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Where Minds Pollinate

Class 12
Mathematics

Chapter 3

Exercise 3.1

Handwritten Solution

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CONCEPTS

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1$$

In General form

$$\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C, \; n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C$$

In General form

$$\int \frac{f'(x)}{f(x)} dx = \ln[f(x)] + C$$

$$\int e^x dx = e^x + c$$

In General form

$$\int e^{f(x)} \cdot f'(x) dx = e^{f(x)} + c$$

$$\int a^x dx = \frac{1}{\ln a} \cdot a^x + c$$

In General form

$$\int a^{f(x)} \cdot f'(x) dx = \frac{1}{\ln a} \cdot a^{f(x)} + c$$

~~Comes with~~

∴ C comes with every integration. but combination of C is also C Hence write single time C.

Q1 $\int (x^2-3x+9)\,dx$

Sol: $\int x^2 dx - 3\int x\,dx + 9\int dx$

$\frac{x^3}{3} - 3\frac{x^2}{2} + 9x + C$ Answer

Q2. $\int (y^2+8y+\sqrt{2})\,dy$

Sol:- $\int y^2 dy + 8\int y\,dy + \sqrt{2}\int dy$

$\frac{y^3}{3} + 8\frac{y^2}{2} + \sqrt{2}y + C$ Answer

Q3 $\int \left(\sqrt{y} + \frac{1}{y^2}\right) dy$

Sol $\int y^{1/2} dy + \int y^{-2} dy$

$= \frac{y^{3/2}}{3/2} + \frac{y^{-1}}{-1} + C$ Answer

$= \frac{2}{3} y^{3/2} - y^{-1} + C$

Q4 $\int (4+x^2)^2 dx$

Sol $\int (16+x^4+8x^2)\,dx$

$= 16\int dx + \int x^4 dx + 8\int x^2 dx$

$= 16x + \frac{x^5}{5} + \frac{8x^3}{3} + C$ Ans

Q5) $\int (1+x)(1-x^2)\,dx$

Sol $\int (1-x^2+x-x^3)\,dx$

$1\int dx - \int x^2 dx + \int x\,dx - \int x^3 dx$

$x - \frac{x^3}{3} + \frac{x^2}{2} - \frac{x^4}{4} + C$ Ans

Q6) $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$

Sol:- $\int x^{1/2} dx + \frac{1}{2}\int x^{-1/2} dx$

$= \frac{x^{3/2}}{3/2} + \frac{1}{2}\frac{x^{1/2}}{1/2} + C$

$= \frac{2}{3} x^{3/2} + x^{1/2} + C$ Ans

Q7 $\int (e^{4x} - e^{-1} + 1)\,dx$

$\frac{1}{4}\int e^{4x} 4\,dx - \frac{1}{e}\int dx + 1\int dx$

$= \frac{1}{4}e^{4x} - \frac{1}{e}x + x + C$ Ans

8 $\int \left(e^{9/2 x} + \frac{1}{x}\right) dx$

Sol $\int e^{9/2} dx + \int \frac{1}{x} dx$

$\frac{1}{9/2}\int e^{9/2} \cdot \frac{9}{2} dx + \ln x$

$= \frac{2}{9} e^{9/2 x} + \ln x + C$

Ans

9) $\int x e^{x^2} dx$

put $x^2 = t$

$2x\,dx = dt$

$\therefore x\,dx = \frac{dt}{2}$

$= \int e^t \frac{dt}{2}$

$= \frac{1}{2}\int e^t dt$

$= \frac{1}{2} e^t + C = \boxed{\frac{1}{2} e^{x^2} + C}$

10) $\int 5^x dx$

$= \frac{5^x}{\ln 5} + C$ Ans

11) $\int 7^{7y} dy$

$= \frac{1}{7}\int 7^{7y} \cdot 7\,dy$

$= \frac{1}{7} \frac{7^{7y}}{\ln 7} + C$ Ans

12) $\int \left(x^3 + \frac{1}{2x} - \frac{1}{x^3}\right) dx$

$= \int x^3 dx + \frac{1}{2}\int \frac{1}{x} dx - \int x^{-3} dx$

$= \frac{x^4}{4} + \frac{1}{2}\ln x - \frac{x^{-2}}{-2} + C$

$= \frac{x^4}{4} + \frac{1}{2}\ln x + \frac{1}{2x^2} + C$

13) $\int \frac{2x+1}{x^2+3} dx$

$= \int \frac{2x}{x^2+3} dx + \int \frac{1}{x^2+3} dx$

$= \ln(x^2+3) + \int \frac{1}{x^2+(\sqrt{3})^2} dx$

$= \ln(x^2+3) + \frac{1}{\sqrt{3}} \tan^{-1}\frac{x}{\sqrt{3}} + C$

Concept: $\frac{d}{dx}\left(\tan^{-1}\frac{x}{a}\right)$

$= \frac{1}{1+\left(\frac{x}{a}\right)^2} \cdot \frac{d}{dx}\left(\frac{x}{a}\right)$

$= \frac{1}{1+\frac{x^2}{a^2}} \cdot \frac{1}{a}$

$= \frac{1}{\frac{a^2+x^2}{a^2}} \cdot \frac{1}{a}$

$= \frac{a^2}{a^2+x^2} \cdot \frac{1}{a}$

$\frac{d}{dx}\left(\tan^{-1}\frac{x}{a}\right) = a \cdot \frac{1}{x^2+a^2}$

$\int d\left(\tan^{-1}\frac{x}{a}\right) = \int a \cdot \frac{1}{x^2+a^2} dx$

$\tan^{-1}\frac{x}{a} = a\int \frac{1}{x^2+a^2} dx$

$\boxed{\frac{1}{a}\tan^{-1}\frac{x}{a} = \int \frac{1}{x^2+a^2} dx}$

(14) $\int \frac{e^{\tan^{-1}z}}{1+z^2}\,dz$

Sol:- $f(z) = \tan^{-1}z$

$f'(z) = \frac{1}{1+z^2}$

$= \int e^{\tan^{-1}z} \cdot \frac{1}{1+z^2}\,dz$

$= \boxed{e^{\tan^{-1}z} + C}$ Ans

(15) $\int (x^{3/2} + e^{3x} + x^0)\,dx$

$\int x^{3/2}\,dx + \int e^{3x}\,dx + 1\int dx$

$= \frac{x^{5/2}}{5/2} + \frac{1}{3}\int e^{3x} \cdot 3\,dx + x$

$= \frac{2}{3}x^{5/2} + \frac{1}{3}e^{3x} + x + C$ Ans

(16) $\int \underbrace{(3x^2+2x)}_{f'(x)}\underbrace{(x^3+x^2+9)}_{f(x)}^5\,dx$

$= \frac{(x^3+x^2+9)^6}{6} + C$ Ans

(17) $\int (5e^{5x} - x^{-3} + 3^{2x})\,dx$

$5\int e^{5x}\,dx - \int x^{-3}\,dx + \int 3^{2x}\,dx$

$= \int e^{5x} \cdot 5\,dx - \frac{x^{-2}}{-2} + \frac{1}{2}\int e^{2x} \cdot 2\,dx$

$= e^{5x} + \frac{1}{2x^2} + \frac{1}{2}\frac{3^{2x}}{\ln 3} + C$

(18) $\int \left(z^{-1/4} + \sqrt{3z} + \frac{4}{z} - \frac{1}{e^z}\right)dz$

$\int z^{-1/4}\,dz + \sqrt{3}\int z^{1/2}\,dz + 4\int \frac{1}{z}\,dz - \int e^{-z}\,dz$

$= \frac{z^{3/4}}{3/4} + \sqrt{3}\,\frac{z^{3/2}}{3/2} + 4\ln z + \int e^{-z} \cdot (-1)\,dz$

$= \frac{4}{3}z^{3/4} + \frac{2\sqrt{3}z^{3/2}}{3} + 4\ln z + e^{-z} + C$

$= \frac{4}{3}z^{3/4} + \frac{2}{\sqrt{3}}z^{3/2} + 4\ln z + \frac{1}{e^z} + C$

Answer ..