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Where Minds Pollinate

Class 12

Mathematics

Chapter 10

Exercise 10.2

Handwritten Solution

Aca

Exercise # 10.2

(numerical Integration)

Trapezoidal Rule:

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\{f(x_1) + f(x_2) + \ldots + f(x_{n-1})\} + f(x_n)\right]$$

(h: difference)

where $h = \frac{b-a}{n}$

Simpson's Rule:

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4\{f(x_1) + f(x_3) + \cdots f(x_{n-1})\} + 2\{f(x_2) + f(x_4) + \cdots f(x_{n-2})\} + f(x_n)\right]$$

Q1

Sol:-

x	0	0.5	1	1.5	2	2.5	3	3.5	4
$f(x)$	1	1.649	2.718	4.482	7.389	12.18	20.09	22.12	45.60
	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$	$f(x_5)$	$f(x_6)$	$f(x_7)$	$f(x_8)$

$h = \frac{4-0}{8} = h = 0.5$. $\int_0^4 f(x)\,dx = ?$

By trapezoid rule:-

$$\int_0^4 f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\{f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7)\} + f(x_8)\right]$$

$$= \frac{0.5}{2}\left[1 + 2\{1.649 + 2.718 + 4.482 + 7.389 + 12.18 + 20.09 + 22.12\} + 45.60\right]$$

$$\int_0^4 f(x)\,dx = 46.96$$ answer.

Now **By Simpson's Rule:-**

$$\int_0^4 f(x)dx = \frac{h}{3}\left[f(x_0) + 4\left\{f(x_1) + f(x_3) + f(x_5) + f(x_7)\right\} + 2\left\{f(x_2) + f(x_4) + f(x_6)\right\} + f(x_8)\right]$$

$$= \frac{0.5}{3}\left[1 + 4\left\{1.649 + 4.482 + 12.18 + 22.12\right\} + 2\left\{2.718 + 7.389 + 20.09\right\} + 45.60\right]$$

$$= \boxed{44.7863}$$ Answer.

Q3. $\int_{a=0}^{b=1} \frac{dx}{2+x^2}$ $n=8$.

Calculator
Shift + mode + 7 + function + Start (0) + end (1) + Step = 0.125

Sol: $h = \frac{b-a}{n}$

$h = \frac{1-0}{8} = \boxed{0.125}$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$f(x)$	0.5	0.4961	0.484	0.4671	0.4444	0.4183	0.3902	0.3615	0.3333
	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$	$f(x_5)$	$f(x_6)$	$f(x_7)$	$f(x_8)$

By Simpsons rule.

$$\int_0^1 \frac{1}{2+x^2}dx = \frac{0.125}{3}\left[0.5 + 4\left\{0.4961 + 0.4671 + 0.4183 + 0.3615\right\} + 2\left\{0.484 + 0.4444 + 0.3902\right\} + 0.3333\right.$$

$$\int_0^1 \frac{1}{2+x^2}dx = 0.4352 \text{ Answer by Simpsons rule.}$$

Now by Trapezoidal Rule:-

$$\int_0^1 \frac{1}{2+x^2}dx = \frac{0.125}{2}\left[0.5+2\{0.4961+0.4844+0.4671+0.4444+0.4183+0.3902+0.3615\}+0.3333\right].$$

$$\int_0^1 \frac{1}{2+x^2}dx = +0.4347 \text{ Answer.}$$

now By analytical way.

$$\int_0^1 \frac{1}{2+x^2}dx = \int_0^1 \frac{1}{(\sqrt{2})^2+x^2}dx$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}\Big|_0^1$$

$$= \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \tan^{-1}(0)\right]$$

$\therefore$ cal in radian

$= 0.4352$ Answer

Q2. $\int_{-1}^{7} f(x)dx$.

x	-1	0	1	2	3	4	5	6	7
$f(x)$	0.9848	1	0.9848	0.9397	0.8660	0.7660	0.6428	0.500	0.3420
	$f(0)$	$f(1)$	$f(2)$						$f(8)$

$h = 1$

$$\int_{-1}^{7} f(x)dx = \frac{1}{2}\left[0.9848+2\{1+0.9848+0.9397+0.8660+0.7666+0.6428+0.500\}+0.3420\right]$$

$= 6.363$ Ans

Using Simpsons rule

$$\int_{-1}^{1} f(x)\,dx = \frac{1}{3}\Big[0.9848 + 4\{1 + 0.9397 + 0.7660 + 0.500\}$$
$$+ 2\{0.9848 + 0.8660 + 0.6428\} + 0.3420\Big]$$

$$= \boxed{6.379}$$

Q4) $\int_0^1 \frac{dx}{1+x}$ $\quad n = 8$

$h = 0.125$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
$f(x)$	1	0.8888	0.8	0.7272	0.6666	0.6153	0.5714	0.5333	0.5	

Trapezoidal rule:-

$$\int_0^1 \frac{dx}{1+x} = \frac{1}{2}\Big[1 + 2\{0.8888 + 0.8 + 0.7272 + 0.6666 + 0.6153 + 0.5333\} + 0.5\Big]$$

$=$

By Simpsons Rule:-

$$= \frac{1}{3}\Big[1 + 4\{0.8888 + 0.7272 + 0.6153 + 0.5333\} + 2\{0.8 + 0.6666 + 0.5714\} + 0.5\Big]$$

$=$

Qs. $\int_{a=0}^{b=\pi/2} \sin x \, dx$; $n=6$

Sol:- $h = \frac{b-a}{n} = \frac{\frac{\pi}{2}-0}{6} = \boxed{\frac{\pi}{12}} = \frac{3.14}{12}$ or $= \boxed{0.2617}$

x	x_0: 0	x_1: $\frac{\pi}{12}$	x_2: $\frac{\pi}{6}$	x_3: $\pi/4$	x_4: $\pi/3$	x_5: $\frac{5\pi}{12}$	x_6: $\pi/2$
$f(x)$	0	0.2588	0.5	0.7071	0.866	0.9659	1

$y = \sin x$

By Trapezoidal Rule:-

$$\int_0^{\pi/2} \sin x \, dx = \frac{\pi/12}{2}\left[0 + 2\{0.2588 + 0.5 + 0.7071 + 0.866 + 0.9659\} + 1\right]$$

$= 0.9942$

By Simpsons Rule:-

$$\int_0^{\pi/2} \sin x \, dx = \frac{\pi/12}{3}\left[0 + 4\{0.2588 + 0.7071 + 0.9659\} + 2\{0.5 + 0.866\} + 1\right]$$

= ~~0.9999 or~~ 1.0000038

Analytical method.

$$\int_0^{\pi/2} \sin x \, dx = -\cos x \Big|_0^{\pi/2}$$

$= -\left(\cos\frac{\pi}{2} - \cos 0\right) = \boxed{1}$ exact value.

$$\text{Error} = \left| \text{exact value} - \text{Approximated value} \right|$$

- **Trapazidal rule:-**

$$\text{Error} = |1 - 0.9942|$$

$$= |0.0058|$$

$$= \underline{0.0058}$$

- **Simpson's rule:**

error = |1 − [illegible]8|

$$= |0.0000038|$$

$$= \underline{0.0000038}$$

Hence Simpsons rule has less error or is more ~~error~~ accurate than trapizoidal rule.

Q6) Sol:- $\displaystyle\int_0^3 \frac{x^2}{1+x}\,dx \qquad n=10$

Sol:- $h = \dfrac{b-a}{n} = \dfrac{3-0}{10} = 0.3$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	
$f(x)$	0	0.0692	0.225	0.4263	0.6545	0.9	1.1571	1.4225	1.6941	1.9702	2.25	

Trapizoidal rule:-

$$\int_0^3 \frac{x^2}{1+x}\,dx = \frac{0.3}{2}\Big[0 + 2\{0.0692 + 0.225 + 0.4263 + 0.6545 + 0.9 + 1.1571 + 1.4225 + 1.6941 + 1.9702\} + 2.25\Big]$$

$$= \underline{\qquad\qquad}$$

Simpsons Rule

$$\int_0^3 \frac{x^2}{1+x}\,dx = \frac{0.3}{3}\Big[0 + 4\{0.0692 + 0.4263 + 0.9 + 1.4225 + 1.9702\} + 2\{0.225 + 0.6545 + 1.1571 + 1.6941\} + 2.25\Big]$$

$$= \underline{\qquad\qquad}$$