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Where Minds Pollinate

Class 12
Mathematics

Chapter 10
Exercise 10.1
Handwritten Solution

Exercise 10.1

Q1) (i) $x^3 - x^2 + x - 7 = 0$, $[2,3]$

Solution:- Bisection method :-

let $f(x) = x^3 - x^2 + x - 7$; $[\underset{a}{2}, \underset{b}{3}]$

$f(a) = f(2) = 2^3 - 2^2 + 2 - 7$

$f(2) = -1 < 0$ -ve

&

$f(b) = f(3) = 3^3 - 3^2 + 3 - 7$

$f(3) = 14 > 0$ +ve.

∴ If same sign the root will not lie in interval

As $f(a) \cdot f(b) < 0$ so root lies in $[2,3]$.

$x_0 = \frac{2+3}{2} = \boxed{2.5}$

So root lies either between $[2, 2.5]$ or $[2.5, 3]$.

$f(2.5) = (2.5)^3 - (2.5)^2 + 2.5 - 7$

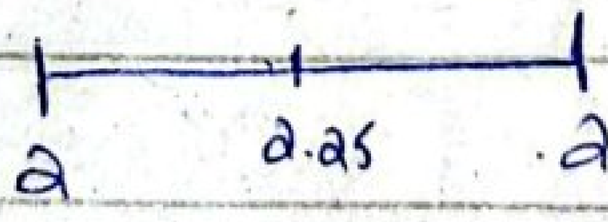
$= 4.875 > 1$

As $\underset{(-ve)}{f(2)} \cdot \underset{(+ve)}{f(2.5)} < 0$

So root lies between interval $[2, 2.5]$

$x_1 = \frac{2+2.5}{2}$

$\boxed{x_1 = 2.25}$ ∴



So root will be ~~between~~ in either $[2, 2.25]$ or $[2.25, 2.5]$

$f(2.25) = (2.25)^3 - (2.25)^2 + (2.25) - 7$

$= 1.5781 > 0$ +ve

As $f(2)\cdot f(2.25) < 0$ so

root lies in $[2, 2.25]$

$$x_2 = \frac{2+2.25}{2}$$

$$\boxed{x_2 = 2.125}$$

So, root will be in either $[2, 2.125]$ or $[2.125, 2.25]$

$$f(2.125) = (2.125)^3 - (2.125)^2 + 2.125 - 7$$

$$= 0.20 > 0 \text{ -ve}$$

As $f(2)\cdot f(2.125) < 0$

So root lies in $[2, 2.125]$

$$x_3 = \frac{2+2.125}{2} = 2.0625$$

So root will be in either $[2, 2.0625]$ or $[2.0625, 2.125]$

$$f(2.0625) = (2.0625)^3 - (2.0625)^2 + 2.0625 - 7$$

$$= -0.41 < 0$$

As $f(2)\cdot f(2.0625) > 0$ so

root will not lie in $[2, 2.0625]$ now

As $f(2.0625)\cdot f(2.125) < 0$

So root lie in $[2.0625, 2.125]$

$$x_4 = \frac{2.0625 + 2.125}{2} = 2.09375$$

So, root will be in either $[2.0625, 2.09375]$ or $[2.09375, 2.125]$

$$f(2.09375) = (2.09375)^3 - (2.09375)^2 + 2.09375 - 7$$

$$= -0.1114$$

As

$f(2.0625)\cdot f(2.09375) > 0$ so

root will not lie in [2.0625, 2.09375].

Now As $f(2.09375) \cdot f(2.125) < 0$ so

root will lie in [2.09375, 2.125].

$$x_5 = \frac{2.09375 + 2.125}{2}$$

$$\boxed{x_5 = 2.109375}$$

So root will be in [2.09375, 2.109375] or [2.109375, 2.12

$f(2.109375) = (2.109375)^3 - (2.109375)^2 + 2.109375 - 7$

$= 0.045 > 0$

As $f(2.09375) \cdot f(2.109375) < 0$

So root lie in [2.09375, 2.109375].

$$x_6 = \frac{2.09375 + 2.109375}{2} = 2.1015625$$

So, root will be in either [2.109375, 2.1015625] or

[2.1015625, 2.109375]

$f(2.1015625) = (2.1015625)^3 - (2.1015625)^2 + (2.1015625) - 7$

$f(2.1015625) = -0.33 < 0$ -ve.

As $f(2.109378) \cdot f(2.1015625) > 0$ so

root will not lie in (2.09375, 2.1015625)

As $f(2.1015625) \cdot f(2.109375) < 0$

so, root will lie in [2.1015625, 2.109375]

So root $= x = 2.10$ (which is correct to two decimal places).

Exercise 4.3

Exercise 10.1

Q2) $x^3 - 2x - 5 = 0 \quad [2,3]$

Sol:- $f(x) = x^3 - 2x - 5$

$f(2) = 2^3 - 2(2) - 5 = -1 < 0$

$f(3) = 3^3 - 2(3) - 5 = 16 > 0$

so $[2,3]$

$x_0 = \frac{2+3}{2} = 2.5$

$f(2.5) = (2.5)^3 - 2(2.5) - 5$

$= 5.625 > 0$

so $[2, 2.5]$

$x_1 = \frac{2+2.5}{2} = 2.25$

$f(2.25) = (2.25)^3 - 2(2.25) - 5$

$= 1.890 > 0$

so $[2, 2.25]$

$x_2 = \frac{2+2.25}{2} = 2.125$

$f(2.125) = (2.125)^3 - 2(2.125) - 5$

$= 0.345 > 0$

so $[2, 2.125]$

$x_3 = \frac{2+2.125}{2} = 2.0625$

$f(x_3) = (2.0625)^3 - 2(2.0625) - 5$

$= -0.35 < 0$

so $[2.0625, 2.125]$

$x_4 = \frac{2.0625 + 2.125}{2} = 2.09375$

$f(x_4) = f(2.09375) = (2.09375)^3 - 2(2.09375) - 5$

$= -0.0089 < 0$

so $[2.09375, 2.125]$

$x_5 = \frac{2.09375 + 2.125}{2} = 2.109375$

$f(x_5) = f(2.109375)$

$= (2.109375)^3 - 2(2.109375) - 5$

$= 0.166 > 0$ so $[2.09375, 2.109375]$

$f(x_6) = (2.1015625)^3 - 2(2.1015625) - 5$

$= 0.078 > 0$

so $[2.09375, 2.1015625]$

$x_7 = \frac{2.09375 + 2.1015625}{2}$

$= 2.09765625$

$f(x_7) = 2.09765625 - 2(2.09765625) - 5$

$= 0.0347 > 0$ so $[2.09375, 2.09765625]$

Upto two decimal root is $\boxed{2.10}$ Answer.

Q3. $x^3 - 4x - 9 = 0$ [2, 3]

sol $f(x) = x^3 - 4x - 9$

$f(2) = 2^3 - 4(2) - 9 = -9 < 0$

$f(3) = 3^3 - 4(3) - 9 = 6 > 0$ root lies in [2, 3]

now bisect $x_0 = \frac{2+3}{2} = 2.5$

$f(x_0) = f(2.5) = (2.5)^3 - 4(2.9) - 9 = -3.37 < 0$

now [2.5, 3]

so $x_1 = \frac{2.5+3}{2} = 2.75$

$f(x_1) = f(2.75)$

$= (2.75)^3 - 4(2.75) - 9$

$= 2.5$ so [2.5, 2.75]

$x_2 = \frac{2.5+2.75}{2} = 2.625$

$f(x_2) = (2.625)^3 - 4(2.625) - 9 = -1.412 < 0$

so [2.625, 2.75]

$x_3 = \frac{2.625+2.75}{2} = 2.6875$

$f(x_3) = (2.6875)^3 - 4(2.6875) - 9$

$= -0.33 < 0$ so [2.6875, 2.75]

$x_4 = \frac{2.6875+2.75}{2} = 2.71875$

$f(x_4) = (2.71875)^3 - 4(2.71875) - 9$

$= 0.220 > 0$ so [2.6875, 2.71875]

$x_5 = \frac{2.6875+2.71875}{2} = 2.703125$

$f(x_5) = (2.703125)^3 - 4(2.703125) - 9$
$= -0.0061 < 0$

so $[2.703125, 2.71875]$

$x_6 = \dfrac{2.703125 + 2.71875}{2} = 2.7109375$

$f(x_6) = (2.7109375)^3 - 4(2.7109375) - 9$
$= 0.079 > 0$ so $[2.703125, 2.7109375]$

$x_7 = \dfrac{2.703125 + 2.7109375}{2} = 2.70703125$

$f(x_7) = (2.70703125)^3 - 4(2.70703125) - 9$
$= 0.009 > 0$

so updated interval is $[2.703125, 2.70703125]$ so upto two decimal places root is 2.71

Q4) $x - \cos x = 0$ $[0,1]$

Solution $f(x) = x - \cos x$

$f(0) = 0 - \cos 0 = -1 < 0$

$f(1) = 1 - \cos 1 = 0.459 > 0$ so $[0,1]$

$x_0 = 0.5 - \cos 0.5$
$= -0.377 < 0$ so $[0.5, 1]$

$x_1 = \dfrac{0.5+1}{2} = 0.75$

$f(0.75) = 0.75 - \cos(0.75)$
$= 0.018 > 0$ so $[0.5, 0.75]$

$x_2 = \dfrac{0.5 + 0.75}{2} = 0.625$

$f(x_2) = (0.625) - \cos(0.625) = -0.186 < 0$

So $[0.625, 0.75]$.

$x_3 = \dfrac{0.625 + 0.75}{2} = 0.6875$

$f(x_3) = 0.6875 - \cos(0.6875) = -0.085 < 0$

so $[0.6875, 0.75]$

$x_4 = \dfrac{0.6875 + 0.75}{2} = 0.71875$.

$f(x_4) = f(0.71875) = (0.71875) - \cos(0.71875)$

$= -0.033 < 0$

Interval $[0.71875, 0.75]$.

$x_5 = \dfrac{0.71875 + 0.75}{2} = 0.734375$.

$f(x_5) = 0.734375 - \cos(0.734375) = -0.007 < 0$

so Interval $[0.734375, 0.75]$.

$x_6 = \dfrac{0.734375 + 0.75}{2} = 0.7421875$.

$f(x_6) = 0.7421875 - \cos(0.7421875) = 0.0051 > 0$.

so $[0.734375, 0.7421875]$ $\quad x_7 = \dfrac{0.734375 + 0.7421875}{2}$

$x_7 = 0.73828125 - \cos(0.73828125)$

$= -0.0013 < 0$. so $[0.73828125, 0.7421875]$.

$x_8 = \dfrac{0.73828125 + 0.7421875}{2} = 0.740234375$.

$f(x_8) = 0.740234375 - \cos(0.740234375)$

$= 0.0019 > 0$

so $[0.73828125, 0.740234375]$.

$x_9 = \frac{0.73828125 + 0.740234375}{2} = 0.7392578125$

$f(x_9) = 0.7392578125 - \cos(0.7392578125)$

$= 0.00028 > 0$

so $[0.73828125, 0.7292578125]$.

upto two decimal places root is 0.74

Q5) $3x - e^x = 0 \quad [0,1]$

Solt $f(x) = 3x - e^x$

$f(0) = 3(0) - e^0 = -1 < 0$

$f(1) = 3(1) - e^1 = 0.281 > 0$.

$x_0 = \frac{0+1}{2} = 0.5$

$f(x_0) = 3(0.5) - e^{0.5} = -0.148 < 0$

so Interval is $[0.5, 1]$.

$x_1 = \frac{0.5+1}{2} = 0.75$

$f(x_1) = 3(0.75) - e^{0.75} = 0.132 > 0$.

so $[0.5, 0.75]$

$x_2 = \frac{0.5+0.75}{2} = 0.625$

$f(x_2) = 3(0.625) - e^{0.625}$

$= 0.006 > 0$ so $[0.5, 0.625]$.

$x_3 = \frac{0.5+0.625}{2} = 0.5625$.

$f(x_3) = 3(0.5625) - e^{0.5625}$

$= -0.067 < 0$ so $[0.5625, 0.625]$

$x_4 = \frac{0.5625 + 0.625}{2} = 0.59375$

$f(x_4) = 3(0.59375) - e^{0.59375} = -0.029 < 0$.

so $[0.59375, 0.625]$.

$x_5 = \frac{0.59375 + 0.625}{2} = 0.609375$.

$f(x_5) = 3(0.609375) - e^{0.609375} = -0.011 < 0$.

so $[0.609375, 0.625]$.

$x_6 = \frac{0.609375 + 0.625}{2} = 0.6171875$.

$f(x_6) = 3(0.6171875) - e^{0.6171875}$
$= -0.0021 < 0$.

so $[0.6171875, 0.625]$.

$x_7 = \frac{0.6171875 + 0.625}{2} = 0.62109375$.

$f(x_7) = 3(0.62109375) - e^{0.62109375}$
$= 0.0023 > 0$. so $[0.6171875, 0.62109375]$.

$x_8 = \frac{0.6171875 + 0.62109375}{2} = 0.619140625$

$f(x_8) = 3(0.619140625) - e^{0.619140625}$
$= 0.000090 > 0$.

so $[0.6171875, 0.619140625]$ so upto decimal places root of equation 0.62.

Exercise 10.1

Q6) $x^3 - 4x - 9 = 0$; $[2,3]$

$f(x) = x^3 - 4x - 9$

$f(2) = 2^3 - 4(2) - 9 = -9 < 0$

$f(3) = 3^3 - 4(3) - 9 = 6 > 0$ Hence root lies in interval $[2,3]$

$$x_0 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$x_0 = \frac{2f(3) - 3f(2)}{f(3) - f(2)} \Rightarrow x_0 = \frac{2(6) - 3(-9)}{6-(-9)} \Rightarrow \boxed{x_0 = 2.6}$$

$f(x_0) = f(2.6)$

$= (2.6)^3 - 4(2.6) - 9$

$= -1.824 < 0$ so $[2.6, 3]$

$$x_1 = \frac{(2.6)f(3) - 3f(2.6)}{f(3) - f(2.6)}$$

$$x_1 = \frac{(2.6)6 - 3(-1.824)}{6-(-1.824)}$$

$\boxed{x_1 = 2.6933}$

$f(x_1) = f(2.6933)$

$= (2.6933)^3 - 4(2.6933) - 9$

$= -0.2364 < 0$ so $[2.6933, 3]$

$$x_2 = \frac{2.6933f(3) - 3f(2.6933)}{f(3) - f(2.6933)}$$

$$= \frac{2.6933(6) - 3(-0.2364)}{6 - (-0.2364)}$$

$f(x_2) = f(2.7049)$

$= -0.0292 < 0$ so $[2.7049, 3]$

$$x_3 = \frac{2.7049(6) - 3(-0.0292)}{6-(-0.0292)}$$

$x_3 = 2.7063$. upto two decimal places the root of equation is 2.71

Q7) $x^3 - 4x - 1 = 0 \quad : [2,3]$

So $f(x) = x^3 - 4x - 1$

$f(2) = 2^3 - 4(2) - 1 = -1 < 0$

$f(3) = 3^3 - 4(3) - 1 = 14 > 0$

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$x_0 = \frac{2f(3) - 3f(2)}{f(3) - f(2)}$$

$$= \frac{2(14) - 3(-1)}{14-(-1)} = \boxed{2.0667}$$

$f(x_0) = (2.0667)^3 - 4(2.0667) - 1$

$= -0.4394 < 0 \qquad [2.0667, 3]$.

$$x_1 = \frac{2.0667(14) - 3(-0.4394)}{14-(-0.4394)}$$

$= 2.0951$

$f(x_1) = (2.0951)^3 - 4(2.0951) - 1$

$= -0.1841 < 0 \qquad [2.0951, 3]$.

$$x_2 = \frac{2.0951(14) - 3(-0.1841)}{14-(-0.1841)}$$

$\boxed{x_2 = 2.1068}$

$f(x_2) = (2.1068)^3 - 4(2.1068) - 1$
$= -0.0759 < 0$ so $[2.1068, 3]$.

$x_3 = \dfrac{2.1068(14) - 3(-0.0759)}{14 - (-0.0759)} = 2.1116$

$f(x_3) = (2.1116)^3 - 4(2.1116) - 1 = -0.0311 < 0$

so $[2.1116, 3]$

$x_4 = \dfrac{2.1116 f(3) - 3 f(2.1116)}{f(3) - f(2.1116)}$

$x_4 = \dfrac{2.1116(14) - 3(-0.0311)}{14 - (-0.0311)} = 2.1136$

Hence correct upto two decimal places 2.11.

Q8) $xe^x = 2 : [0,1]$.

Sol:- $f(x) = xe^x - 2$

$f(0) = 0e^0 - 2 = -2 < 0$

$f(1) = 1e^1 - 2 = 0.7183 > 0$. so $[0,1]$.

$x_0 = \dfrac{0 - (-2)}{0.7183 - (-2)} = 0.7358$.

$f(x_0) = 0.7358e^{0.7358} - 2$
$= -0.4643 < 0$. $[0.7358, 1]$

$x_1 = \dfrac{0.7358(0.7183) - (-0.4643)}{0.7183 - (-0.4643)}$

$\boxed{x_1 = 0.8395}$

$f(x_1) = 0.8395e^{0.8395} - 2$

$= -0.0564 < 0$ $[0.8395, 1]$

$$x_2 = \frac{0.8395(0.7183)-(-0.4643)}{0.7183-(-0.4643)} = \boxed{0.8512}$$

$f(x_2) = 0.8512e^{0.8512} - 2 = -0.0061 < 0.$

$[0.8512, 1]$.

$$x_3 = \frac{0.8512(0.7183)-(-0.0061)}{0.7183-(0.0061)}$$

$= \boxed{0.8524}$ correct upto two decimal places

root = 0.85

(Q9) $e^x \sin x = 1$ $[0,1]$

Sol: $e^x \sin x = 1$

$e^x \sin x - 1 = 0$

$f(x) = e^x \sin x - 1$

$f(0) = e^0 \sin 0 - 1 = -1 < 0$

$f(1) = e \sin 1 - 1 = 1.2874 > 0$ so $[0,1]$

$$x_0 = \frac{0-(-1)}{1.2874-(-1)} = 0.4372$$

$f(x_0) = e^{0.4372}\sin(0.4372) - 1 = -0.3444 < 0$

$[0.4372, 1]$.

$$x_1 = \frac{0.4372(1.2874)-(-0.3444)}{(1.2874)-(-0.3444)} = 0.5560.$$

$f(x_1) = e^{0.5560}\sin(0.5560) - 1 = -0.0797 < 0.$

$$x_2 = \frac{0.5560(1.2874)-(-0.0797)}{1.2874-(-0.0797)}$$

$= 0.5819$

Concept   Cal in radian

• Regula falsi Method or method of false position

$$C \text{ or } x_0 = \frac{af(b) - bf(a)}{f(b) - f(a)}.$$

Newton Raphson formula :-

x_0 (Initial guess)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad n = 0, 1, 2, 3 \ldots$$

[x_0 will be given]

Q10) $x = \cos x$; $[0, 1]$ (a, b)

for:- $x - \cos x = 0$

Let $f(x) = x - \cos x$

$f(a) = f(0) = 0 - \cos 0 = -1$ (-ve)

$f(b) = f(1) = 1 - \cos 1 = 0.45$ (+ve)

As $f(a) \cdot f(b) < 0$ so root lies in $[0, 1]$.

1st iteration: $[0, 1]$

$$x_0 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$x_0 = \frac{0(0.45) - 1(-1)}{0.45 - (-1)}$$

$x_0 = 0.6896$

So root will be in $[0, 0.6896]$ or $[0.6896, 1]$.

Ignore

$f(0) = -1$ (-ve)

$f(0.6896) = 0.6896 - \cos(0.6896)$

$= -0.0819$ (-ve)   same signs.

$f(1) = 0.45$ (+ve)

So, root will be in $[0.6896, 1]$.

2nd iteration

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$x_1 = \frac{0.6896(0.45) - 1(-0.0819)}{0.45 - (-0.0819)}$$

$$x_1 = 0.7368$$

So root will be in $[0.6896, 0.7368]$ or $[0.7368, 1]$

Ignore

$f(0.6896) = -0.0819$ (-ve)

$f(0.7368) = 0.7368 - \cos(0.7368)$

$= -3.822$ (-ve)

So root lies in $[0.7368, 1]$

3rd iteration:- $[0.7368, 1]$

$$x_2 = \frac{0.7368(0.45) - 1(-0.0038)}{0.45 - (-0.0038)}$$

$$\boxed{x_2 = 0.739}$$. 2 digit repeat.

As upto 2 decimal places $x_1 = 0.73$

$x_2 = 0.73$

So approximated root is ≈ 0.73

Answer.

$f(x_2) = e^{0.5819\sin(0.5819)} - 1 = -0.0165 < 0$

$[0.5819, 1]$.

$$x_3 = \frac{0.5819(1.2874) - (-0.0165)}{1.2874 - (-0.0165)} = 0.5871$$

Hence correct upto two decimal places 0.58.

Q 11) $x^3 - 3x - 5 = 0$.

Solution:- $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(x_n) = x_n^3 - 3x_n - 5$

$f'(x_n) = 3x_n^2 - 3$.

$$x_{n+1} = x_n - \frac{(x_n^3 - 3x_n - 5)}{3x_n^2 - 3}$$

$$x_{n+1} = \frac{3x_n^3 - 3x_n - x_n^3 + 3x_n + 5}{3x_n^2 - 3}$$

$$\boxed{x_{n+1} = \frac{2x_n^3 + 5}{3x_n^2 - 3}}$$

put n=0

$$x_1 = \frac{2x_0^3 + 5}{3x_0^2 - 3} = \frac{2(2)^3 + 5}{2(2)^2 - 3} = 2.3333$$

put n=1

$$x_2 = \frac{2(2.3333)^3 + 5}{3(2.3333)^2 - 3} = 2.2806$$

put n=2

$$x_3 = \frac{2(2.2806)^3 + 5}{3(2.2806)^2 - 3} = 2.2790$$

put $n=3$

$$= \frac{2(2.2790)^3+5}{2(2.2790)^2-3} = \boxed{2.2790}$$

Hence correct upto three decimal places 2.279.

Q12) $x^4-x-10=0$

Sol: $f(x_n)= x_n^4-x_n-10$

$f'(x_n)= 4x_n^3-1$

$$x_{n+1} = x_n - \frac{x_n^4-x_n-10}{4x_n^3-1}$$

$$x_{n+1} = \frac{x_n(4x_n^3-1)-[x_n^4-x_n-10]}{4x_n^3-1}$$

$$x_{n+1}=\frac{4x_n^4-x_n-x_n^4+x_n+10}{4x_n^3-1}$$

$$x_{n+1} = \frac{3x_n^4+10}{4x_n^3-1}$$

$n=0$

$$x_1 = \frac{3x_0^4+10}{4x_0^3-1}$$

$$= \frac{3(2)^4+10}{4(2)^3-1} = 1.8710$$

$n=1$

$$x_2 = \frac{3(1.8710)^4+10}{4(1.8710)^3-1}$$

$$\boxed{x_2 = 1.8558}$$

$n=2$

$$x_3 = \frac{3(1.8558)^4+10}{4(1.8558)^3-1}$$

$$\boxed{x_3 = 1.8556}$$

Correct upto three decimal places 1.855

Q13) $e^x = 1 + 2x$

Solution:- $f(x_n) = e^{x_n} - 2x_n - 1$

$f'(x_n) = e^{x_n} - 2$

$$x_{n+1} = x_n - \frac{e^{x_n} - 2x_n - 1}{e^{x_n} - 2}$$

$$x_{n+1} = \frac{x_n e^{x_n} - 2x_n - e^{x_n} + 2x_n + 1}{e^{x_n} - 2}$$

$$\boxed{x_{n+1} = \frac{(x_n - 1)e^{x_n} + 1}{e^{x_n} - 2}}$$

n=0

$$x_1 = \frac{(2-1)e^2 + 1}{e^2 - 2} = \boxed{1.5567}$$

n=1

$$x_2 = \frac{(1.5567 - 1)e^{1.5567} + 1}{e^{1.5567} - 2}$$

$\boxed{x_2 = 1.3271}$

n=2

$$x_3 = \frac{(1.3271 - 1)e^{1.3271} + 1}{e^{1.3271} - 2}$$

$\boxed{x_3 = 1.2616}$

n=3

$$x_4 = \frac{(1.2616 - 1)e^{1.2616} + 1}{e^{1.2616} - 2}$$

$= 1.2565$

n=4

$$x_5 = \frac{(1.2565 - 1)e^{1.2565} + 1}{e^{1.2565} - 2}$$

$x_5 = 1.2564$

Hence correct upto 3 decimal place

$\boxed{1.256}$

Q14) $3x-1=\cos x$

Sol $f(x_n)=3x_n-1$

$f'(x_n)=3+\sin x_n$

$$x_{n+1}=x_n-\frac{3x_n-\cos x_n-1}{3+\sin x_n}$$

$$x_{n+1}=\frac{3x_n+x_n\sin x_n-3x_n+\cos x_n+1}{3+\sin x_n}$$

$$x_{n+1}=\frac{x_n\sin x_n+\cos x_n+1}{3+\sin x_n}$$

$n=0$

$$x_1=\frac{2\sin 2+\cos 2+1}{3+\sin 2}$$

$x_1=0.6145$

$n=1$

$$x_2=\frac{0.6145\sin 0.6145+\cos(0.6145)+1}{3+\sin(0.6145)}$$

$x_2=0.6071$

$n=2$

$$x_3=\frac{(0.6071)\sin(0.6071)+6(0.6071)+1}{3+\sin(0.6071)}$$

$\boxed{x_3=0.6071}$

Hence correct upto 3 decimal places so

0.607

)

Q15) $\sin(x) = 1 - x$

Sol: let $\sin(x) - 1 + x = f(x)$

$f(x_n) = \sin x_n + x_n - 1$

$f'(x) = \cos x + 1$

$f' x_n = \cos x_n + 1$

By Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{\sin x_n + x_n - 1}{\cos x_n + 1}$$

$$x_{n+1} = \frac{x_n \cos x_n + \cancel{x_n} - \sin x_n - \cancel{x_n} + 1}{\cos x_n + 1}$$

$$x_{n+1} = \frac{x_n \cos x_n - \sin x_n + 1}{\cos x_n + 1}$$

First iteration:- $n=0$; $x_0 = 2$

$$x_1 = \frac{x_0 \cos x_0 + \sin x_0 + 2x_0 - 1}{\cos x_0 + 1}$$

$$x_1 = \frac{2\cos 2 - \sin 2 + 1}{\cos 2 + 1}$$

$x_1 = -1.27016$

2nd iteration: $n=1$, $x_1 = -1.2702$

$$x_2 = \frac{(-1.2702)\cos(-1.2702) - \sin(-1.2702) + 1}{\cos(-1.2702) + 1}$$

$x_2 = 1.2183$

3rd Iteration: $n=2$, $x_2 = 1.2183$

$$x_3 = \frac{(1.2183)\cos(1.2183) - \sin(1.2183) + 1}{\cos(1.2183) + 1}$$

$x_3 = 0.3584$

4th iteration: $n=3$, $x_3 = 0.3584$

$$x_4 = \frac{(0.3584)\cos(0.3584) - \sin(0.3584) + 1}{\cos(0.3584) + 1}$$

$x_4 = 0.5086$

5th iteration: $n=4$, $x_4 = 0.5086$

$$x_5 = \frac{(0.5086)\cos(0.5086) - \sin(0.5086) + 1}{\cos(0.5086) + 1}$$

$x_5 = 0.5109$

6th iterath $n=5$, $x_5 = 0.5109$

$$x_6 = \frac{(0.5109)\cos(0.5109) - \sin(0.5109) + 1}{\cos(0.5109) + 1}$$

$x_6 = 0.5109$.

Hence correct upto three decimal places the root of equation is 0.5109.

Q16) $x^2 + 4\sin x = 0$; $x_0 = 2$

Sol: let $f(x) = x^2 + 4\sin x$; $f(x_n) = x_n^2 + 4\sin x_n$

$f'(x) = 2x + 4\cos x$; $f'(x_n) = 2x_n + 4\cos x_n$

By Newton Raphson method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^2 + 4\sin x_n}{2x_n + 4\cos x_n}$$

$$= \frac{x_n(2x_n + 4\cos x_n) - x_n^2 + 4\sin x_n}{2x_n + 4\cos x_n}$$

$$= \frac{2x_n^2 + x_n 4\cos x_n - x_n^2 + 4\sin x_n}{2x_n + 4\cos x_n}$$

$$x_{n+1} = \frac{x_n^2 + 4x_n\cos x_n - 4\sin x_n}{2x_n + 4\cos x_n}$$

first iteration: $n = 0$; $x_0 = 2$

$$x_1 = \frac{x_0^2 + 4x_0\cos x_0 - 4\sin x_0}{2x_0 + 4\cos x_0}$$

$$= \frac{2^2 + 4(2)\cos 2 - 4\sin 2}{2(2) + 4\cos 2}$$

$x_1 = -1.2701$

Calculator
$f(x) = \frac{x^2 + 4x\cos x - 4\sin x}{2x + 4\cos x}$
+ Calc
put value of x

2nd iteration: $n = 1$; $x_1 = -1.2701$

$$x_2 = \frac{x_1^2 + 4x_1\cos x_1 - 4\sin x_1}{2x_1 + 4\cos x_1}$$

$$x_2 = \frac{(-1.2701)^2 + 4(-1.2701)\cos - 4\sin(-1.2701)}{2(-1.2701) + 4\cos(-1.2701)} = -2.8986$$

3rd iteration

put $n=2$, $x_2 = -2.8986$

$$x_3 = \frac{x_2^2 + 4x_2\cos x_2 - 4\sin x_2}{2x_2 + 4\cos x_2}$$

$$= \frac{(-2.8986)^2 + 4(-2.8986)\cos(-2.8986) - 4\sin(-2.8986)}{2(-2.8986) + 4\cos(-2.8986)}$$

$x_3 = -2.1300$

4^{th} Iteration put $n=3$ $x_3 = -2.1300$

$$x_4 = \frac{(-2.1300)^2 + 4(-2.1300)\cos(-2.1300) - 4\sin(-2.1300)}{2(-2.1300) + 4\cos(-2.1300)}$$

$x_4 = -1.9504$

5^{th} Iteration $n=4$ $x_4 = -1.9504$

$$x_5 = \frac{(-1.9504)^2 + 4(-1.9504)\cos(-1.9504) - 4\sin(-1.9504)}{2(-1.9504) + 4\cos(-1.9504)}$$

$x_5 = -1.9339$

6th Iteration: $n=5$ $x_5 = -1.9339$.

$$x_6 = \frac{(-1.9339)^2 + 4(-1.9339)\cos(-1.9339) - 4\sin(-1.9339)}{2(-1.9339) + 4\cos(-1.9339)}$$

$x_6 = -1.9337$ As upto 3 decimal places

$x_5 = -1.9339$

$x_6 = -1.9337$ approximated root $x = -1.933$

Q17) Sol:- $A = 2500m^2$

$l = 20 + w$

$x_0 = 20$

let width $= x$

length $= x + 20$

As Area of rectangle $= l \times w$

$2500 = (x+20)(x)$

$0 = x^2 + 20x - 2500$

$x^2 + 20x - 2500 = 0$

let $f(x) = x^2 + 20x - 2500$; $x_0 = 20$

Solve by Newton Raphson method:-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^2 + 20x_n + 2500}{2x_n + 20}$$

$$x_{n+1} = \frac{2x_n^2 + \cancel{20x_n} - x_n^2 \cancel{+ 20x_n} + 2500}{2x_n + 20}$$

$$x_{n+1} = \frac{2x_n^2 - x_n^2 + 2500}{2x_n + 20}$$

$$x_{n+1} = \frac{x_n^2 + 2500}{2x_n + 20}$$

put $n=0$ $x_0 = 20$

$$x_1 = \frac{(20)^2 + 2500}{2(20) + 20}$$

$x_1 = 48.3333$

put $n=1$ $x_1 = 48.3333$

$$x_2 = \frac{(48.3333)^2 + 2500}{2(48.3333)+20} \Rightarrow x_2 = 41.4524$$

put $n=2$ $x_2 = 41.4524$

$$x_3 = \frac{(41.4524)^2 + 2500}{2(41.4524)+20} \Rightarrow x_3 = 40.9922$$

put $n=3$ $x_3 = 40.9922$

$$x_4 = \frac{(40.9922)^2 + 2500}{2(40.9922)+20} \Rightarrow x_4 = 40.9901$$

Hence correct upto two decimal places width = 40.9901

Q18) Solution:-

$P(x) = x^3 - 4x^2 - 7x + 10$

Interval $[-1, 1.5]$.

$P(-1) = (-1)^3 - 4(-1)^2 - 7(-1) + 10 = 12$ (+ve)

$P(1.5) = (1.5)^3 - 4(1.5)^2 - 7(1.5) + 10 = -6.125$ (-ve)

$x_1 = \frac{-1+1.5}{2} = 0.25$ 1st Iteration

$f(x_1) = f(0.25) = (0.25)^3 - 4(0.25)^2 - 7(0.25) + 10$

$= 8.0156 > 0$

2nd Iteration $[0.25, 1.5]$

$x_2 = \frac{0.25+1.5}{2} = 0.875$

$f(x_2) = f(0.875) = (0.875)^3 - 4(0.875)^2 - 7(0.875) + 10$

$= 1.4824 > 0$

$[0.875, 1.5]$

3rd Iteration: $[0.875, 1.5]$

$x_3 = \frac{0.875+1.5}{2} = 1.1875$

$f(1.1875) = (1.1875)^3 - 4(1.1875)^2 - 7(1.18 + 10$

$f(1.1875) = -2.278 < 0$

$[0.875, 1.875]$.

4th Iteration:-

$[0.875, 1.875]$

$x_4 = \frac{0.875 + 1.1875}{2}$

$x_4 = 1.03125$

approximated root upto few iterations