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Where Minds Pollinate

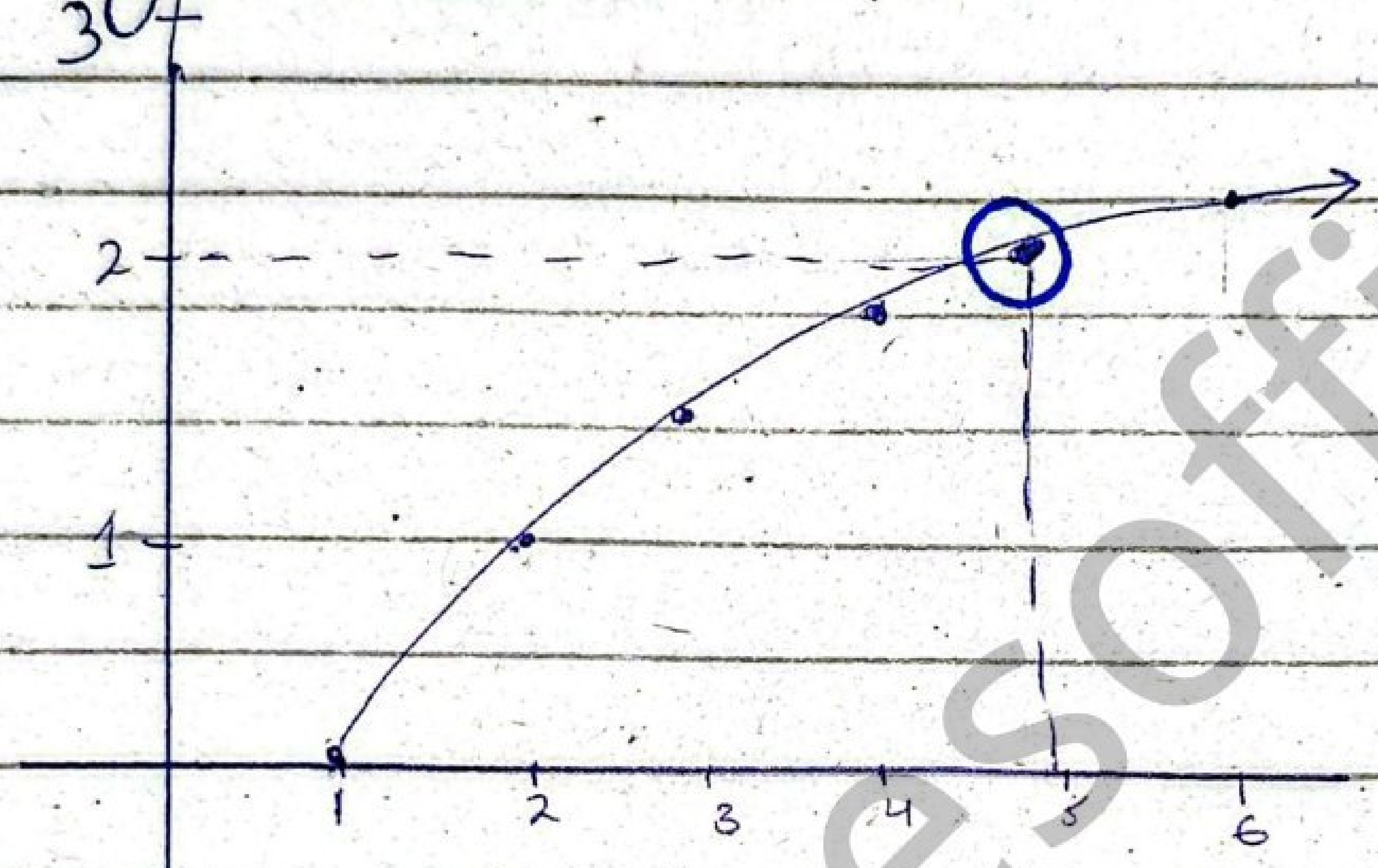
Class 12
Mathematics

Chapter 2
Exercise 2.1
Handwritten Solution

Exercise 2.1

Q1. a) $\lim_{x \to 5} \sqrt{x-1}$ Dom: $[0, \infty)$

x	1	2	3	4	4.9	5	5.1	6
y	0	1	1.4	1.7	1.9	2	2.02	2.2



$L.H.L = \lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} \sqrt{x-1}$

$= \boxed{2}$

$R.H.L = \lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} \sqrt{x-1} = \boxed{2}$

Hence $\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = 2$

Limit exist $\boxed{\lim_{x \to 5} \sqrt{x-1} = 2}$ Answer.

Concept

Leftside 5 Rightside

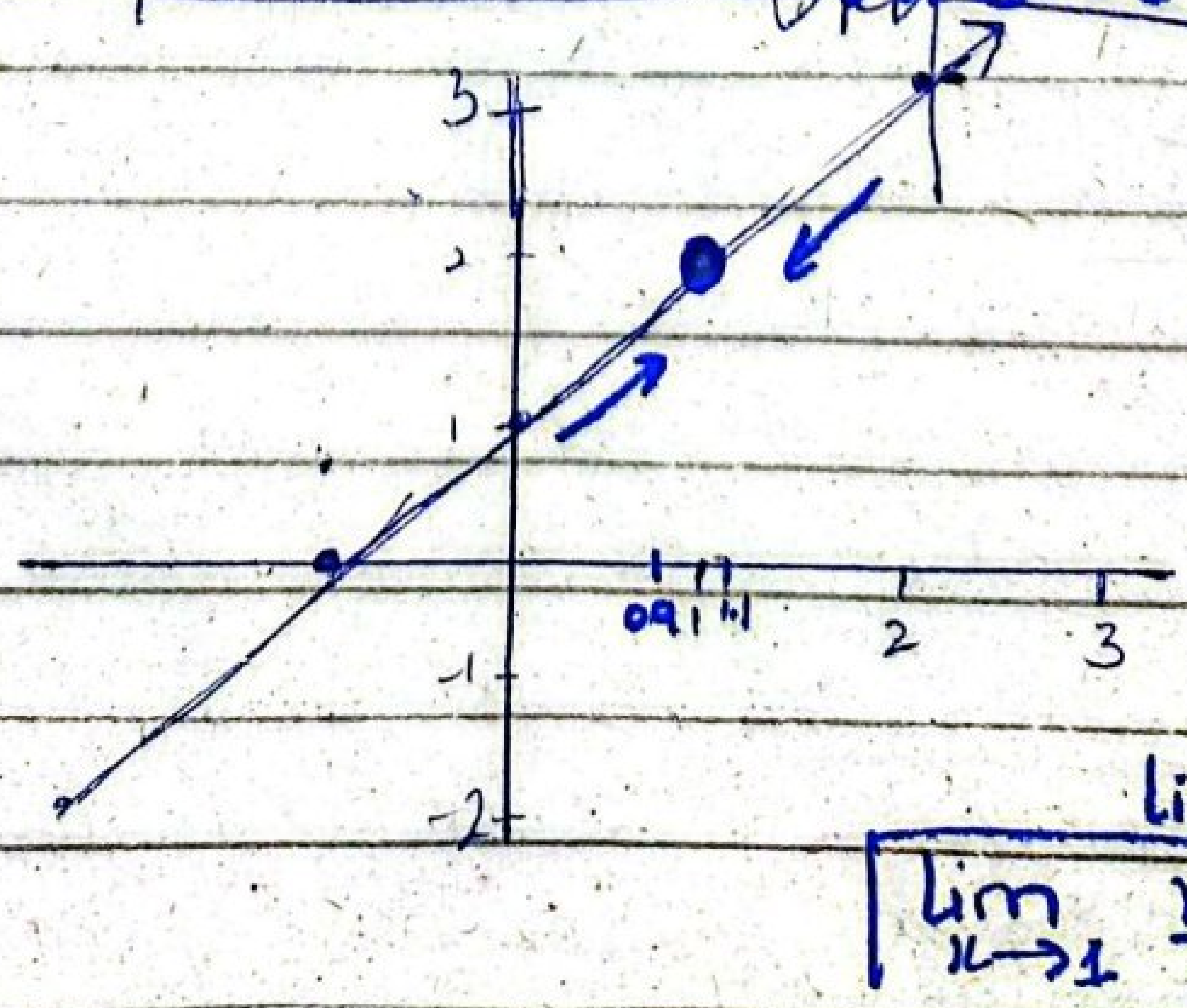
$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x)$

b) $\lim_{x \to 1} \frac{x^2 - 1^2}{(x-1)}$

$\lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$\lim_{x \to 1} (x+1)$

$f(x) = x+1$



$L.H.L = \lim_{x \to 1^-} f(x) = 2$

$R.H.L$

$= \lim_{x \to 1^+} f(x) = 2$

$LHL = RHL$

Limit exist

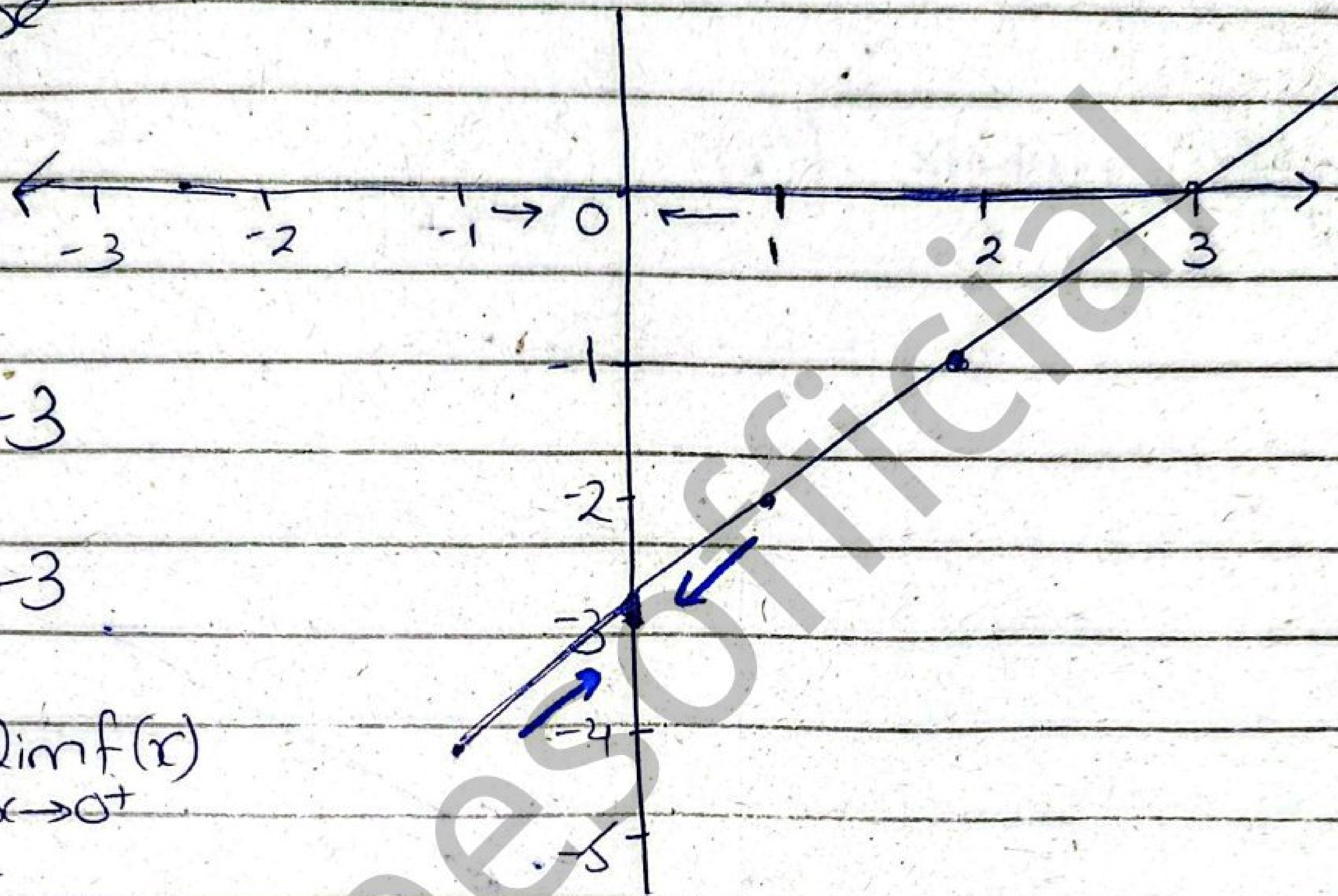
$\boxed{\lim_{x \to 1} \frac{x^2 - 1}{x-1} = 2}$ Ans

(c) $\lim_{x \to 0} \frac{x^2 - 3x}{x}$ $\qquad f(x) = x - 3$

$y = f(x)$

Sol:- $\lim_{x \to 0} \frac{\cancel{x}(x-3)}{\cancel{x}}$

$\lim_{x \to 0} (x-3)$



$\lim_{x \to 0^-} f(x) = -3$

$\lim_{x \to 0^+} f(x) = -3$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

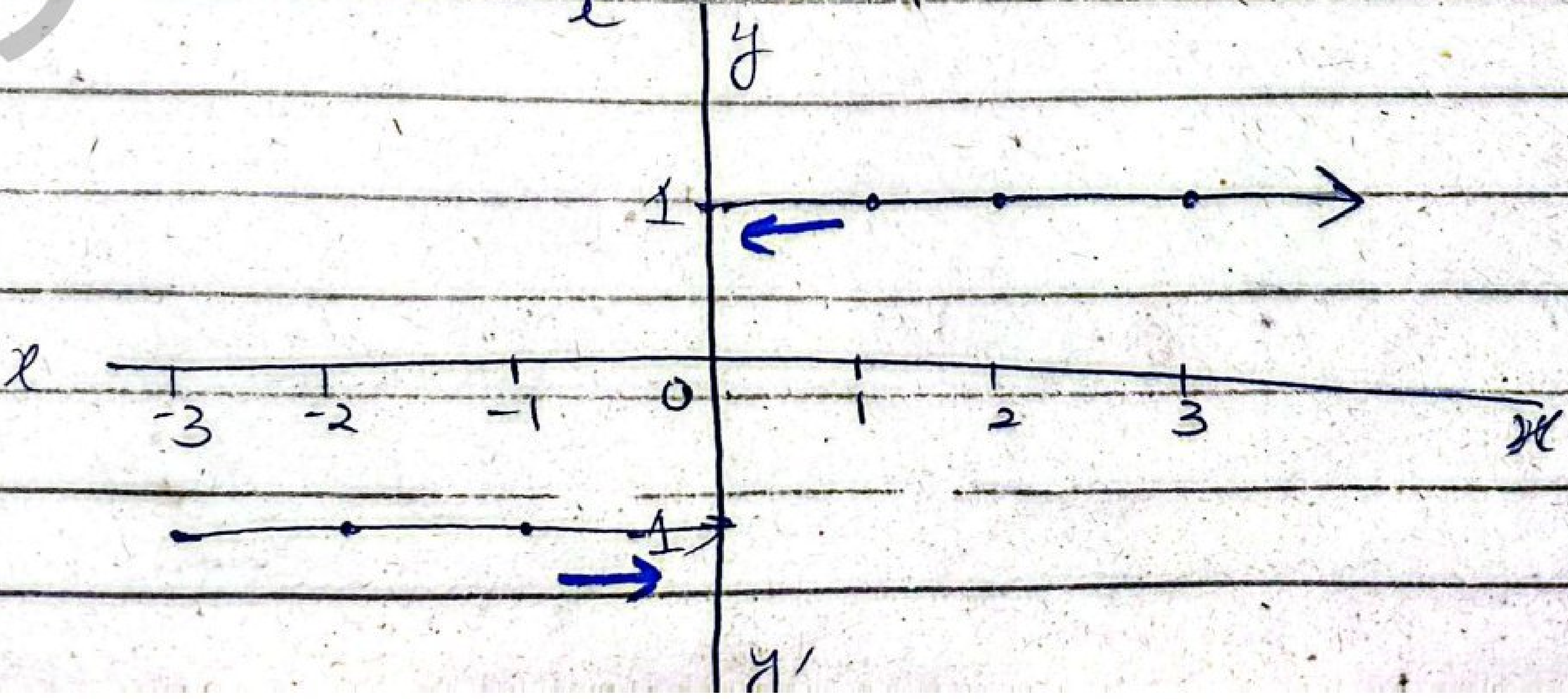
limit exist

$\boxed{\lim_{x \to 0} \frac{x^2 - 3x}{x} = -3}$

(d) $\lim_{x \to 0} \frac{|x|}{x}$

if $x > 0 \Rightarrow \lim_{x \to 0^+} \frac{\cancel{x}}{\cancel{x}} = \underline{1}$

if $x < 0 \Rightarrow \lim_{x \to 0^-} \frac{-x}{x} = -\underline{1}$



LHL:-

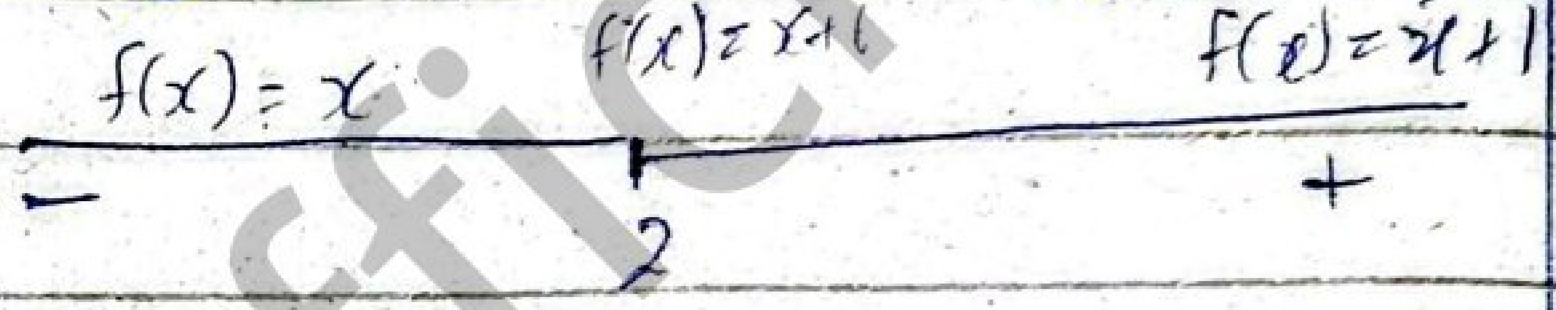
$$\lim_{x\to 0^-} f(x) = -1$$

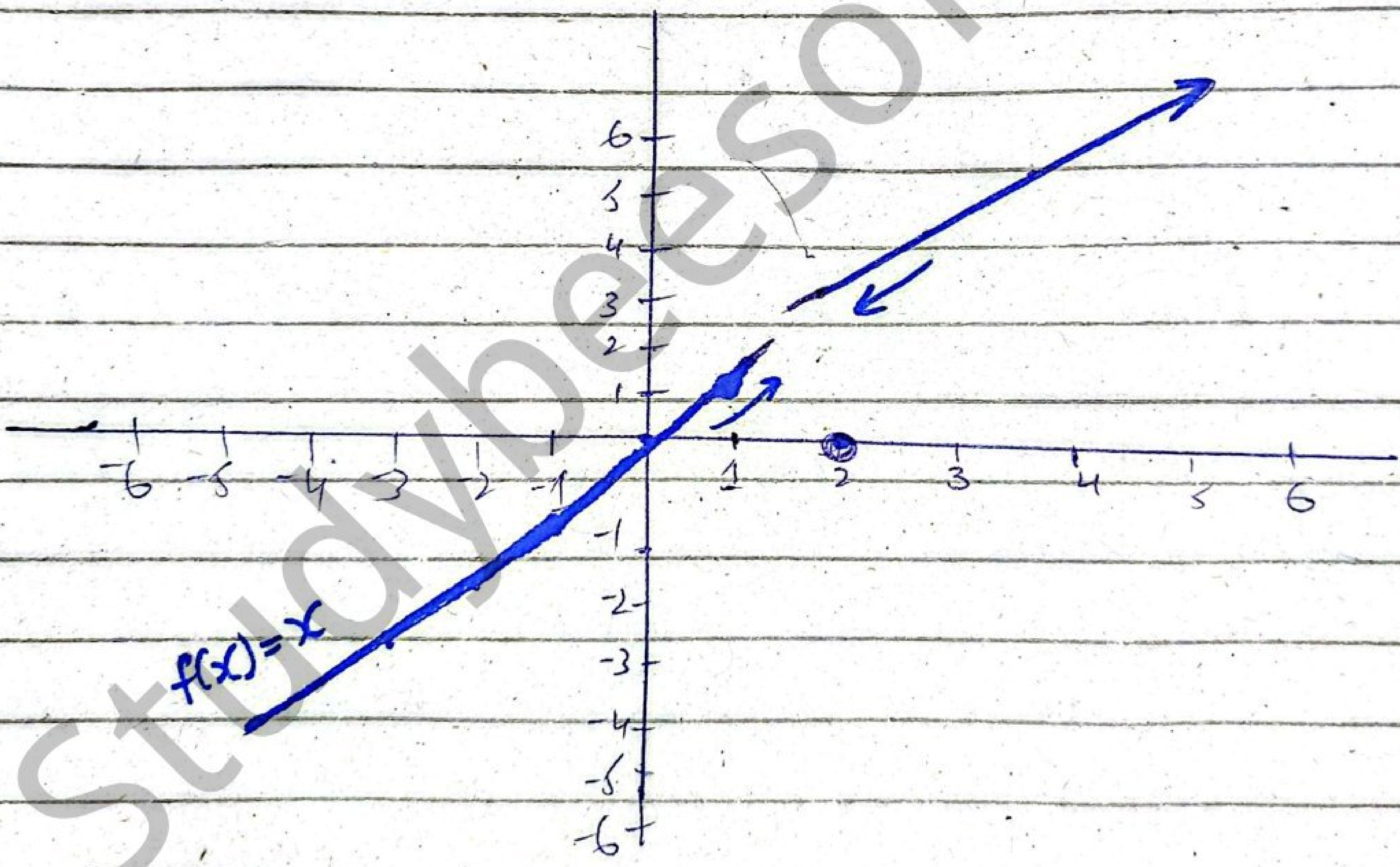
RHL $\lim_{x\to 0^+} f(x) = 1$

LHL ≠ RHL

Limit doesn't exist.

(e) $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} x & x<2 \\ x+1 & x\geq 2 \end{cases}$





LHS: $\lim_{x\to 2^-} f(x) = \lim_{x\to 2} (x)$

$= 2$

RHL:-

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x+1)$

$= 3$

LHL ≠ RHL

Hence limit

doesn't ex

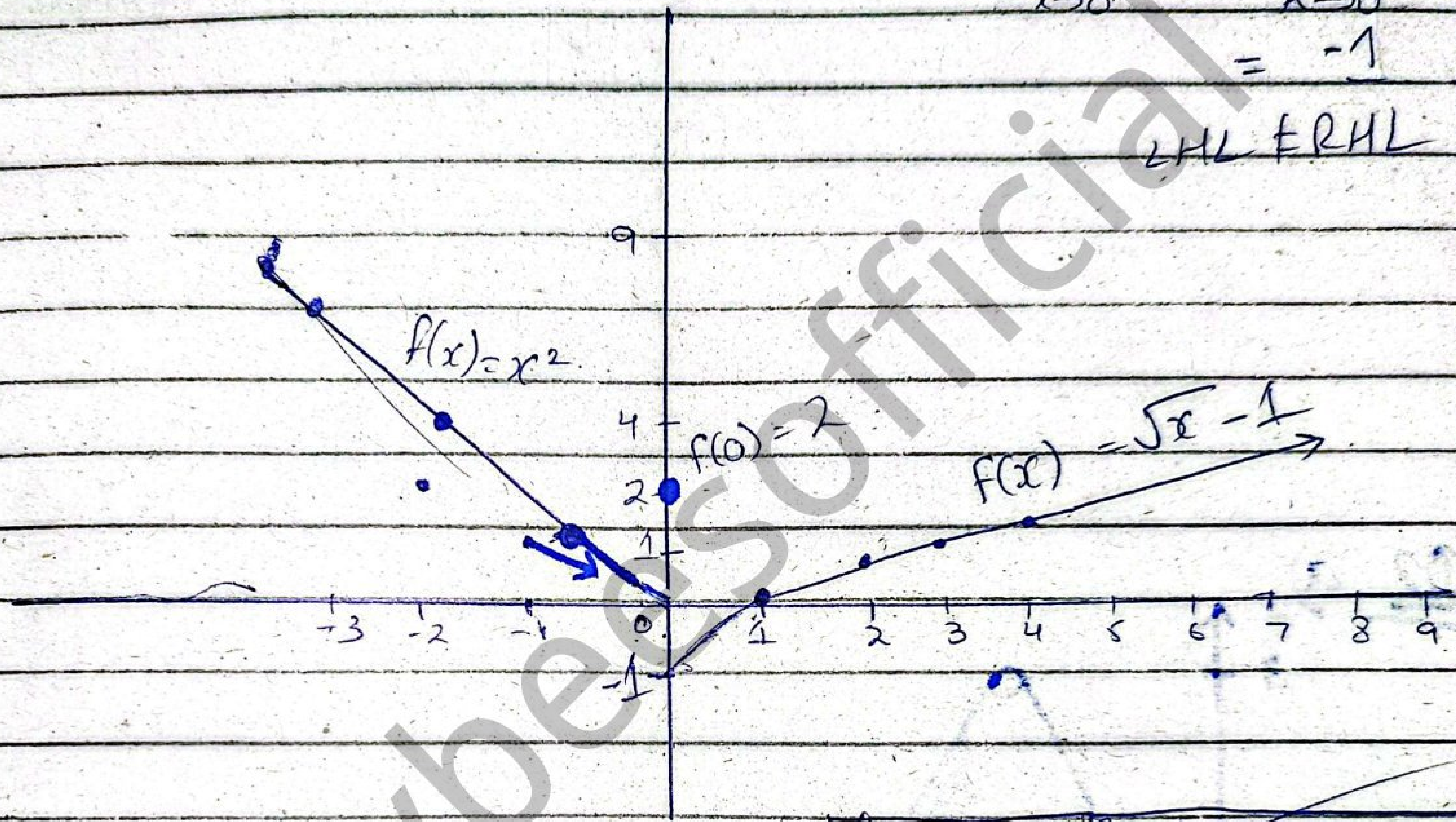
(f) $\lim_{x \to 0} f(x)$, where $f(x) = \begin{cases} x^2 & x < 0 \\ 2 & x = 0 \\ \sqrt{x} - 1 & x > 0 \end{cases}$

Sol: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 = 0$

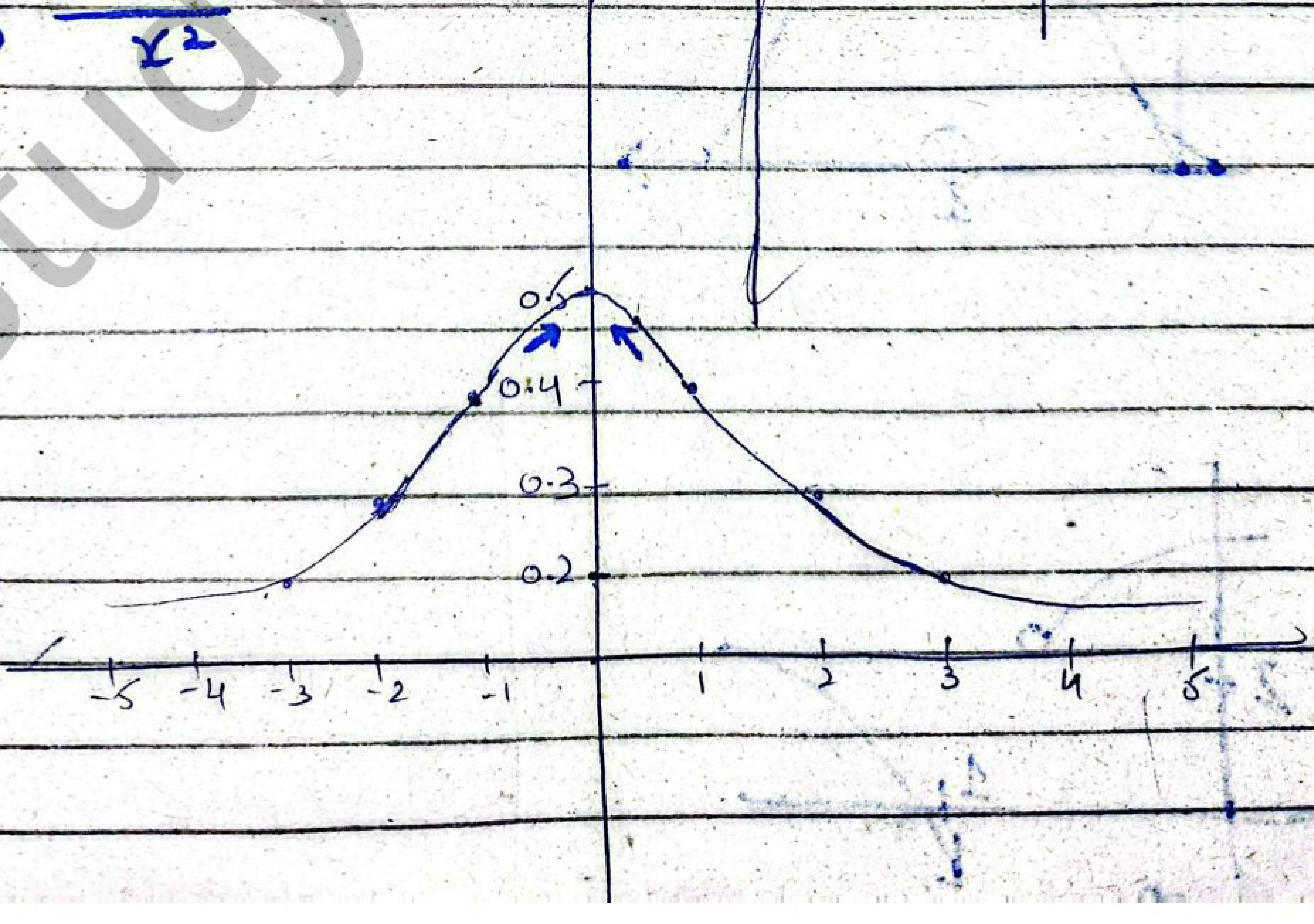
R.H.L $\Rightarrow \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \sqrt{x} - 1$

$= -1$

LHL $\neq$ RHL



(g) $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$



$x \to 0^-$	$\frac{1-\cos x}{x^2}$
-0.1	0.499583
-0.01	0.499995
-0.001	0.499999

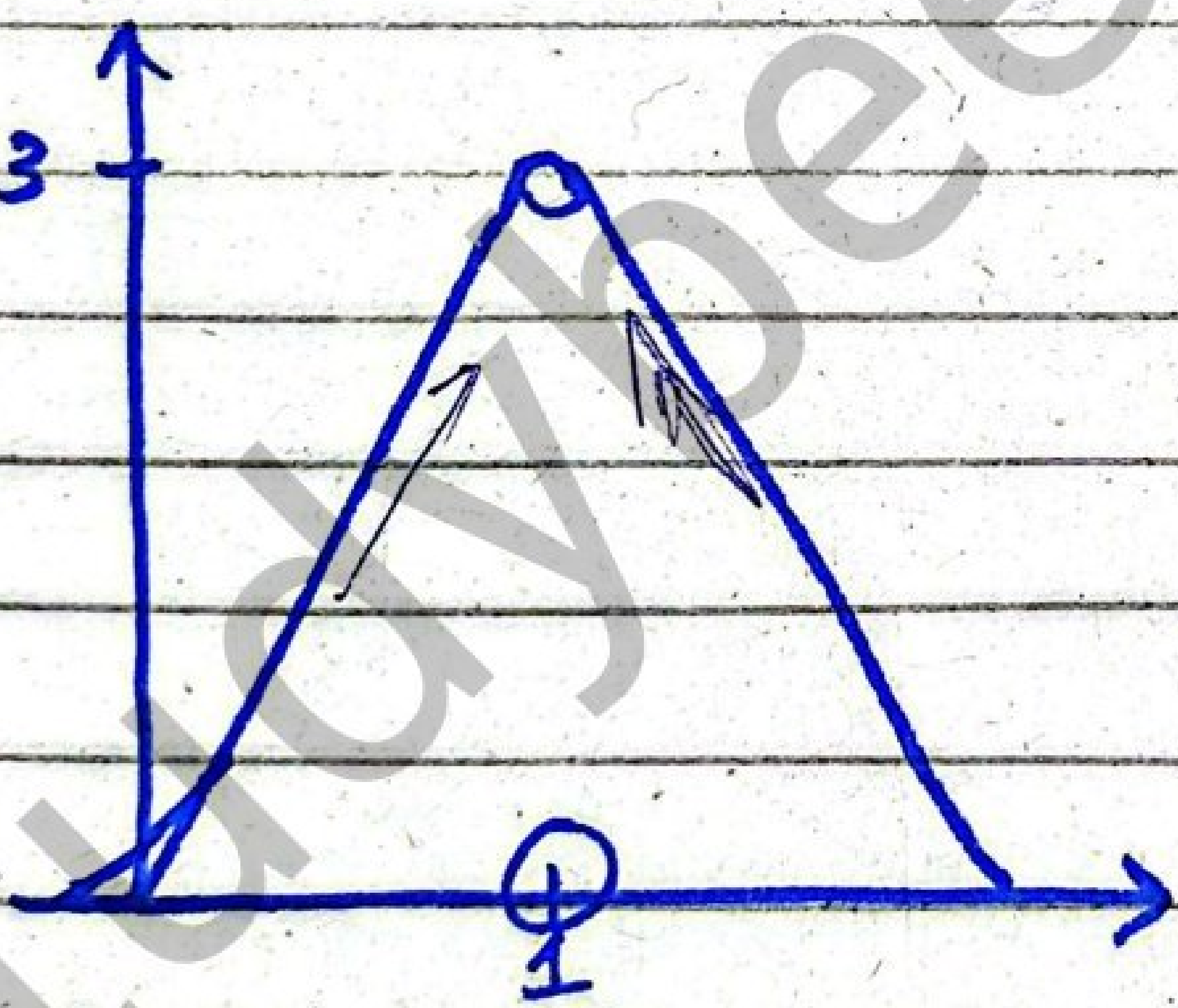
$x \to 0^+$	$\frac{1-\cos x}{x^2}$
0.1	0.499583
0.01	0.499995
0.001	0.499999

$$LHL = \lim_{x\to 0^-} f(x) = \lim_{x \to 0^-} \frac{1-\cos x}{x^2} = 0.5$$

$$RHL = \lim_{x \to 0^+} f(x) = \lim_{x\to 0^+} = \frac{1-\cos x}{x^2} = 0.5$$

LHL = RHL so limit exist

Q2 a)

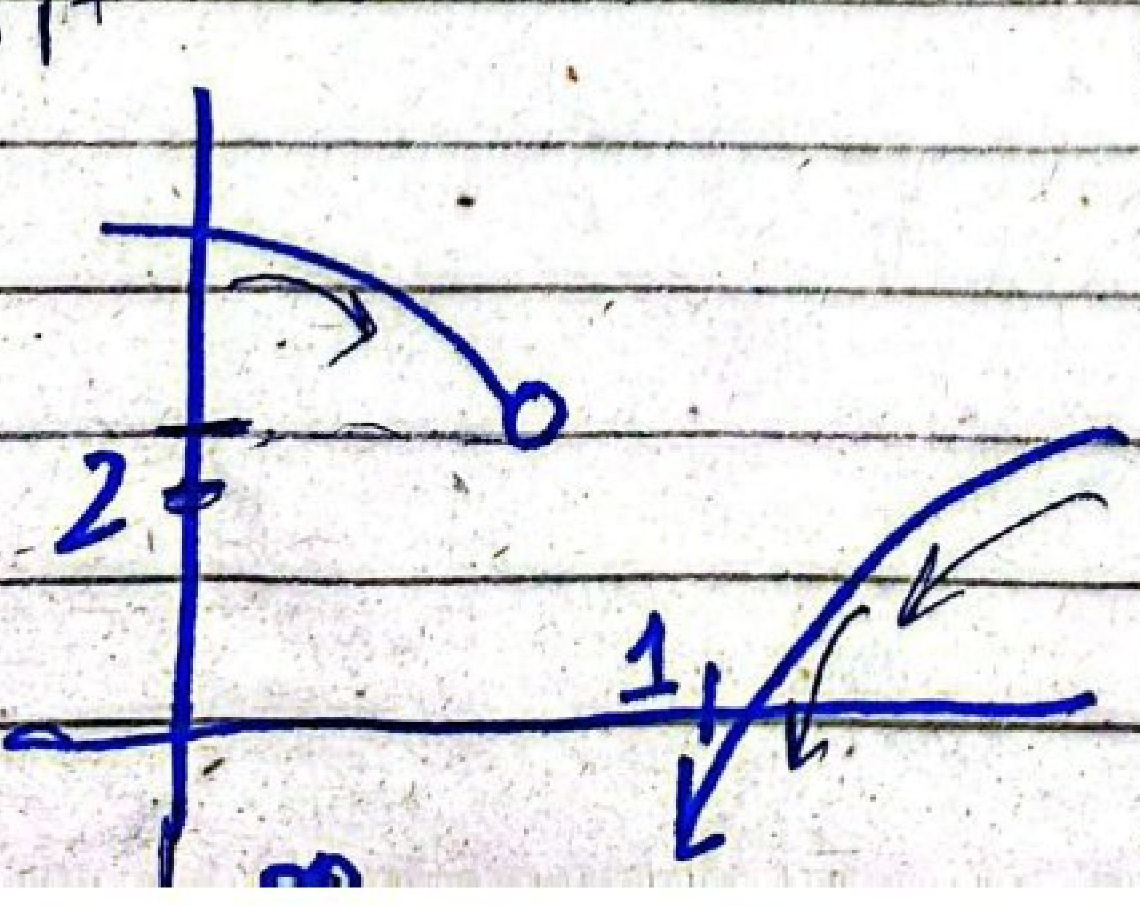


$\lim_{x\to 1^-} f(x) = 3$

$\lim_{x\to 1^+} f(x) = 3$

LHL = RHL

(b)



LHL

$\lim_{x\to 1^-} f(x) = 2$

$RHL = \lim_{x\to 1^+} f(x) = -\infty$

LHL ≠ RHL

Concept:- Undefined values:

1) $\frac{0}{0}$ 2) $\frac{x}{0}$ 3) $\frac{\infty}{\infty}$ 4) ∞^{+} 5) ∞^{-}

Q3) $\lim\limits_{x\to -2} \dfrac{x^3+8}{x+2}$

Sol:- $\lim\limits_{x\to -2} \dfrac{(x)^3+(2)^3}{x+2}$

$= \lim\limits_{x\to -2} \dfrac{\cancel{(x+2)}(x^3-2x+4)}{\cancel{(x+2)}}$

$= (-2)^2 - 2(-2)+4$

$= \boxed{12}$ Answer.

(4) $\lim\limits_{x\to 4} \dfrac{\sqrt{x}-2}{x-4}$

Sol:- $\lim\limits_{x\to 4} \dfrac{\sqrt{x}-2}{x-4} \times \dfrac{\sqrt{x}+2}{\sqrt{x}+2}$

$= \lim\limits_{x\to 4} \dfrac{(\sqrt{x})^2-(2)^2}{(x-4)(\sqrt{x}+2)}$

$= \lim\limits_{x\to 4} \dfrac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x}+2)}$

$= \lim\limits_{x\to 4} \dfrac{1}{\sqrt{x}+2}$

$= \dfrac{1}{\sqrt{4}+2} = \dfrac{1}{4}$ Answer

5) $\lim\limits_{x\to 7} \dfrac{x^2-21}{x+2}$

Apply limit

$= \dfrac{(7)^2-21}{9}$

$= \dfrac{28}{9}$ Answer

6) $\lim\limits_{x\to 0} \dfrac{x^2-6x}{x^2-7x+6}$

Sol:- Apply limit

$= \dfrac{0^2-6(0)}{(0^2)-7(0)+6}$

$= \dfrac{0}{6} = 0$ Ans

(7) $\lim\limits_{y\to 1} \dfrac{y^3-1}{y-1}$

Sol:- $\lim\limits_{y\to 1} \dfrac{(y^3)-1}{(y-1)}$

$\lim\limits_{y\to 1} \dfrac{\cancel{(y-1)}(y^3+1+2y)}{\cancel{(y-1)}}$

Apply Limit

$\lim\limits_{y\to 1} (y^3+1+2y)$

$\lim\limits_{y\to 1} 1^3+1+2(1)$

$= 4$ Ans

8) $\lim_{x\to 3^+} \frac{(x+3)^2}{\sqrt{x-3}}$

Sol:- $\sqrt{3-3} = 0$

$\Rightarrow$ Denominator $= 0$

$\Rightarrow \frac{(x+3)^2}{0} = \infty$

$\Rightarrow$ Hence Limit doesn't exist

9) $\lim_{x\to 2} (x-4)^4(x^2-3)^{10}$ Apply limit

Sol: $= (2-4)^4(2^2-3)^{10}$

$= 16$

10) $\lim_{x\to 0}\left(x-\frac{1}{x-2}\right)$

= Apply limit

$= 0-\frac{1}{0-2}$

$= +\frac{1}{2}$ Answer.

12) $\lim_{x\to 0}\frac{\tan x}{x}$

Sol:- $\lim_{x\to 0}\frac{\sin x}{(\cos x)x}$

$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{\cos x}$

$= \left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\frac{1}{\cos x}\right)$

$= 1\cdot\frac{1}{\cos(0)} \Rightarrow \boxed{1}$ Answer

Concept

Sandwich theorm

$\lim_{x\to 0}\frac{\sin x}{x} = 1$

Sine law:-

$\lim_{x\to 0}\frac{\sin 3x}{3x} = 1$

11) $\lim_{x\to -3}\frac{2x+6}{4x^2-36}$

$\lim_{x\to -3}\frac{2(x+3)}{4(x^2-9)}$

$\lim_{x\to -3}\frac{1(x+3)}{2(x^2-(3)^2)}$

$=$ $\lim_{x\to -3}\frac{1\cancel{(x+3)}}{2\cancel{(x+3)}(x-3)}$

$\lim_{x\to -3}\frac{1}{2(x-3)}$

Apply limit

$\frac{1}{2(-3-3)} = \frac{1}{-12}$

13) $\lim_{x\to 0}\frac{x}{\sin 3x}$

Sol: $\frac{1}{3}\lim_{x\to 0}\frac{3x}{\sin 3x}$ $\therefore$ 3x S ÷

$\frac{1}{3}\lim_{x\to 0}\frac{1}{\left(\frac{\sin 3x}{3x}\right)}$

Put $3x=t$

if $x\to 0 \Rightarrow 3x\to 0$

$3x\to 0$ then $t\to 0$

$\frac{1}{3}\left(\frac{1}{\lim_{t\to 0}\frac{\sin t}{t}}\right) = \frac{1}{3}\left(\frac{1}{1}\right) = \frac{1}{3}$

OR

$\lim_{x\to 0}\frac{3x}{3(\sin 3x)}$

$\lim_{x\to 0}\frac{1}{3\left(\frac{\sin 3x}{3x}\right)}$

$\lim_{x\to 0}\frac{1}{3}\left(\frac{1}{\lim_{x\to 0}\frac{\sin 3x}{3x}}\right)$

$= \frac{1}{3}\left(\frac{1}{1}\right)$

$= \frac{1}{3}$