrac{3y}{2}\right)\right] + 0$

$-\dfrac{2}{3}\sec^2\left(\dfrac{3y}{2}\right)\cdot\dfrac{3}{2}$

$= -\sec^2\left(\dfrac{3y}{2}\right)$

Proved.

③ $\int [1 - 8\,\text{cosec}^2(2x)]\,dx$

Sol:- $1\int dx - 8\int \text{cosec}^2(2x)\cdot dx$

$= \int dx + 8[-\text{cosec}^2(2x)\cdot dx]$

$= x + 8\left[\frac{\cot 2x}{2}\right.$

$= x + 4\cot(2x) + C$ **Answer.**

Recheck

$\frac{d}{dx}[x + 4\cot(2x) + C]$

$1 + 4[-\text{cosec}^2 2x \times 2$

$\boxed{1 - 8\,\text{cosec}^2 2x}$ verified.

④ $\int \frac{1}{2}(\text{cosec}^2 x - \text{cosec}\,x \cot x)\,dx$

Sol: $\frac{1}{2}\int \text{cosec}^2 x\,dx - \int \text{cosec}\,x \cot x \cdot dx$

$\frac{1}{2}[-\cot x - [-\text{cosec}\,x)] + C$

$\boxed{-\frac{1}{2}\cot x + \frac{1}{2}\text{cosec}\,x + C}$

Recheck

$-\frac{1}{2}\frac{d}{dx}\cot x + \frac{1}{2}\frac{d}{dx}\text{cosec}\,x + \frac{d}{dx}(C)$

$-\frac{1}{2}[\cancel{-}\text{cosec}^2 x + \frac{1}{2}(-\text{cosec}\,x \cot x)] + 0$

$+\frac{1}{2}\text{cosec}^2 x - \frac{1}{2}\text{cosec}\,x \cot x$

$\frac{1}{2}[\text{cosec}^2 x - \text{cosec}\,x \cot x]$ Ans

Q5 $\int \frac{\cos^2 z}{7}\,dz$

$= \frac{1}{7}\int \cos^2 z\,dz$

$= \frac{1}{7}\left[\frac{1}{2}z - \frac{1}{4}\sin 2z\right] + C$

$\boxed{\frac{z}{14} + \frac{\sin 2z}{28} + C}$ **Answer**

Recheck:

$\frac{1}{14}\frac{d}{dz}(z) + \frac{1}{28}\frac{d}{dz}(\sin 2z)\cdot\frac{d}{dz}(C)$

$\frac{1}{14} + \frac{1}{28}[2\cos 2z]$

$\frac{1}{14}[1 + \cos 2z]$

$= \frac{1}{7}\left[\frac{1 + \cos 2z}{2}\right]$

$= \boxed{\frac{1}{7}\cos^2 z}$ proof.

Q6) $\int (1+\tan^2\theta)\,d\theta$

Sol $\because 1+\tan^2\theta = \sec^2\theta$

$\int \sec^2\theta\, d\theta$

$= \boxed{\tan\theta + C}$ Answer

Recheck

$\frac{d}{d\theta}\tan\theta \cdot \frac{d}{d\theta}(\theta)$

$\frac{d}{d\theta}\sec^2\theta$

$\boxed{1+\tan^2\theta}$ Verified.

(7) $\int \frac{1+\cos 4t}{2}\,dt$

$= \frac{1}{2}\left[\int 1 dt + \int \cos 4t \cdot dt\right]$

$= \frac{1}{2}\left[t + \frac{\sin 4t}{4}\right] + C$ Answer

Recheck

$\frac{1}{2}\frac{d}{dt}t + \frac{1}{8}\frac{d}{dt}\sin 4t$

$\frac{1}{2} + \frac{1}{8}[\cos 4t \cdot 4]$ verified

$\frac{1}{2}[1+\cos 4t]$ or $\boxed{\frac{1+\cos 4t}{2}}$

(8) $\int \sec^2(5x-1)\,dx$

Sol $\frac{\tan(5x-1)}{\frac{d}{dx}(5x-1)}$

$= \boxed{\frac{\tan(5x-1)}{5}}$ Ans

Recheck

$\frac{1}{5}\frac{d}{dx}\tan(5x-1)$

$\frac{1}{\cancel{5}}\sec^2(5x-1)\cancel{5}$

$\sec^2(5x-1)$ Answer verified

(9) $\int(\tan 5x + \cos 7x)\,dx$

$= \int \tan 5x \cdot dx + \int \cos 7x \cdot dx$

$= \frac{\ln \sec 5x}{5} + \frac{\sin 7x}{7} + C$ Answer

$\tan 5x + \cos 7x$ verified

Recheck

$\frac{1}{5}\frac{d}{dx}\ln\sec 5x + \frac{1}{7}\frac{d}{dx}(\sin 7x) + 0 \Rightarrow \frac{1}{\cancel{5}}\left[\frac{1}{\sec 5x}\cdot \sec 5x \tan 5x \cdot \cancel{5}\right]$

$+ \frac{1}{\cancel{7}}\left[\cancel{7}\cos 7x\right]$

Recheck Q13

$\frac{1}{11}\int 11\sec 11x \cdot \frac{\sec 11x + \tan 11x}{\sec 11x + \tan 11x}$

(10) $\int(\cot 9y - 3)\,dy$

Sol:- $\int \cot 9y - 3\int dy$

$= \boxed{\frac{\ln \sin 9y}{9} - 3y + C}$

Recheck $\frac{1}{9}\frac{d}{dy}(\ln \sin 9y) - 3\frac{d}{dy}(y) + 0$

$\frac{1}{9}\cdot\left[\frac{1}{\sin 9y}\cdot\frac{d}{dy}(\sin 9y)\right] - 3(1)$

$\frac{1}{9}\left[\frac{1}{\sin 9y}\cdot 9\cos 9y\right] - 3 \Rightarrow \boxed{\cot 9y - 3}$

Q11) $\int(\tan^2 2\theta + \cot^2 2\theta)\,d\theta$

$\because 1 + \tan^2\theta = \sec^2\theta$

Sol:- $\because 1 + \cot^2\theta = \csc^2\theta$

$= \int(\sec^2 2\theta - 1 + \csc^2 2\theta - 1)\,d\theta$

$= \int \sec^2 2\theta\,d\theta + \int \csc^2 2\theta\,d\theta - 2\int d\theta$

$= \frac{\tan 2\theta}{2} - \frac{\cot 2\theta}{2} - 2\theta + C$

Q12) $\int \sin^2\left(\frac{11}{2}y\right)dy$

Sol:- $\int \frac{\left(1 - \cos 2\left(\frac{11y}{2}\right)\right)}{2}dy$

$= \frac{1}{2}\left[\int dy - \int \cos 11y\,dy\right]$

$= \frac{1}{2}\left[y - \frac{\sin 11y}{11}\right] + C$

$\frac{y}{2} - \frac{\sin 11y}{22} + C$ **Answer**

Short cut

$\frac{1}{2}y - \frac{1}{4}\sin 11y$ ... $\frac{y}{2} - \frac{\sin 11y}{22}$

Q13) $\int \csc 11x \tan 11x\,dx$

Sol:- $\int\left(\frac{1}{\sin 11x}\cdot\frac{\sin 11x}{\cos 11x}\right)dx$

$\int\left(\frac{1}{\cos 11x}\right)dx$ $\quad \frac{1}{11}\int 11\sec 11x \times \frac{\sec 11x + \tan 11x}{\sec 11x + \tan 11x}$

$\int \sec 11x\,dx$ or $\frac{1}{11}\int \frac{11(\sec^2 11x + \sec 11x\tan 11x)}{\tan 11x + \sec 11x}$

$\frac{\ln|\sec 11x + \tan 11x|}{11} + C$

Q14) $\int \cos\theta(\tan\theta + \sec\theta)\,d\theta$

$= \int\left(\cos\theta\cdot\frac{\sin\theta}{\cos\theta} + \cos\theta\cdot\frac{1}{\cos\theta}\right)d\theta$

$\int(\sin\theta + 1)\,d\theta$

$\int \sin\theta\,d\theta + \int 1\,d\theta$ Recheck

$= -\cos\theta + \theta + C$ Answer

Short trick

$\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

$\frac{y}{2} - \frac{\sin 11y}{22} + C$

(15) $\int \operatorname{cosec}^2\left(\frac{x-1}{3}\right)dx$

Solution: $\dfrac{-\cot\left(\frac{x-1}{3}\right)}{\frac{d}{dx}\left(\frac{x-1}{3}\right)}$

$= \dfrac{-\cot\left(\frac{x-1}{3}\right)}{1/3}$

$= -3\cot\left(\frac{x-1}{3}\right) + c$ Answer.

(16) $\int (\cos x)^{1/5} \sin x\, dx$

Sol. $-\int (\cos x)^{1/5}(-\sin x)\, dx$

$= \dfrac{-(\cos x)^{6/5}}{6/5}$

$= \frac{-5}{6}(\cos x)^{6/5} + C$ Answer

(17) $\int e^y \sin e^y\, dy$

Sol. put $e^y = t \Rightarrow \frac{d}{dy}(e^y) = \frac{d}{dy}(t)$

$\boxed{dy\, e^y = dt}$

$= \int \sin t\, dt$

$= -\cos t + c$

$= \boxed{-\cos e^y + C}$ Ans

(18) $\int 9\tan(x+7)dx$

Sol. $9\int \tan(x+7)\, dx$

$\dfrac{9\ln|\sec(x+7)|}{1} + C$

$= 9\ln|\sec(x+7)| + C$.

OR

Sol. $-9\int \frac{-\sin(x+7)}{\cos(x+7)}dx$

$= -9\ln\cos(x+7) + C$

$= 9\ln(\cos(x+7))^{-1} + C$

$= 9\ln\left(\frac{1}{\cos(x+7)}\right) + C$

$= \boxed{9\ln\sec(x+7) + C}$

Ans

$\int e^y \sin e^y\, dy$

OR $\int \sin e^y\, e^y dy$

$-\cos e^y + C$

Ans