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Where Minds Pollinate

Class 12

Mathematics

Chapter 2

Exercise 2.4

Handwritten Solution

Exercise 24

Concept: $f'(x_0) = \lim_{\Delta x \to 0} \dfrac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$

is called the derivative of f with respect to x_0

Newton used by y' and f'.

Leibniz used by $\dfrac{d}{dx}$

Rule of Derivative:

(1) **Power rule:** $\dfrac{d}{dx}(x^n) = nx^{n-1}$ (depend; independent)

(2) **Derivative of constant:** $\dfrac{d}{dx}(c) = 0$ where C is any constant number.

Q. State and prove that power rule of derivative.

Proof: Statement: $\dfrac{d}{dx}(x^n) = nx^{n-1} \cdot \dfrac{d}{dx}(x)$

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}} \qquad \because \boxed{\frac{dx}{dx} = 1}$$

Proof $\boxed{f(x) = x^n}$ Given function

$f(x+\Delta x) = (x+\Delta x)^n$

$\because (a+b)^n = a^n + na^{n-1}b + \dfrac{n(n-1)}{2!}a^{n-2}b^2 + \ldots\ldots + b^n$ ∴ binomial theorem

$$f'(x^n) = \lim_{\Delta x \to 0} \frac{\left[\cancel{x^n} + nx^{n-1}\Delta x + \frac{n(n-1)}{2}x^{n-2}(\Delta x)^2 + \ldots\ldots(\Delta x)^n\right] - \cancel{x^n}}{\Delta x}$$

$$f'(x^n) = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\left[nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}\Delta x) + \ldots\ldots(\Delta x)^{n-1}\right]}{\cancel{\Delta x}}$$

$$\boxed{f'(x^n) = nx^{n-1}}$$

Proved.

a) $y = x^9$

Solution: $\boxed{\frac{dy}{dx} = 9x^8}$ Answer

(b) $f(x) = 4x^{1/3}$

Sol:- $f'(x) = 4\frac{d}{dx}(x)^{1/3}$

$f'(x) = 4\cdot\frac{1}{3}x^{1/3-1} = \boxed{4/3\, x^{-2/3}}$ Ans

(c) $f(x) = 9$

Sol:- $\boxed{f'(x) = 0}$

(d) $6x^3+3x^2-10$

Sol:- $f'(x) = 6\frac{d}{dx}(x^3)+3\frac{d}{dx}(x^2)-\frac{d}{dx}(10)$

$= 6(3x^2)+3(2x)-0$

$\boxed{f'(x) = 18x^2+6x}$ Answer

Q2) a) $f(x) = \sqrt{5}$

Sol:- (a) $f'(x) = 0$ Answer

(b) $f(x) = \sqrt{5}x$

$f'(x) = \sqrt{5}\frac{d}{dx}(x) = \sqrt{5}(1) = \boxed{\sqrt{5}}$

(c) $f(x) = 5\sqrt{x}$

Sol: $f'(x) = 5\frac{d}{dx}(x^{1/2})$

$= 5\left(\frac{1}{2}x^{1/2-1}\right) \Rightarrow \boxed{f'(x) = \frac{5}{2}x^{-1/2}}$ Ans

(d) $f(x) = \sqrt{5x}$

$f'(x) = \frac{d}{dx}(5x)^{1/2}$

$f'(x) = \frac{1}{2}(5x)^{1/2-1}\cdot\frac{d}{dx}(5x)$

$f'(x) = \frac{1}{2}(5x)^{-1/2}\cdot 5\frac{d}{dx}(x)$

$\boxed{f'(x) = \frac{5}{2}(5x)^{-1/2}}$ Ans

Second method (d)

$f'(x) = \sqrt{5}\cdot\sqrt{x}$

$f'(x) = \sqrt{5}\frac{d}{dx}(x)^{1/2}$

$= \sqrt{5}\left(\frac{1}{2}x^{1/2-1}\right)$

$\boxed{f'(x) = \frac{\sqrt{5}}{2}x^{-1/2}}$ Ans

(c) $f(x) = (x^2+x^3)^3$

Sol: $f'(x) = 3(x^2+x^3)^{3-1}\cdot\frac{d}{dx}(x^2+x^3)$

$\boxed{f'(x) = 3(x^2+x^3)^2\cdot(2x+3x^2)}$ Answer

Q3) (a) $f(x) = x^2(x^3+5)$

Sol: $f(x) = x^5+5x^2$

$f'(x) = 5x^4+5(2x)$

$\boxed{f'(x) = 5x^4+10x}$ Answer

(b) $f(x) = (x+9)(x-9)$

Sol: $f(x) = x^2-9^2$

$f'(x) = 2x-0$

$\boxed{f'(x) = 2x}$ Answer.

(d) $f(x) = -3x^{-8}+2\sqrt{x}$

Sol: $f'(x) = -3\frac{d}{dx}(x^{-8})+2\frac{d}{dx}(x^{1/2})$

$f'(x) = -3(-8x^{-9})+2\left(\frac{1}{2}x^{-1/2}\right)$

$\boxed{f'(x) = 24x^{-9}+x^{-1/2}}$ Answer.

(e) $f(x) = ax^3 + bx^2 + cx + d$ where a, b, c, d are constant.

Sol: $f'(x) = a\left(\frac{d}{dx}(x^3)\right) + b\frac{d}{dx}(x^2) + c\frac{d}{dx}(x) + \frac{d}{dx}(d)$

$f'(x) = a(3x^2) + b(2x) + c(1) + 0$

$\boxed{f'(x) = 3ax^2 + 2bx + c}$ Answer

(f) $f(x) = x^{24} + 2x^{1/2} + 3x^8 + 9x^4$

$f'(x) = 24x^{23} + 2\frac{d}{dx}(x^{1/2}) + 3\frac{d}{dx}(x^8) + 9\frac{d}{dx}(x^4)$

$f'(x) = 24x^3 + 2\left(\frac{1}{2}x^{-1/2}\right) + 3(8x^7) + 9(4x^3)$

$\boxed{f'(x) = 24x^3 + x^{-1/2} + 24x^7 + 36x^3}$

Q4 (a) $y = \frac{x + 2x^{3/2}}{\sqrt{x}}$

Sol: $y = \frac{x}{\sqrt{x}} + \frac{2x^{3/2}}{\sqrt{x}}$

$y = x^{1-1/2} + 2x^{3/2 - 1/2}$

$y = x^{1/2} + 2x^1$

$\boxed{\frac{dy}{dx} = \frac{1}{2}x^{-1/2} + 2(1)}$ Ans

(c) $y = (4x^2 - 3)(7x^2 + x)$

Sol:- $y = 4x^2(7x^2 + x) - 3(7x^2 + x)$

$y = 28x^4 + 4x^3 - 21x^2 - 3x$

$\frac{dy}{dx} = 28(4x^3) + 4(3x^2) - 21(2x) - 3(1)$

$\boxed{\frac{dy}{dx} = 112x^3 + 12x^2 - 42x - 3}$

(b) $y = (x^3 - 5)(2x + 3)$

Sol $y = x^3(2x + 3) - 5(2x + 3)$

$y = 2x^4 + 3x^3 - 10x - 15$

$\frac{dy}{dx} = 2(4x^3) + 3(3x^2) - 10(1) - 0$

$\boxed{\frac{dy}{dx} = 8x^3 + 9x^2 - 10}$ Answer

Q5 (a) $f(x) = x^2 + 3x$

$f'(x) = 2x + 3$ at $x = 1$.

$f'(1) = 2(1) + 3 = \boxed{5}$

(b) $f(x) = x^4 - x^2$

$f'(x) = 4x^3 - 2x$

at $x = 1$

$f'(1) = 4(1)^3 - 2(1)$

$\boxed{f'(1) = 2}$ Answer