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Where Minds Pollinate

Class 12
Mathematics

Chapter 1
Exercise 1.4
Handwritten Solution

Ex 1.4

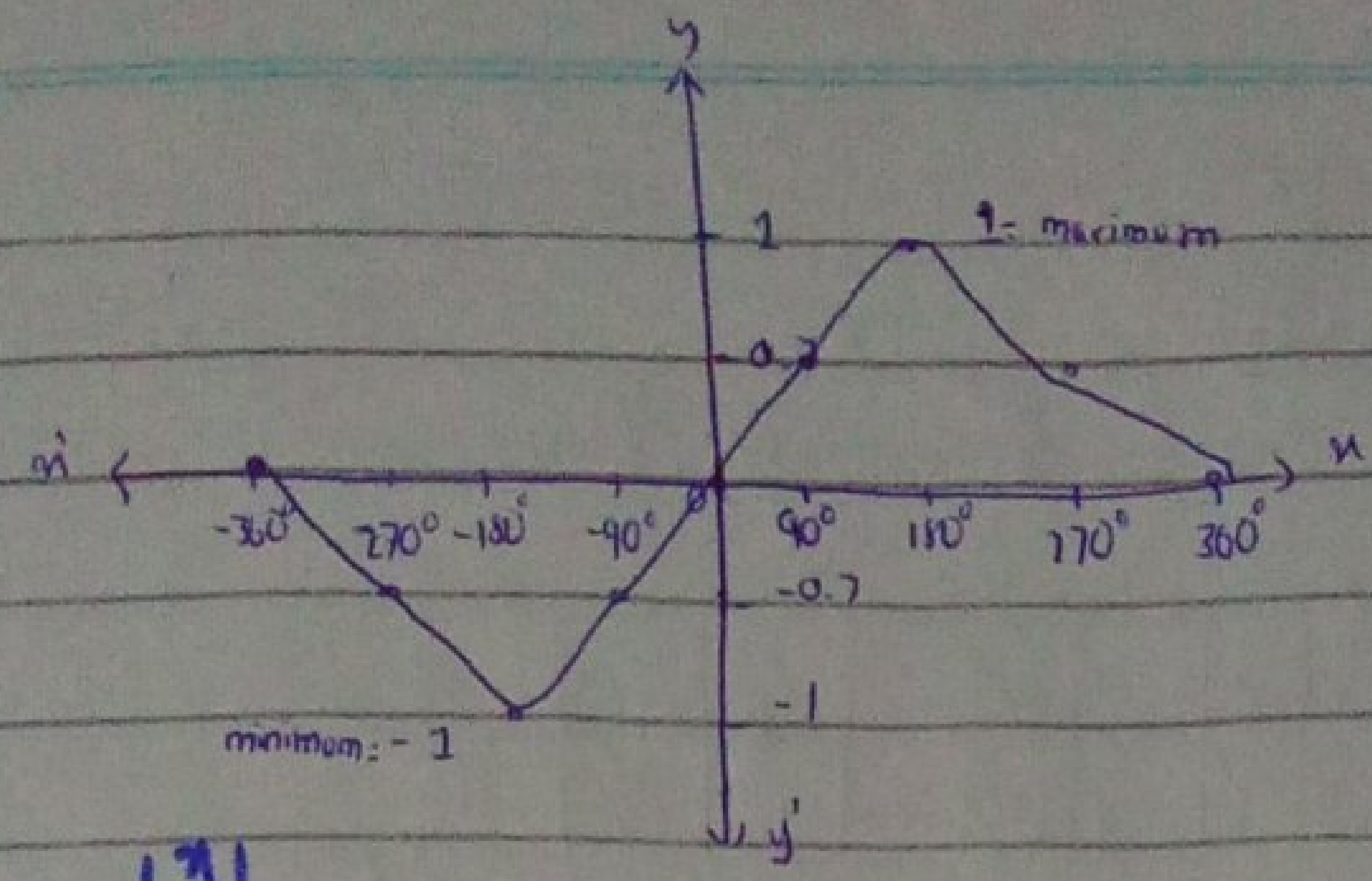
Q1

(i) $f(x) = \sin\left(\frac{x}{2}\right)$

Dom $f(x) = \mathbb{R}$ or $(-\infty, +\infty)$

Range ~~of~~ $f(x) = [-1, 1]$

x	-360°	-270°	-180°	-90°	0	90°	180°	270°	360°
y	0	-0.7	-1	-0.7	0	0.7	1	0.7	0

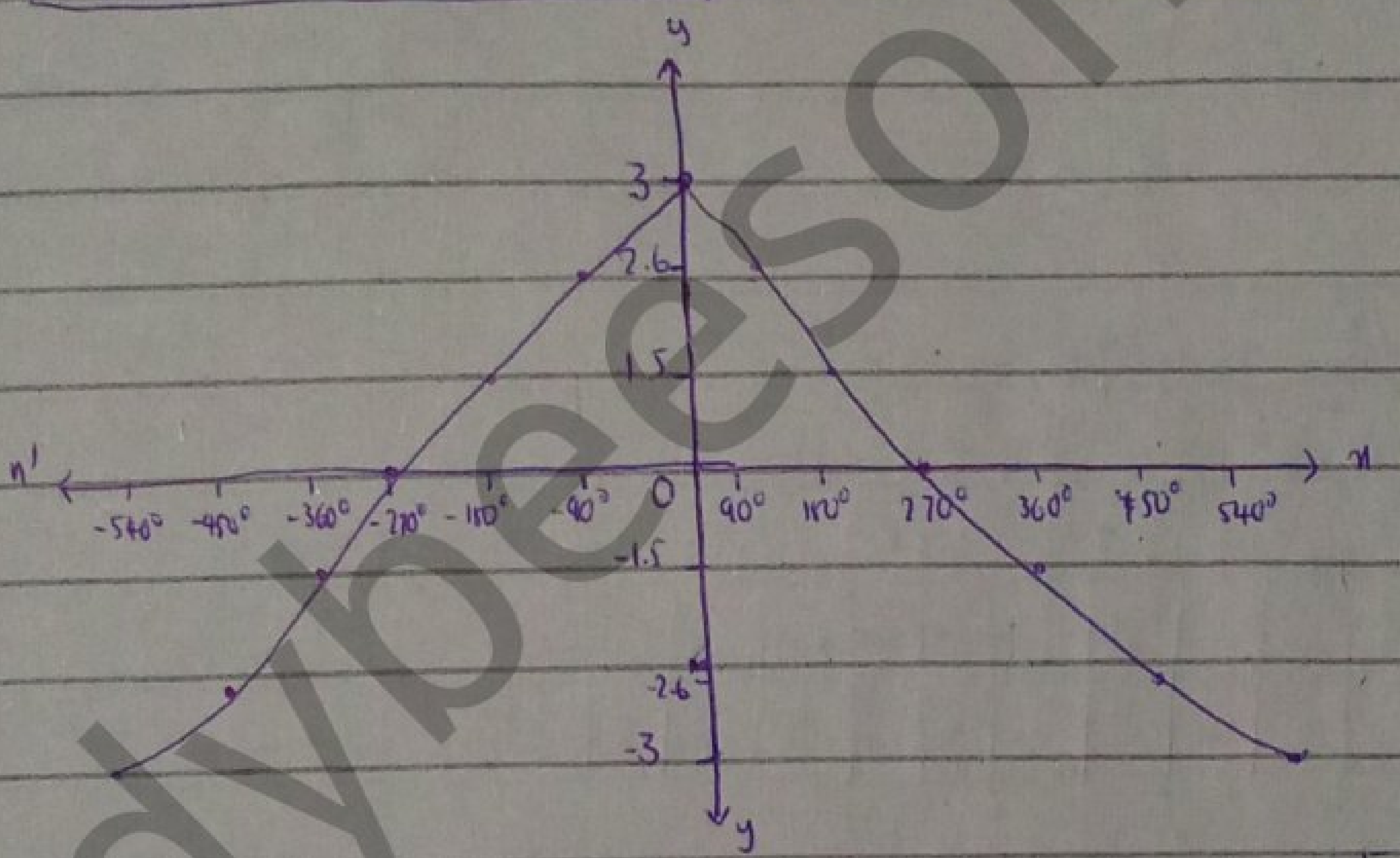


(ii) $g(x) = 3\cos\left(\frac{x}{3}\right)$

Dom $g(x) = (-\infty, +\infty)$

Range $g(x) = [-3, 3]$

x	-450°	-360°	-270°	-180°	-90°	0	90°	180°	270°	360°	450°	540°
y	-2.6	-1.5	0	1.5	2.6	3	2.6	1.5	0	-1.5	-2.6	-3



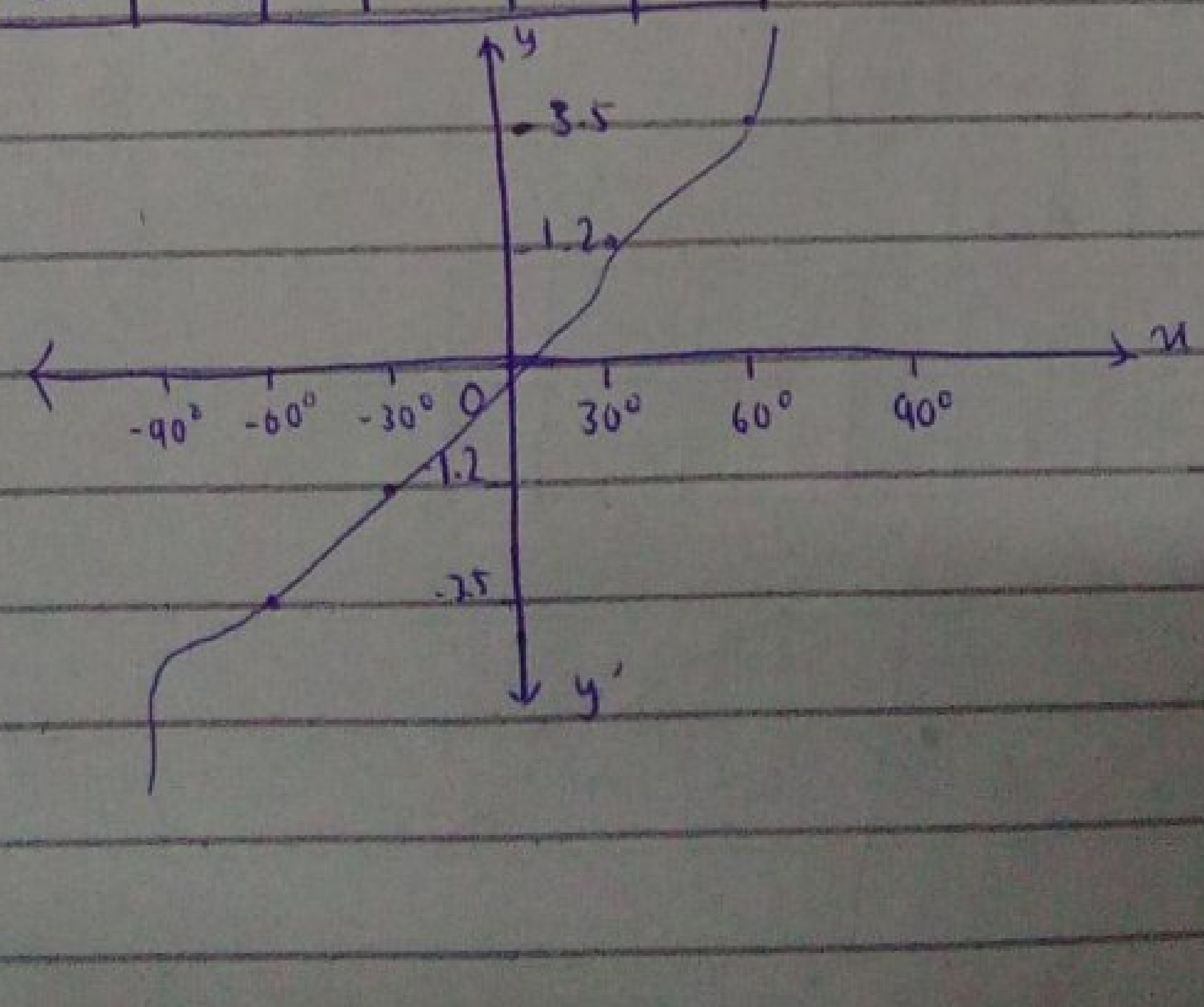
$2\tan x = \frac{2\sin x}{\cos x}$

(iii) $h(x) = 2\tan x$

Dom $h(x)$: $\mathbb{R} - \left\{ x \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z} \right\}$

Range $h(x) = (-\infty, +\infty)$

x	-90°	-60°	-30°	0	30°	60°	90°
y	∞	-3.5	-1.2	0	1.2	3.5	∞

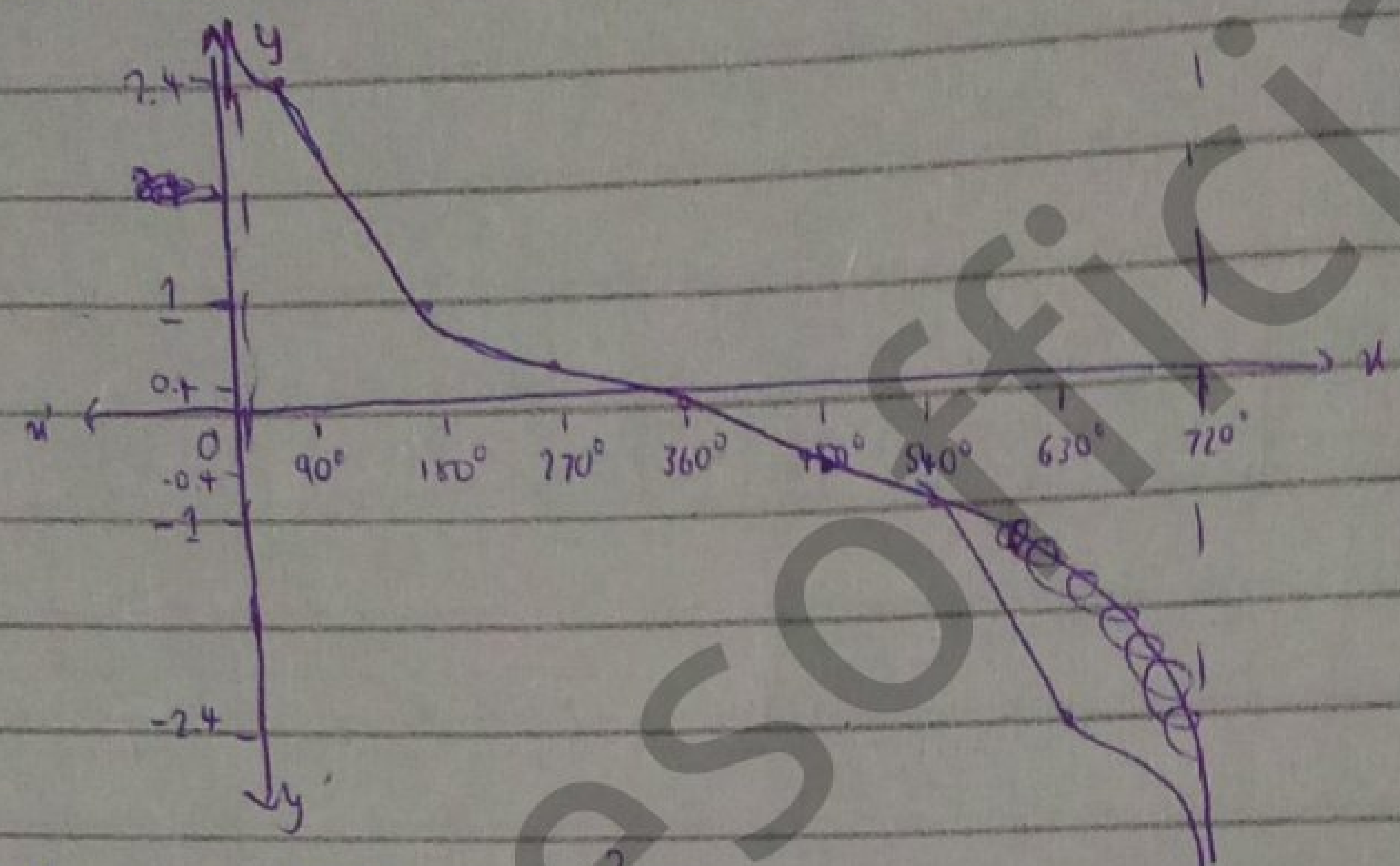


(iv) $f(x) = y = \cot\left(\frac{x}{4}\right) = \frac{\cos\frac{x}{4}}{\sin\frac{x}{4}}$

Dom $f(x) = \mathbb{R} - \{x \neq 4n\pi : n \in \mathbb{Z}\}$

Range $f(x) = (-\infty, +\infty)$

x	0°	90°	180°	270°	360°	450°	540°	630°	720°
y	∞	2.4	1	0.4	0	-0.4	-1	-2.4	∞

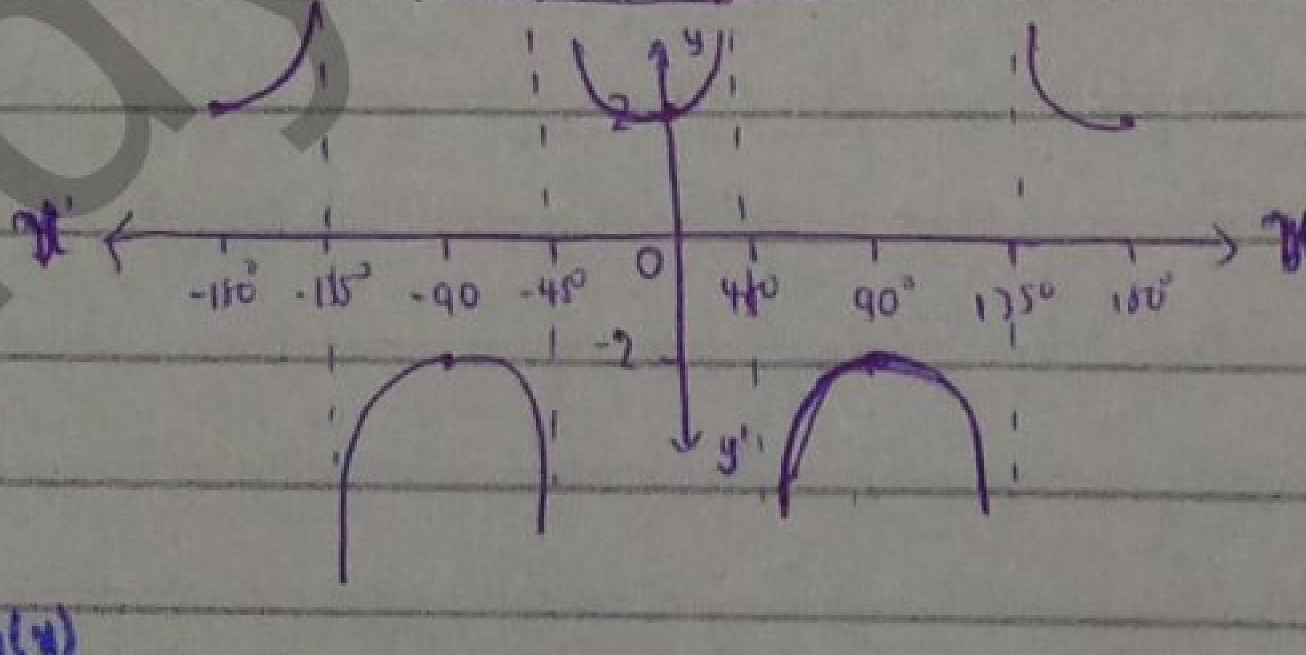


(v) $g(x) = y = 2\sec(2x) = \frac{2}{\cos(2x)}$

Dom $g(x) = \mathbb{R} - \{x \neq (2n+1)\frac{\pi}{4}, n \in \mathbb{Z}\}$

Range $g(x) = y \leq -2, y \geq 2$ or $(-\infty, -2] \cup [2, +\infty)$

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
y	2	∞	-2	∞	2	∞	-2	∞	2

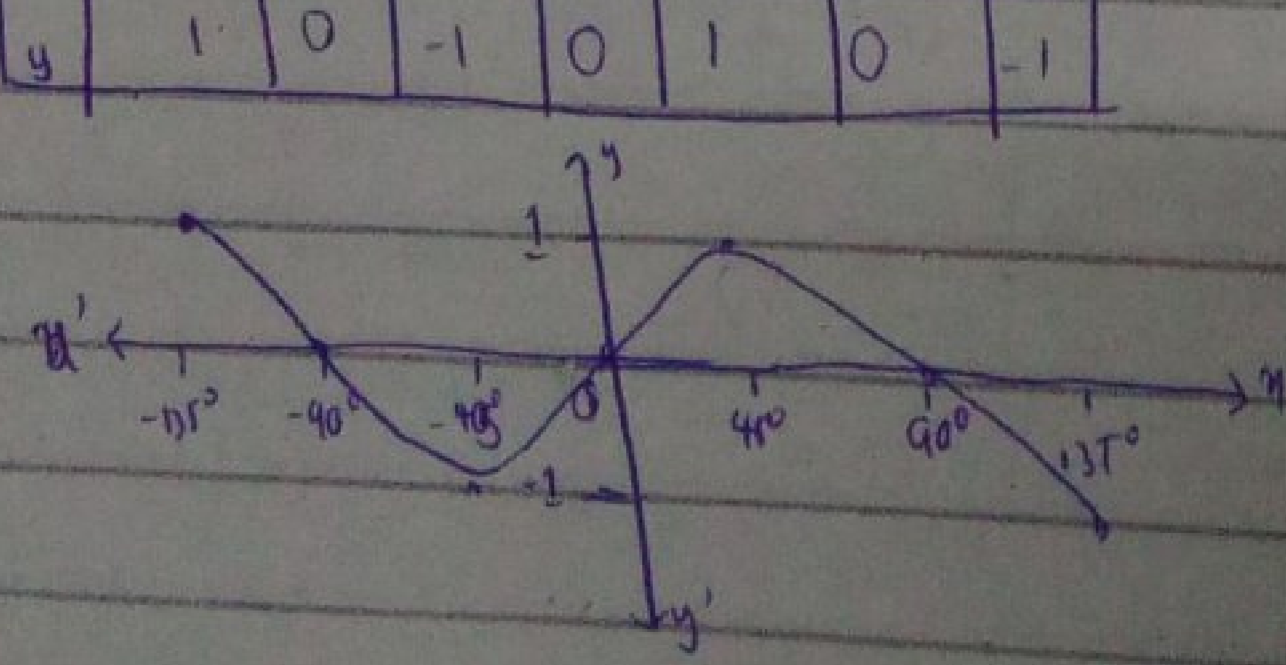


(vi) $h(x) = y = \sin(2x)$

Dom $h(x) = (-\infty, +\infty)$

Range $h(x) = [-1, 1]$

x	-135°	-90°	-45°	0	45°	90°	135°
y	1	0	-1	0	1	0	-1



(linear function)

* if the horizontal line touches only one point then it is a one-to-one function

Q2) (i) $f(x) = \frac{1}{3}x + 3$

let $f(x) = y \Rightarrow x = f^{-1}(y)$ [for inverse function]

$y = \frac{1}{3}x + 3$

x-intercept :- Put $y = 0$

$0 = \frac{x}{3} + 3$

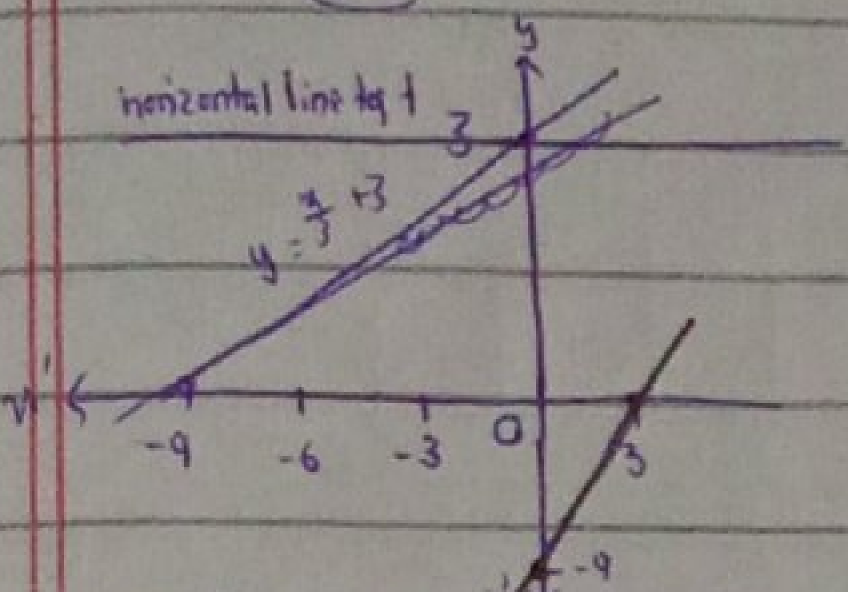
$-3 = \frac{x}{3}$

$\boxed{x = -9}$ $(-9, 0)$

y-intercept :- Put $x = 0$

$y = 0 + 3$

$\boxed{y = 3}$ $(0, 3)$



Hence the given function is one-to-one

→ Now finding the inverse function

$y = \frac{x}{3} + 3$

$y - 3 = \frac{x}{3}$

$3(y-3) = x$

$x = 3y - 9$

$f^{-1}(y) = 3y - 9$

replace y with x,

$\boxed{f^{-1}(x) = 3x - 9}$

→ Graph of the inverse :

x-intercept :- Put $y = 0$

$0 = 3x - 9$

$9 = 3x$

$\frac{9}{3} = x$

$x = 3$ $(3, 0)$

y-intercept :- Put $x = 0$

$y = -9$ $(0, -9)$

→ The point at which the two lines are crossing.

Comparing $f(x)$ & $f^{-1}(x)$,

$\frac{x}{3} + 3 = 3x - 9$

$3 - 9 = 3x - \frac{x}{3}$

$12 = \frac{9x - x}{3}$

$12(3) = 9x - x$

$36 = 8x$

$36/8 = x$ $x = 4.5$

Putting any eq to get the value of y,

$y = 3(4.5) - 9$

$y = 4.5$

So the point of intercept will be $(4.5, 4.5)$

(ii) $g(x) = x(x-5)$ (Quadratic function) [means it will have a parabolic shape]

~~let $g(x) = y$~~

~~$y = x(x-5)$~~

~~x-intercept :- Put $y = 0$~~

~~$0 = x(x-5)$~~

~~$0 = x^2 - 5x$~~

$g(x) = x^2 - 5x$

→ The nature of the parabola :

$a > 1$ (upward parabola)

→ x-intercept :- Put $y = 0$

$0 = x^2 - 5x$

$0 = x(x-5)$

$\boxed{x = 0}$ | $x - 5 = 0 \Rightarrow \boxed{x = 5}$

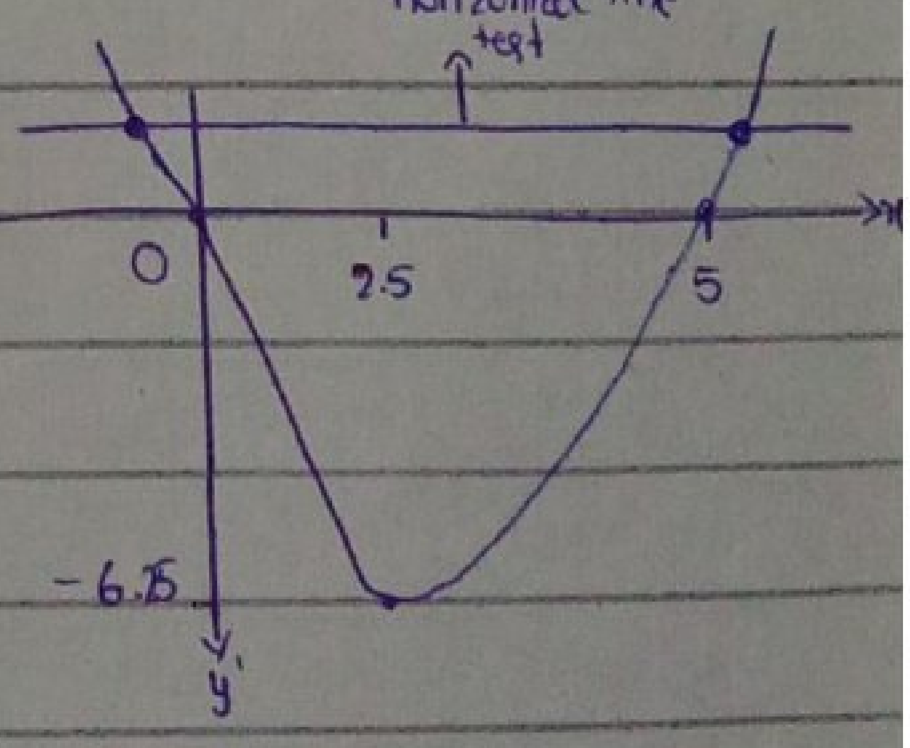
$(0, 0)$ $(5, 0)$

→ Finding the max & min point of parabola:

$x = \frac{-b}{2a} = \frac{-(-5)}{2(1)} = \frac{5}{2} = 2.5$

$y = (2.5)^2 - 5(2.5) = -6.25$

So $(2.5, -6.25)$



Hence given function is not one-to-one.

(iii) $h(x) = (x+1)^2$

$h(x) = y$

$y = x^2+2x+1$ (Quadratic eq. so parabolic shape)

→ <u>Finding the nature of the parabola:</u>

$a > 1$ (so upward parabola)

→ <u>x-intercept:</u> Put $y=0$,

$0 = (x+1)^2$

$\sqrt{0} = \sqrt{(x+1)^2}$

$0 = x+1$

$x = -1$

$(-1, 0)$

→ <u>y-intercept:</u> Put $x=0$,

$y = 0^2+2(0)+1$

$y = 0+0+1$

$y = 1$

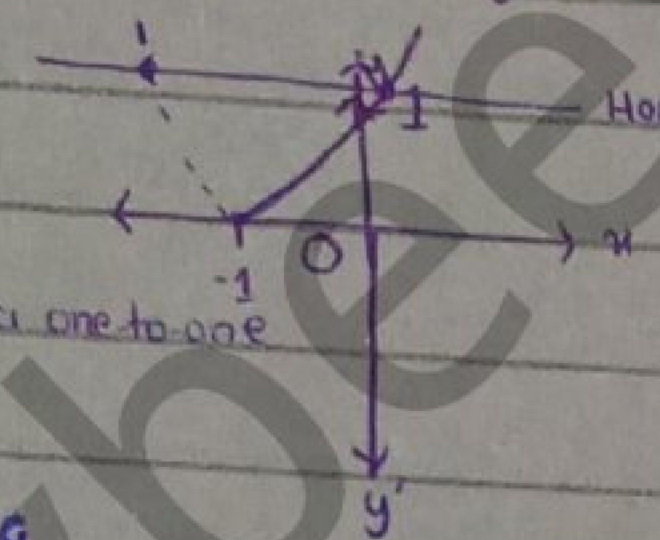
$(0, 1)$

→ <u>Finding the max & min points of the parabola:</u>

$x = \frac{-b}{2a}$ $\quad x = \frac{-(2)}{2(1)}$ $\quad x = \frac{-2}{2}$ $\quad x = -1$

$y = (-1)^2+2(-1)+1$ $\quad y = 1-2+1$ $\quad y = 2-2$ $\quad y = 0$

So $(-1, 0)$



Hence it's not a one-to-one function.

(iv) $f(x) = x^3-8$

$f(x) = y$ → $x = f^{-1}(y)$

$y = x^3-8$

→ <u>x-intercept:</u> Put $y=0$,

$0 = x^3-8$

$\sqrt[3]{8} = \sqrt[3]{x^3}$

$x = 2$ $\quad (2, 0)$

→ <u>y-intercept:</u> Put $x=0$,

$y = (0)^3-8$

$y = 0-8$

$y = -8$

$(0, -8)$

Put $x=3$,

$y = (3)^3-8$

$y = 27-8$

$y = 19$

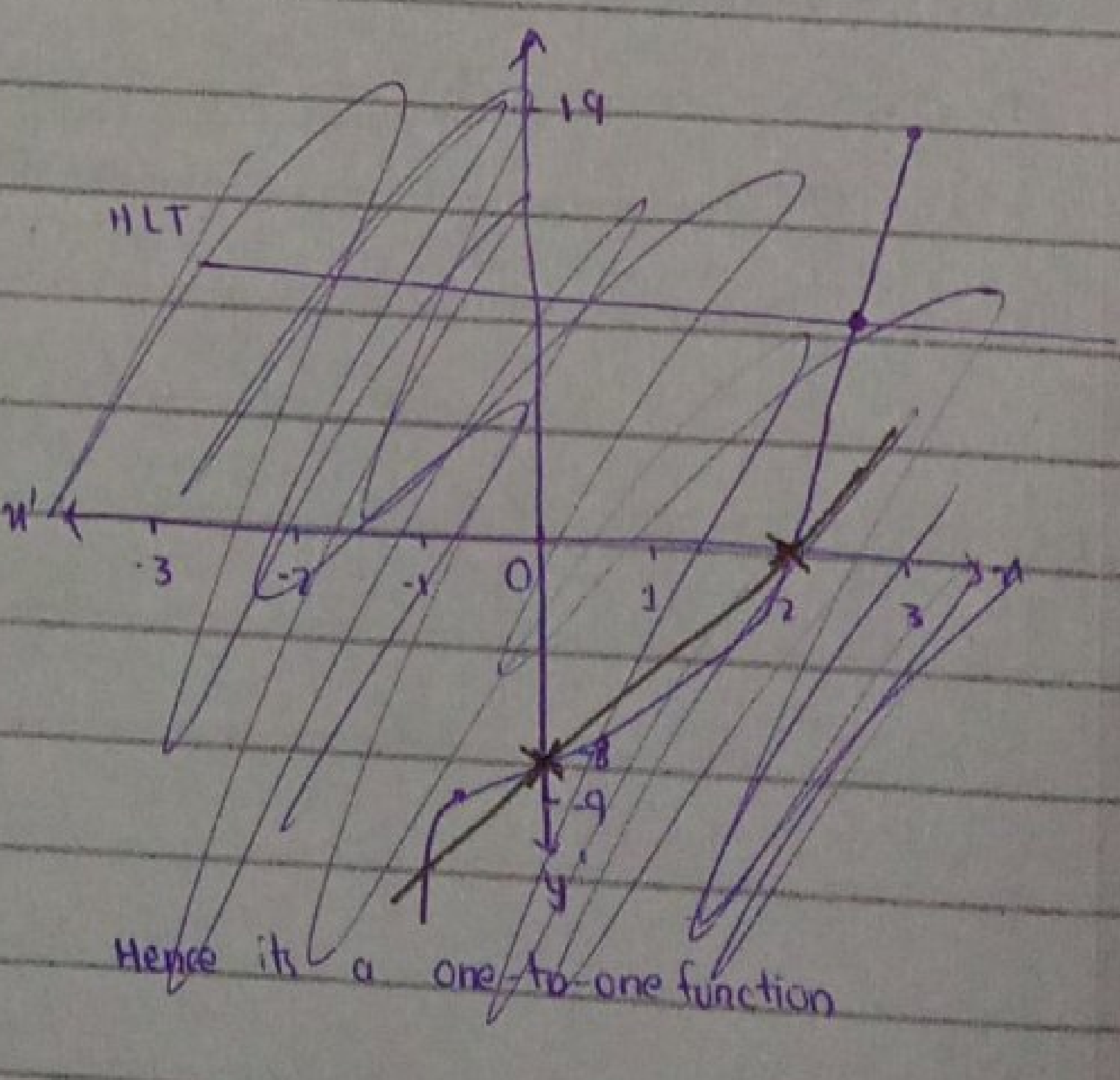
$(3, 19)$

Put $x=1$,

$y = (-1)^3-8$

$y = -1-8$

$y = -9$ $\quad (-1, -9)$



Hence it's a one-to-one function

→ <u>Finding inverse function:</u>

$y = x^3-8$

$\sqrt[3]{y+8} = \sqrt[3]{x^3}$

$x = \sqrt[3]{y+8}$

$f^{-1}(y) = \sqrt[3]{y+8}$

Replace y with x

$f^{-1}(x) = \sqrt[3]{x+8}$

$y = \sqrt[3]{x+8}$

→ **x-intercept:** Put $y=0$ | $(0)^3 = (\sqrt[3]{x+8})^3$ $\quad 0 = x+8 \quad x=-8 \quad (-8,0)$

→ **y-intercept:** Put $x=0$, | $y=\sqrt[3]{0+8} \quad y=\sqrt[3]{8} \quad y=2 \quad (0,2)$

Put $x=4$,

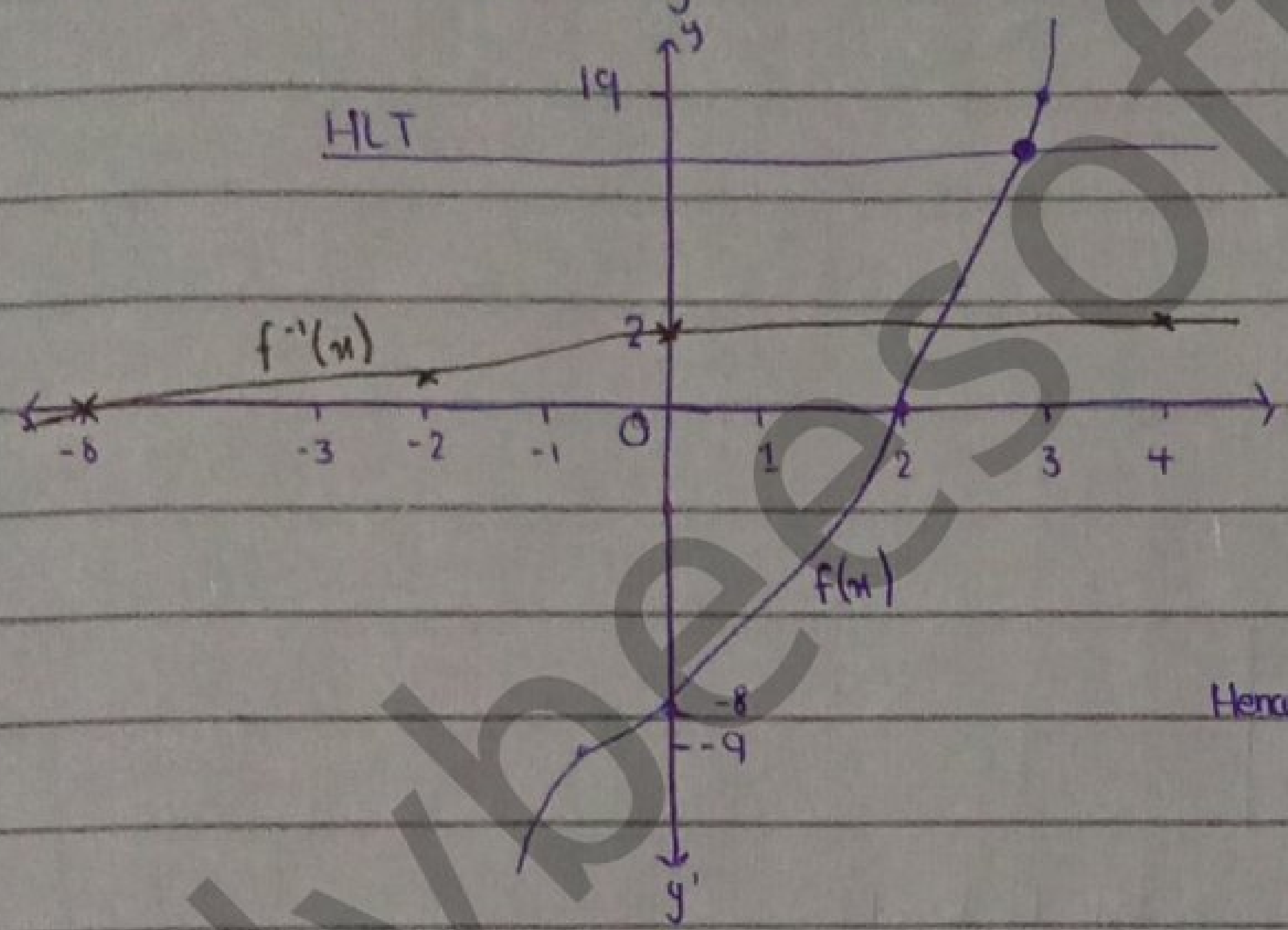
$y=\sqrt[3]{4+8} \quad y=\sqrt[3]{12} \quad y=2.3 \quad (4,2.3)$

~~Put $x=19$,~~

~~$y=\sqrt[3]{19+8} \quad y=\sqrt[3]{27} \quad y=3 \quad (19,3)$~~

Put $x=-2$,

$y=\sqrt[3]{-2+8} \quad y=\sqrt[3]{6} \quad y=1.8 \quad (-2,1.8)$



Hence its a one-to-one function

(v) $g(x) = 4+x$

$g(x)=y \Rightarrow x=f^{-1}(y)$

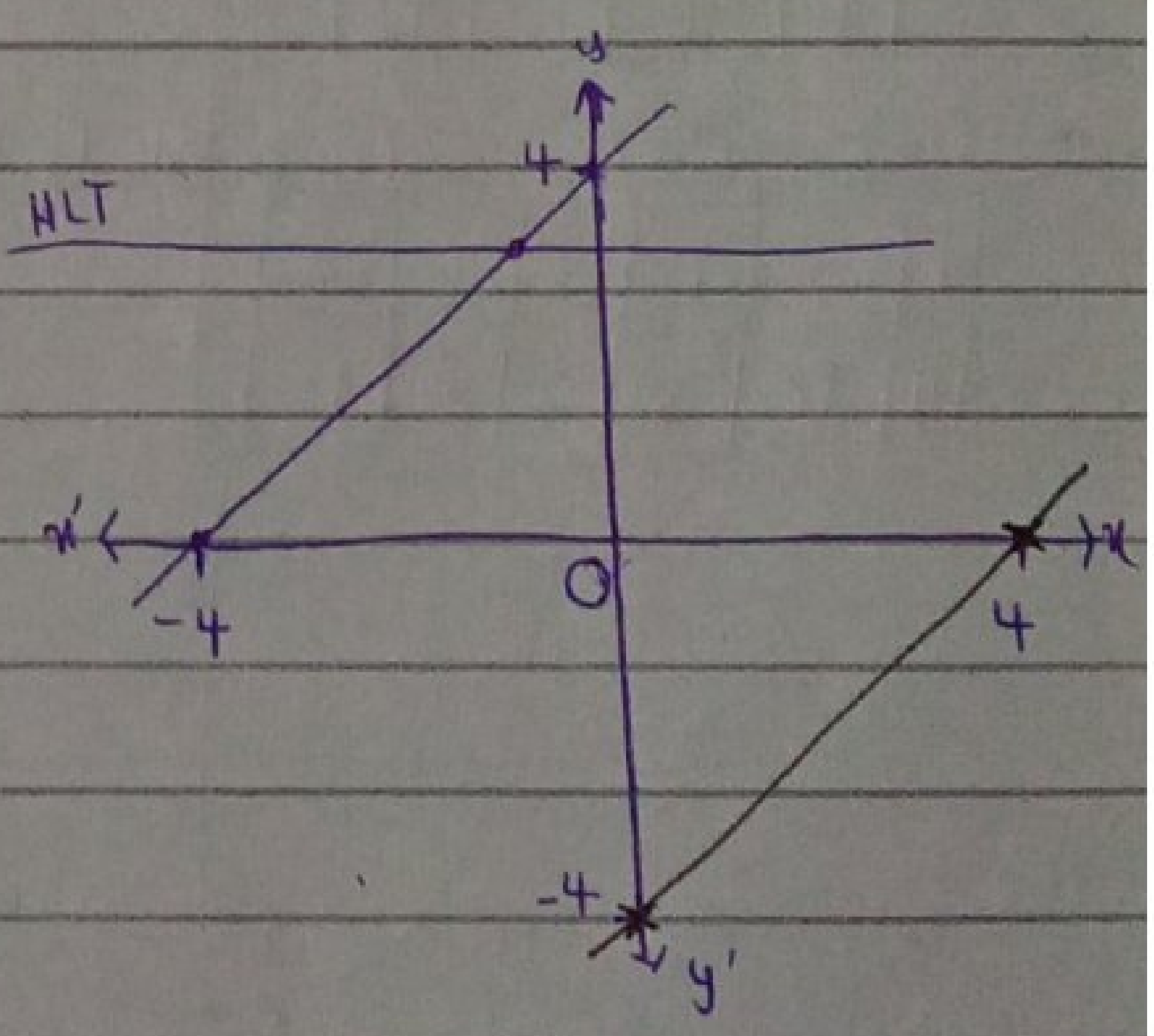
$y=x+4$

→ **x-intercept:** Put $y=0$,

$0=x+4 \quad x=-4 \quad (-4,0)$

→ **y-intercept:** Put $x=0$,

$y=0+4 \quad y=4 \quad (0,4)$



Hence its a one-to-one function

→ **Finding the inverse:**

$y=x+4$

$x=y-4$

$f^{-1}(y)=y-4$

Replacing y with x

$f^{-1}(x)=x-4 \Rightarrow y=x-4$

→ **x-intercept:** Put $y=0$,

$0=x-4$

$x=4 \quad (4,0)$

→ **y-intercept:** Put $x=0$,

$y=0-4$

$y=-4 \quad (0,-4)$

(vi) $h(x) = \frac{1}{3x+5}$

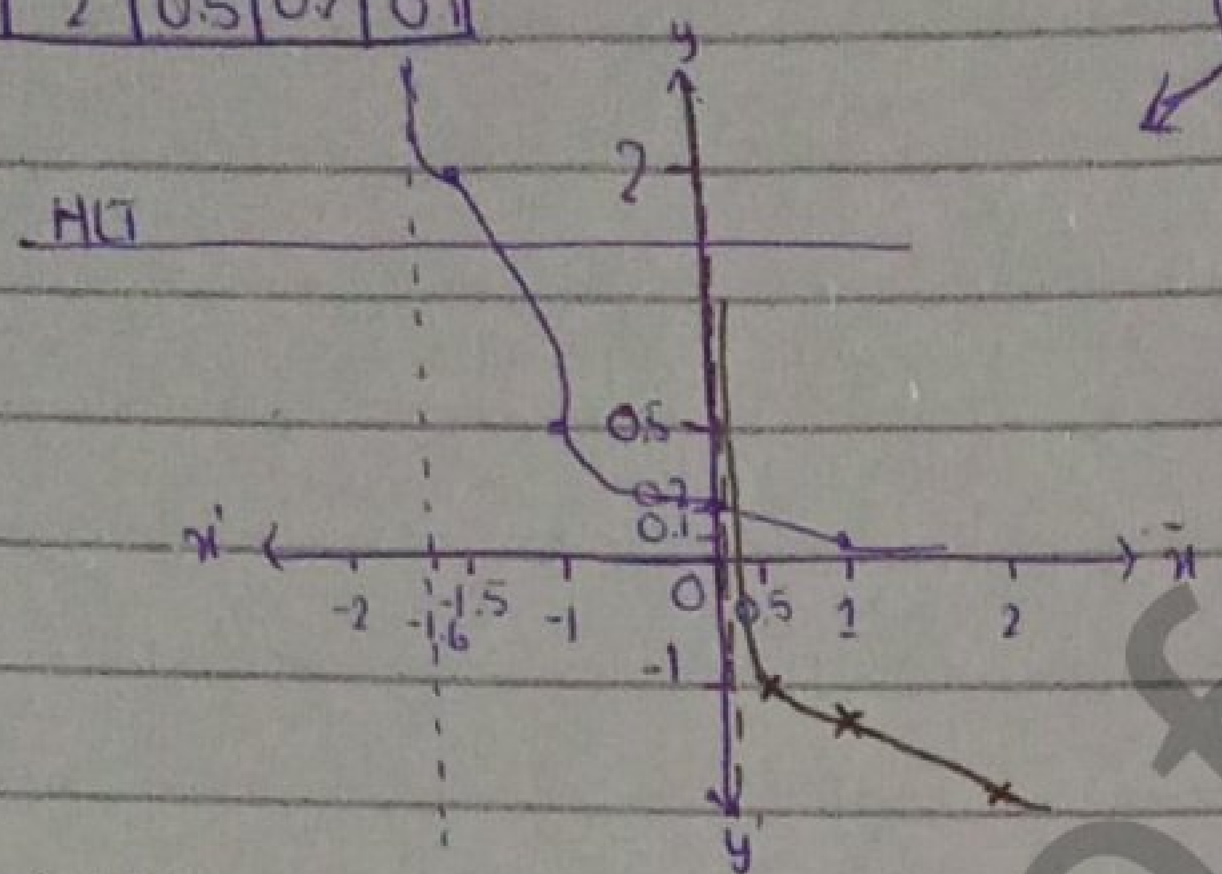
→ Finding at which point the function will become undefined / The point of vertical asymptote:

$3x+5=0$
$3x=-5$

$x = \frac{-5}{3}$
$\boxed{x = -1.66\ldots}$

→ Table:

x	-1.5	-1	0	1
y	2	0.5	0.2	0.1



Hence its a one-to-one function

→ Finding inverse:

$y = \frac{1}{3x+5}$ $\quad y(3x+5)=1 \quad 3x+5 = \frac{1}{y} \quad 3x = \frac{1}{y} - 5 \quad x = \frac{1}{3y} - \frac{5}{3}$

$x = \frac{1-5y}{3y} \quad f^{-1}(y) = \frac{1-5y}{3y}$ Replacing y with x,

$f^{-1}(x) = \frac{1-5x}{3x}$

→ The point of vertical asymptote:

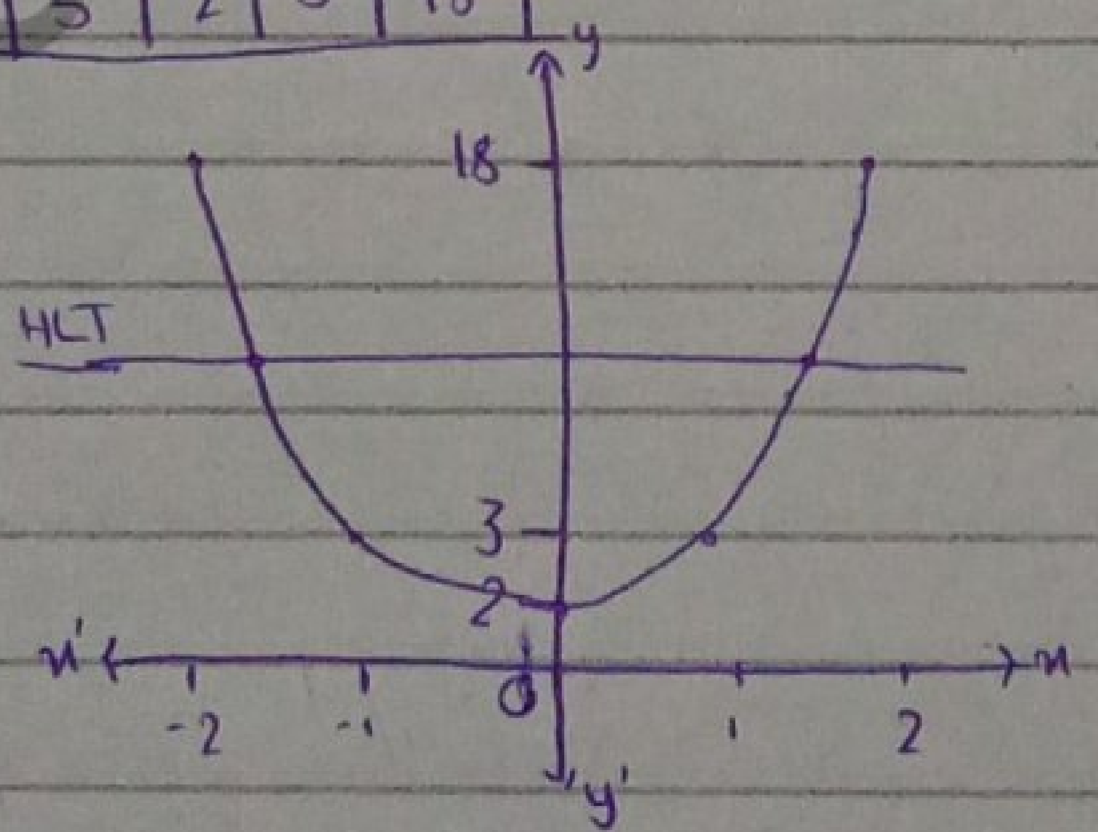
$3x=0 \quad x=\frac{0}{3} \quad x=0$

→ Table:

x	0.5	1	2
y	-1	-1.3	-1.5

(vii) $f(x) = x^4+2$ (biquadratic function)

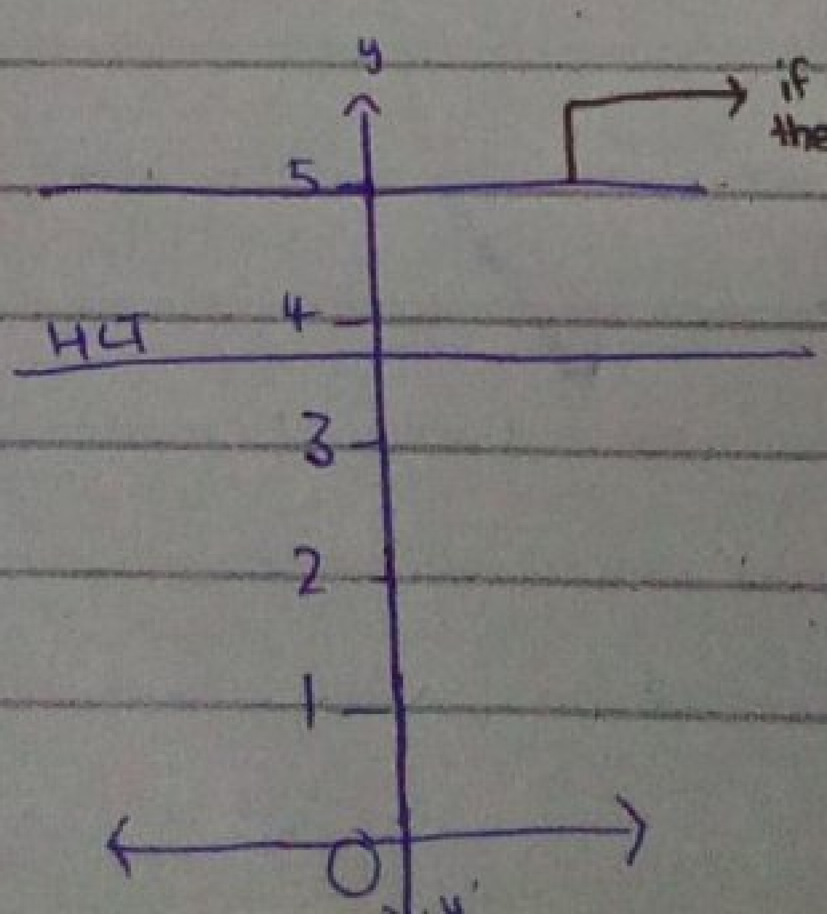
x	-2	-1	0	1	2
y	18	3	2	3	18



not a one-to-one function

(viii) $g(x) = 3$

$y=5$

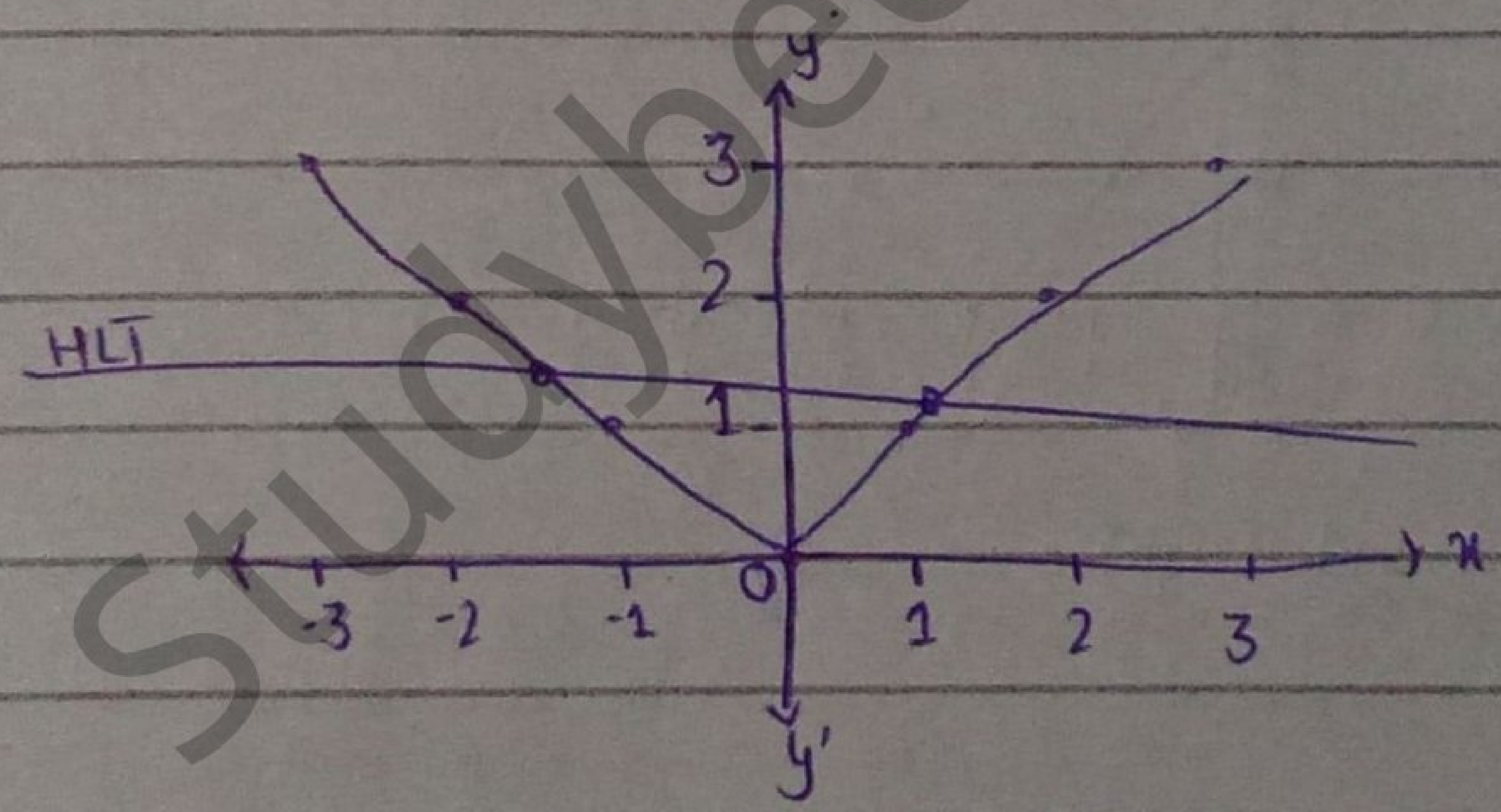


→ if the horizontal line falls on this line then, it will meet the line on infinite points.

not a one-to-one function

(ix) $h(x) = |x|$

x	-3	-2	-1	0	1	2	3
y	3	2	1	0	1	2	3



Hence not a one-to-one function