Studybeesofficial

Where Minds Pollinate

Class 12
Mathematics

Chapter 1

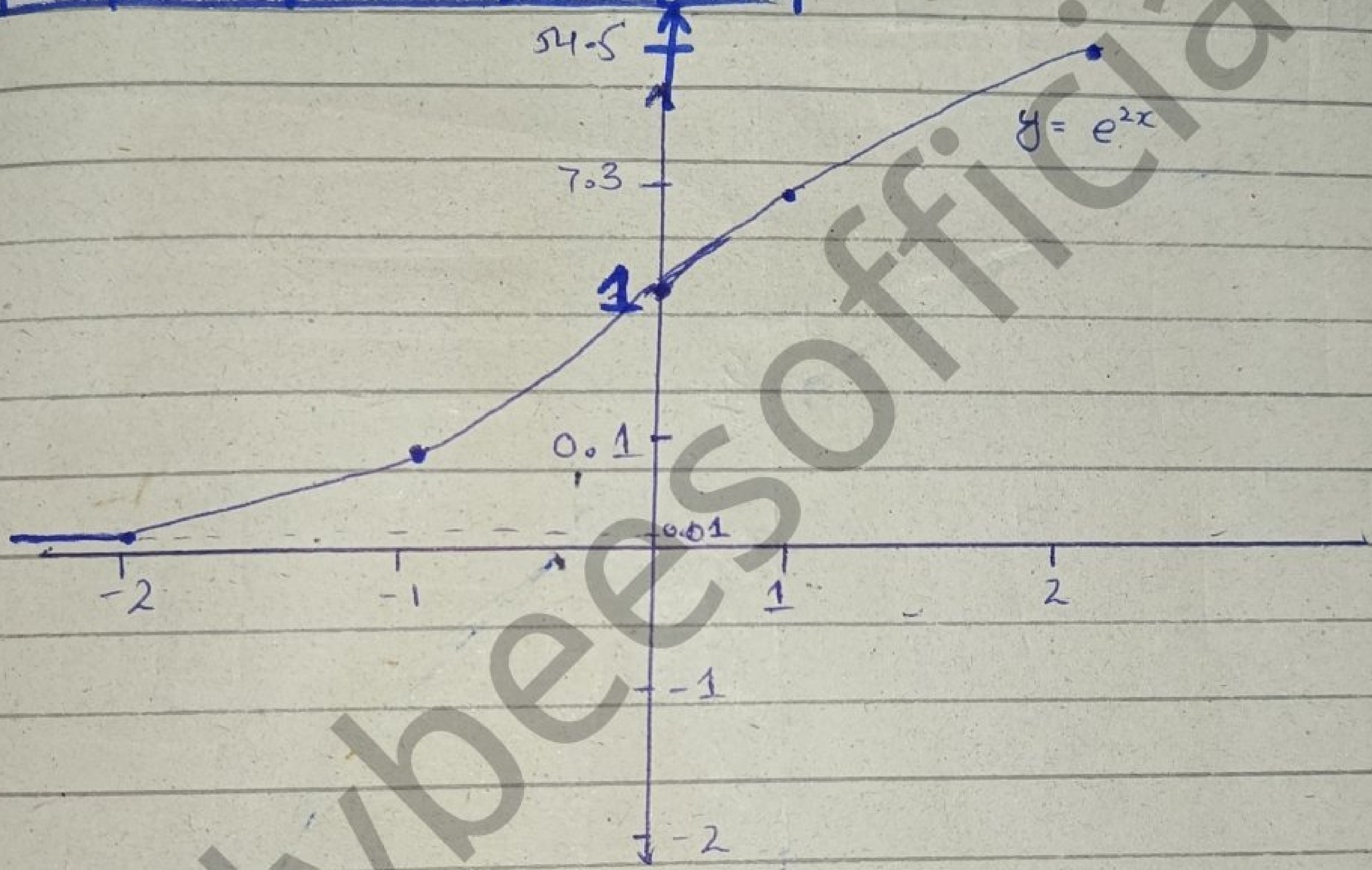
Exercise 1.3

Handwritten Solution

Exercise 1.3.

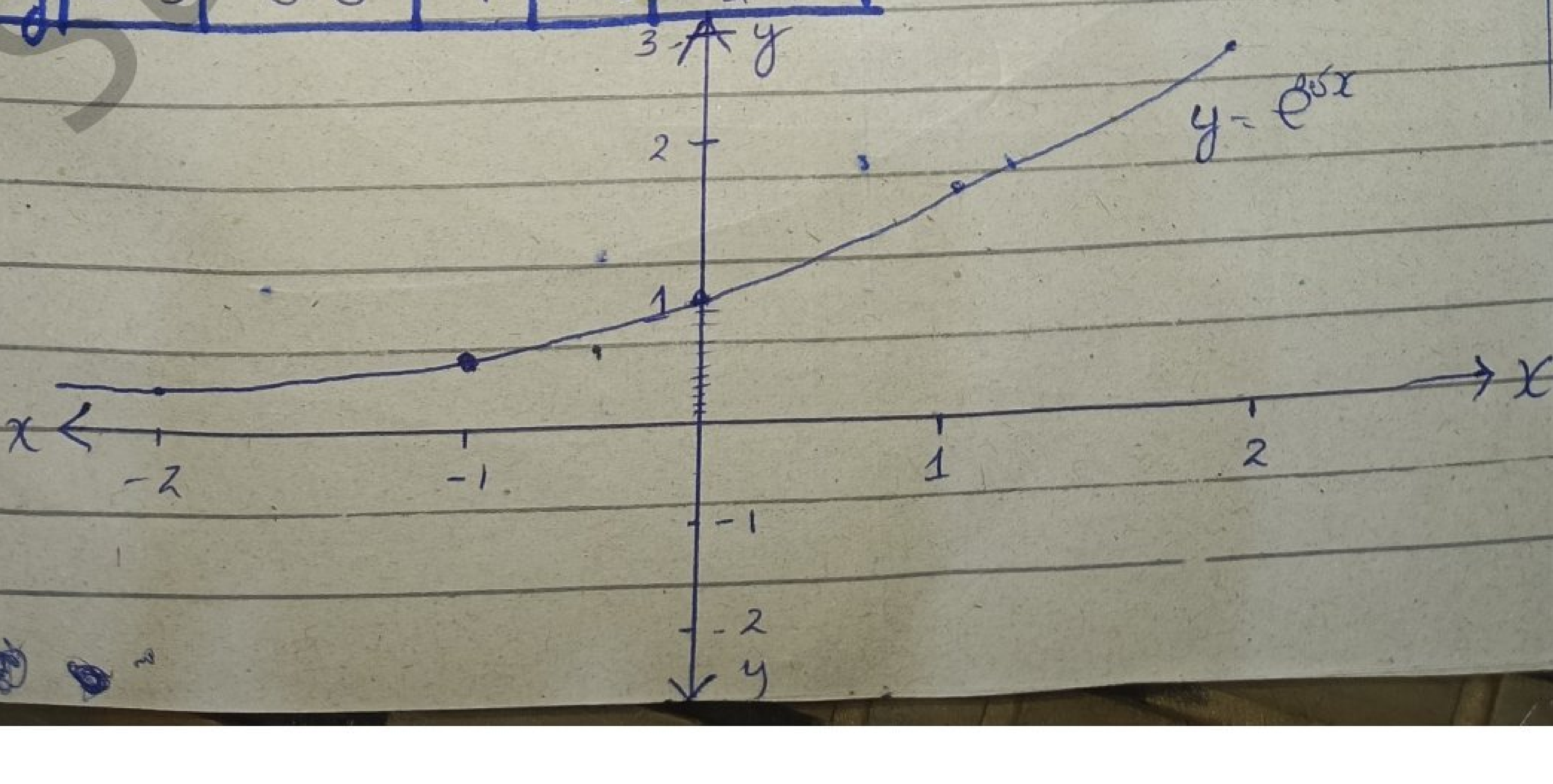
Q1 (1) $f(x) = e^{2x}$ Domain = $\mathbb{R}$

x	-2	-1	0	1	2
y	0.01	0.1	1	7.3	54.5



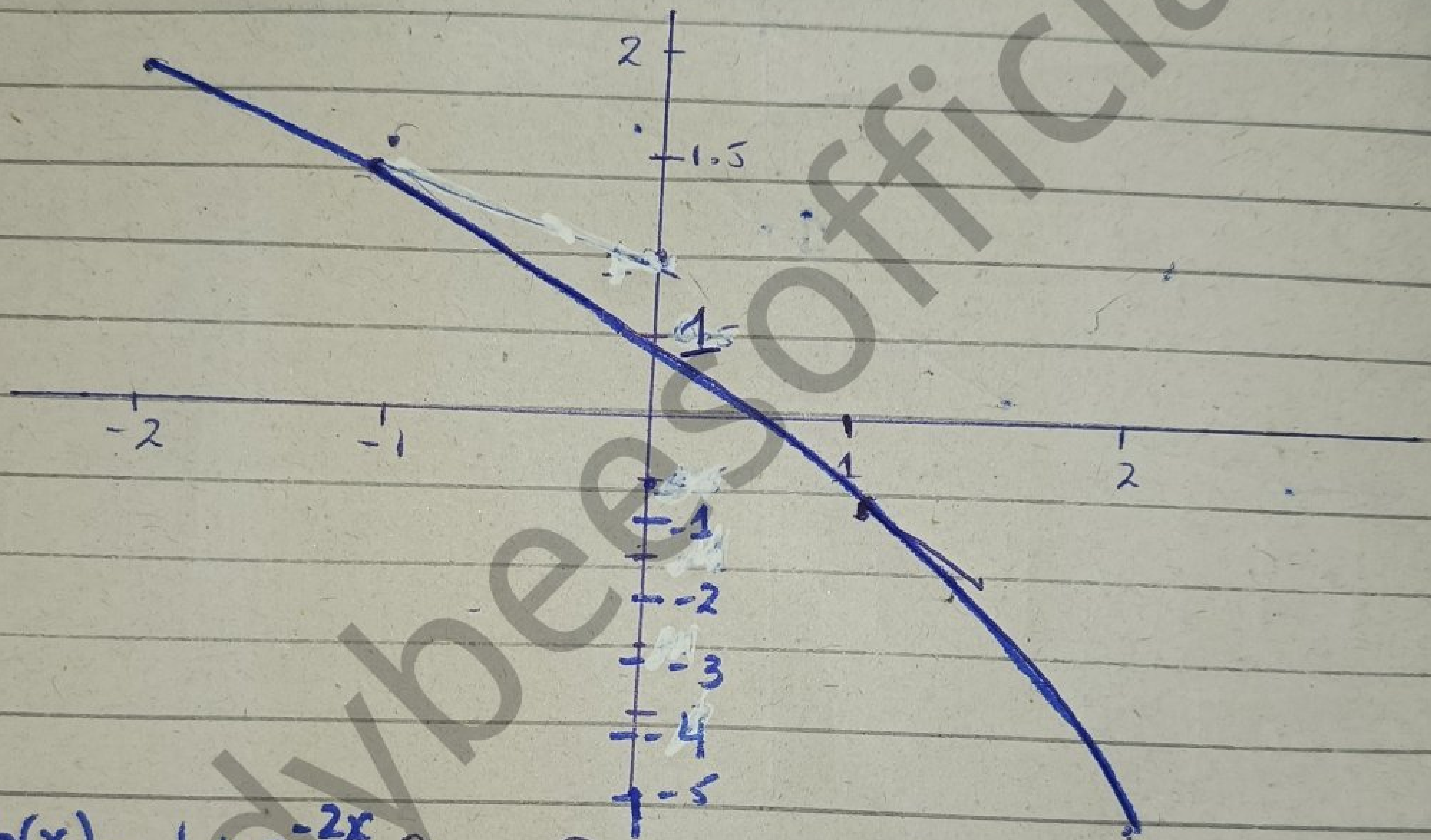
(2) $g(x) = e^{0.5x}$ Dom $(-\infty, +\infty)$

x	-2	-1	0	1	2
y	0.3	0.6	1	1.6	2.71



(iii) $h(x) = 2 - e^{x}$ Dom $= \mathbb{R}$

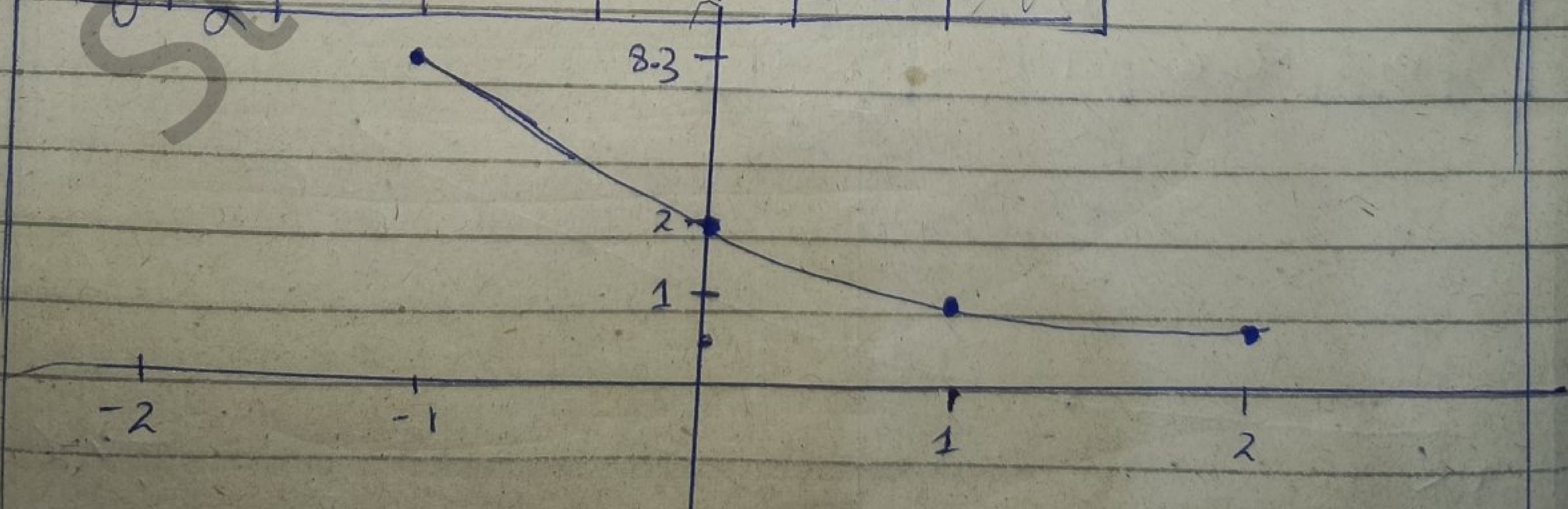
x	-2	-1	0	1	2
y	1.8	1.6	1	-0.7	-5.3



(iv) $h(x) = 1 + e^{-2x}$ Dom $= \mathbb{R}$

fa

x	-2	-1	0	1	2	
y	55.5	8.3	2	1.1	1.0	

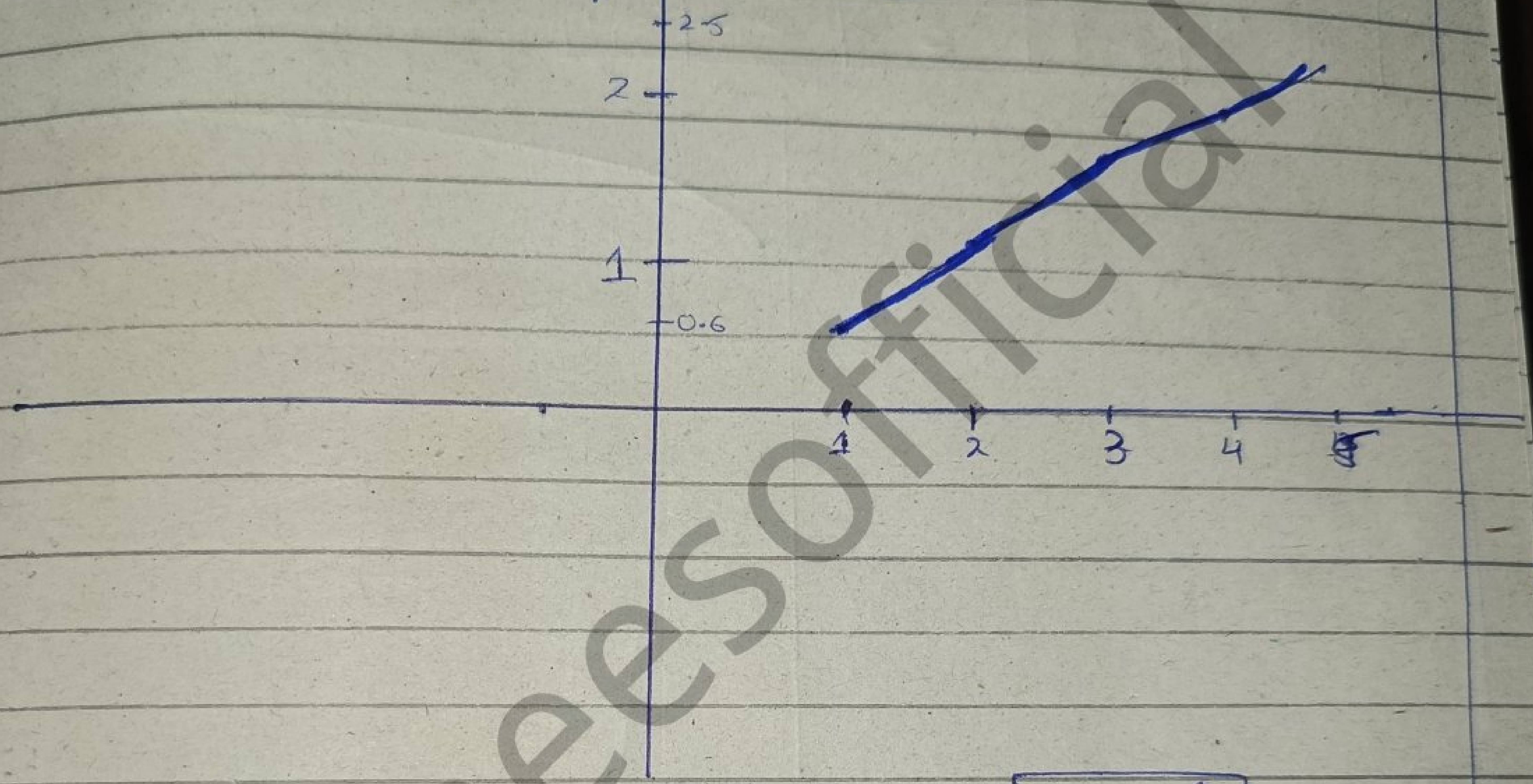


$2x > 0$
$\boxed{x > 0}$

(v) $f(x) = \ln(2x)$ Dom f(x) cant put -ve

x	1	2	3	4	5	
y	0.6	1.3	1.7	2.0	2.3	

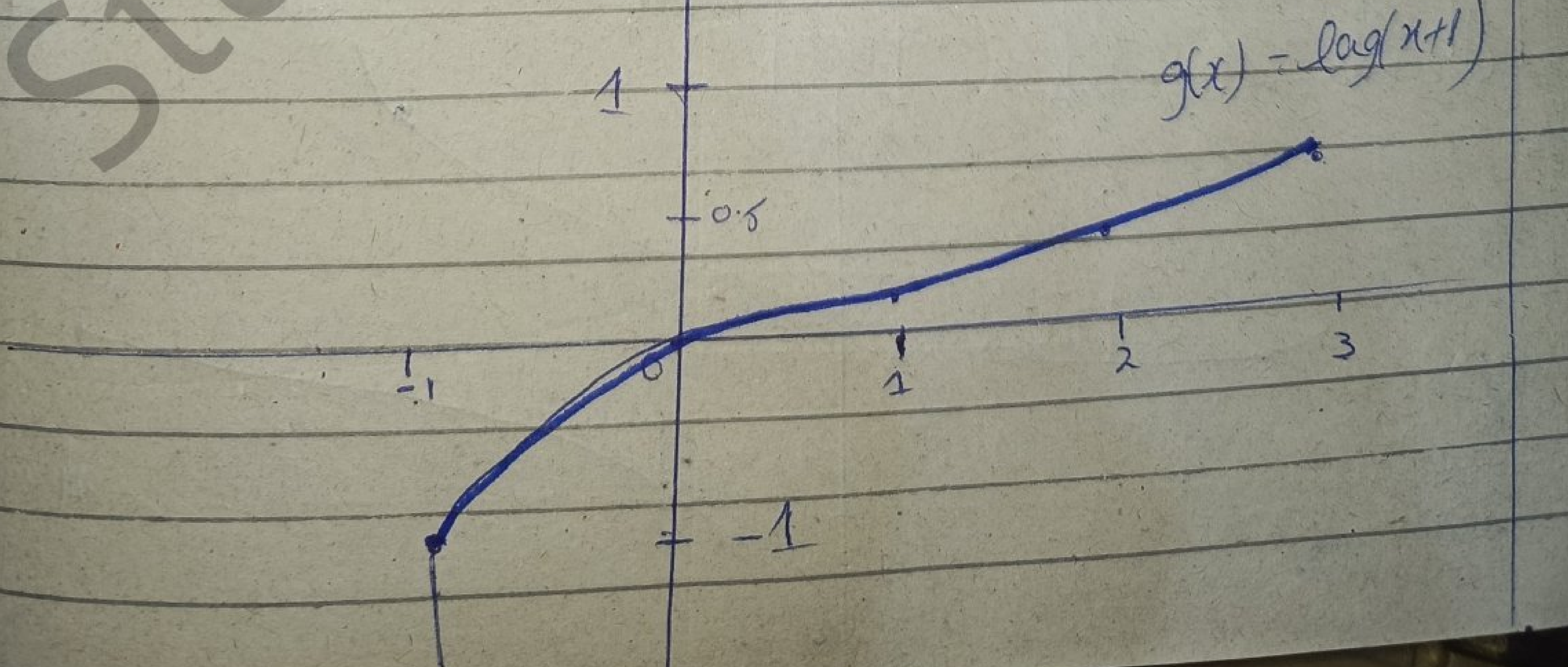
Dom = $(0, \infty)$
dot include -



(vi) $g(x) = \log(x+1)$ $x+1 > 0$ $\boxed{x > -1}$

Domain = $(-1, \infty)$

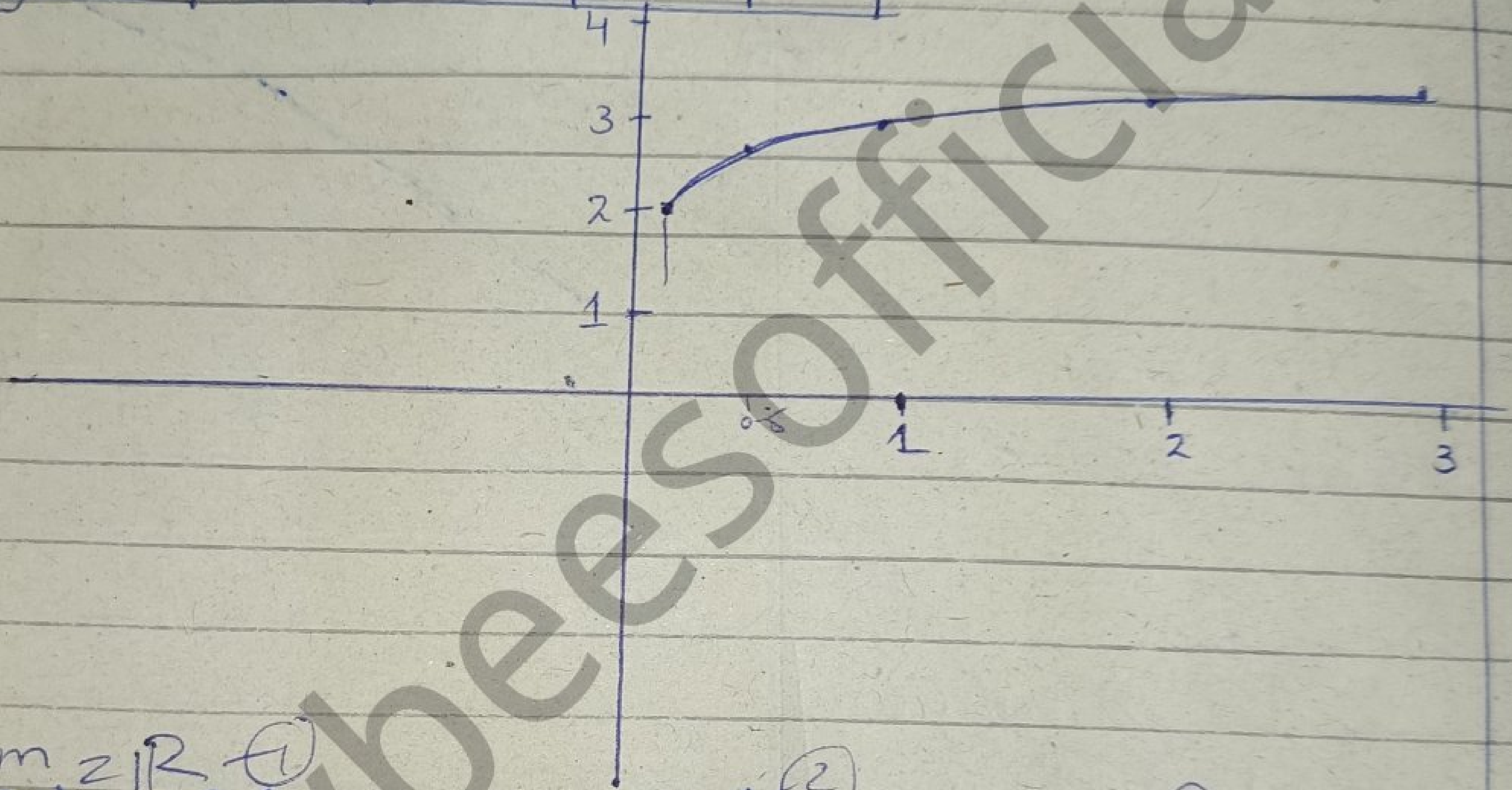
x	-0.9	0	1	2	3
y	-1	0	0.3	0.4	0.6



(viii) $h(x) = 3 + \log(x)$ $\quad x > 0$

Dom $h(x) = (0, \infty)$

x	0.1	0.5	1	2	3
y	2	2.7	3	3.3	3.4



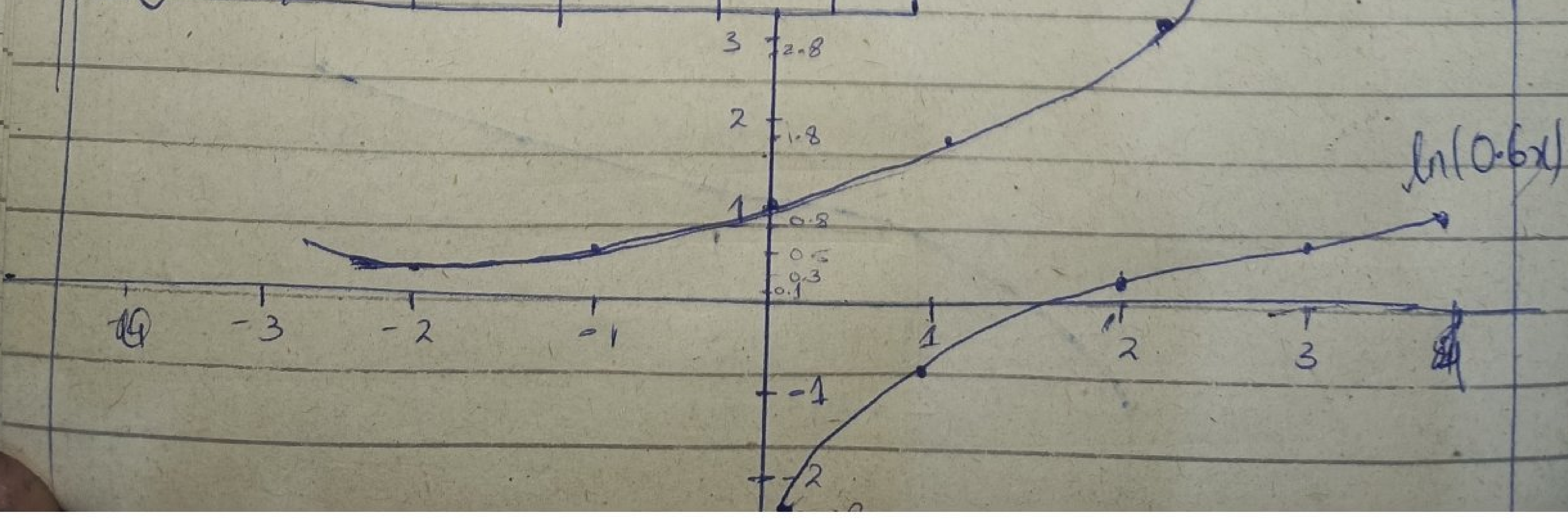
Dom = $\mathbb{R}$ (1)

(viii) $f(x) = e^{0.6x}$ & $g(x) = \ln(0.6x)$ (2) Dom = $(0, \infty)$

x	-2	-1	0	1	2	
y	0.3	0.5	1	1.8	3.3	

from (2)

x	0.1	1	2	3	4	
y	-2.8	-0.5	0.1	0.5	0.8	



Q2) Sol: $t = 0$ in 2024 ; $\boxed{N(0) = 7.5 \text{ million}}$

a) $\Rightarrow$ $\boxed{1 \text{ billion} = 1000 \text{ million}}$

$N(t) = 7.5(6)^{0.5t}$

$1000 = 7.5(6)^{0.5t}$

$\frac{1000}{7.5} = (6)^{0.5t}$

Taking natural log on b/s.

$\ln\left(\frac{1000}{7.5}\right) = \ln(6)^{0.5t}$

$\ln\left(\frac{1000}{7.5}\right) = 0.5t \ln(6)$

$\frac{\ln\left(\frac{1000}{7.5}\right)}{0.5 \ln 6} = t \Rightarrow t \approx 5.4 \text{ years}$

1 billion CD sold in $2024 + 5.4 \approx 2029$

b) N_0 = Initial = 7.5

duble = 7.5×2 = 15 million

$15 = 7.5(6)^{0.5t}$

Taking natural log on b/s

$\ln 2 = (0.5)t \ln(6)$

$\frac{\ln 2}{0.5 \ln 6} = t$ $\quad \boxed{t \approx 0.77}$

hence dubling CD sold in 2024

Q3) Sol: **a)** $A(t) = 50{,}000(1.06)^t$

$450{,}000 = 50{,}000(1.06)^t$

$\frac{450{,}000}{50{,}000} = (1.06)^t$

$9 = (1.06)^t$

Taking ln on b/s

$\ln 9 = \ln(1.06)^t$

$\ln 9 = t \ln(1.06)$

$\frac{\ln 9}{\ln(1.06)} = t \Rightarrow \boxed{t \approx 37.7}$

b) Initial amount at $t = 0$

$A(0) = 50{,}000$

duble $= 50{,}000 \times 2 = 100{,}000$

$100{,}000 = 50{,}000(1.06)^t$

$2 = (1.06)^t$

Taking ln

$\frac{\ln(2)}{\ln(1.06)} = t \Rightarrow t \approx 11.89 \text{ years}$

Q4) Sol: let initial population = P_0

let population at any time = $P(t)$

$\Rightarrow$ growth rate population model $\Rightarrow \boxed{P(t) = P_0 e^{rt}}$ $\quad \therefore r$ = growth rate

Given: $r = 1\% = \frac{1}{100} = \boxed{0.01 \text{ per year}}$

duble population $P(t) = 2P_0$ $\quad t = ?$

$2P_0 = P_0 e^{(0.01)t}$

$\frac{2P_0}{P_0} = e^{(0.01)t} \Rightarrow 2 = e^{(0.01)t}$

Taking $\ln$ on both sides

$\ln 2 = \ln e^{(0.01)t}$

$\ln 2 = (0.01)t$

$\frac{\ln 2}{0.01} = t \Rightarrow t \approx 69.3$ years

Q5) Sol let initial population $= P_0 = 5.2$ billion

and population at any time $= P(t)$

$\Rightarrow$ growth population function $= P(t) = P_0 e^{rt}$ ∴ r is growth rate

(a) $\boxed{P(t) = 5.2^{(0.016)t}}$ $r = 1.6\% = \frac{1.6}{100} = 0.016$

(b) time difference $= 2000 - 1990 = \boxed{10 \text{ years}}$

$P(10) \approx 5.2\, e^{(0.016)(10)}$

$\boxed{P(10) \approx 6.1 \text{ billion}}$

(c) $P(t) = 5.2 e^{(0.016)t}$

$\frac{8}{5.2} = e^{(0.016)t}$

Taking $\ln$ on both sides

$\ln\left(\frac{8}{5.2}\right) = \ln e^{(0.016)t}$

$t = \frac{0.4307}{0.016} \Rightarrow \boxed{t \approx 26.9 \text{ years}}$

Total time

$\approx 1990 + 26.9$ years

≈ 2016.9

In 2016.9 in this year population was 8 billion

Q6) Sol $S(t) = 68 - 20\log(t+1)$

a) Put $t=0 \Rightarrow S(0) = 68 - 20\log(1)$

$\boxed{S(0) = 68}$

b) (i) $t = 4$.

$S(4) = 68 - 20\log(5)$

$\boxed{S(4) = 54.02}$

(ii) $t = 24$

$S(24) = 68 - 20\log(25)$

$\boxed{S(24) \approx 40}$

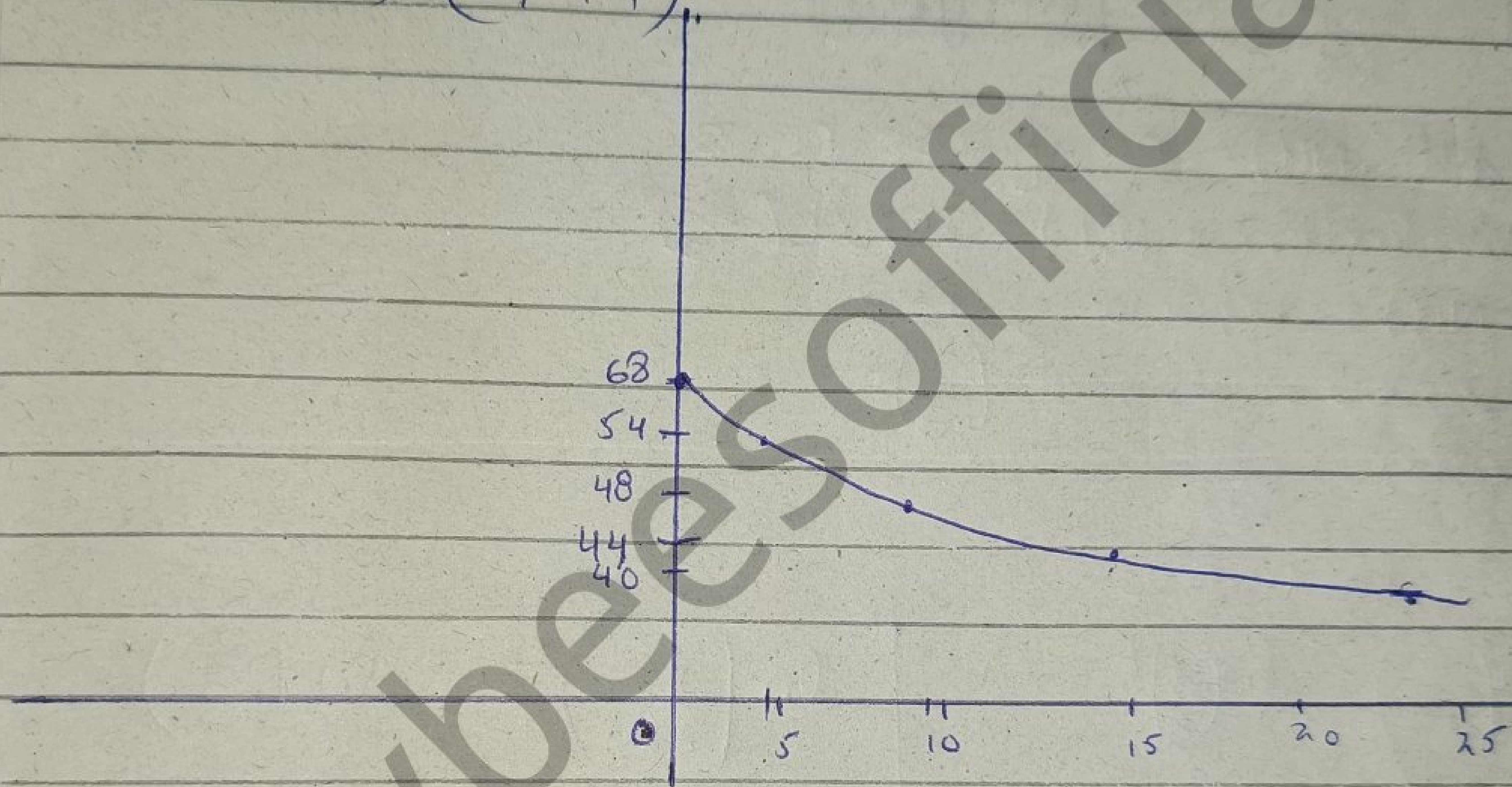
c) points :- (0, 68), (4, 54.00), (24, 40)

for some more points put t = 9 ⇒ S(9) = 68 − 20 log(10)

S(9) = 48 ⇒ (9, 48)

put $\boxed{t = 15}$ ⇒ S(15) = 68 − 20 log(16)

S(15) = 44 ⇒ (15, 44)



(d) $50 = 68 - 20\log(t+1)$

$50 - 68 = -20\log(t+1)$

$+\frac{18}{20} = \log(t+1)$

Taking antilog on b/s

$\text{antilog}\left(\frac{18}{20}\right) = t+1$

$t \simeq \text{antilog}\left(\frac{18}{20}\right) - 1$

$\boxed{t \simeq 6.94}$ Ans

Q7) Sol initial demand = P_0

double demand = $P(t) = 2P_0$

⇒ Given $P(t) = P_0 e^{kt}$

$2P_0 = P_0 e^{(0.1)t}$ $\quad k = 10\% = \frac{10}{100} = 0.1$

Taking ln on b.s

$\ln 2 = \ln e^{(0.1)t}$

$\ln 2 = (0.1)t$

$t = \frac{\ln 2}{0.1} \Rightarrow \approx \boxed{6.9 \text{ yeas}}$

Q 8) Sol:- $N(t)$ = no of use cans

Given function $\Rightarrow N(t) = 250{,}000(\frac{2}{3})^t$

a) $N(t) = 60{,}000 \; ; \; t = ?$

$60{,}000 = 250{,}000\left(\frac{2}{3}\right)^t$

$\frac{60{,}000}{250{,}000} = \left(\frac{2}{3}\right)^t$

Taking ln on b.s

$\ln\left(\frac{6}{25}\right) = \ln\left(\frac{2}{3}\right)^t$

$\ln\left(\frac{6}{25}\right) = t\ln\left(\frac{2}{3}\right)$

$\frac{\ln\left(\frac{6}{25}\right)}{\ln(2/3)} = t$

$\boxed{t \approx 3.5 \text{ years}}$

b) $N(t) = 1000, \; t = ?$

$1000 = 250{,}000\left(\frac{2}{3}\right)^t$

$\frac{1000}{250{,}000} = \left(\frac{2}{3}\right)^t$

Taking ln on b.s

$\ln\left(\frac{1}{250}\right) = t\ln\left(\frac{2}{3}\right)$

$t \approx \frac{\ln\left(\frac{1}{250}\right)}{\ln(2/3)}$

$\boxed{t \approx 13.6}$ years