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Where Minds Pollinate

Class 12
Mathematics

Chapter 2

Exercise 2.9

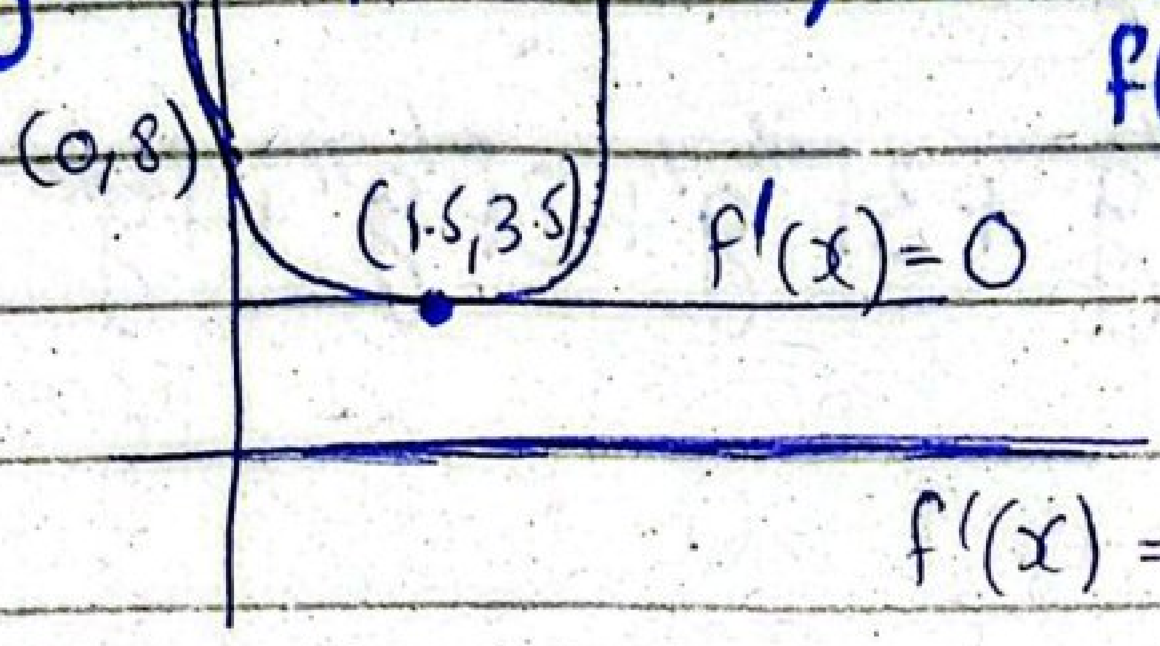
Handwritten Solution

Exercise 29

Concept: Critical Points: Critical values occur where the first derivative is **zero or undefined**.

Q) why we put $f'(x)=0$ on critical point.

Ans)



$f(x) = 2x^2 - 6x + 8$

$f'(x) = 0$

$f'(x) = \tan x = \tan(0^\circ) = \boxed{0}$

$\boxed{f'(x)=0}$

→ Absolute (Global) Extrema

→ **Concavity** Concave up if $f''(x) > 0$ ☺

Concave down if $f''(x) < 0$ ☹

• Point of Inflection: A point where concavity change

→ Relative (local) Extrema.

Q1 (i) $f(x) = 2x^2 - 6x + 8$

Solution: $\frac{d}{dx} f(x) = \frac{d}{dx}[2x^2 - 6x + 8]$

$f'(x) = 4x - 6$

put $f'(x) = 0$

$0 = 4x - 6$

$6 = 4x$

$\boxed{x = \frac{3}{2}}$ answer

(ii) $x^3 + x - 2$ Taking derivative w.r.t x

Solution:- $f'(x) = 3x^2 + 1$

$0 = 3x^2 + 1$

$x = -\frac{1}{3}$

$\boxed{x = \frac{-1}{\sqrt{3}}}$ Hence no critical value.

③ $f(x) = \dfrac{x}{x^2+2}$

Sol:- $f'(x) = \dfrac{(x^2+2)1 - x(2x)}{(x^2+2)^2}$

$f'(x) = \dfrac{x^2+2-2x^2}{(x^2+2)^2}$

put $f'(x) = 0$

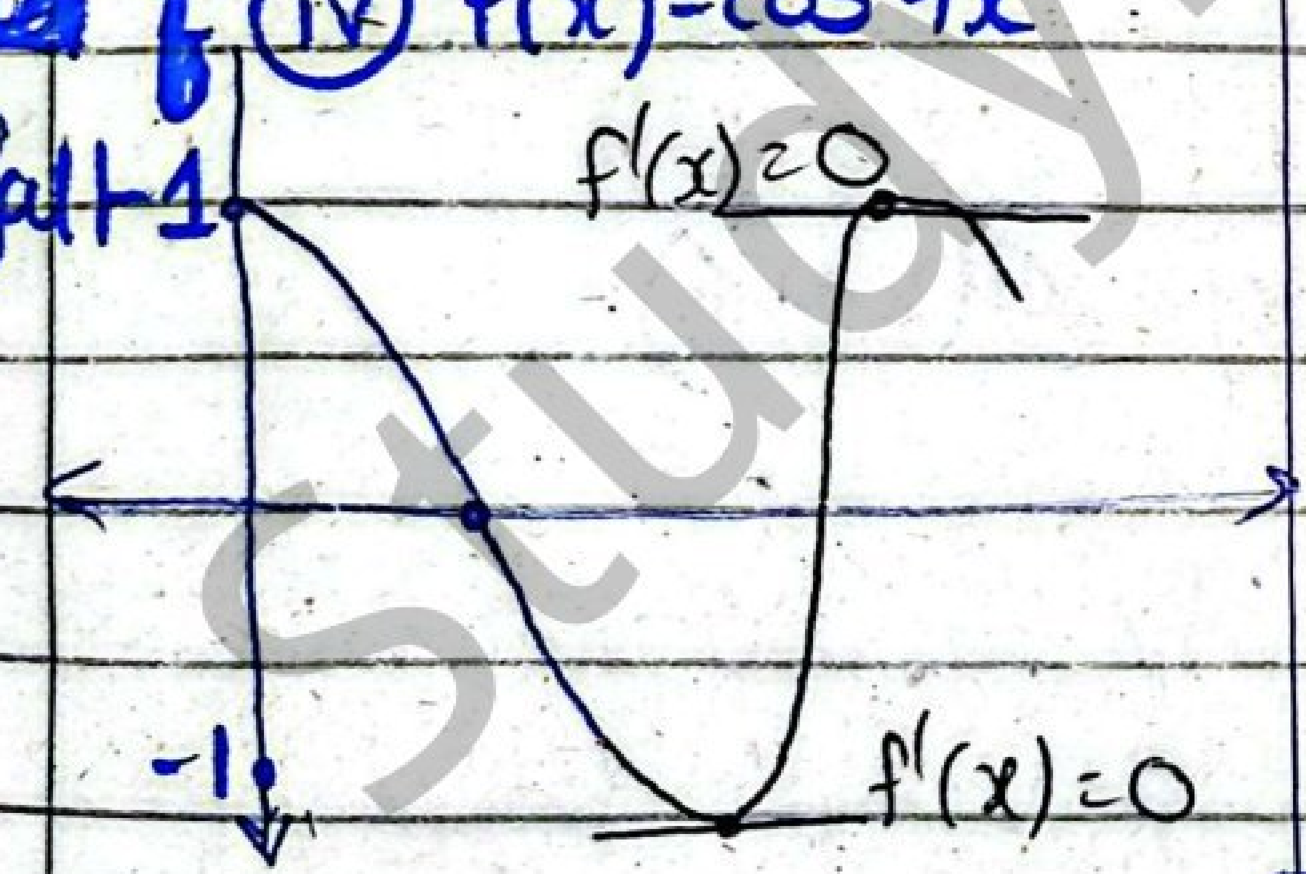
$\dfrac{-x^2+2}{(x^2+2)^2} = 0$

$-x^2+2 = 0$

$\boxed{x = \pm\sqrt{2}}$ Answer

(iv) $f(x) = \cos 4x$

Sol:- 1



Sol:- $f'(x) = -\sin 4x \cdot 4$

$f'(x) = -4\sin 4x$

put $f'(x) = 0$

$-4\sin 4x = 0$

$4x = \sin^{-1}(0) \Rightarrow 4x = \pi$ $\quad 4x = n\pi \Rightarrow \boxed{x = \frac{n\pi}{4}}$ $\quad n \in Z$

(v) $f(x) = (4x-3)^{1/3}$

Sol:- $f'(x) = \frac{1}{3}(4x-3)^{-2/3}(4)$

$f'(x) = \dfrac{4}{3(4x-3)^{2/3}}$

put $f'(x) = 0$

$\dfrac{4}{3(4x-3)^{2/3}} = 0$ or $\dfrac{4}{3(4x-3)^{2/3}} = (0)(3)$

put $(4x-3)^{2/3} = 0$

$\left[(4x-3)^{2/3}\right]^{3/2} = (0)^{3/2}$

$4x - 3 = 0$

$\boxed{x = \frac{3}{4}}$ Answer

(vi) $f(x) = x^2(x+1)^3$

Sol:- $f'(x) = x^2\left[3(x+1)^2(1)\right] + (x+1)^3(2x)$

$f'(x) = 3x^2(x+1)^2 + 2x(x+1)^3$

$f'(x) = x(x+1)^2\left[3x + 2(x+1)\right]$

$f'(x) = x(x+1)^2(5x+2)$

put $f'(x) = 0$

$x(x+1)^2(5x+2) = 0$

$\boxed{x = 0}$ $\quad (x+1)^2 = 0$ $\quad 5x+2 = 0$

$\boxed{x = -1}$ $\quad \boxed{x = -\frac{2}{5}}$

Q no 2 :- (i) $f(x) = -x^2 + 6x \; ; [1, 4]$

~~Solution~~ $f'(x) = -2x + 6$

put $f'(x) = 0$

$-2x + 6 = 0$

$\cancel{2x} = \cancel{6}\,3$

$\boxed{x = 3}$

Hence $3 \in [1, 4]$

$f(1) = -1 + 6 = 5$

$f(3) = -9 + 18 = 9$

$f(4) = -16 + 24 = 8$

max value $= 9$ at $x = 3$

min value $= 5$ at $x = 1$

(ii) $f(x) = (x-1)^2 \; ; [2, 5]$

Sol $f'(x) = 2(x-1)(1)$

put $f'(x) = 0$

$2(x-1) = 0$

$\boxed{x = 1} \notin [2, 5]$

$f(2) = (2-1) = \boxed{1}$

$f(5) = (5-1)^2 = \boxed{16}$

max value. 16 at $x = 5$

min value: 1 at $x = 2$

(iii) $f(x) = x^{2/3} \; ; [-1, 8]$

Sol:- $f'(x) = \frac{2}{3} x^{-1/3}$

$f'(x) = \frac{2}{3x^{1/3}}$

put $f'(x) = 0$

$\frac{2}{3x^{1/3}} = 0$

$\frac{2}{x^{1/3}} = (0)(3)$

now $x^{1/3} = 0$ for undefine

$\boxed{x = 0}$ critical value $\in [-1, 8]$

~~max value:~~

~~max at~~

$f(-1) = [(-1)^2]^{1/3} = (1)^{1/3} = 1$

$f(0) = 0$

$f(8) = (8)^{2/3} = 4$

Max Value: 4 at $x = 8$

Min Value: 0 at $x = 0$

(iv) $f(x) = x^3 - 6x^2 + 2$; $[-3, 2]$

Sol: $f'(x) = 3x^2 + 12x$

put $f'(x) = 0$

$0 = 3x^2 - 12x$

$3x(x-4) = 0$

$\boxed{x = 0}$ $\boxed{x = 4}$

$\boxed{f(-3) = -79}$

$\boxed{f(0) = 2}$

$\boxed{f(2) = -14}$

Max value 2 at $x = 0$

Min Value: -79 at $x = -3$

(v) $f(x) = 1 + 5\sin 3x$; $[0, \pi/2]$

Sol: $f'(x) = 5\cos 3x \cdot 3$

$f'(x) = 15\cos 3x$

$f'(x) = 0$

$15\cos 3x = 0$

$\cos 3x = 0$

$3x = \cos^{-1}(0)$

$3x = \frac{\pi}{2}$

$\boxed{x = \pi/6} \in \left[0, \frac{\pi}{2}\right]$

$f(0) = 1 + 5\sin 3(0) = 1$

$f\left(\frac{\pi}{6}\right) = 1 + 5\sin 3\left(\frac{\pi}{6}\right) = 6$

$f(\pi/2) = 1 + 5\sin\left(\frac{3\pi}{2}\right) = -4$

Max value: 6 at $x = \pi/6$

Min value: -4 at $x = \pi/2$

(vi) $f(x) = 2\cos 2x - 4\cos x$

$[0, 2\pi]$

Sol: $f'(x) = -2 \cdot 2\sin 2x + 4\sin x$

put $f'(x) = 0$

$-4\sin 2x + 4\sin x = 0$

$4(-\sin 2x + \sin x) = 0$

$-2\sin x\cos x + \sin x = 0$

$\sin x[-2\cos x + 1] = 0$

$\sin x = 0$

$x = \sin^{-1}(0)$

$x = \boxed{0}, \boxed{\pi}, \boxed{2\pi}$

$-2\cos x + 1 = 0$

$\cos x = \frac{1}{2}$

$x = \cos^{-1}\left(\frac{1}{2}\right)$

$x = \boxed{\pi/3}, \boxed{5\pi/3}$

$f(0) = 2\cos(0) - 4\cos(0) = -2$

$f(\pi) = 2\cos(2\pi - 4\cos\pi) = 6$

$f(2\pi) = 2\cos(4\pi) - 4\cos(2\pi) = -2$

$f(\pi/3) = 2\cos\left(\frac{2\pi}{3}\right) - 4\cos\left(\frac{\pi}{3}\right) = \boxed{-3}$

$f\left(\frac{5\pi}{3}\right) = 2\cos\left(\frac{10\pi}{3}\right) - 4\cos\left(\frac{5\pi}{3}\right) = \boxed{-3}$

Max value is 6 at $x = \pi$

Min value is -3 at

$x = \pi/3, 5\pi/3$ Ans

Q3. (i) $f(x) = -x^2 + 7x$

Sol:- $f'(x) = -2x + 7$

$f''(x) = -2 < 0$

Concave downward Ans

(ii) $f(x) = -x^3 + 6x^2 + x^{-1}$

Sol:- $f'(x) = -3x^2 + 12x + 1$

$f''(x) = -6x + 12$

put $f''(x) = 0$

$0 = -6x + 12$

$12 = 6x$

$\boxed{x = 2}$

if $x < 2$ let $x = 1$

$f''(1) = -6(1) + 12 = 6 > 0$

Concave upward.

if $x > 2$ let $x = 3$

$f''(3) = -18 + 12 = -6 < 0$

Concave downward.

Concave up $(-\infty, 2)$

Concave down $(2, \infty)$.

(iii) $f'(x) = (x+5)^5$

Sol:- $f'(x) = 5(x+5)^4$

$f''(x) = 20(x+5)^3$

put $f''(x) = 0$

$20(x+5)^3 = 0$

(iii) $((x+5)^3)^{1/3} = (0)^{1/3}$

$x + 5 = 0$

$\boxed{x = -5}$

$x < -5$ let $x = -6$

$f''(-6) = 20(-6+5)^3 = -20 < 0$

Concave down

if $x > -5$

$f''(-4) = 20(-4+5)^3 = 20 > 0$

Concave upward.

Concave up $(-5, +\infty)$

Concave down $(-\infty, -5)$ Ans

(iv) $f(x) = x(x-4)^3$

Sol:- $f'(x) = x[3(x-4)^2] + (x-4)^3(1)$

$f'(x) = 3x(x-4)^2 + (x-4)^3$

$f''(x) = 3[x\{2(x-4)\} + (x-4)^2(1)] + 3(x-4)^2$.

$f''(x) = 3(x-4)[2x + x - 4 + x - 4]$

$\boxed{f''(x) = 3(x-4)(4x-8)}$

$f''(x) = 0$

$0 = 3(x-4)(4x-8)$

$x - 4 = 0$ or $4x - 8 = 0$

$\boxed{x = 4}$; $\boxed{x = 2}$

$(-\infty,2),(2,4),(4,\infty)$

Check $(-\infty,2)$

let $x=1$

$f''(1)=3(-3)(-4)=36>0$ Concave upward.

Check $(2,4)$

let $x=3$

$f''(3)=3(-1)(4)=-12<0$ Concave down

Check $(4,\infty)$.

let $x=5$

$f''(5)=3(1)(12)=36>0$ Concave upward

$(-\infty,2)$ and $(4,\infty)$ concave upward and $(2,4)$ concave downward.

(v). $f(x)=x^{1/2}+2x$

Sol $f'(x)=\frac{1}{2}x^{-1/2}+2$

$f''(x)=\frac{1}{2}\left[-\frac{1}{2}x^{-3/2}\right]=\boxed{\frac{-1}{4x^{3/2}}}$

Domain $f''(x)$ $(0,\infty)$ let $x=1$.

$f''(1)=\frac{-1}{4\sqrt{(1)^3}}=-\frac{1}{4}<0$

Concave downward.

Hence, $(0,\infty)$ concave downward

(vi) $f(x)=x+\frac{9}{x}$ or $x+9x^{-1}$

Sol $f'(x)=1+9\left[-1x^{-2}\right]=1-9x^{-2}$

$f''(x) = -9[-2x^{-3}]$

$$\boxed{f''(x) = \frac{18}{x^3}}$$

Domain $f''(x)$ $(-\infty, 0)$ & $(0, \infty)$
$R-[0]$

Check $(-\infty, 0)$
let $x = -1$

$f''(-1) = \frac{18}{(-1)^3} = -18 < 0$ Concave down

Check $(0, +\infty)$ let $x = 1$
$f''(1) = \frac{18}{(1)^3} = 18 > 0$ Concave up

Hence $(-\infty, 0)$ concave downward and $(0, +\infty]$ concave upward

Q4) (i) $f(x) = x^4 - x^3 + 2x^2 + x - 1$

Sol:- $f'(x) = 4x^3 - 3x^2 + 4x + 1$

$f''(x) = 12x^2 - 6x + 4$

put $f''(x) = 0$

$2(6x^2 - 3x + 2) = 0$

$6x^2 - 3x + 2 = 0$

$a = 6 \quad b = -3 \quad c = 2$

$x = \frac{3 \pm \sqrt{9 - 4(12)}}{2(6)}$

$x = \frac{3 \pm \sqrt{-39}}{12}$ no real point of Inflection.

(ii) $f(x) = x^{5/3} + 4x$

$f'(x) = \frac{5}{3} x^{2/3} + 4$

$f''(x) = \frac{5}{3}\left(\frac{2}{3}\right) x^{-1/3}$

$f''(x) = \frac{10}{9x^{1/3}}$

Hence no point of Inflection

(iii) $f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

put $f''(x) = 0$

$-\sin x = 0$

$\sin x = 0$

$\boxed{x = n\pi \;;\; n \in Z}$

Hence Point of Inflection

(iv) $f(x) = \cos x$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

put $f''(x) = 0$

$-\cos x = 0$

$\cos x = 0$

$x = \cos^{-1}(0)$

$\boxed{x = (2n+1)\frac{\pi}{2} \;,\; n \in Z}$ Answer

(v) $f(x) = x - \sin x$

$f'(x) = 1 - \cos x$

$f''(x) = -(-\sin x)$

$f''(x) = \sin x$

put $f''(x) = 0$

$\sin x = 0$

$\boxed{x = n\pi \,,\, n \in Z}$

(vi) $f(x) = \tan x$

$\tan x$ is undefined

$x = (2n+1)\frac{\pi}{2} \;,\; n \in Z$

$f'(x) = \sec^2 x$

$f''(x) = 2\sec x \cdot (\sec x \tan x)$

$f''(x) = 2\sec^2 x \tan x$

$f''(x) = \dfrac{2\tan x}{\cos^2 x}$

put $f''(x) = 0$

$\cos^2 x \neq 0$

$2\tan x = 0$

$\tan x = 0 \rightarrow \dfrac{\sin x}{\cos x} = 0$, $\cos x \neq 0$

$x = n\pi \;;\; n \in Z$

Q5 (i) $f(x) = -(-2x+5)^2$

Sol: $f'(x) = -2[(-2x-5)(-2)]$

$f'(x) = 4(-2x-5)$

$f'(x) = -8x-20$

put $f'(x) = 0$

$0 = -8x-20$

$x = -\frac{20}{8} \Rightarrow \boxed{x = -\frac{5}{2}}$

$f''(x) = -8(1) = \boxed{-8}$

$f''\left(-\frac{5}{2}\right) = -8 < 0$ **Relative maximum**

at $x = -\frac{5}{2}$ Answer

(ii) $f(x) = x^3+3x^2+3x+1$

Sol: $f'(x) = 3x^2+6x+3$

$f''(x) = 6x+6$

put $f'(x) = 0$

$3(x^2+2x+1) = 0$

$x^2+2x+1 = 0$

$(x+1)^2 = 0$

$\boxed{x = -1}$ critical value

$f''(-1) = 6(-1)+6$

$\boxed{f''(-1) = 0}$

Inclusive at $x = -1$.

(iii) $f(x) = 6x^5-10x^2$

Sol: $f'(x) = 30x^4-20x$

$f''(x) = 120x^3-20$

$f'(x) = 0$

$30x^4-20x = 0$

$10x(3x^3-2) = 0$

$10x = 0$; $3x^3-2 = 0$

$\boxed{x = 0}$ $\boxed{x = \sqrt[3]{\frac{-2}{3}}}$

$f''(0) = -20 < 0$ **max at** $\boxed{x = 0}$.

$f''\left(\sqrt[3]{\frac{2}{3}}\right) = 120\left(\sqrt[3]{\frac{2}{3}}\right)^3 - 20$

$= 60 > 0$ **minimum at $x = \sqrt[3]{\frac{2}{3}}$**

(iv) $f(x) = x^2 + \frac{1}{x^2} = x^2 + x^{-2}$

Sol: $f'(x) = 2x + (-2x^{-3})$

$f''(x) = 2 + 6x^{-4} = 2 + \frac{6}{x^4}$

put $f'(x) = 0$

$2x - \frac{2}{x^3} = 0$ $x^3 \neq 0$.

$2x^4 - 2 = 0$

$2(x^4-1) = 0$

$x^4 = 1$

$\boxed{x = 1}$ critical value.

$f''(1) = 2 + \frac{6}{(1)^4} = \boxed{8 > 0}$

(v) $f(x) = \cos 3x$, $[0, 2\pi]$.

Sol: $f'(x) = -3\sin 3x$

$f''(x) = -3.3\cos 3x = -9\cos 3x$.

put $f'(x) = 0$

$\sin 3x = 0$

$3x = \sin^{-1}(0)$

$3x = n\pi$

$x = \frac{n\pi}{3}, n \in Z$.

Critical values $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$.

$f''(0) = -9\cos 3(0) = -9 < 0$ local maximum

$f''(\frac{\pi}{3}) = -9\cos 3(\frac{\pi}{3}) = 9 > 0$ local minimum.

$f''(\frac{2\pi}{3}) = -9\cos 3(\frac{2\pi}{3}) = -9 < 0$ local max

$f''(\pi) = -9\cos 3(\pi) = 9 > 0$ local min

$f''(\frac{4\pi}{3}) = -9\cos 3(\frac{4\pi}{3}) = -9 < 0$ local maximum

$f''(\frac{5\pi}{3}) = -9\cos 3(\frac{5\pi}{3}) = 9 > 0$ local min.

$f''(2\pi) = -9\cos 3(2\pi) = -9 < 0$ local max

(vi) $f(x) = \cos x + \sin x$, $[0, 2\pi]$.

Sol: $f'(x) = -\sin x + \cos x$

$\boxed{f''(x) = -\cos x - \sin x}$

put $f'(x)=0 \Rightarrow -\sin x+\cos x=0$

$\frac{\sin x}{\cos x}=\frac{\cos x}{\cos x} \Rightarrow \tan x=1$

$x=\tan^{-1}(1)$

$\boxed{x=\frac{\pi}{4},\frac{5\pi}{4}}$.

$f''\left(\frac{\pi}{4}\right)=-\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{4}\right)=-\sqrt{2}$

$f''\left(\frac{\pi}{4}\right)=-\sqrt{2}<0$ local max.

$f''\left(\frac{5\pi}{4}\right)=-\cos\left(\frac{5\pi}{4}\right)-\sin\left(\frac{5\pi}{4}\right)=\sqrt{2}>0$ local min

Q6. (i) $f(x)=\cos x\sin x$; $x=\frac{\pi}{4}$

Sol: $f'(x)=\cos x[\cos x]+\sin x[-\sin x]$

$f'(x)=\cos^2 x-\sin^2 x$

$\boxed{f'(x)=\cos 2x}$

put $x=\frac{\pi}{4}$

$f'\left(\frac{\pi}{4}\right)=\cos 2\left(\frac{\pi}{4}\right)=0$

Hence $x=\frac{\pi}{4}$ is critical point.

$f''(x)=-2\sin 2x$

$f''\left(\frac{\pi}{4}\right)=-2\sin 2\left(\frac{\pi}{4}\right)=-2<0$

local maximum at $x=\frac{\pi}{4}$

Ans

(ii) $f(x) = x\sin x \quad , x=0$

Sol $f'(x) = x\cos x + \sin x(1)$

put $x=0$

$f'(0) = 0 + \sin(0)$

$\boxed{f'(0) = 0}$

$\boxed{x=0}$ is critical point.

$f''(x) = x[-\sin x] + \cos x(1) + \cos x$

$f''(x) = -x\sin x + 2\cos x$.

$f''(0) = 0 + 2\cos(0)$

$f''(0) = 2(1) = 2 > 0$

local min at $x=0$.

(iii) $f(x) = \tan^2 x \; ; x = \pi$.

Sol $f'(x) = 2\tan x \sec^2 x$

put $x=\pi$

$f'(\pi) = \dfrac{2\tan\pi}{\cos^2\pi}$

$\tan(\pi) = 0$

$f'(\pi) = 0 \Rightarrow \boxed{x=\pi}$ is critical point.

$f''(x) = 2[\tan x\{2\sec x \cdot \sec x \tan x\} + \sec^2 x\{\sec^2 x\}]$

$f''(\pi) = 2[\tan\pi\{2\sec^2\pi \tan\pi\} + \sec^4\pi]$

$f''(\pi) = 2\left[\dfrac{1}{\cos^4\pi}\right] = 2\left[\dfrac{1}{(-1)^4}\right]$

$f''(\pi) = 2 > 0$ local minimum

at $x=\pi$.

(iv) $f(x) = (1+\sin x)^3$, $x = \frac{\pi}{8}$

Sol: $f'(x) = 3(1+\sin x)^2(0+\cos x)$

$f'(x) = 3\cos x(1+\sin x)^2$

put $x = \frac{\pi}{8}$

$f'\left(\frac{\pi}{8}\right) = 3\cos\frac{\pi}{8}\left(1+\sin\left(\frac{\pi}{8}\right)\right)^2$

$= 3(0.92)(1+0.38)^2 \neq 0$

$f'\left(\frac{\pi}{8}\right) \neq 0$ Hence $\frac{\pi}{8}$ is not critical value. So we cannot find local extrema at $\boxed{x = \frac{\pi}{8}}$