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Where Minds Pollinate

Class 12
Mathematics

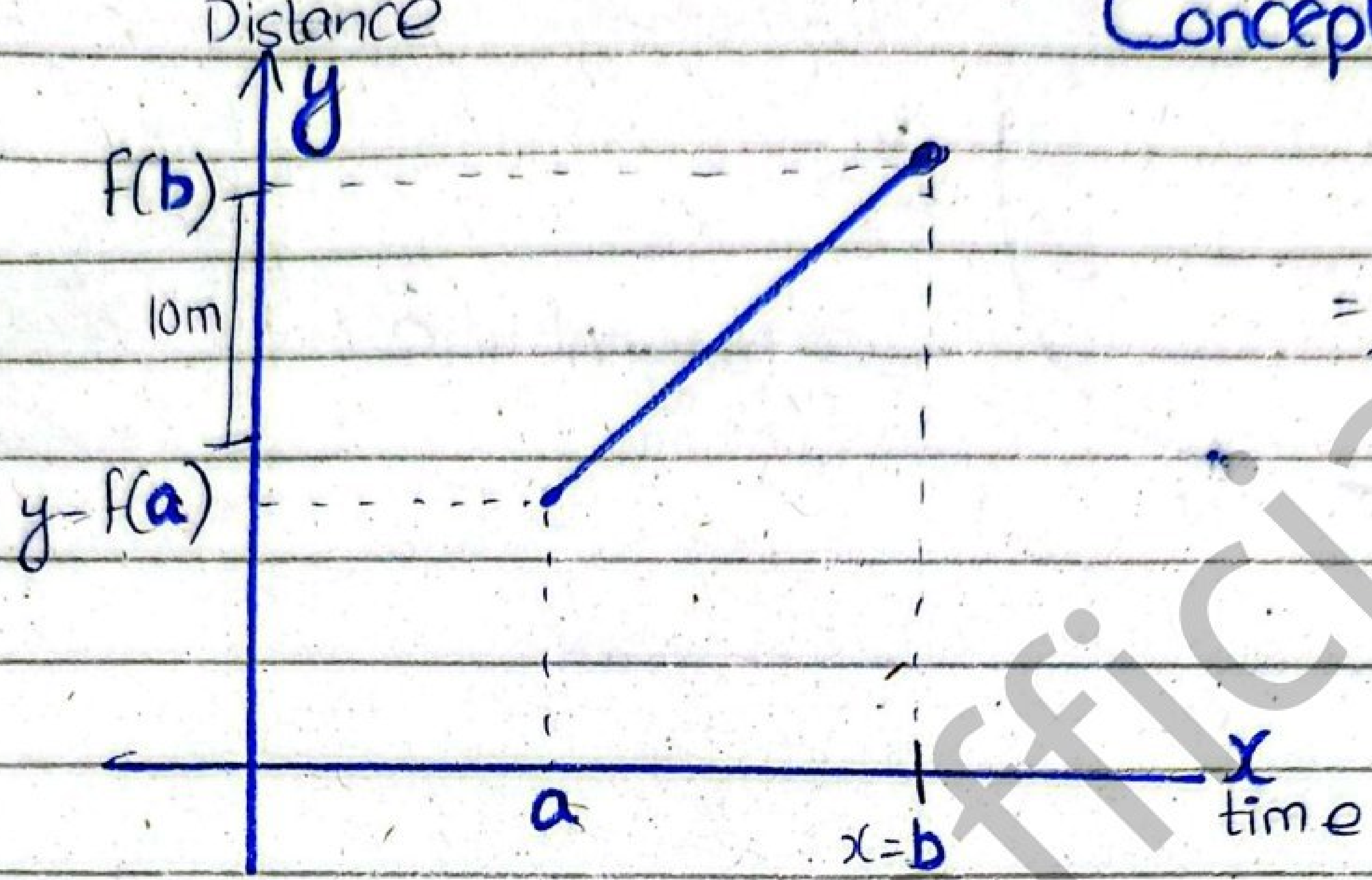
Chapter 2

Exercise 2.3

Handwritten Solution

Exercise 2.3

Concept



$= \dfrac{\text{Distance}}{\text{time}} = \dfrac{\Delta y}{\Delta x}$

$s = \dfrac{10}{10}$

$\boxed{s = 1\,ms^{-1}}$

Δx = veryvery small value

$y = f(x) = f(a)$

if $x = a + \Delta x$

$\boxed{y = f(a + \Delta x)}$

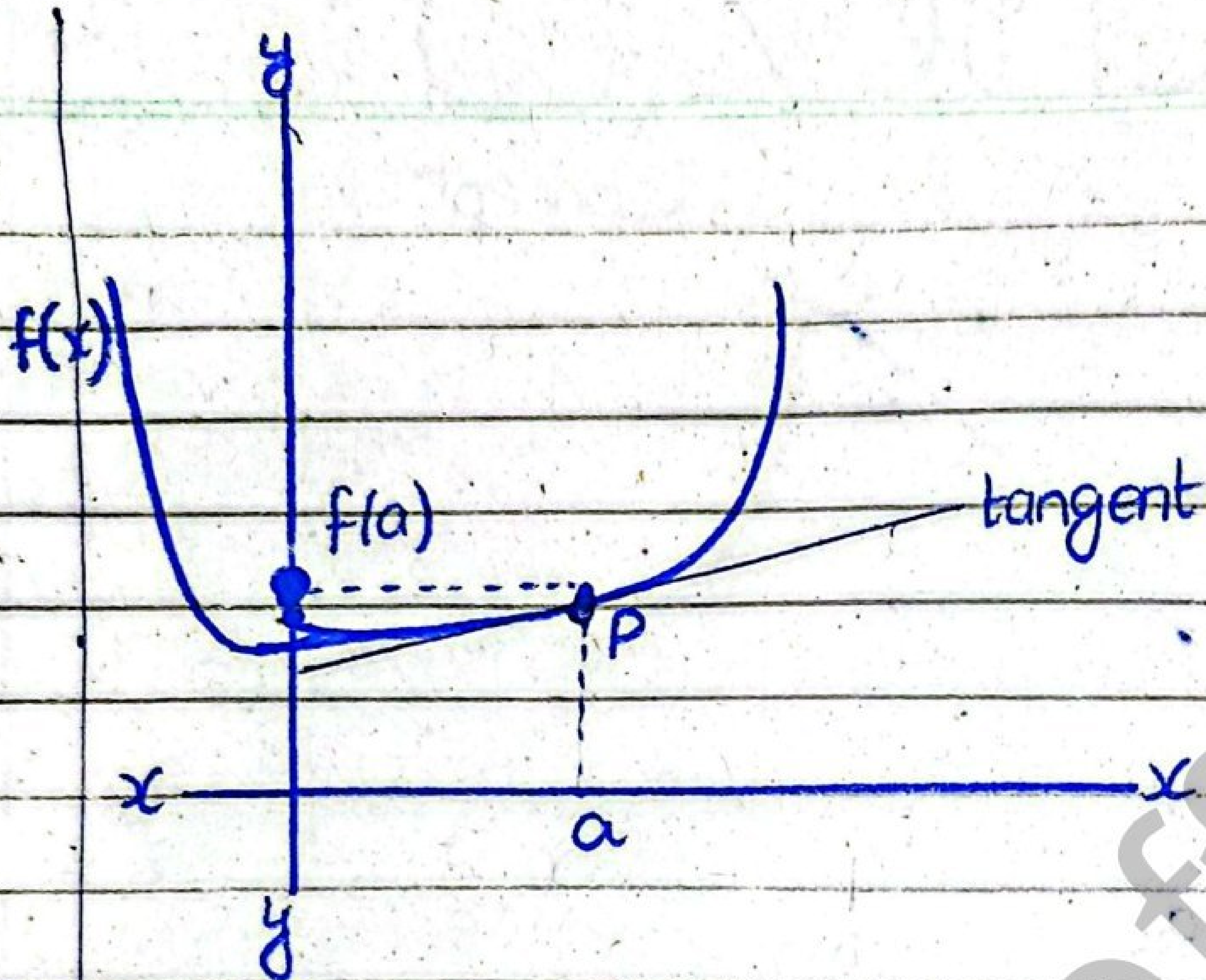
Average rate of Change if a function $f(x)$ the average rate of change from $x = a$ to $x = b$ is:

$$A.R.C = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} = m_{sec}$$ also called **slope of secant line.**

- **Slope of tangent line or Instantaneous rate of change**

The instantaneous rate of change of a function at a specific point is how fast the function is changing at that exact point.

$$m_{tan} = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

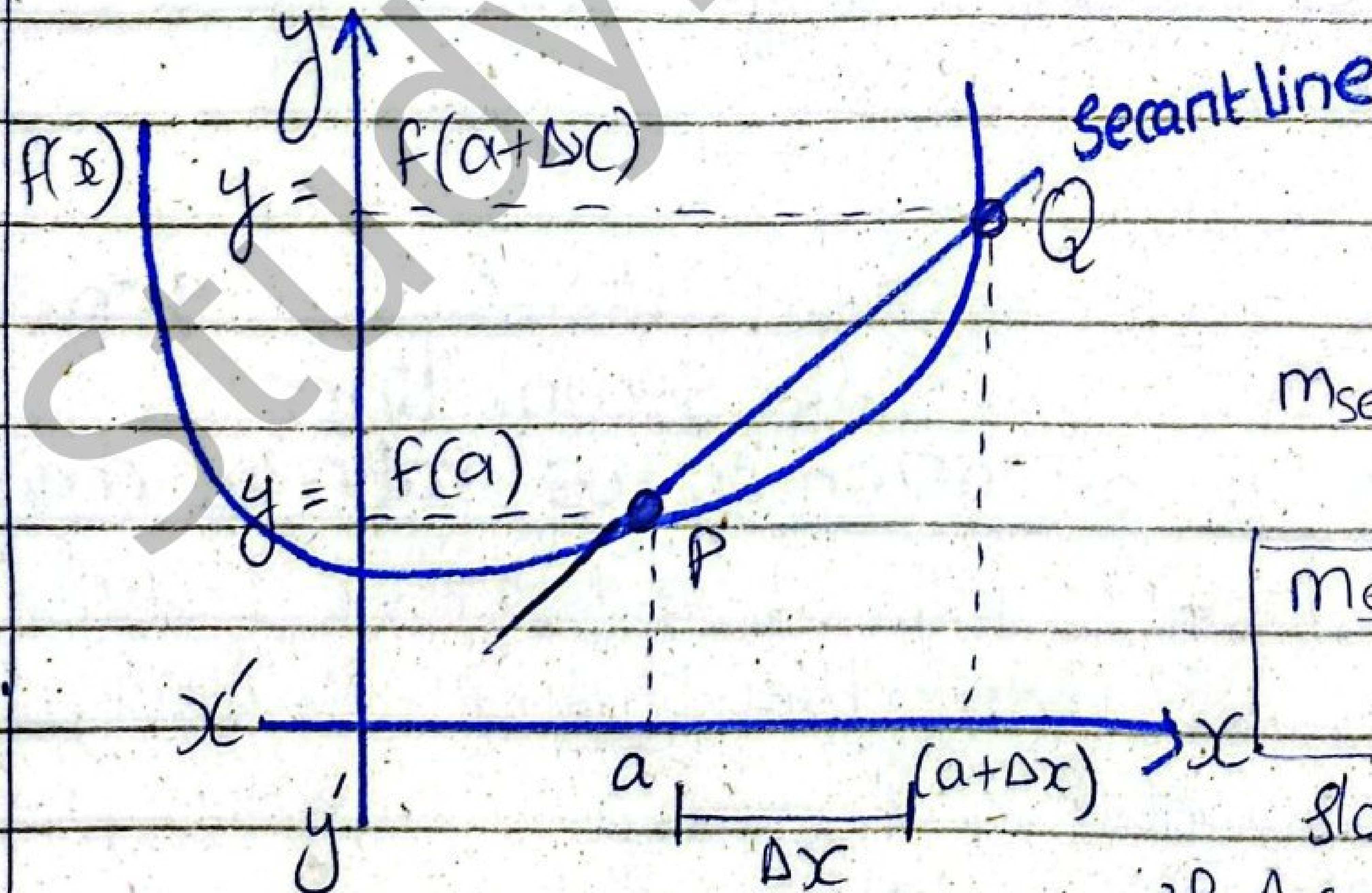


Let P be the point on the function f(x) then coordinate of P is

$$P(x,y) = P(a, f(a))$$

Now find point Q

$$Q(x,y) = Q((a+\Delta x), f(a+\Delta x))$$



Slope of PQ

$$m_{sec} = \frac{f(a+\Delta x) - f(a)}{a+\Delta x - a}$$

$$\boxed{m_{sec} = \frac{f(a+\Delta x) - f(a)}{\Delta x}}$$

slope of secant line

if $\Delta x \to 0$ then Q point reach at point P

$$m_{tan} = \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$$ proved

$y = f(x)$

Q1 $f(x) = 2x-1 \; ; \; (\overset{x}{x}, \overset{y}{f(x)}) = (4,7)$;

Solution:- $m_{tan} = \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$

$f(a+\Delta x) = 2(a+\Delta x)-1$
$= \boxed{2a + 2\Delta x - 1}$

$\boxed{f(a) = 2a - 1}$

putting values.

$m_{tan} = \lim_{\Delta x \to 0} \frac{2a + 2\Delta x - 1 - (2a-1)}{\Delta x}$

$m_{tan} = \lim_{x \to 0} \frac{2a + 2\Delta x - 1 - 2a + 1}{\Delta x}$

$m_{tan} = \lim_{\Delta x \to 0} \frac{2\cancel{\Delta x}}{\cancel{\Delta x}}$

$\boxed{m_{tan} = 2}$ Answer

Q2) $f(x) = \frac{-1}{2}x + 3$, $(a, f(a))$

$f(a+\Delta x) = \frac{-1}{2}(a+\Delta x) + 3$

$f(a) = \frac{-1}{2}a + 3$

$m_{tan} = \lim_{x \to 0} \frac{\cancel{-\frac{1}{2}a} - \frac{1}{2}\Delta x + \cancel{3} + \cancel{\frac{1}{2}a} - \cancel{3}}{\Delta x}$

$m_{tan} = \lim_{x \to 0} \frac{\frac{-1}{2}\cancel{\Delta x}}{\cancel{\Delta x}}$

$\boxed{m_{tan} = \frac{-1}{2}}$

Q3) $f(x) = x^2+4$, $(\overset{a}{-1}, \overset{b}{5})$

Solution:- $f(a+\Delta x) = (a+\Delta x)^2+4$

$f(a+\Delta x) = a^2+(\Delta x)^2+2a\Delta x+4$

$\boxed{f(a) = a^2+4}$

$$m_{tan} = \lim_{\Delta x \to 0} \frac{a^2+(\Delta x)^2+2a\Delta x+4-\cancel{a^2}-4}{\Delta x}$$

$$m_{tan} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(\Delta x+2a)}{\cancel{\Delta x}}$$

Apply limit

$m_{tan} = 0+2a \Rightarrow m_{tan} = 2(-1) = \boxed{-2}$ Answer.

Q4) $f(x) = x^2-5x+4$, $(2,-2)$

$a=2$.

$f(2+\Delta x) = (2+\Delta x)^2-5(2+\Delta x)+4$

$f(2+\Delta x) = 4+\Delta x^2+4\Delta x-10-5\Delta x+4$

$\boxed{f(2+\Delta x) = \Delta x^2-\Delta x-2}$

$f(2) = (2)^2-5(2)+4 = -2$.

$$m_{tan} = \lim_{\Delta x \to 0} \frac{\Delta x^2-\Delta x-2-(-2)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x^2-\Delta x-\cancel{2}+\cancel{2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(\Delta x-1)}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} \Delta x-1$$

apply limit

$= \boxed{-1}$ Answer.

Q5) $f(x) = x^3$; $(1, f(x))$

$a = 1$

Sol $f(1+\Delta x) = (1+\Delta x)^3 = a^3+b^3+3a^2b+3ab^2$

$f(1+\Delta x) = 1+\Delta x^3+3\Delta x+3\Delta x^2$

$f(1) = 1$

$$\lim_{\Delta x \to 0} \frac{\cancel{1}+\Delta x^3+3\Delta x+3\Delta x^2-\cancel{1}}{\Delta x}$$

$$\Rightarrow m_{tan} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(\Delta x^2+3+3\Delta x)}{\cancel{\Delta x}}$$

Apply limit

$m_{tan} = 0+3+3(0)$

$\boxed{m_{tan} = 3}$ Answer

Q6) $f(x) = \frac{1}{x}$, $\left(\frac{1}{3}, f\left(\frac{1}{3}\right)\right)$

$a = 1/3$

Sol: $f\left(\frac{1}{3}+\Delta x\right) = \frac{1}{\frac{1}{3}+\Delta x} = \frac{1}{\frac{1+3\Delta x}{3}} = \frac{3}{1+3\Delta x}$

$f(1/3) = \frac{1}{1/3} = \boxed{3}$

$$m_{tan} = \lim_{\Delta x \to 0} \frac{\frac{3}{1+3\Delta x} - 3}{\Delta x} = \frac{\frac{3-(3+9\Delta x)}{1+3\Delta x}}{\frac{\Delta x}{1}}$$

$$m_{tan} = \lim_{\Delta x \to 0} \frac{3-3-9\Delta x}{1+3\Delta x} \times \frac{1}{\Delta x}$$

$$m_{tan} = \lim_{\Delta x \to 0} \frac{-9\Delta x}{(1+3\Delta x)\Delta x}$$

apply limit

$m_{tan} = \frac{-9}{1+0} = \boxed{-9}$

Answer

Q7) $f(x) = x^3+2x^2-4x$; $[-1,2]$.

Sol:- $a=-1,\ b=2$

$f(b)=f(2)=(2)^3+2(2)^2-4(2) = \boxed{8}+\cancel{8}-\cancel{8}$.

$f(a)=f(-1)=(-1)^3+2(-1)^2-4(-1)=$

$A.R.C = \frac{f(b)-f(a)}{b-a} = \frac{8-5}{2-(-1)} = \frac{3}{3} = \boxed{1}$ Answer

Q8) $f(x) = \cos x$, $[-\pi,\pi]$

$a=-\pi \quad b=\pi$

$f(b)=f(\pi)=\cos\pi$

$f(a)=f(-\pi)=\cos(-\pi)=\cos\pi$

$A.R.C = \frac{f(b)-f(a)}{b-a} = \frac{\cos\pi-\cos\pi}{\pi+\pi} = 0$ Ans $\boxed{A.R.C = 0}$.

Q9) $f(t) = -4t^2+10t+6$; $t=3$

$\lim_{\Delta t\to 0} \frac{f(t+\Delta t)-f(t)}{\Delta t} \Rightarrow \lim_{\Delta t\to 0} \frac{f(3+\Delta t)-f(3)}{\Delta t}$ — (1)

$\lim_{\Delta t\to 0} f(3+\Delta t) = -4(3+\Delta t)^2+10(3+\Delta t)+6$

$= -4(9+\Delta t^2+6\Delta t)+30+10\Delta t+6$

$= -30-4\Delta t^2-24\Delta t+30+10\Delta t$

$\boxed{f(3+\Delta t) = -4\Delta t^2-14\Delta t}$

$f(3) = -4(3)^2+10(3)+6 = -36+36 = \boxed{0}$

$\lim_{\Delta t\to 0} = \frac{\Delta t(-4\Delta t-14)-0}{\Delta t}$

$\boxed{\lim_{\Delta t\to 0} = -14}$ Answer

Q10) $f(t) = t^2+\frac{1}{5t+1}$; $t=0$

Sol $\lim_{\Delta t\to 0} \frac{f(0+\Delta t)-f(0)}{\Delta t}$

$f(\Delta t) = (\Delta t)^2 + \frac{1}{5(\Delta t)+1} = \frac{5(\Delta t)^3 + \Delta t^2 + 1}{5\Delta t + 1}$

$f(0) = 0 + 1 \Rightarrow \boxed{f(0) = 1}$

$= \lim_{\Delta t \to 0} \frac{\left(\frac{5\Delta t^3 + \Delta t^2 + 1}{5\Delta t + 1}\right) - 1}{\Delta t}$

$= \lim_{\Delta t \to 0} \frac{\frac{5\Delta t^3 + \Delta t^2 + 1 - 5\Delta t - 1}{(5\Delta t + 1)}}{\Delta t / 1} = \lim_{\Delta t \to 0} \frac{\cancel{\Delta t}(5\Delta t^2 + \Delta t - 5)}{5\Delta t + 1} \times \frac{1}{\cancel{\Delta t}}$

apply limit $= \frac{-5}{1} = \boxed{-5}$ Answer

Q11 Sol:- $\lim_{\Delta t \to 0} \frac{S\left(\frac{1}{2} + \Delta t\right) - S\left(\frac{1}{2}\right)}{\Delta t} = ?$

(i)

$S\left(\frac{1}{2} + \Delta t\right) = 122.5 - 4.9\,(0.5 + \Delta t)^2$

$= 122.5 - 4.9\,(0.25 + \Delta t^2 + \Delta t)$

$= 122.5 - 1.225 - 4.9\Delta t^2 - 4.9\Delta t$

$S(0.5 + \Delta t) = 121.275 - 4.9\Delta t^2 - 4.9\Delta t$

$S(0.5) = 122.5 - 4.9(0.5)^2 = 121.275$

$= \lim_{\Delta t \to 0} \frac{(\cancel{121.275} - 4.9\Delta t^2 - 4.9\Delta t) - \cancel{121.275}}{\Delta t}$

$= \lim_{\Delta t \to 0} \cancel{\Delta t} \left(\frac{-4.9\Delta t - 4.9}{\cancel{\Delta t}}\right)$

$=$ apply limit $= \boxed{-4.9\,ms^{-1}}$

(ii) $S(t) = 0$

$0 = 122.5 - 4.9t^2$

$4.9t^2 = 122.5$

$\sqrt{t^2} = \sqrt{\frac{122.5}{4.9}}$ $\quad \boxed{t = 5\text{ sec}}$ Ans

(iii) $\lim_{\Delta t \to 0} \dfrac{S(5+\Delta t) - S(5)}{\Delta t}$

$S(5+\Delta t) = 122.5 - 4.9(5+\Delta t)^2$

$S(5+\Delta t) = 122.5 - 4.9(25+\Delta t^2+10\Delta t)$

$S(5+\Delta t) = 122.5 - 122.5 - 4.9\Delta t^2 - 4.9\Delta t$

$\boxed{S(5+\Delta t) = -4.9\Delta t^2 - 4.9\Delta t}$

$S(5) = 122.5 - 4.9(5)^2$

$= 122.5 - 122.5 = \boxed{0}$

$= \lim_{\Delta t \to 0} \dfrac{-4.9\Delta t^2 - 49\Delta t - 0}{\Delta t}$

$= \lim_{\Delta t \to 0} \dfrac{\cancel{\Delta t}(-4.9\Delta t - 49)}{\cancel{\Delta t}} \Rightarrow \boxed{-49ms^{-1}}$ Ans.

Q6 (i) $S(2) = -16(2)^2 + 256(2)$

$\boxed{S(2) = 448ft}$

$S(6) = -16(6)^2 + 256(6)$

$\boxed{S(6) = 960ft}$

$S(9) = 16(9)^2 + 256(9)$

$\boxed{S(9) = 1008ft}$

$S(10) = -16(10)^2 + 256(10)$

$\boxed{S(10) = 960ft}$

(ii) $A.V = \dfrac{S(b) - S(a)}{b-a}$

$b = 5, a = 2$

$A.V = \dfrac{S(5) - S(2)}{5-2}$

$A.V = \dfrac{880 - 448}{3}$

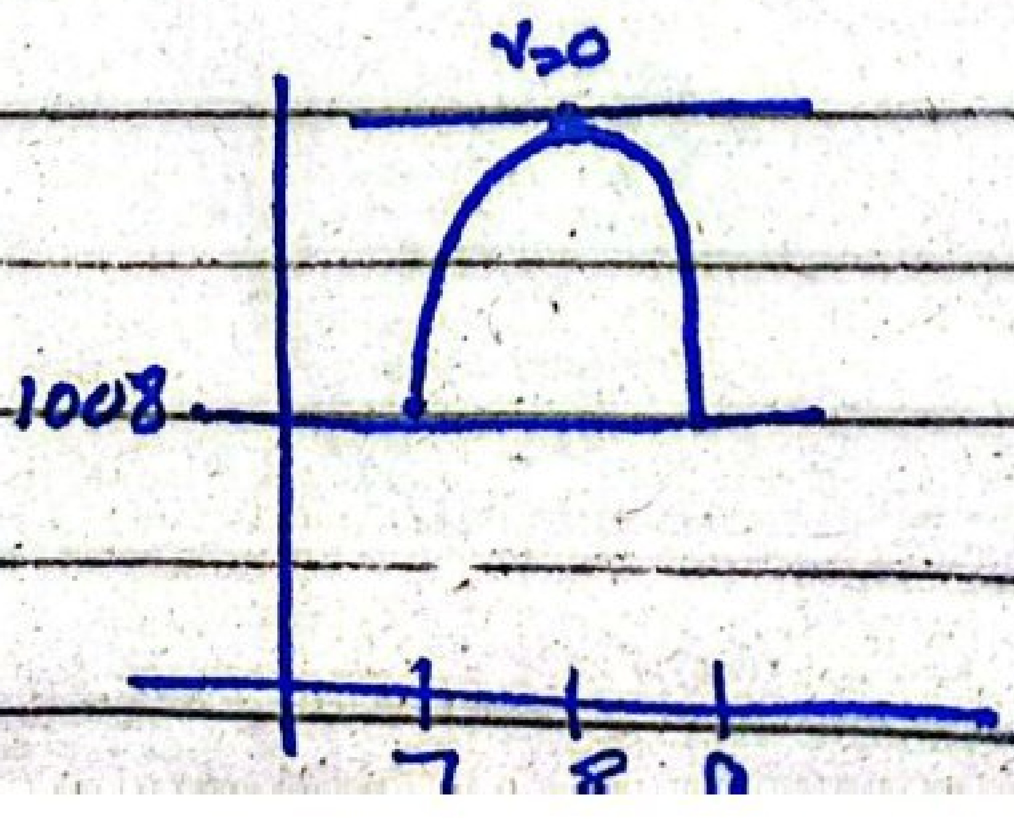
$\boxed{A.V = 144ft/s}$

(iii) $a = 7, b = 9$

$A.V = \dfrac{S(9) - S(7)}{9-7}$

$A.V = \dfrac{1008 - 1008}{2}$

$\boxed{AV = 0}$



(iv) $S(t)=0$

$-16t^2+256t=0$

$t(-16t+256)=0$

$\boxed{t=0}$ $\quad +16t=256$

$t=\frac{256}{16}$

$\boxed{t=16sec}$

(v) $m_{tan}=\lim_{\Delta t\to 0}\frac{S(8+\Delta t)-S(8)}{\Delta t}$

$S(8+\Delta t)=-16(8+\Delta t)^2+256(8+\Delta t)$

$=-16(64+\Delta t^2+16\Delta t)+2048+256\Delta t$

$=-1024-16\Delta t^2-256\Delta t+2048+256\Delta t$

$=\boxed{-16\Delta t^2+1024}$ $\quad \boxed{S(8)=1024}$

$m_{tan}=\lim_{\Delta t\to 0}\frac{-16\Delta t^2+\cancel{1024}-\cancel{1024}}{\Delta t}=\lim_{\Delta t\to 0}-16\Delta t$

$m_{tan}=-16(0) \Rightarrow \boxed{V=0}$

(vi) $\boxed{S(8)=1024}$ Answer