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Where Minds Pollinate

Class 12
Mathematics

Chapter 2

Review Exercise 2

Handwritten Solution

Review Exercise # 2

Q2. $\lim_{x\to 0}\frac{x^3}{\sin^2 3x}$

Sol: ∵ $\lim_{x\to 0}\frac{\sin x}{x}=1$ Sandwich Theorm

$= \lim_{x\to 0}\frac{1}{9}\left(\frac{9x^3}{\sin^2 3x}\right)$ ∵ × { ÷ by 9

$= \frac{1}{9}\lim_{x\to 0}\left(\frac{x\cdot(3x)^2}{\sin^2 3x}\right)$

$= \frac{1}{9}\lim_{x\to 0} x\left(\frac{x}{\frac{\sin^2 3x}{(3x)^2}}\right) \Rightarrow \frac{1}{9}\lim_{3x\to 0} x \times \frac{1}{\left(\lim_{x\to 0}\frac{\sin 3x}{(3x)}\right)^2}$ $x\to 0$ then $3x\to$

$= \frac{1}{9}(0)\frac{1}{1} = \boxed{0}$ Answer

Q3. $y=\frac{1+\sin x}{x\cos x}$

Sol:- $\frac{dy}{dx}=\frac{(x\cos x)(\cos x)-(1+\sin x)\frac{d}{dx}(x\cos x)}{(x\cos x)^2}$

∴ $\frac{d}{dx}x\cos x = x[-\sin x]+\cos x(1)$

So

$= \frac{x\cos^2 x-(1+\sin x)(-x\sin x+\cos x)}{(x\cos x)^2}$

$= \frac{x\cos^2 x-\left[-x\sin x+\cos x-[x\sin^2 x+\sin x\cos x]\right]}{(x\cos x)^2}$

$= \frac{x\cos^2 x+x\sin x-\cos x+x\sin^2 x-\sin x\cos x}{(x\cos x)^2}$

$= \frac{x+x\sin x-\cos x-\sin x\cos x}{(x\cos x)^2}$ Answer

Q4. If $f'(0) = -1$ & $g(0) = 6$ what is $\frac{d^2}{dx^2}[xf(x) + xg(x)]$ at $x=0$?

Solution $\frac{x(1+\sin x) - \cos x(1+\sin x)}{(x\cos x)^2}$

$\frac{dy}{dx} = \frac{(x-\cos x)(1+\sin x)}{(x\cos x)^2}$ Answer. $\therefore x(\sin^2 x + \cos^2 x) = x(1) = x$

now $\frac{d}{dx}[xf(x) + xg(x)]$

$= xf'(x) + f(x)(1) + xg'(x) + g(x)(1)$

$= x[f'(x) + g'(x)] + f(x) + g(x)$

Again Diff w.r.t x

$= \frac{d^2}{dx^2}[xf(x) + xg(x)]$

$= \frac{d}{dx}[x(f'(x) + g'(x)] + f(x) + g(x)]$

$= x[f''(x) + g''(x)] + [f'(x) + g'(x)](1) + f'(x) + g'(x)$

$= f'(0) + g'(0) + f'(0) + g'(0)$

$= -1 + 6 - 1 + 6 = 10$ Answer

Q5) Solution: $x^3 + y^3 = 27$

$3x^2 + 3y^2\frac{dy}{dx} = 0$

$3(x^2 + y^2\frac{dy}{dx}) = 0$

$x^2 + y^2\frac{dy}{dx} = 0$

$y^2\frac{dy}{dx} = -x^2$

$$\boxed{\frac{dy}{dx} = \frac{-x^2}{y^2}}$$

$\frac{d^2y}{dx^2} = \frac{y^2(-2x) - (-x^2)\,2y\left(\frac{dy}{dx}\right)}{(y^2)^2}$

$\frac{d^2y}{dx^2} = \frac{-2xy^2 + 2x^2y\left(\frac{-x^2}{y^2}\right)}{y^4}$

$\frac{d^2y}{dx^2} = \frac{-2xy^3 - 2x^4}{\frac{y}{y^4/1}}$

$$\frac{d^2y}{dx^2} = \frac{-2xy^3 - 2x^4}{y^5}$$

Answer

$$\frac{d^2y}{dx^2} = \frac{-2x(y^3+x^3)}{y^5} \Rightarrow \frac{d^2y}{dx^2} = \frac{-2x(27)}{y^5} \Rightarrow \boxed{\frac{d^2y}{dx^2} = \frac{-54x}{y^5}}$$

Q6. Solution:- $f(x+\Delta x) \simeq f(x) + dy$

$$\frac{dy}{dx} = f'(x) \Rightarrow \boxed{dy = f'(x)dx}$$

$\boxed{f(x+\Delta x) \approx f(x) + f'(x)dx}$ formula

$f(x+\Delta x) = \sqrt{64+1} \Rightarrow f(x) = \sqrt{x}$ $\quad dx = \Delta x = 1$.

$\boxed{x = 64}$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$$

$$f(x+\Delta x) \approx \sqrt{x} + \frac{1}{2\sqrt{x}}(1)$$

$$f(64+1) \approx \sqrt{64} + \frac{1}{2\sqrt{64}}$$

$$f(64+1) \approx \sqrt{64} + \frac{1}{2\sqrt{64}}$$

$$\sqrt{65} \approx 8 + \frac{1}{16}$$

$$\frac{128+1}{16} = \frac{129}{16}$$

$\boxed{\sqrt{65} \approx 8.0625}$ Answer

Q7. Sol $h = 5m$

$r = 8m$

$dr = \Delta r = \pm 0.25m$

Volume $= \pi r^2 h$

$\frac{dV}{dr} = \pi h(2r)$

$\boxed{dV = 2\pi r h \, dr}$

$dV = 2\pi(8)(5)(0.25)$

$\boxed{dV = 20\pi \, m^3}$ Answer

Relative error $= \frac{dV}{V}$

$= \frac{2\pi r h \, dr}{\pi r^2 h}$

$= \frac{2dr}{r}$

$= \frac{2(\pm 0.25)}{8}$

$= \boxed{\pm 0.0625}$ Answer

Percentage error $=$

$= 0.0625 \times 100\%$

$= \boxed{6.25\%}$

~~Ans~~ Answer

8.) Solution: $l^2 = x^2 + y^2$

$(15)^2 = (5)^2 + y^2$

$y^2 = \sqrt{225 - 25}$

$y = \sqrt{200}$

$\boxed{y = 10\sqrt{2} \text{ ft}}$

$(15)^2 = x^2 + y^2$

$225 = x^2 + y^2$

Diff w.r.t '$\cancel{t}$'

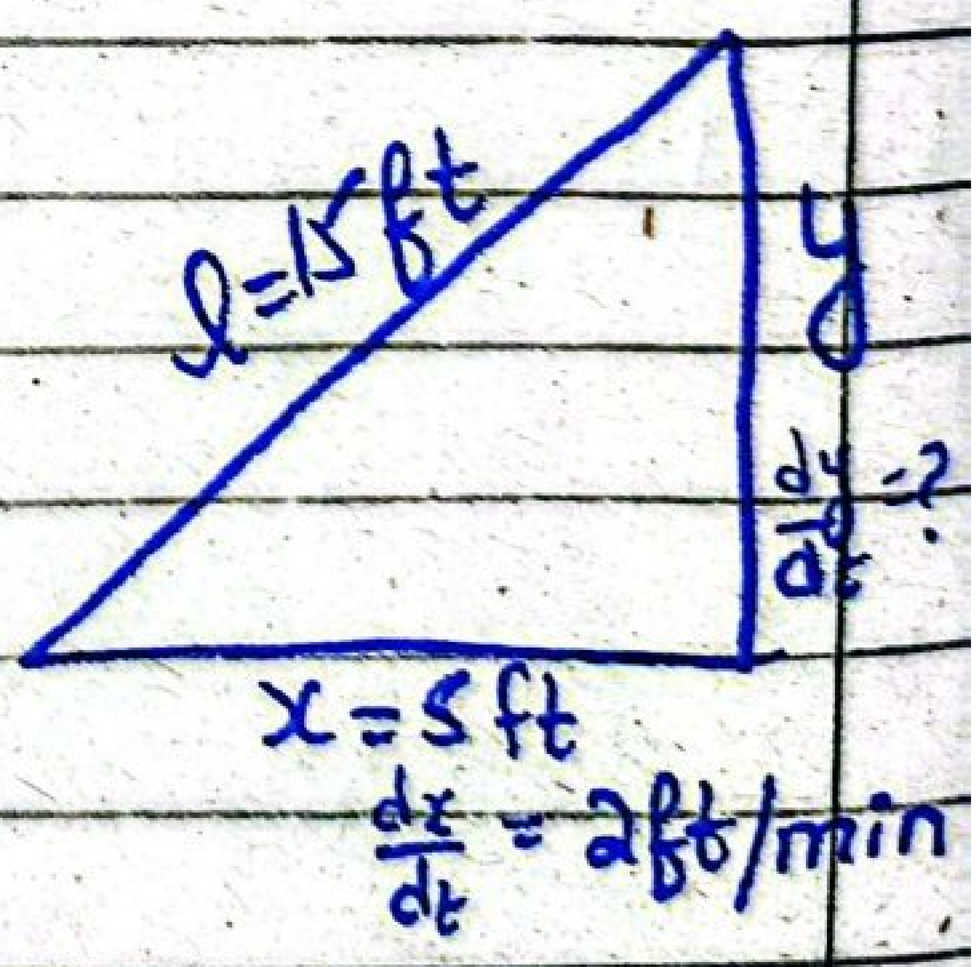
$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{0}{2} = x\frac{dx}{dt} + y\frac{dy}{dt}$

$0 = (5)(2) + 10\sqrt{2}\frac{dy}{dt}$

$\frac{-10}{10\sqrt{2}} = \frac{dy}{dt}$

$\boxed{\frac{dy}{dt} = \frac{-1}{\sqrt{2}} \text{ ft/min}}$ Ans



Q9. a) $[-3, 1]$.

$f'(x) = 3x^2 - 6x - 24$

$0 = 3(x^2 - 2x - 8)$

$0 = x^2 - 2x - 8$

$0 = x^2 - 4x + 2x - 8$

$0 = x(x-4) + 2(x-4)$

$0 = (x+2)(x-4)$

$x+2 = 0$; $x - 4 = 0$

$\boxed{x = -2}$ $\quad$ $\boxed{x = 4}$

$\in [-3, 1]$ $\quad$ $\notin [-3, 1]$

$f(-3) = (-3)^3 - 3(-3)^2 - 24(-3) + 2$

$= 20$

$f(-2) = 30$ max value at $x = -2$.

$f(1) = -24$ min value at $x = 1$.

Answer.

(b) $[-3, 8]$

for $-2 \in [-3, 8]$ & $4 \in [-3, 8]$

$f(-3) = 20$

$f(-2) = 30$

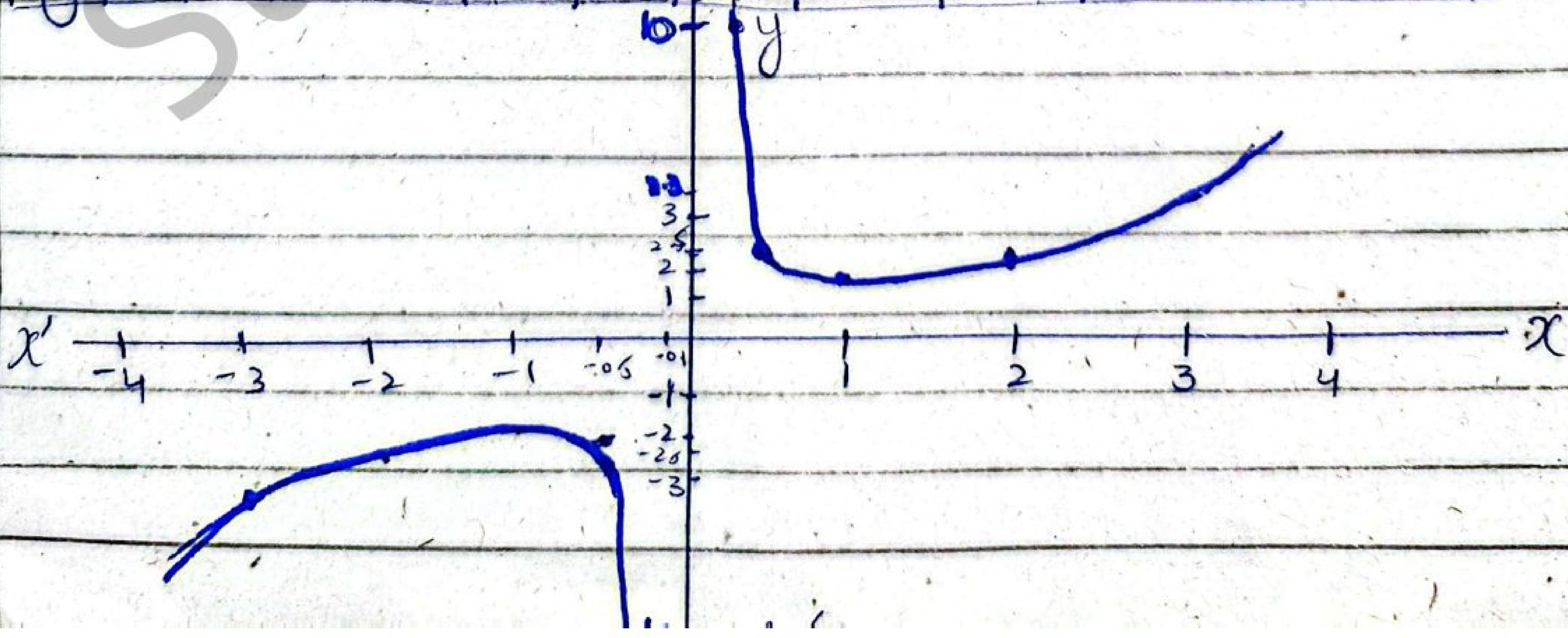
$f(4) = -78$ min

$f(8) = 130$ max.

Answer

(10) $f(x) = x + \frac{1}{x}$

x	-3	-2	-1	-0.5	-0.1	0.1	0.5	1	2	3		
y	-3.3	-2.5	-2	-2.5	-10.1	10.1	2.5	2	2.5	3.3		



SLO

$y = x^x$

$\frac{d}{dx}(x^x) = ?$

$y = x^x$

Taking ln b.S

$\ln y = \ln x^x$

$\ln y = x \cdot \ln x$

Diff w.r.t x

$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln(1)$

$\frac{dy}{dx} = y[1 + \ln x]$

$\boxed{\frac{dy}{dx} = x^x\left[1 + \ln x\right]}$ Answer

$\frac{d}{dx}(2^x) = 2^x \cdot \ln 2$ Answer. $\therefore \frac{d}{dx}(a^x) = a^x \cdot \ln a$

$\frac{d}{dx}\left(2^{x^2+x}\right) = 2^{x^2+x} \cdot \ln 2 \cdot \frac{d}{dx}(x^2+x)$

$= 2^{x^2+x} \cdot \ln 2 \cdot (2x+1)$ Answer.