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Where Minds Pollinate

Class 12
Mathematics

Chapter 2

Exercise 2.8

Handwritten Solution

Exercise 2.8

Concepts: First Order Derivative.

$$f'(x) = \frac{dy}{dx}$$

- **Second Order Derivative:**

$$f''(x) = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

- **Third Order Derivative.**

$$f'''(x) = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$$

- **n^{th} Order Derivative:**

$$f^n(x) = \frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right) = \frac{d^ny}{dx^n}$$

Q1) $y = -x^3 + 6x + 9 \longrightarrow$ Diff w.r.t x.

Sol:- $\frac{dy}{dx} = -3x^2 + 6$

$$\boxed{\frac{d^2y}{dx^2} = -6x}$$

Q2) $y = 30x^2 - x^3$

Sol: Diff w.r.t x

$$\frac{d}{dx}y = 30\frac{d}{dx}(x^2) - \frac{d}{dx}(x^3)$$

$$\frac{dy}{dx} = 30[2x] - [3x^2]$$

$$\frac{dy}{dx} = 60x - 3x^2$$

Again diff w.r.t x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = 60\frac{d}{dx}x - 3\frac{d}{dx}(x^2)$$

$$\frac{d^2y}{dx^2} = 60(1) - 3(2x)$$

$$\frac{d^2y}{dx^2} = 60 - 6x$$

$\boxed{\text{Answer}}$

Q3 $y=(-5x+9)^2$

Sol Diff w.r.t "x"

$$\frac{d}{dx}(y)=\frac{d}{dx}(-5x+9)^2$$

$$\frac{d}{dx}y=2(-5x+9)^{2-1}\cdot\frac{d}{dx}(-5x+9)$$

$$\frac{dy}{dx}=2(-5x+9)[-5(1)+0]$$

$$\frac{dy}{dx}=-10(-5x+9)$$

$$\frac{dy}{dx}=50x-90$$

again diff w.r.t x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)=50\frac{d}{dx}x-\frac{d}{dx}(90)$$

$$\frac{d^2y}{dx^2}=50(1)-0$$

$$\boxed{\frac{d^2y}{dx^2}=50}$$ Answer

follow same pattern on others questions as well. Here directly steps are done.

Q4) $y=2x^6+5x^3-6x^2$

Sol $\frac{dy}{dx}=12x^5+15x^2-6x^2$

$$\boxed{\frac{d^2y}{dx^2}=60x^4+30x-12}$$ Answer

Q5 $y=20x^{-3}$

$$\frac{dy}{dx}=-60x^{-4}$$

$$\boxed{\frac{d^2y}{dx^2}=240x^{-5}}$$ A

Q6) $y = \frac{2}{x^4}$ or $2x^{-4}$

Sol $\frac{dy}{dx} = -8x^{-5}$

$\boxed{\frac{d^2y}{dx^2} = +40x^{-6}}$ ans

Q7) $f(x) = x^2(3x-4)^3$

Sol $f'(x) = x^2[3(3x-4)^2(3(1)-0)] + (3x-4)^3(2x)$

$f'(x) = 9x^2(3x-4)^2 + 2x(3x-4)^3$

$f''(x) = 9[x^2(2(3x-4)(3)) + (3x-4)^2(2x)] + 2[x(3(3x-4)^2(3)) + (3x-4)^3(1)]$

$f''(x) = 9[6x^2(3x-4) + 2x(3x-4)^2] + 2[9x(3x-4)^2 + (3x-4)^3]$

Answer

Q8) $f(x) = (x^2+5x-1)^4$

Sol: $f'(x) = 4(x^2+5x-1)^3(2x+5)$

$f''(x) = 4[(x^2+5x-1)^3(2) + (2x+5)[3(x^2+5x-1)^2(2x+5)]]$

$f''(x) = 4[2(x^2+5x-1)^3 + 3(2x+5)^2(x^2+5x-1)^2]$ Ans

Q9) $f(x) = \cos 10x$

Sol $f'(x) = -\sin 10x(10)$

$f''(x) = -10[\cos 10x(10)]$

$\boxed{f''(x) = -100\cos 10x}$

Q10) $f(x) = \tan\frac{x}{2}$

Sol:- $f'(x) = \sec^2\frac{x}{2}\left(\frac{1}{2}\right)$

$f''(x) = \frac{1}{2}\left[2\sec\frac{x}{2}\frac{d}{dx}\left(\sec\frac{x}{2}\right)\right]$

$f''(x) = \sec\frac{x}{2}\left[\sec\frac{x}{2}\tan\frac{x}{2}\left(\frac{1}{2}\right)\right]$

$\boxed{f''(x) = \frac{1}{2}\sec^2\frac{x}{2}\tan\frac{x}{2}}$

Ans

Q11) $f(\theta) = \sin^2 5\theta$

Sol:- $f'(\theta) = 2\sin 5\theta \frac{d}{d\theta}(\sin 5\theta)$

$f'(\theta) = 2\sin 5\theta \cdot \cos 5\theta (5)$

$f'(\theta) = 10\sin 5\theta \cos 5\theta$

$f''(\theta) = 10[\sin 5\theta \cdot (-5\sin 5\theta) + \cos 5\theta (5\cos 5\theta)]$

$f''(\theta) = 10[-5\sin^2 5\theta + 5\cos^2 5\theta]$

$f''(\theta) = 50[\cos^2 5\theta - \sin^2 5\theta]$ Answer.

Q12. $f(\theta) = \frac{1}{3+2\cos\theta}$

Sol:- $f(\theta) = (3+2\cos\theta)^{-1}$

$f'(\theta) = -1(3+2\cos\theta)^{-2}\frac{d}{d\theta}(3+2\cos\theta)$

$f'(\theta) = (3+2\cos\theta)^{-2}(2\sin\theta)$

$f''(\theta) = (3+2\cos\theta)^{-2}[2\cos\theta] + 2\sin\theta[-2(3+2\cos\theta)^{-3}(-2\sin\theta)]$

$f''(\theta) = 2(3+2\cos\theta)^{-2}[\cos\theta + 4\sin^2\theta(3+2\cos\theta)^{-1}]$ Answer

Q13 $f(x) = e^{2x}(x^2+1)$

Sol:- $f'(x) = e^{2x}(2x) + (x^2+1)e^{2x} \cdot 2$

$f'(x) = 2e^{2x}[x + x^2 + 1]$

$f''(x) = 2[e^{2x}(1+2x) + (x^2+x+1)(e^{2x} \cdot 2)]$

$f''(x) = 2[e^{2x} + 2xe^{2x} + 2e^{2x}(x^2+x+1)]$

$f''(x) = 2e^{2x}[1+2x+2(x^2+x+1)]$

$f''(x) = 2e^{2x}[1+2x+2x^2+2x+2]$

$f''(x) = 2e^{2x}[2x^2+4x+3]$ Answer.

(14) $f(x) = (x^2+1)\ln(x^2+1)$

Sol:- $f'(x) = (x^2+1)\dfrac{1}{(x^2+1)}(2x) + \ln(x^2+1)\cdot(2x)$

$$f'(x) = 2x\left[1+\ln(x^2+1)\right]$$

$$f''(x) = 2\left\{x\left(\frac{1}{x^2+1}(2x)\right) + \left(1+\ln(x^2+1)\right)\cdot(1)\right]$$

$$f''(x) = 2\left[\frac{2x^2}{x^2+1} + 1+\ln(x^2+1)\right]$$ Answer.

Q15 $y = 4x^7 + x^6 - x^4$; $\dfrac{d^4y}{dx^4}$

Sol:- $\dfrac{dy}{dx} = 28x^6 + 6x^5 - 4x^3$

$$\frac{d^2y}{dx^2} = 168x^5 + 30x^4 - 12x^2$$

$$\frac{d^3y}{dx^3} = 840x^4 + 120x^3 - 24x$$

$$\frac{d^4y}{dx^4} = 3360x^3 + 360x^2 - 24$$ Ans

Q16) $y = \dfrac{2}{x}$; $\dfrac{d^5y}{dx^5}$

Sol: $y = 2x^{-1}$

$$\frac{dy}{dx} = -2x^{-2}$$

$$\frac{d^2y}{dx^2} = 4x^{-3}$$

$$\frac{d^3y}{dx^3} = -12x^{-4}$$

$$\frac{d^4y}{dx^4} = 48x^{-5}$$

$$\boxed{\frac{d^5y}{dx^5} = -240x^{-6}}$$

Q17 $f(x) = \cos\pi x$; $f'''(x)$

$f'(x) = -\sin\pi x \cdot (\pi(1))$

$f''(x) = -\pi[\cos(\pi x)\cdot\pi]$

$f'''(x) = -\pi^2[-\sin(\pi x)\cdot\pi]$

$f'''(x) = \pi^3 \sin(\pi x)$ Answer

Q18 $f(x) = \dfrac{1}{\sec(2x+1)}$; $f^{(4)}(x)$

Sol $f(x) = \cos(2x+1)$

$f'(x) = -\sin(2x+1)\cdot(2(1)+0)$

$f'(x) = -2\sin(2x+1)$

$f''(x) = -2\cdot 2\cos(2x+1)$

$f'''(x) = -4[-\sin(2x+1)\cdot 2]$

$f'''(x) = +8\sin(2x+1)$

$f^4(x) = 8\cos(2x+1)\cdot 2$

$f^4(x) = 16\cos(2x+1)$ Ans

Q19 Sol a) $f(x) = x^3+2x$

$\boxed{f'(x) = 3x^2+2}$

$\boxed{f''(x) = 6x}$

$f''(x+\Delta x) = 6(x+\Delta x)$

$\boxed{f''(x+\Delta x) = 6x+6\Delta x}$

b) $f'''(x) = \lim\limits_{\Delta x \to 0} \dfrac{f''(x+\Delta x) - f''(x)}{\Delta x}$

$f'''(x) = \lim\limits_{\Delta x \to 0} \dfrac{6x+6\Delta x-6x}{\Delta x}$

$f'''(x) = \lim\limits_{\Delta x \to 0} \dfrac{6\cancel{\Delta x}}{\cancel{\Delta x}}$

$\boxed{f'''(x) = 6}$

A

20) Proof: (a) $\frac{d}{dx}(fg) = fg' + gf'$

$$\frac{d^2}{dx^2}(fg) = [fg'' + g'f'] + [gf'' + f'g']$$

$$\boxed{\frac{d^2}{dx^2}(fg) = f''g + 2f'g' + fg''}$$ Proved.

(b) $\frac{d^3}{dx^3}(fg) = f'''g + 3f''g' + 3f'g'' + fg'''$

$$\frac{d^3}{dx^3}(fg) = \frac{d}{dx}\left[\frac{d^2}{dx^2}(fg)\right] = \frac{d}{dx}\left[f''g + 2f'g' + fg''\right]$$

$$= \left[f''g' + gf'''\right] + 2\left[f'g'' + g'f''\right] + \left[fg''' + g''f'\right]$$

$$= \boxed{f'''g + 3f''g + 3f'g'' + fg'''}$$ Ans.