Studybeesofficial

Where Minds Pollinate

Class 12
Mathematics

Chapter 2

Exercise 2.6

Handwritten Solution

Exercise 2.6

Rules: (i) $\frac{d}{dx}(\sin x) = \cos x \frac{dx}{dx}$

(ii) $\frac{d}{dx}(\cos x) = -\sin x$

(iii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(iv) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$.

(v) $\frac{d}{dx}(\sec x) = \sec \cdot \tan x$

(vi) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

Inverse function:-

(i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$.

(ii) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$ for $-1 < x 1$.

(iii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$ for $x \in \mathbb{R}$

(iv) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$ for $x \in \mathbb{R}$

(v) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$ for $|x| > 1$

(vi) $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}$ for $|x| > 1$

Prove that $\frac{d}{dx}(\sin x) = \cos x$;

Sol:- $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$f(x) = \sin x$

$f(x+\Delta x) = \sin(x+\Delta x)$

now

$$\frac{d}{dx}(\sin x) = \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin x}{\Delta x}$$

$\because \sin p - \sin q = 2\cos\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$

$$= \lim_{\Delta x \to 0} \frac{2\cos\left(\frac{x+\Delta x + x}{2}\right) \cdot \sin\left(\frac{\cancel{x}+\Delta x - \cancel{x}}{2}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cos\left(\frac{2x+\Delta x}{2}\right)\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$= \lim_{\Delta x \to 0} \cos\left(\frac{2x+\Delta x}{2}\right) \cdot \lim_{\Delta x \to 0} \frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

if $\Delta x \to 0$ then $\frac{\Delta x}{2} \to 0$

$$\boxed{\lim_{\Delta x \to 0} \frac{\sin x}{x} = 1}$$

$$\lim_{\Delta x \to 0} \cos\left(\frac{2x+\Delta x}{2}\right) \cdot 1$$

Apply limit

$$\frac{d}{dx}(\sin x) = \cos\left(\frac{\cancel{2}x}{\cancel{2}}\right)$$

$$\boxed{\frac{d}{dx}(\sin x) = \cos x}$$

Prove: $\frac{d}{dx}(\tan x) = \sec^2 x$

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right).$$

$$= \frac{\cos x \cdot (\cos x) - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$$

$$\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x}$$

$$\boxed{\frac{d}{dx}(\tan x) = \sec^2 x}$$ Proved

Prove $\frac{d}{dx}(\sec x) = \sec x \tan x$

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$= \frac{\cos x(0) - 1(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$\frac{d}{dx}(\sec x) = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$\boxed{\frac{d}{dx}(\sec x) = \sec x \tan x}$$ Proved

Trick

$\tan x \longrightarrow \sec x \longleftarrow \sec x$

$\cot x \longrightarrow \operatorname{cosec} x \longleftarrow \operatorname{cosec} x.$

Prove that: $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

Sol: let $y = \cos^{-1}x$

$\cos y = x$

Diff w.r.t x

$-\sin y \cdot \frac{d(y)}{dx} = \frac{d}{dx}(x)$

$-\sin y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{-1}{\sin y}$

$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

$\because \sin^2 y + \cos^2 y = 1$

$\sqrt{\sin^2 y} = \sqrt{1-\cos^2 y}$

$\sin y = \sqrt{1-\cos^2 y}$

$\boxed{\sin y = \sqrt{1-x^2}}$

Prove $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

let $y = \cot^{-1}x$

$\cot y = x$

Diff w.r.t x

$-\operatorname{cosec}^2 y \frac{d}{dx}(y) = \frac{d}{dx}(x)$

$-\operatorname{cosec}^2 y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{-1}{\operatorname{cosec}^2 y}$

$\boxed{\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}}$

$\therefore \boxed{1+\tan^2 y = \sec^2 y}$

$\therefore \boxed{1+\cot^2 y = \operatorname{cosec}^2 y}$

$\boxed{1+x^2 = \operatorname{cosec}^2 y}$

Q Prove $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}$

Sol: let $y = \operatorname{cosec}^{-1}x$

$\operatorname{cosec} y = x$

Diff w.r.t x

$-\operatorname{cosec} y \cot y \frac{d}{dx}(y) = \frac{d}{dx}(x)$

$\frac{dy}{dx} = \frac{1}{-\operatorname{cosec} y \cot y}$

$\boxed{\frac{dy}{dx} = \frac{-1}{|x|\sqrt{x^2-1}}}$ Proved

$\because \sqrt{\cot^2 y} = \sqrt{\operatorname{cosec}^2 y - 1}$

$\therefore \cot y = \sqrt{x^2-1}$

Q1 $y = x^2 - \cos x$

Sol: $\frac{dy}{dx} = 2x - (-\sin x)$

$\frac{dy}{dx} = 2x + \sin x$

Q2. $y = 4x^3 + x + \sin x$

Sol: $\frac{dy}{dx} = 4(3x^2) + 1 + \cos x$

$\boxed{\frac{dy}{dx} = 12x^2 + 1 + \cos x}$ Ans

Q3 $y = 3\cos x - 5\cot x$

Sol $\frac{dy}{dx} = 3(-\sin x) - 5(-\operatorname{cosec}^2 x)$

$\boxed{\frac{dy}{dx} = -3\sin x + 5\operatorname{cosec}^2 x}$ Ans

Q4) $y = \sin x - \cos x$

Sol $\frac{dy}{dx} = \sin x \cdot \frac{d}{dx}(\cos x)$

$+ \cos x \cdot \frac{d}{dx}(\sin x)$

$\frac{dy}{dx} = \sin x(-\sin x) + \cos x(\cos x)$

$\frac{dy}{dx} = -\sin^2 x + \cos^2 x$

$\frac{dy}{dx} = \cos^2 x - \sin^2 x$

$\boxed{\frac{dy}{dx} = \cos 2x}$

Proved

Q5) $y = (x^2 + \sin x)\sec x$

Sol: $\frac{dy}{dx} = (x^2+\sin x)\cdot(\sec x \tan x) + \sec x(2x + \cos x)$

$\frac{dy}{dx} = x^2 \sec x \tan x + \sin x \sec x \tan x + 2x\sec x + \cos x \cdot \sec x$

$= x^2 \cdot \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} + \sin x \cdot \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} + 2x\sec x + \cos x \cdot \frac{1}{\cos x}$

$\frac{dy}{dx} = \frac{x^2 \sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} + 2x\sec x + 1$

$$\boxed{\frac{dy}{dx} = \frac{x^2 \sin x}{\cos^2 x} + \tan^2 x + 2x\sec x + 1}$$

Q6. $y = \frac{5-\cos x}{5+\sin x}$

$\frac{dy}{dx} = \frac{(5+\sin x)(0-(-\sin x)) - (5-\cos x)(0+\cos x)}{(5+\sin x)^2}$

$\frac{dy}{dx} = \frac{5\sin x + \sin^2 x - (5\cos x - \cos^2 x)}{(5+\sin x)^2}$

$\frac{dy}{dx} = \frac{5\sin x + \sin^2 x - 5\cos x + \cos^2 x}{(5+\sin x)^2}$

$$\boxed{\frac{dy}{dx} = \frac{1 + 5\sin x - 5\cos x}{(5+\sin x)^2}}$$

(7) $y = \frac{\sec x}{1+\tan x}$

Sol $\frac{dy}{dx} = \frac{(1+\tan x)(\sec x \tan x) - (\sec x)(0+\sec^2 x)}{(1+\tan x)^2}$

$$= \frac{\sec x \tan x + \sec x \tan^2 x - \sec^3 x}{(1+\tan x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{\sec[\tan x + \tan^2 x - \sec^2 x]}{(1+\tan x)^2}}$$ Answer

(8) $y = \dfrac{\sin x}{x^2 + \sin x}$

Sol $\dfrac{dy}{dx} = \dfrac{(x^2+\sin x)(\cos x) - \sin x(2x + \cos x)}{(x^2+\sin x)^2}$

$$= \frac{x^2\cos x + \sin x\cos x - 2x\sin x - \sin x\cos x}{(x^2+\sin x)^2}$$

$$\frac{dy}{dx} = \boxed{\frac{x^2\cos x - 2x\sin x}{(x^2+\sin x)^2}}$$ Answer

(9) $y = \dfrac{\cot x}{x+1}$

Sol:- $\dfrac{dy}{dx} = \dfrac{(x+1)(-\operatorname{cosec}^2 x) - \cot x(1)}{(x+1)^2}$

$$\boxed{\frac{dy}{dx} = \frac{-x\operatorname{cosec}^2 x - \operatorname{cosec}^2 x - \cot x}{(x+1)^2}}$$

(10) $y = (1+\cos x)(x - \sin x)$

Sol $\dfrac{dy}{dx} = (1+\cos x)(1-\cos x) + (x - \sin x)(0 + (-\sin x))$

$$\frac{dy}{dx} = 1 - \cos^2 x + (-x\sin x + \sin^2 x)$$

$$\boxed{\frac{dy}{dx} = 1 - x\sin x + \sin^2 x - \cos^2 x}$$ Answer

(11) $y = \sin^{-1}(5x-1)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(5x-1)^2}} \cdot \frac{d}{dx}(5x-1)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(25x^2+1-10x)}} \cdot (5(1)-0)$$

$$\frac{dy}{dx} = \frac{5}{\sqrt{1-25x^2-1+10x}}$$

$$\boxed{\frac{dy}{dx} = \frac{5}{\sqrt{10x-25x^2}}}$$ Answer

(12) $y = 4\cot^{-1}\frac{x}{2}$

$$\frac{dy}{dx} = 4\left[\frac{-1}{1+\left(\frac{x}{2}\right)^2}\right] \cdot \frac{d}{dx}\left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = 4\left[\frac{-1}{1+\frac{x^2}{4}}\right] \cdot \left(\frac{1}{2}\right)$$

$$\frac{dy}{dx} = 2\left[\frac{-1}{\frac{4+x^2}{4}}\right]$$

$$\frac{dy}{dx} = 2\left(\frac{-4}{4+x^2}\right)$$

$$\boxed{\frac{dy}{dx} = \frac{-8}{x^2+4}}$$

(13) $y = \frac{\sin^{-1}x}{\sin x}$

for $\frac{dy}{dx} = \frac{\sin x \cdot \left(\frac{1}{\sqrt{1-x^2}}\right) - \sin^{-1}x(\cos x)}{(\sin x)^2}$

$$\frac{dy}{dx} = \boxed{\frac{\frac{\sin x}{\sqrt{1-x^2}} - \sin^{-1}x\cos x}{(\sin x)^2}}$$ Ans

Q14) $y = \frac{\sec^{-1}x}{x}$

$$\frac{dy}{dx} = \frac{x\left(\frac{1}{x\sqrt{x^2-1}}\right) - \sec^{-1}x\,(1)}{x^2}$$

$$\frac{dy}{dx} = \boxed{\frac{\frac{1}{\sqrt{x^2-1}} - \sec^{-1}x}{x^2}}$$ Answer

or $\frac{dy}{dx} = \frac{1 - \sec^{-1}x\sqrt{x^2-1}}{x^2\sqrt{x^2-1}}$

(15) $y = x\sin^{-1}x + x\cos^{-1}x$

$$y = x\left[\sin^{-1}x + \cos^{-1}x\right]$$

$$\frac{dy}{dx} = x\left[\frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}}\right] + (\sin^{-1}x + \cos^{-1}x)(1)$$

$$\boxed{\frac{dy}{dx} = \sin^{-1}x + \cos^{-1}x}$$

(16) $y = \frac{1}{\tan^{-1}x^2}$

fa $$\frac{\tan^{-1}x^2(0) - 1\left[\frac{1}{1+(x^2)^2}\right]\cdot\frac{d}{dx}(x^2)}{(\tan^{-1}x^2)^2}$$

$$\frac{dy}{dx} = \frac{\frac{-2x}{1+x^4}}{\frac{(\tan^{-1}x^2)^2}{1}}$$

$$\frac{dy}{dx} = \frac{-2x}{1+x^4} \times \frac{1}{(\tan^{-1}x^2)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{(1+x^4)(\tan^{-1}x^2)^2}}$$ Br