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Where Minds Pollinate

Class 12
Mathematics

Chapter 2

Exercise 2.2

Handwritten Solution

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Exercise 2.2

Defination of Limits :- $\boxed{\lim_{x\to a} f(x) = L}$ — real no

A function is said to be continuous at a number **a** if:

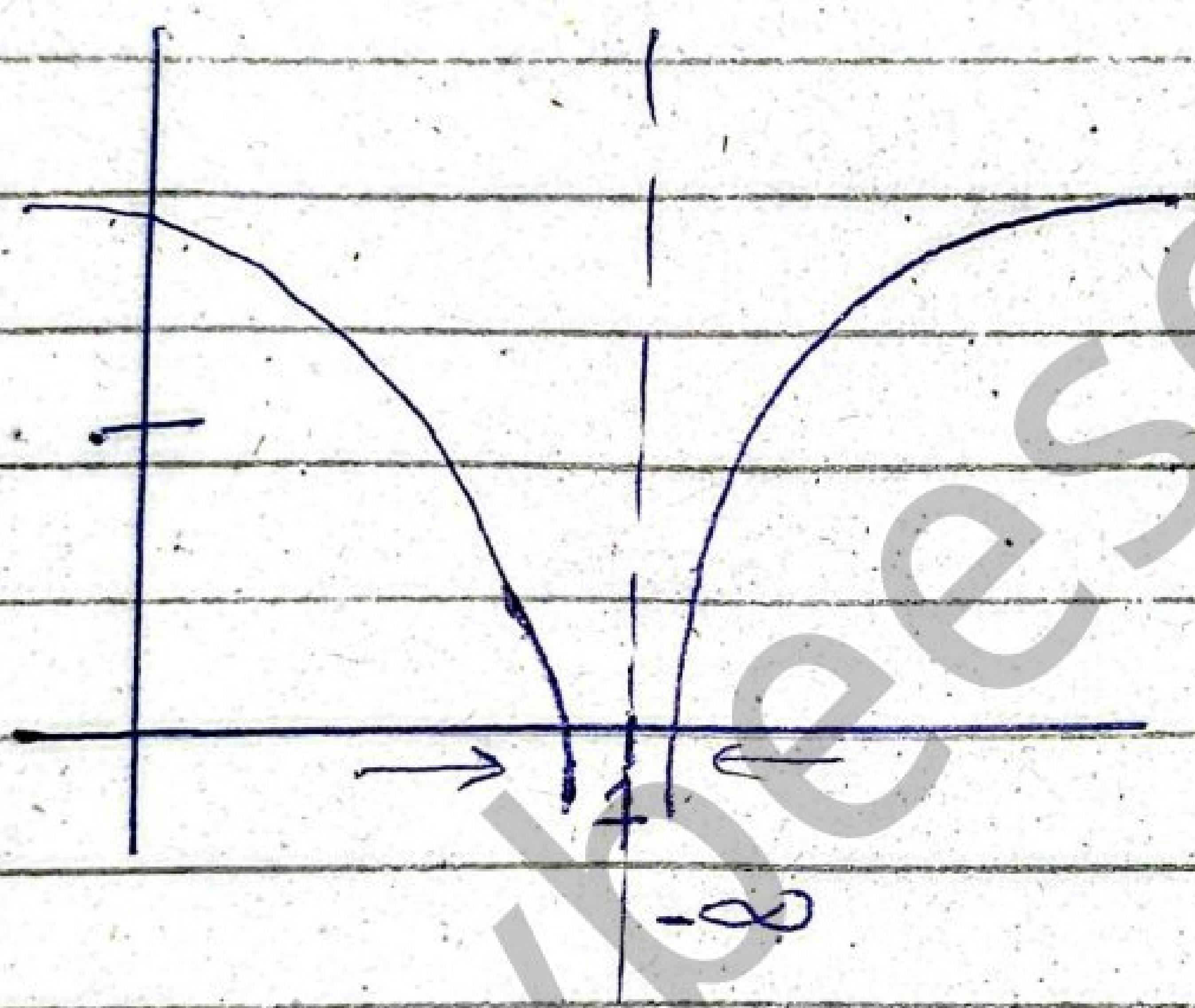
(i) $f(a)$ is defined

(ii) $\lim_{x\to a} f(x)$ exist

(iii) $\lim_{x\to a} f(x) = f(a)$

Concept

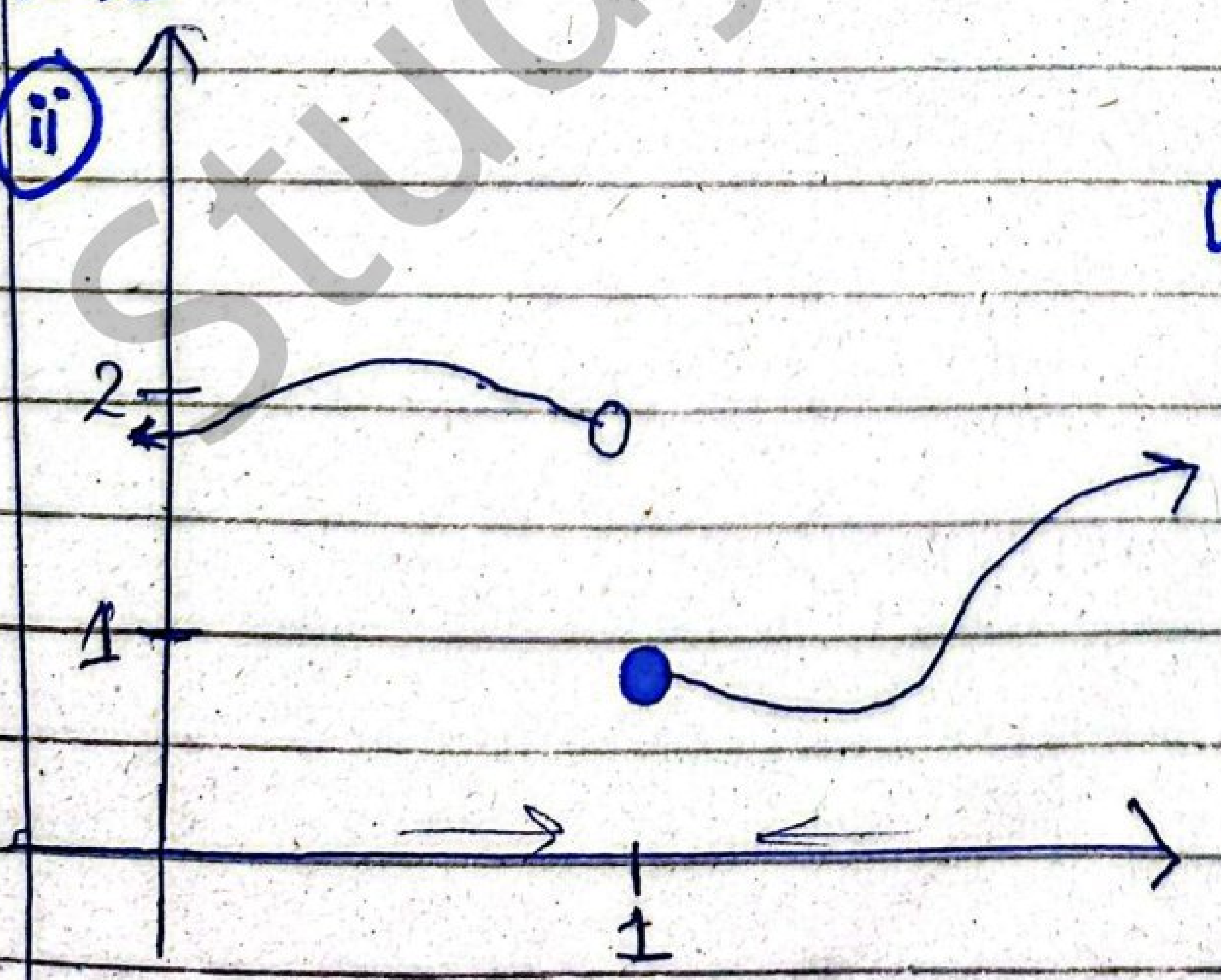
(i)



Dis Continous

$\lim_{x\to 1} f(x)$ does not exist & $f(1)$ is not defined.

(ii)



Discontinous

$\lim_{x\to 1} f(x)$ doesn't exist & $f(1)$ is defined

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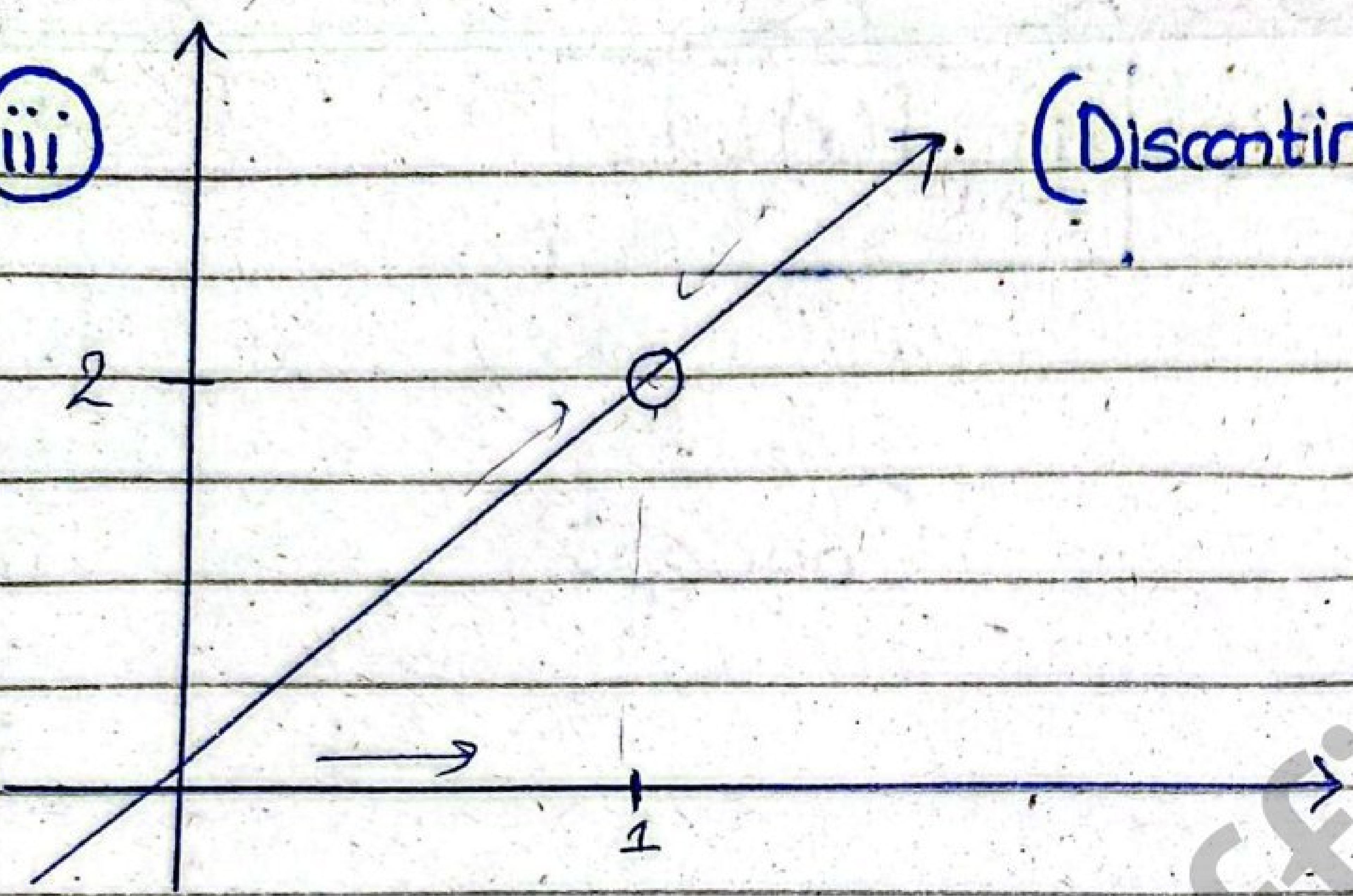
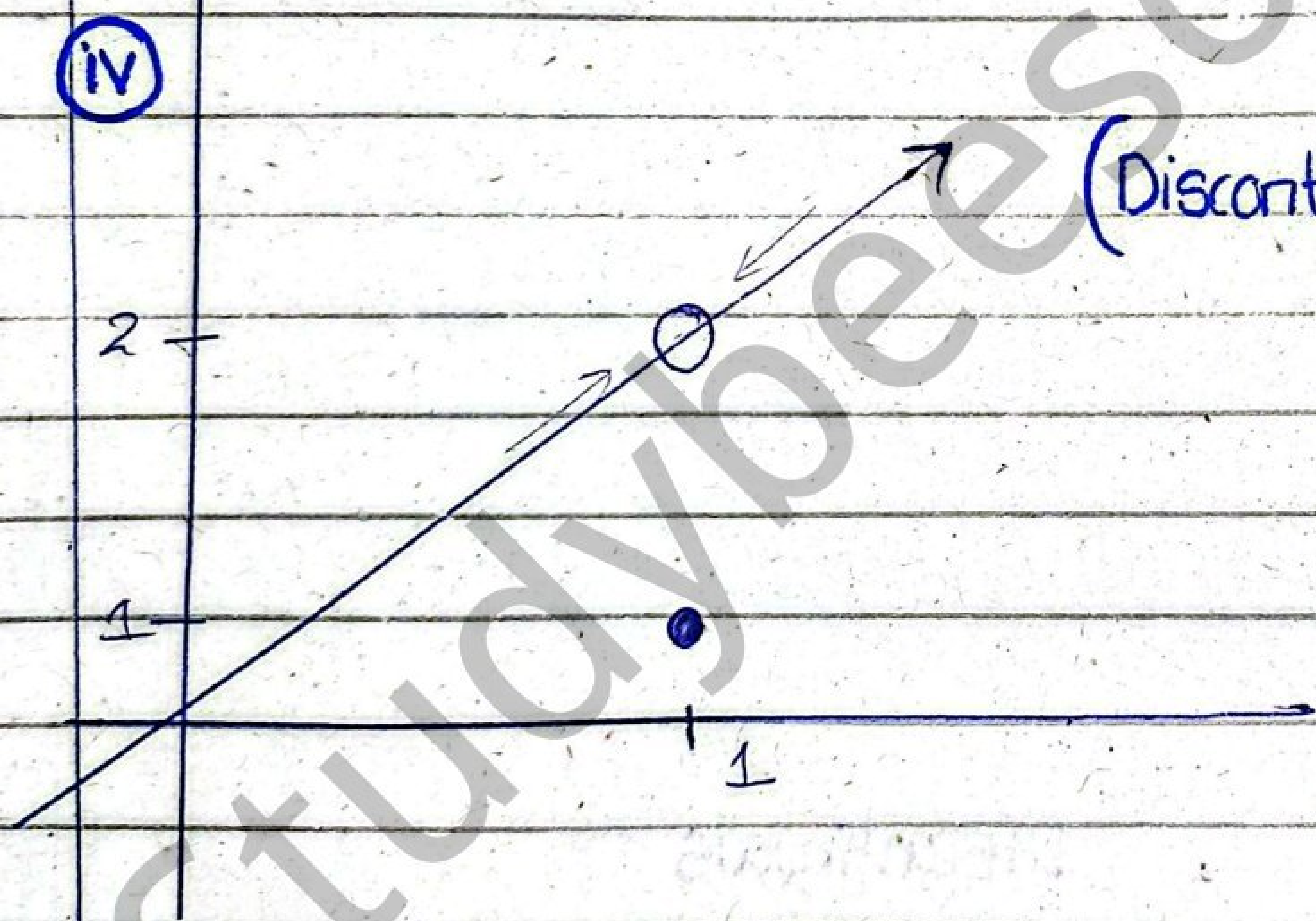
(iii) (Discontinuous).



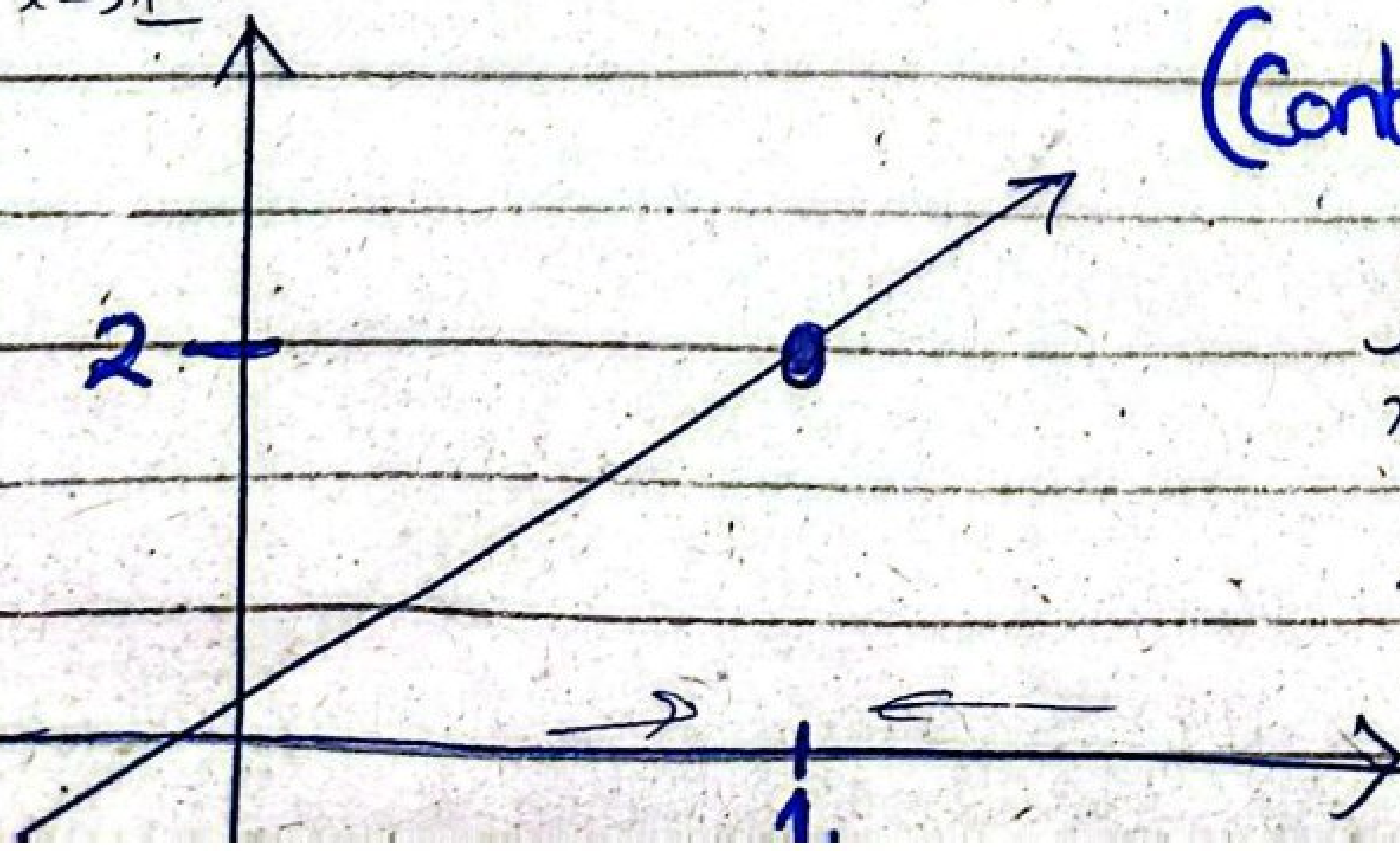
$\lim_{x\to 1} f(x)$ exist but $f(1)$ is not defined

(iv) (Discontinuous.)



$\lim_{x\to 1}$ it $f(x)$ exist & $f(1)$ is defined but $\lim_{x\to 1} f(x) \neq f(1)$

(v) (Continuous)



$\lim_{x\to 1} f(x)$ exist & $f(1)$ is defined

$\Rightarrow \lim_{x\to 1} f(x) = f(1)$

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0 is denominator the function is discontinuous.

① $f(x) = x^2 - 5x + 6$

Sol:- This is a polynomial function

→ Polynomial functions are continuous everywhere so no discontinuity in this function.

② $f(x) = \dfrac{2x}{x^2+5}$

Sol The denominator $x^2+5 \neq 0$ for any real number.

⇒ Given rational function is continuous.

So, No discontinuity in function for any real number.

③ $f(x) = \dfrac{1}{x^2-9x+8}$

Sol:- put $x^2-9x+8=0$

$x^2-8x-x+8=0$

$x(x-8)-1(x-8)=0$

$\boxed{x=+1}$ $\boxed{x=8}$

put $x=+1$

so $\dfrac{1}{1^2-9(1)+8} = \dfrac{1}{0} = \infty$

put $x=8$

$\dfrac{1}{(8)^2-9(8)+8} = \dfrac{1}{0} = \infty$

Hence $\boxed{x=1}$ & $\boxed{x=8}$ are the points or number where function is Discontinuous.

④ $f(x) = \dfrac{x^2-1}{x^4-1}$

Sol:- we see that at $x=1$ & -1 given function is undefined.

$f(x) = \dfrac{\cancel{x^2-1}}{(\cancel{x^2-1})(x^2+1)}$

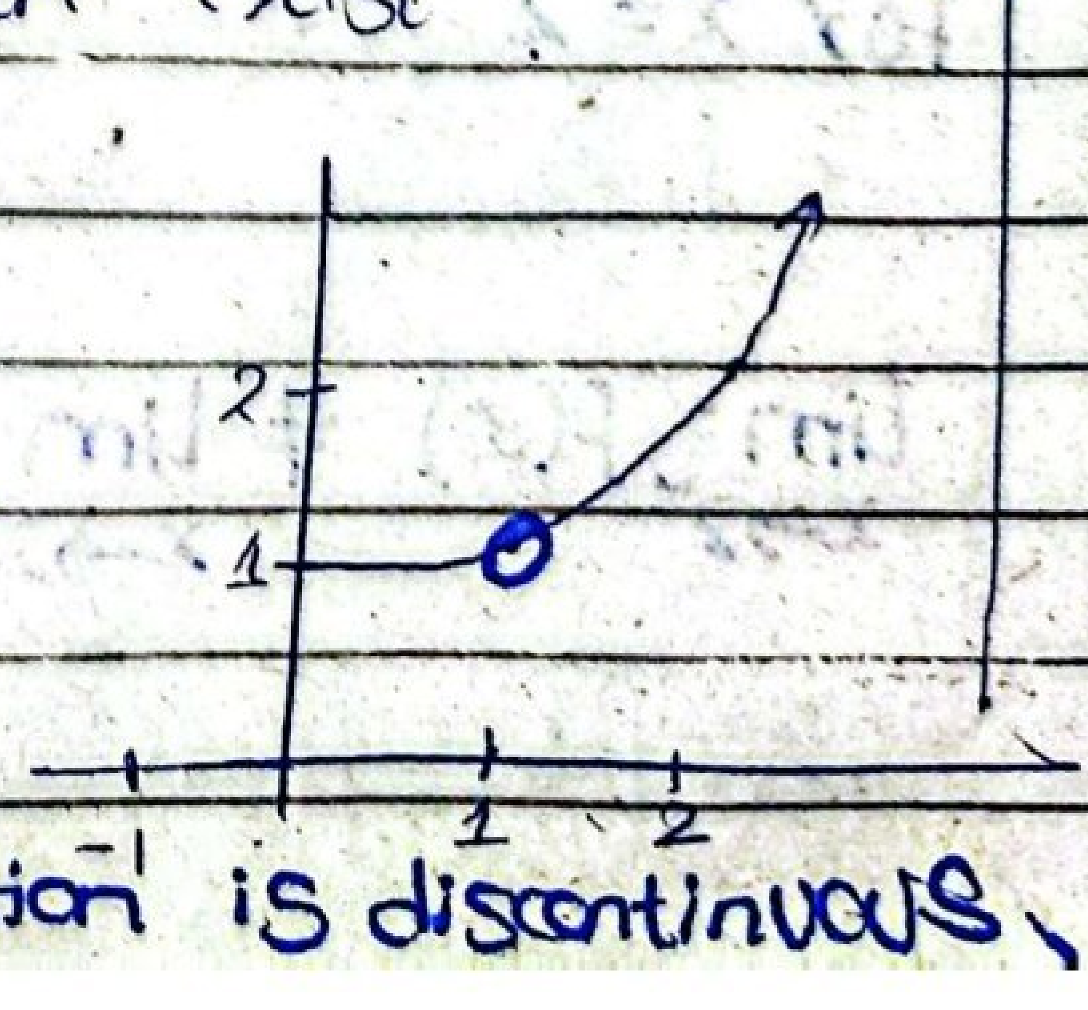
$f(x) = \dfrac{1}{x^2+1}$

$\lim_{x\to 1} = \dfrac{1}{1+1} = \boxed{\tfrac{1}{2}}$ defined.

$\lim_{x\to -1} f(x) = \lim_{x\to -1} \dfrac{1}{x^2+1} = \dfrac{1}{(-1)^2+1} = \boxed{\dfrac{1}{2}}$

⇒ Limit of function exist

Hence here is removeable discontinuity at $x=1$ & $x=-1$.



Hence $x=1$ & $x=-1$ function is discontinuous.

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(5) $f(x) = \dfrac{x-1}{\sin 2x}$

Sol put $\sin 2x = 0$
for discontinuity

$2x = 0, \pi, 2\pi, 3\pi, 4\pi \ldots$

$2x = n\pi, \ n \in \mathbb{Z}$

$$\boxed{x = \frac{n\pi}{2}}$$

Hence $x = \dfrac{n\pi}{2}, n \in \mathbb{Z}$

given function is discontinuous.

6) $f(x) = \begin{cases} x & x < 0 \\ x^2 & 0 \le x \le 2 \\ x & x > 2 \end{cases}$

$f(x) = x \quad x^2 \quad x^2 \quad x^2 \quad = x$

$0 \qquad 2$

For 0

$\Rightarrow \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} x = 0$

$\Rightarrow \lim\limits_{x \to 0^+} = \lim\limits_{x \to 0^+} x^2 = 0$

$\Rightarrow f(x) = x^2 \Rightarrow f(0) = 0$

$\Rightarrow \lim\limits_{x \to 0} f(x) = f(0)$

Hence function is continuous at $\boxed{x = 0}$

For $x = 2$

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} x^2 = 4$; $\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} x = 2$

$\lim\limits_{x \to 2^-} f(x) \neq \lim\limits_{x \to 2^+}$

Hence $\lim\limits_{x \to 2}$ doesnot exist.

Therefore given function is discontinuous.

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(7) $f(x) = \begin{cases} \frac{\sin x}{x} & , x \neq 0 \\ \frac{1}{2} & , x = 0 \end{cases}$

Sol: $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin x}{x} = 1.$

$\Rightarrow \lim_{x \to 0}$ exist

Now $f(0) = \frac{1}{2}$ There fore $\lim_{x \to 0} f(x) \neq f(0)$ function is Discontinuous.

(Q8) $f(x) = \begin{cases} \frac{x^2 - 36}{x - 6} & ; x \neq 6 \\ 12 & ; x = 6 \end{cases}$

Sol: $\lim_{x \to 6} f(x) = \lim_{x \to 6} \frac{x^2 - 36}{x - 6}$

$= \lim_{x \to 6} \frac{x^2 - 6^2}{x - 6}$

$= \lim_{x \to 6} (x + 6)$

apply limit

$= 6 + 6 = 12.$

Therefore $\boxed{\lim_{x \to 6} f(x) = 12}$ — (1)

$\boxed{f(6) = 12}$ — (2)

From (1) & (2)

$\lim_{x \to 6} f(x) = f(6)$

Hence given function continuous at $x = 6$

So no discontinuity in the given fu.

Q9) $f(x) = x^2+1$

a) $[-1, 3]$ b) $[3, \infty)$

Solution:- The given function $f(x)$ is continuous everywhere so given both interval function is continuous Ans

Q10) $f(x) = \frac{1}{x}$

a) $(-3, 3)$ b) $(0, 10]$

Sol: In $(-3,3)$ interval contain $-2, -1, \boxed{0}, 1, 2$.

$\Rightarrow$ In $(-3,3)$ interval given function is discontinuous.

(b) In $(0, 10]$ interval contain $0.1 \ldots\ldots 10$

In this interval given function is continuous

Q11) $f(x) = \frac{1}{\sqrt{x}}$

a) $[1, 4)$ (b) $[1, 9]$

Sol Given function is continuous for $x > 0$

for $[1,4)$ given function is continuous for $[1, 9]$ the given function is continuous. Ans

Q12 $f(x) = \sqrt{x^2-9}$

a) $[-3, 3]$ b) $[3, \infty)$

Sol:- a) $[-3, 3]$ this interval contain $-3, \check{-2}, \check{-1}, \check{0}, \check{1}, \check{2}, 3$

therefore given function is discontinuous at $[-3, 3]$

b) $[3, \infty)$ ~~given~~ In this interval given function is continuous for $[3, \infty)$ Answer

Q13) $f(x) = \frac{x}{x^3+8}$

a) $[-4, -3]$ (b) $[-10, 10]$

Sol: In denominator $x^3+8\neq 0$

$x = -2 \Rightarrow (-2)^3+8=0$

a) In this Interval $[-4,-3]$ given function is continuous.

(b) $[-10,10]$ In this interval given function contain -2 so it is discontinuous.

Q14) $f(x) = \sin\frac{1}{x}$

a) $[\frac{1}{\pi}, 5)$ (b) $[\frac{\pi}{2}, \frac{3\pi}{2}]$

Sol Given function is undefined at $x=0$.

a) In this Interval $[\frac{1}{\pi}, 5)$ given function is continuous.

b) In this Interval $[\frac{\pi}{2}, \frac{3\pi}{2}]$ given function is also continuous

Q15 $f(x) = \begin{cases} mx, & x<4 \\ x^2, & x\geq 4 \end{cases}$

$f(x)=mx \quad f(x)=(x^2) \quad f(x)=x^2$

4

Sol:

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} mx = \boxed{4m}$

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} x^2 = \boxed{16}$

$f(x) = x^2$

$f(4) = (4)^2 = \boxed{16}$.

By defination of continuity.

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = f(4)$

$4m = 16 = 16$

$\frac{4m}{4} = \frac{16}{4}$

$\boxed{m=4}$ Answer

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(16) $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & x \neq 2 \\ m & x = 2 \end{cases}$

Sol: $\lim_{x\to 2} f(x) = \lim_{x\to 2} \dfrac{x^2-4}{x-2}$

$= \lim_{x\to 2} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$

$= \lim_{x\to 2} (x+2)$

Apply limit

$\lim_{x\to 2} f(x) = 2+2 = \boxed{4}$

$\boxed{f(2) = m}$

$\lim_{x\to 2} f(x) = f(2) \Rightarrow \boxed{m=4}$

(17) $f(x) = \begin{cases} mx, & x<3 \\ n, & x=3 \\ -2x+9, & x>3 \end{cases}$

$f(x)=mx$ | n | $f(x)=-2x+9$ at 3

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (mx) = 3m$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (-2x+9) = -6+9 = \boxed{3}$

At function:- $f(3) = n$

By defination of Continuity

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

$3m = 3 = n$

$\cancel{3}m = \cancel{3}$; $\boxed{n=3}$

$\boxed{m=1}$

(Q18) $f(x) = \begin{cases} mx-n & ; x<1 \\ 5 & ; x=1 \\ 2mx+n & ; x>1 \end{cases}$

Sol

$f(x)=mx-n$ | 5 | $f(x)=2mx+n$ at 1

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (mx-n) = \boxed{m-n}$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2mx+n) = \boxed{2m+n}$

At function:- $f(1) = 5$

As By defination

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$m-n = 2m+n = 5$

$m-n=5$ —(1) $\quad 2m+n=5$ —(2)

Solve (1) & (2)

$2m+\cancel{n} = 5$

$m-\cancel{n} = 5$

$3m = 10$

$\boxed{m = \dfrac{10}{3}}$

$\dfrac{10}{3} - n = 5$

$n = \dfrac{10-15}{3}$

$\boxed{n = -\dfrac{5}{3}}$ Answer

Q19 Prove that the equation $\frac{x^2+1}{x+3} + \frac{x^4+1}{x-4} = 0$ has a solution in the interval $(-3,4)$.

Proof (i) The given function

$f(x) = \frac{x^2+1}{x+3} + \frac{x^4+1}{x-4}$ is continuous in interval $(-3, 4)$

(ii) let $a = 0$, $b = 3$

$f(a) = \frac{0+1}{0+3} + \frac{0+1}{0-4} = \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12} > 0}$

$f(b) = \frac{(3)^2+1}{3+3} + \frac{3^4+1}{3-4} = \frac{10}{6} - 82 = \boxed{\frac{-250}{3} < 0}$

Therefore

$f(a) > 0$ & $f(b) < 0$ have opposite sign by (IVT) there is at least one number $c \in (a,b)$ such that $\boxed{f(c)=0}$

Intermediate value Theorm (IVT)

If a function $f(x)$ is:

1) Continuous on a closed interval $[a,b]$

2) And if $f(a)$ and $f(b)$ have opposite signs then

there ~~exists~~ is at least one number $c \in (a,b)$ such that $f(c)=0$

Q20) $f(x) = \begin{cases} 1, & x \text{ rational} \\ 0, & x, \text{ irrational} \end{cases}$

Proof:- This function is a classic example known as the **Dirichlet function**.

we want to check whether this function is continuous at any real numbers $c \in \mathbb{R}$

by defination of continuity:

$\lim_{x \to c} f(x) = f(c)$

Between two rational numbers there is always an irrational number & viceversa

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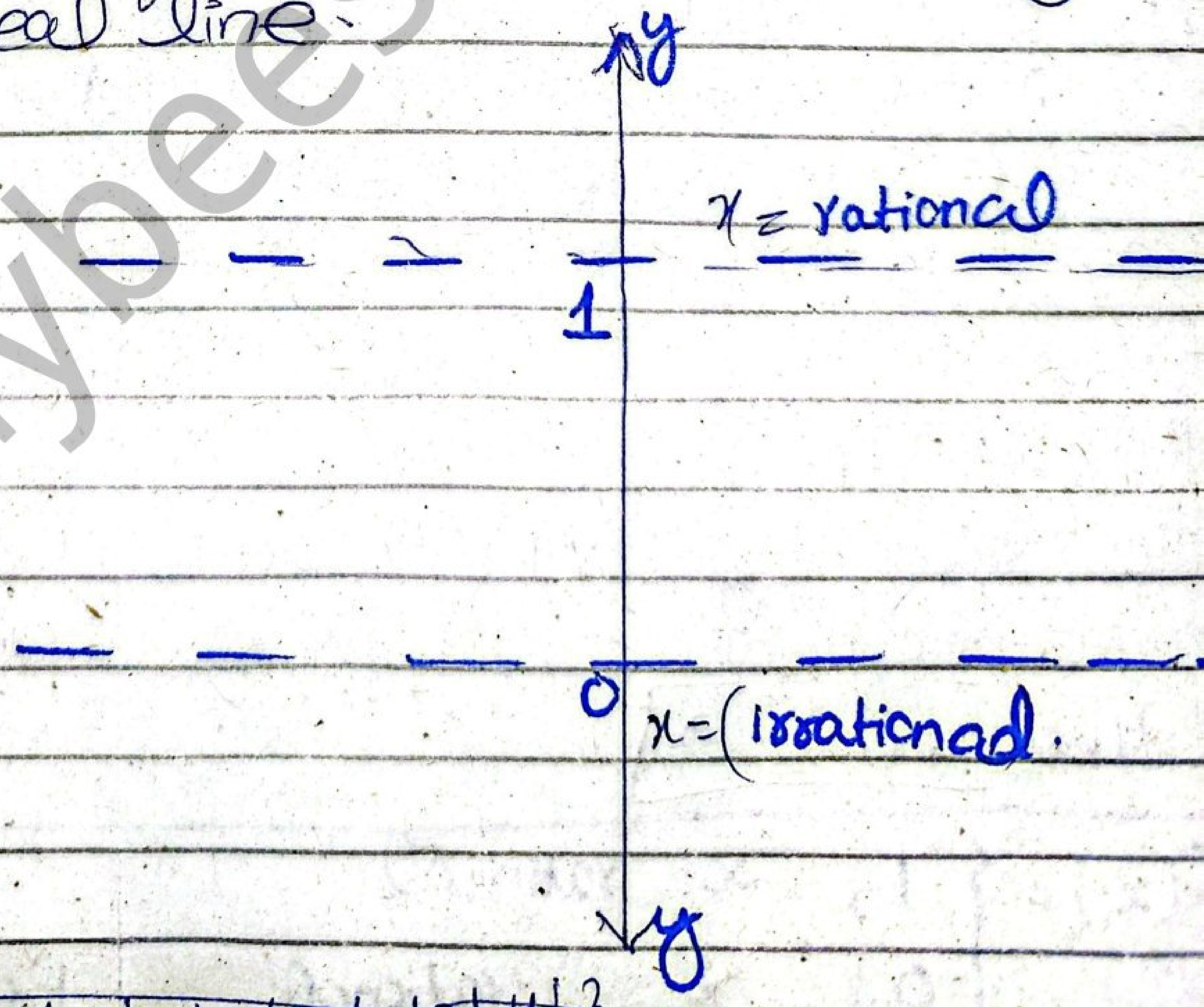
Take any real number c, around c there are:

(i) As $x \to c$, if x is rational then $f(x) = 1$.

(ii) As $x \to c$ if x is irrational then $f(x) = 0$.

Hence limit doesnt exist because $\lim_{x \to c} f(x)$ doesnt approach a single value

Hence function discontinuous at every point on the real line.



Exercise 1.4

[illegible]

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