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Where Minds Pollinate

Class 12
Mathematics

Chapter 2

Exercise 2.10

Handwritten Solution

Exercise # 2.10

Q1. Solution:- $m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\lim_{v \to c^-} m = \lim_{v \to c^-} \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m = \frac{m_0}{\sqrt{1-\frac{c^2}{c^2}}} \Rightarrow m = \frac{m_0}{\sqrt{1-1}} \Rightarrow m = \frac{m_0}{0} \Rightarrow \boxed{m = \infty}$$

Q2. $f(x) = \begin{cases} kx+1 & , x \le 3 \\ 2-kx & , x > 3 \end{cases}$ is Continuous at 3. find is k?

$(kx+1)$

$(kx+1)$ 3 $(2-kx)$

L.H.L $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (kx+1)$

$$\boxed{\lim_{x \to 3^-} f(x) = 3k+1}$$

RHL: $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (2-kx) = \boxed{2-3k}$

At function: $f(x) = kx+1$

$$\boxed{f(3) = 3k+1}$$

Given function is continuous

$$L.H.L = R.H.L = f(3)$$

Compare: $3k+1 = 2-3k = 3k+1$

$3k+1 = 2-3k$

$6k = 1$

$$\boxed{k = \frac{1}{6}}$$ Answer.

Q3. Solution:- $V = \frac{4\pi}{3} r^3$, $\frac{d}{dr} V = S$ = surface area = ?

$$\frac{dv}{dr} = \frac{4\pi}{3}\left[3r^2\right]$$

$\boxed{S = 4\pi r^2}$ Answer.

Q4. Solution:- $S(t) = \frac{1}{2} g t^2 + V_0 t + S_0$

$$\frac{dS}{dt} = S(t)$$

$$\frac{dS}{dt} = \frac{1}{2} g\left[2t\right] + V_0\left[1\right] + 0$$

$$\boxed{\frac{ds}{dt} = gt + V_0}$$

for $t = 4$

$$\left.\frac{ds}{dt}\right|_{t=4} = 4g + V_0 \quad \text{Answer}$$

Q5. Solution: length $= x = 10cm$

$\Delta x = dx = \pm 0.3cm$

$x = 10cm$

Area of square = A = length × width

$$A = x^2$$

$$\frac{dA}{dx} = 2x$$

$$dA = 2x\,dx$$

$$dA = 2 \times 10 \times (\pm 0.3cm)$$

$\boxed{dA = \pm 6cm^2}$ Maximum error in area

Relative error $= \frac{\text{Max (Absolute) error}}{\text{exact value}}$

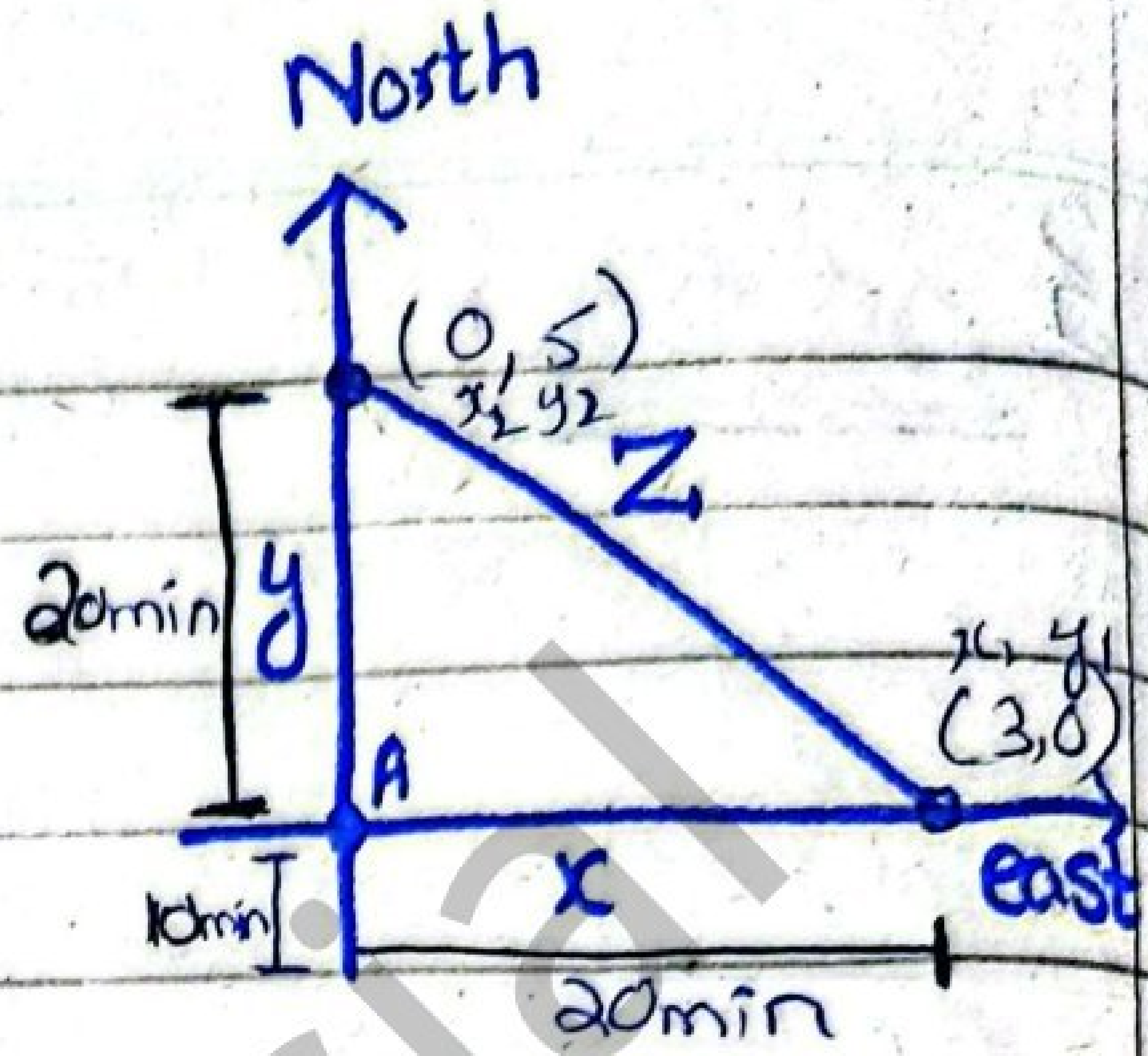
$A = (10)^2 = \boxed{100cm^2}$ ← exact area

Relative error $= \frac{6}{100} = 0.06$

Relative error $= 0.006 \times 100\%$

$= 6\%$

Q6. Solution: $\frac{dz}{dt} = ?$ $t_{woman} = 30min = \frac{30}{60} = \frac{1}{2}$ hour



$t_{man} = 20min = \frac{20}{60} = \frac{1}{3}$ hour

Speed = $\frac{\text{distance}}{\text{Time}}$

$\boxed{\text{distance} = \text{speed} \times \text{time}}$

$y = 10 \times \frac{1}{2} = \boxed{5km}$, $x = 9 \times \frac{1}{3} = \boxed{3km}$

Distance between two points $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$Z = \sqrt{(0-3)^2+(5-0)^2} = \boxed{\sqrt{34}}$

$Z^2 = x^2+y^2$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

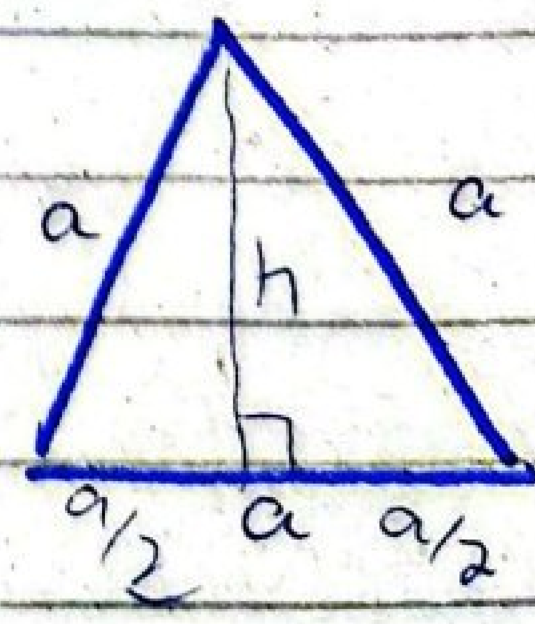
$2(z)\frac{dz}{dt} = 2\left[3(9)+5(10)\right]$

$z\frac{dz}{dt} = 27+50 = 77$

$\frac{dz}{dt} = \frac{77}{\sqrt{34}}$ $\boxed{\frac{dz}{dt} \simeq 13.2}$ Answer.

Q7. $\frac{da}{dt} = 2cm/hr$, $a = 8cm$

$\frac{dA}{dt} = ?$



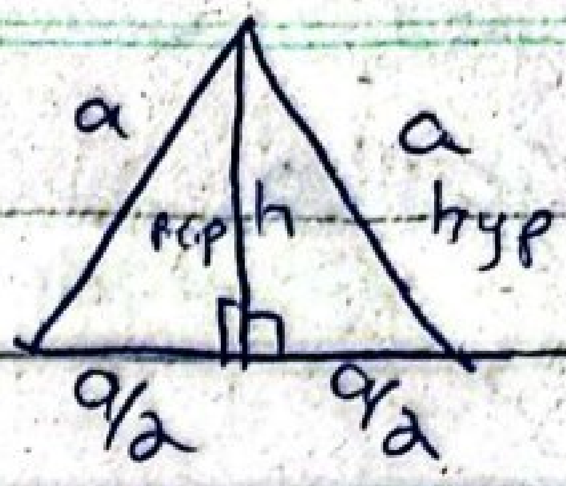
Area of equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$A = \frac{\sqrt{3}}{4}a^2$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a \times \frac{da}{dt}$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2 \times 8 \times 2 \Rightarrow \frac{dA}{dt} = 8\sqrt{3}\ cm^2/hr$ Ans

Concept to find area of triangle



Area of triangle

$A = \frac{1}{2}$ base × height

$a^2 = \left(\frac{a}{2}\right)^2 + h^2$

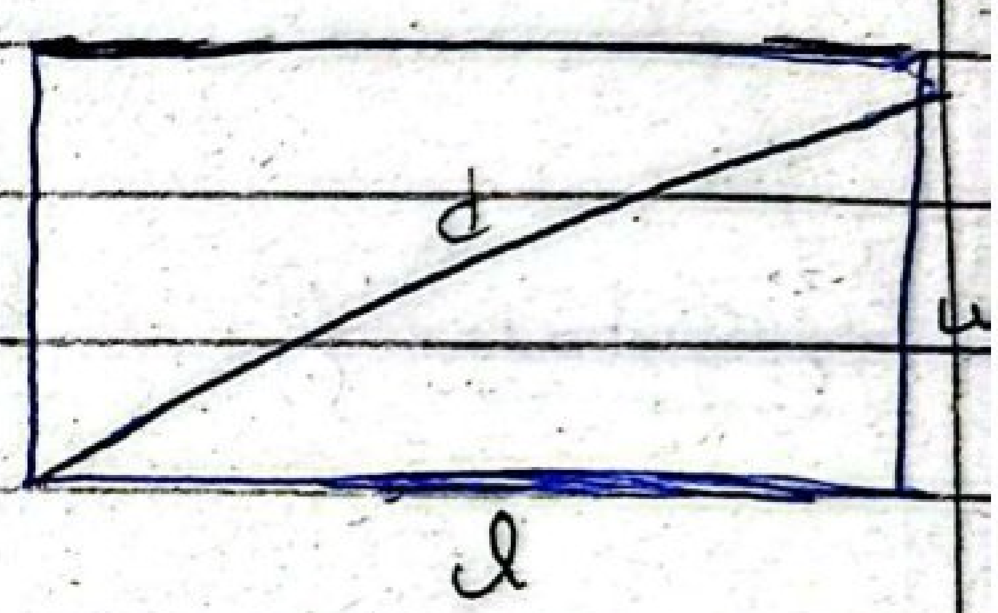
$a^2 - \frac{a^2}{4} = h^2$

$\frac{3a^2}{4} = h^2 \Rightarrow \boxed{h = \frac{\sqrt{3}}{2} a} \Rightarrow$

$A = \frac{1}{2} a \times \frac{\sqrt{3}}{2} a$

$\boxed{A = \frac{\sqrt{3}}{4} a^2}$

Q8. Solution:



$\frac{dd}{dt} = 1$ in/hr

$\frac{dl}{dt} = \frac{1}{4}$ in/hr

$\frac{dw}{dt} = ?$, $w = 6$ inch $l = 8$ inch

$d^2 = l^2 + w^2$ —— (1)

$\sqrt{d^2} = \sqrt{8^2 + 6^2}$

$d = 10$ in

from (1)

$2d \cdot \frac{dd}{dt} = 2l \cdot \frac{dl}{dt} + 2w \frac{dw}{dt}$

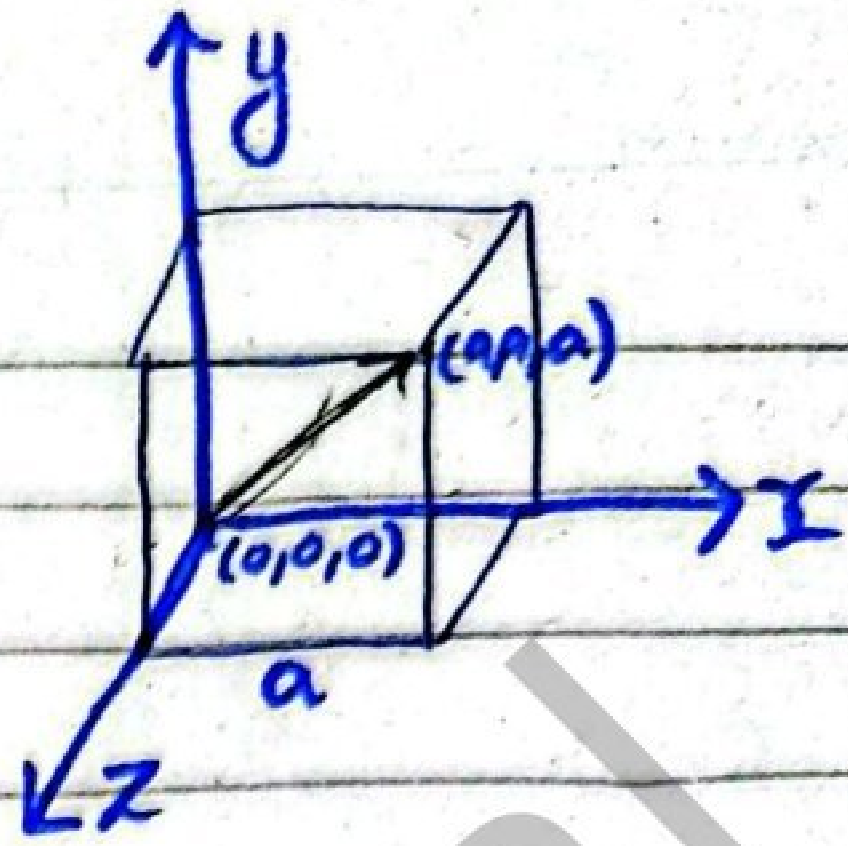
$2d \cdot \frac{dd}{dt} = \left[l \frac{dl}{dt} + w \frac{dw}{dt} \right] 2$

$10 \cdot (1) = 8 \cdot \left(\frac{1}{4}\right) + 6 \frac{dw}{dt}$

$10 - 2 = 6 \frac{dw}{dt}$

$\frac{dw}{dt} = \frac{4}{3}$ in/hr Ans

Q9. **Solution:** $\frac{da}{dt} = 5cm/hr$



$\frac{d}{dt} d = ?$

formula:- Diagonal of a cube

$$\boxed{d = \sqrt{3}a}$$

$$\frac{d}{dt} d = \sqrt{3} \cdot \frac{da}{dt}$$

$$\frac{d}{dt} d = \sqrt{3}(5) = \boxed{5\sqrt{3} \text{ cm/hr}} \text{ Answer}$$

Proof

$$d = \sqrt{(a-0)^2 + (a-0)^2 + (a-0)^2}$$

$$d = \sqrt{3a^2}$$

$$\boxed{d = \sqrt{3}a}$$

Q10) **Solution:-** $y^2 = x+1$; $\frac{dx}{dt} = 4x+1$

$$\left.\frac{dx}{dt}\right|_{x=8} = 4(8)+4$$

SLO

$$\boxed{\left.\frac{dx}{dt}\right|_{x=8} = 36} \text{ Answer}$$

if Question asked in exam find also $\frac{dy}{dt} = ?$ then

Sol $y^2 = x+1$

diff w.r.t $\cancel{x}$ t

$$2y \cdot \frac{dy}{dt} = \frac{dx}{dt} + 0$$

$$\frac{dy}{dt} = \frac{\cancel{36}^{18}}{\cancel{2}y}$$

$$\frac{dy}{dt} = \frac{18}{y} \quad \text{—(1)}$$

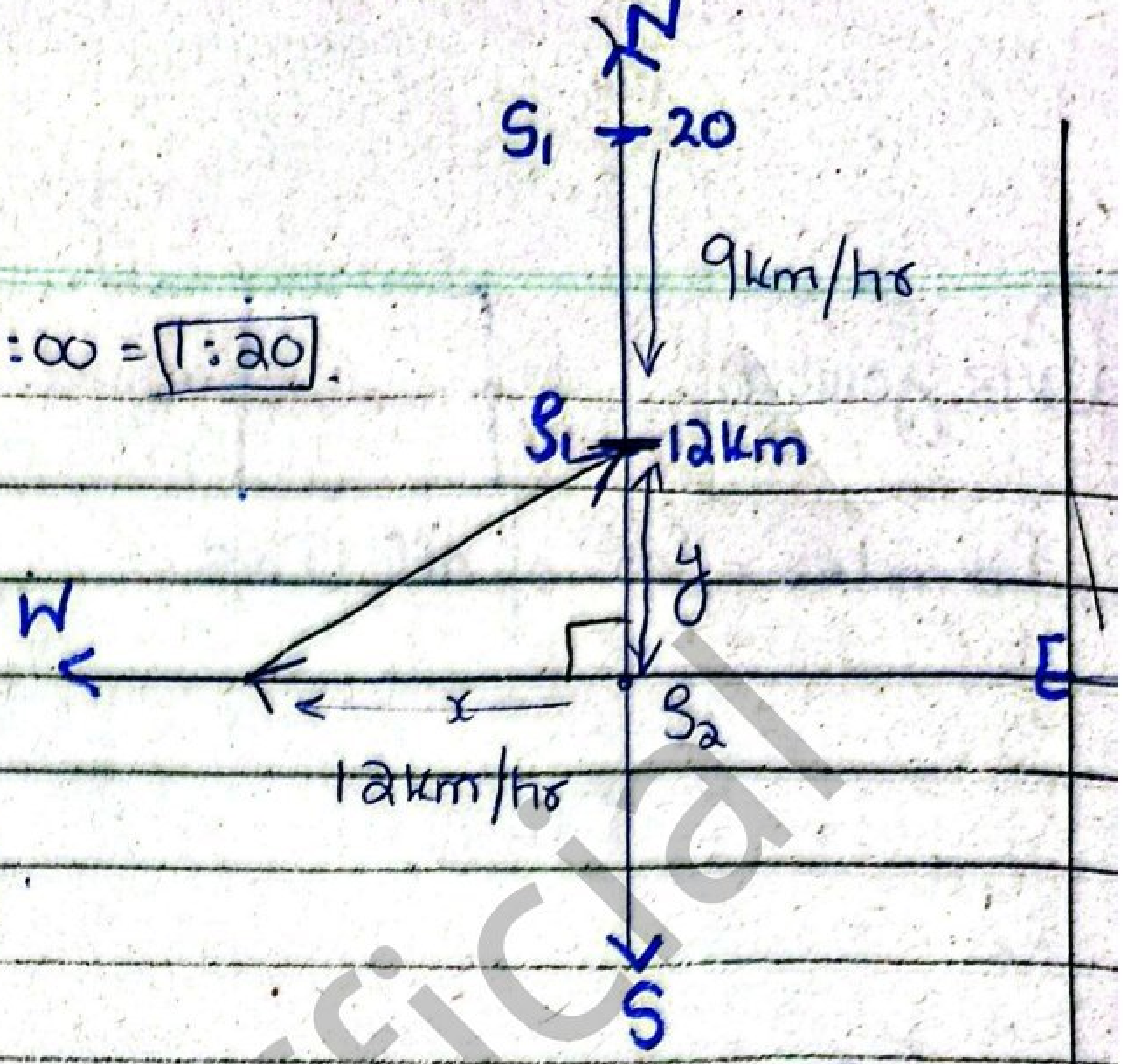
$$\sqrt{y^2} = \sqrt{8+1}$$

$$y = \sqrt{9}$$

$\boxed{y = \pm 3}$ put in (1)

$$\frac{dy}{dt} = \frac{\cancel{18}^{6}}{\pm \cancel{3}}$$

$$\boxed{\frac{dy}{dt} = \pm 6} \text{ Ans}$$



Q11) Solution:- time $= 9:20 - 8:00 = \boxed{1:20}$.

$= 1 + \frac{20}{60} = 1 + \frac{1}{3} = \boxed{\frac{4}{3}\,hr}$

S_1 (distance) $= 9 \times \frac{4}{3} = 12Km$.

$y = 20 - 12 = 8Km$

S_2 (distance) $= x = 12 \times \frac{4}{3} = \boxed{16Km}$.

$D^2 = x^2 + y^2$

$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$D = \sqrt{x^2+y^2}$

$D = \sqrt{(16)^2 + (8)^2} = \boxed{8\sqrt{5}}$.

$\boxed{\frac{dx}{dt} = 12Km/hr}$

$\frac{dy}{dt} = -9Km/hr$

$2\left[8\sqrt{5}\ \frac{dD}{dt}\right] = 2\left[16(12) + 8(-9)\right]$

$\frac{dD}{dt} = \frac{120}{8\sqrt{5}}$

$\boxed{\frac{dD}{dt} = 6.7\,Km/hr}$

Q12. Sol:- Let x & y be the two non-negative numbers.

Condition:- $x + y = 60$ —(1) & $P = xy$ —(2).

$\boxed{y = 60 - x}$ put in (2).

$P(x) = x(60 - x)$

$P(x) = 60x - x^2$

$\frac{dP}{dx} = 60 - 2x$

$\frac{d^2P}{dx^2} = -2 < 0$

Maximum

Put $\frac{dP}{dx} = 0$

$60 - 2x = 0$

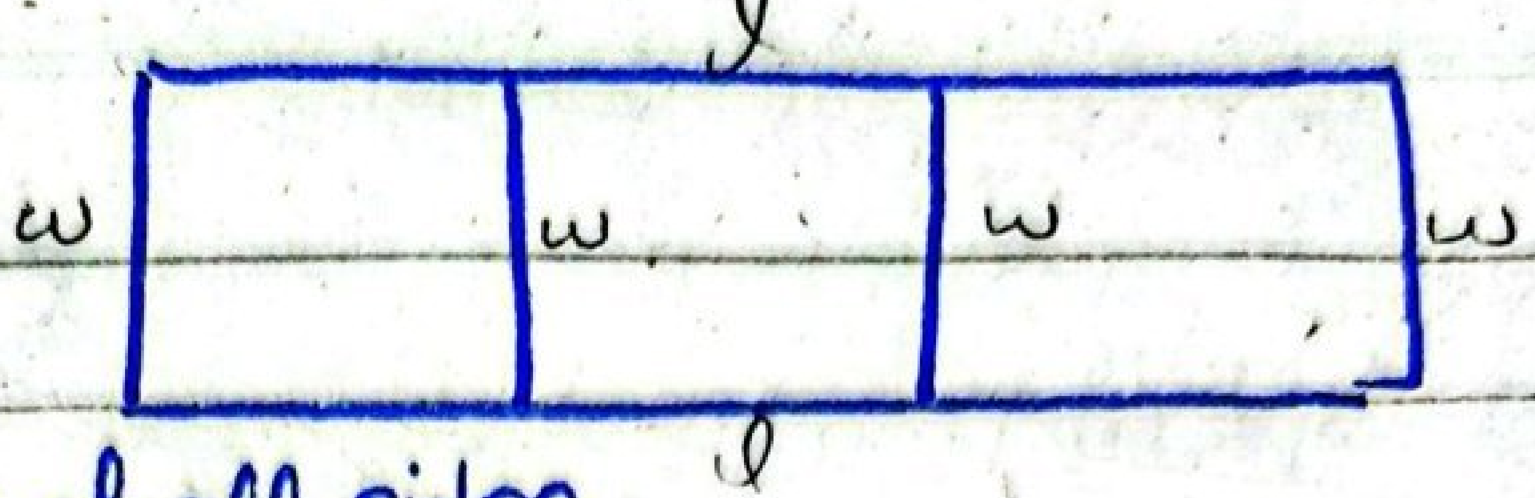
$60 = 2x$

$\boxed{x = 30}$

$y = 60 - 30$

$\boxed{y = 30}$

Q13 Solution:-



Perimeter = Sum of all sides

$P = 2l + 4w$

$8000 = 2(l + 2w)$

$\boxed{4000 - 2w = l}$ — (1).

Area $= l \times w$

using (1).

$A = w(4000 - 2w)$

$A = 4000w - 2w^2$

$\frac{dA}{dw} = 4000 - 2(2w)$

$\frac{d^2A}{dw^2} = -4 < 0$ maximum area or greatest area.

put $\frac{dA}{dw} = 0$

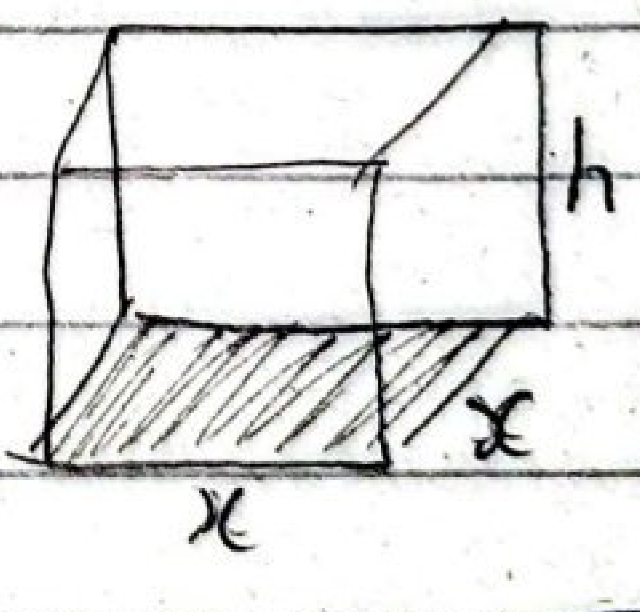
$4000 - 4w = 0$

$4000 = 4w$

$\frac{4000}{4} = w \Rightarrow \boxed{w = 1000m}$ & $\boxed{l = 2000m}$

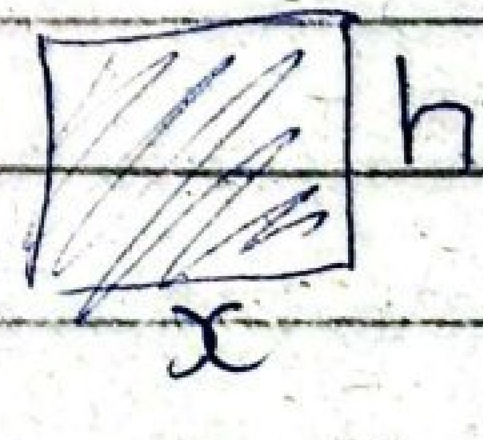
Q14) Solution:- Volume $= V = x \times x \times h$

$32000 = x^2 h$



$\boxed{h = \frac{32000}{x^2}}$

Total Surface area $= x^2 + 4xh$



$A = x^2 + 4x\left(\frac{32000}{x^2}\right)$

$A = x^2 + 128000x^{-1}$

$A = xh$

$\frac{dA}{dx} = 2x - 128000x^{-2}$

$\frac{d^2A}{dx^2} = 2 + 256000x^{-3} > 0$ Hence minimum (least).

put $\frac{dA}{dx} = 0$

$2x - 128000x^{-2} = 0$

$2x - \frac{128000}{x^2} = 0$

$\frac{2x^3 - 128000}{x^2} = 0$

$2[x^3 - 64000] = 0$

$x^3 - (40)^3 = 0$

$(x-40)\underbrace{(x^3+40x+40^2)}_{\times \text{ complex}} = 0$

$\boxed{x = 40cm}$

$h = \frac{32000}{(40)^2} \Rightarrow h = \frac{32000}{1600} = \boxed{20cm}$

Q15 Solution:- Profit $p(x) = R(x) - G(x)$

a) $P(x) = -3x^2 + 970x - 2x^2 - 500$

$\boxed{P(x) = -5x^2 + 970x - 500}$

$\frac{dp}{dx} = -10x + 970$

$\frac{d^2p}{dx^2} = -10 < 0$ maximum

put $\frac{dp}{dx} = 0$

$-10x + 970 = 0$

$\boxed{x = 97}$

$P(97) = -5(97)^2 + 970(97) - 500$

$\boxed{P(97) = 46,545}$ Answer.

(b) $A \cdot C = \frac{G(x)}{x} = \frac{2x^2+500}{x}$

$\boxed{A \cdot C = 2x + 500x^{-1}}$

$\frac{d(A.C)}{dx} = 2 - 500x^{-2}$

$\frac{d^2(A.C)}{dx^2} = +1000x^{-3} > 0$ minimum

put $\frac{d(A.C)}{dx} = 0$

$2 - \frac{500}{x^2} = 0$

$2 = \frac{500}{x^2}$

$2x^2 = 500$

$x^2 = 250$

$\boxed{x = \sqrt{250}}$

$A \cdot C = 2\sqrt{250} + \frac{500}{\sqrt{250}}$

$A \cdot C = 20\sqrt{10}$

$\boxed{A \cdot C = 63.24}$

Q16) Solution:- $r = 4\% = \frac{4}{100} = 0.04$

$P_0 = \$50$

formula $P(t) = P_0 e^{rt}$

(a) $\boxed{P(t) = 50e^{0.04t}}$ Answer

(b) $P(8) = 50e^{(0.04)8}$

$\boxed{P(8) = 68.85}$ Answer

(c) $\frac{dp}{dt} = P_0 e^{rt} \cdot r$

$\left.\frac{dp}{dt}\right|_{t=8} = 50\, e^{(0.04)(8)} \cdot (0.04) \Rightarrow \boxed{\left.\frac{dp}{dt}\right|_{t=8} = 2.75}$

Q17:- **Solution:-** $C(t) = 500e^{0.04t} - 100t$

ⓐ $\frac{dc}{dt} = 500(0.04)e^{0.04t} - 100$

$\boxed{\frac{dc}{dt} = 20e^{0.04t} - 100}$ Answer.

ⓑ $\frac{d^2c}{dt^2} = 20(0.04)e^{0.04t} = \frac{4}{5}e^{0.04t} > 0 \quad \forall t$

Hence cost is minimal func

Q18. **Solution:-** $P(t) = 150(1+0.05t)^2$

a) $\frac{dp}{dt} = 150[2(1+0.05t)](0.05)$

$\frac{dp}{dt} = 15(1+0.05t)$

$\frac{dp}{dt}\Big|_{t=3} = 15 + 15(0.05)(3)$

$\boxed{P'(3) = \frac{dp}{dt}\Big|_{t=3} = 17.25}$

ⓑ Inflation rate

$= \frac{P'(3)}{P(3)}$

$P(3) = 150(1+0.05(3))^2$

$P(3) = 198.375$

Inflation rate $= \frac{17.25}{198.375}$

$= 0.0869 \times 100\%$

$= 8.6\%$ Answer

Q19) Sol:- $d(t) = 15t$ $\quad S = \frac{\Delta d}{\Delta t}$

(a) speed $= \frac{dd}{dt} = \frac{d}{dt}(15t) = 15$ nautical miles

(b) $d(3) = 15(3) = 45$ nautical miles

(c) Here slope is 15 which show slope is constant.

20) Solution. $S(t) = 5t^2 + 3t$

(a) $\frac{d}{dt} S =$ speed $(V) = \frac{d}{dt}(5t^2 + 3t)$

$\boxed{V(t) = 10t + 3}$ — slope

(b) $V(4) = 10(4) + 3 = 43 ms^{-1}$ Ans

(c) Slope is increases if increase in time