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Where Minds Pollinate

Class 12
Mathematics

Chapter 1

Exercise 1.1

Handwritten Solution

Ch# 1 Exercise: 1.1

· Concept:

Indeterminate form:

(i) $\frac{1}{0} = \infty$

(ii) $\sqrt{-} = i$ (complex)] mathersor

∴ (Domain Range real)

Q1) (i) $f(x) = x^2 - 6$

Sol: Dom $f(x) = \mathbb{R}$ or $(-\infty, \infty)$

(ii) $g(x) = \frac{x}{x+3}$

Sol: Dom $g(x) = \mathbb{R} - \{-3\}$

or Dom $g(x) = (-\infty, -3) \cup (-3, \infty)$

$x+3=0$

$\boxed{x=-3}$ cant put -3

(iii) $h(x) = \frac{x+4}{x^2-9}$

Sol:- Dom $h(x) = \mathbb{R} - [-3, 3]$

OR

Dom $h(x) = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

$x^2 - 9 \neq 0$

$x^2 = 9$

we cant put +3 & -3 ∴ $\boxed{x = \pm 3}$

(iv) $i(x) = \frac{x}{5x+2}$

Sol: Dom $i(x) = \mathbb{R} - \{-\frac{2}{5}\}$

OR

$(-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, \infty)$

$5x + 2 \neq 0$

$5x = -2$

$\boxed{x = -\frac{2}{5}}$

v) $j(x) = \dfrac{x}{x^2+4}$

Sol:- $\text{Dom } j(x) = R - \{-4\}$

OR

$\text{Dom } j(x) = (-\infty, -4) \cup (-4, \infty)$

$x^2+4=0$

$x = \pm 4$

(vi) $K(x) = \sqrt{x+1}$

Sol: $x+1 \geq 0 \qquad x \geq -1$

$\text{Dom } K(x) = x \geq -1$

OR

$[-1, \infty)$

$\sqrt{x+1} = 0$

$x+1 \neq 0$

$\boxed{x \neq -1}$

Q2 (i) $f(x) = x+7$

Sol: $\text{Dom } f(x) = (-\infty, \infty)$

Range $= (-\infty, \infty)$

(ii) $f(x) = 2x^2+1$

Sol:- $\text{Dom } f(x) = (-\infty, \infty) = R$

Range

$x^2 \geq 0$

$\times 2 \qquad 2x^2 \geq 0$

add $+1 \qquad 2x^2+1 \geq 1$

Range $= [1, \infty)$

(iii) $f(x) = 2\sqrt{x-5}$

Range $= [0, \infty)$

Sol: $x-5 \geq 0$

$x \geq 5$

$\text{Dom } f(x) = [5, \infty)$

$\therefore e^x$ can never be zero

(iv) $f(x) = |x-2| - 3$

Sol: $\text{Dom } f(x) = (-\infty, \infty)$

Range $|x-2| \geq 0$

add -3.

$y = |x-2| - 3 \geq -3$

Range $= [-3, \infty)$.

(v) $f(x) = 1 + \sin x$

Sol:- $\text{Dom } f(x) = (-\infty, +\infty)$

Range of $\sin x$

$-1 \leq y \leq 1$

adding 1.

$1-1 \leq y \leq 1+1$

$0 \leq y \leq 2$

Range $= [0, 2]$

(vi) $f(x) = 3 + \sqrt{x-2}$

~~Dom~~ $x - 2 \geq 0$

$x \geq 2$

Dom $= [2, \infty)$.

Range: $\sqrt{x-2} \geq 0$

$\sqrt{x-2} \geq 0$

$3 + \sqrt{x-2} \geq 3$

Range $= [3, \infty)$

(vii) $f(x) = \frac{3e^x}{7}$

Sol:- $\text{Dom } f(x) = (-\infty, +\infty)$.

Range

$e^x > 0$

$\therefore e^0 = 1$

$\therefore e^1 = 2.73$

$e^{-1} = \frac{1}{e} = \frac{1}{2.73} =$ +ve.

Range $= (0, \infty)$.

~~because~~ $e^x > 0$

$\frac{3}{7}e^x > 0$

(viii) $f(x) = \frac{x^2-16}{x+4}$

Sol:

Domain $= R - [-4]$

OR

Domain $= (-\infty, -4) \cup (-4, \infty)$

Range:

$y = \frac{x^2 - 4^2}{x+4} = \frac{(x-4)(x+4)}{x+4}$

$y = x - 4$

$y = -4 - 4$

$y \neq -8$

Range $= R - [-8]$

(ix) $f(x) = (x-1)^2 + 1$.

Sol: Domain = Real no $(-\infty, +\infty)$.

Range:- $\therefore (x-1)^2 \geq 0$

$(x-1)^2 + 1 \geq 1$

Range = $[1, \infty)$.

$\therefore$ if $x^2 - 2x + 2$ Range = ?

completing square

$(x^2 - 2x + 1) + 1$

$(x-1)^2 + 1 \leftarrow$

(x) $f(x) = \dfrac{1}{x-1}$

Sol: Dom = $R - [1]$

Range

$y = \dfrac{1}{x-1}$; $x = ?$

$y(x-1) = 1$

$xy - y = 1$

$xy = 1 + y$

$\boxed{x = \dfrac{1+y}{y}}$ $y \neq 0$

so

Range = $R - [0]$

(xi) $f(x) = \dfrac{x-2}{x+3}$

Sol Dom = $\mathbb{R} - \{-3\}$

Range

$y = \dfrac{x-2}{x+3}$

$xy + 3y = x - 2$.

$3y + 2 = x - xy$.

$\boxed{\dfrac{3y+2}{1-y} = x}$ $y \neq 1$.

Range:- $R - \{1\}$

(xii) $f(x) = \dfrac{x^2 - x - 6}{x - 3}$

Sol Domain :- $R - \{3\}$

Range:-

$y = \dfrac{x^2 - 3x + 2x - 6}{x-3}$

$y = \dfrac{x(x-3) + 2(\cancel{x-3})}{\cancel{x-3}}$

$y = \dfrac{(x+2)\cancel{(x-3)}}{\cancel{(x-3)}}$

$\boxed{y = x + 2}$

$y \neq \overset{\text{Domain}}{3} + 2 = \boxed{5}$

Range = $R - \{5\}$

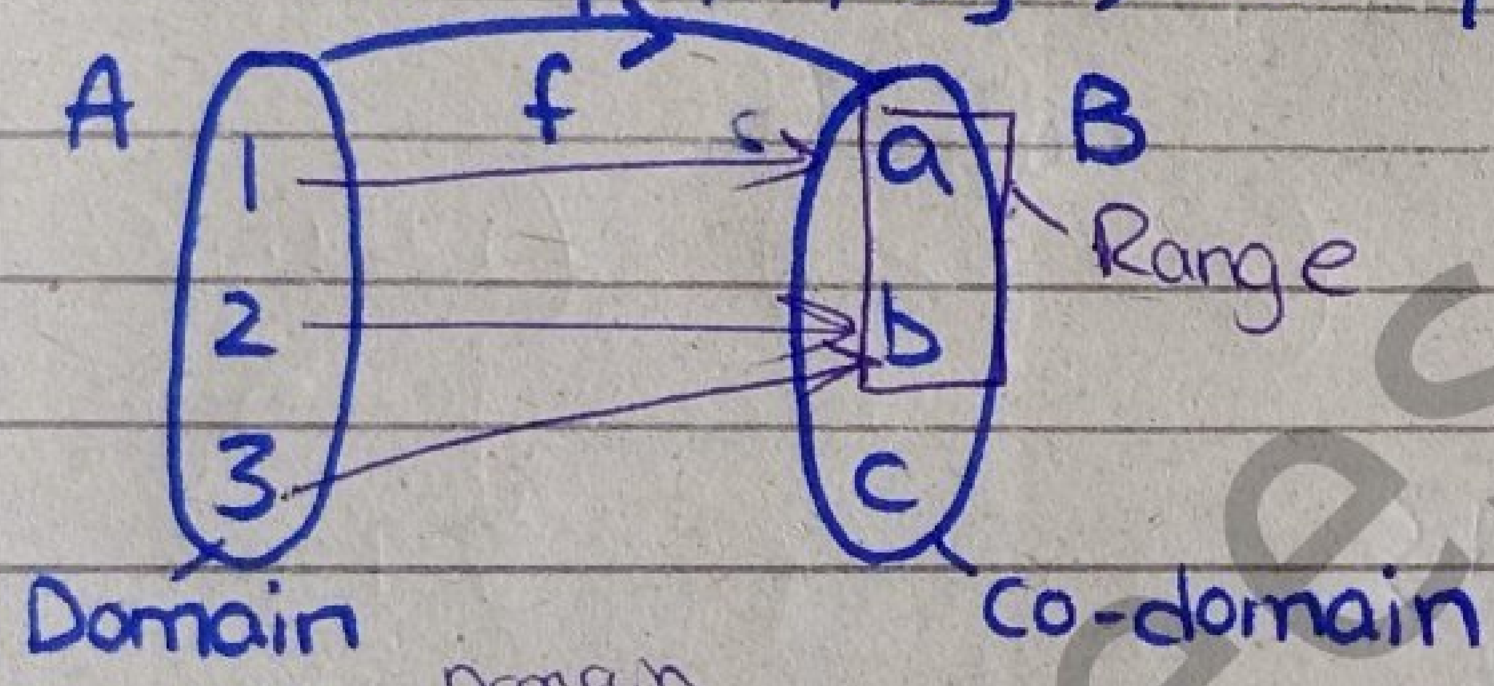
Concept :

For Function

- All domains of function should have image
- Elements of domain should not be repeated

1) Into - function :-

let $A = \{1, 2, 3\}$, $B = \{a, b, c\}$



Range is output

$f = \{(1, a), (2, b), (3, b)\}$ (Domain)

Domain $f = \{1, 2, 3\}$

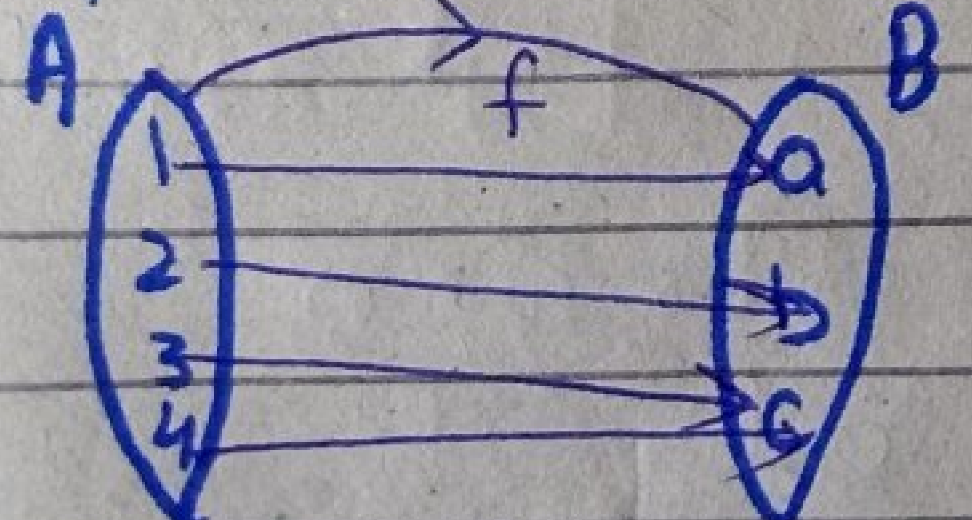
Range $= \{a, b\} \subseteq$ co-domain

- Range should not be equal to co-domain

2) On-to function :-

let $A = \{1, 2, 3, 4\}$

$B = \{a, b, c\}$

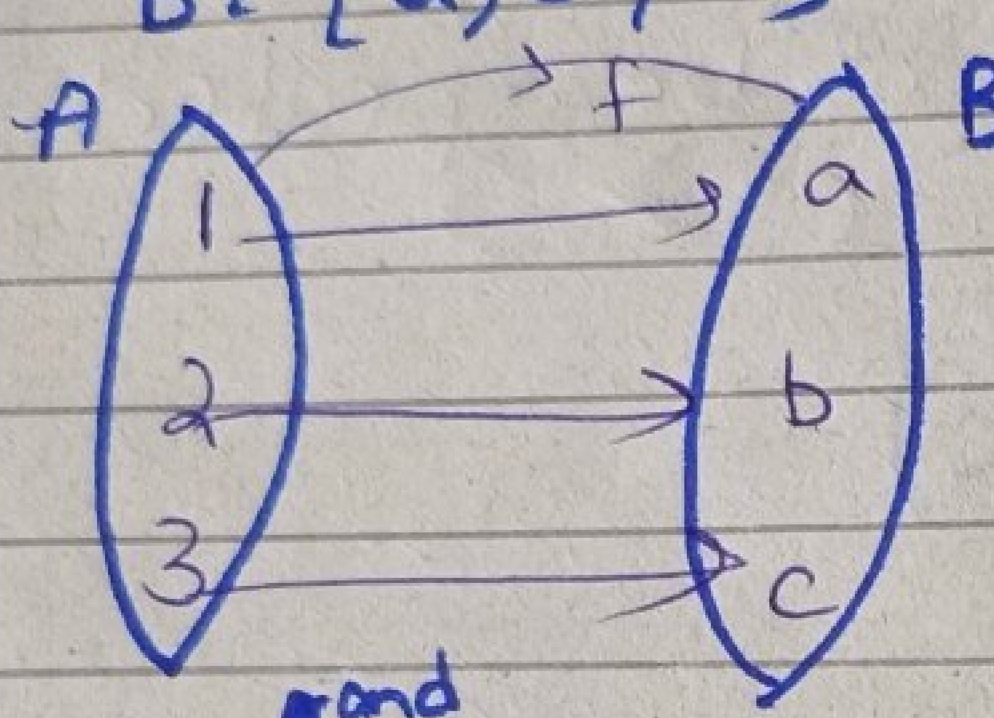


if Range = co-domain then its onto function

Range $= \{a, b, c\} =$ co-domain

3- One - One function .

let $A = \{1, 2, 3\}$
$B = \{a, b, c\}$



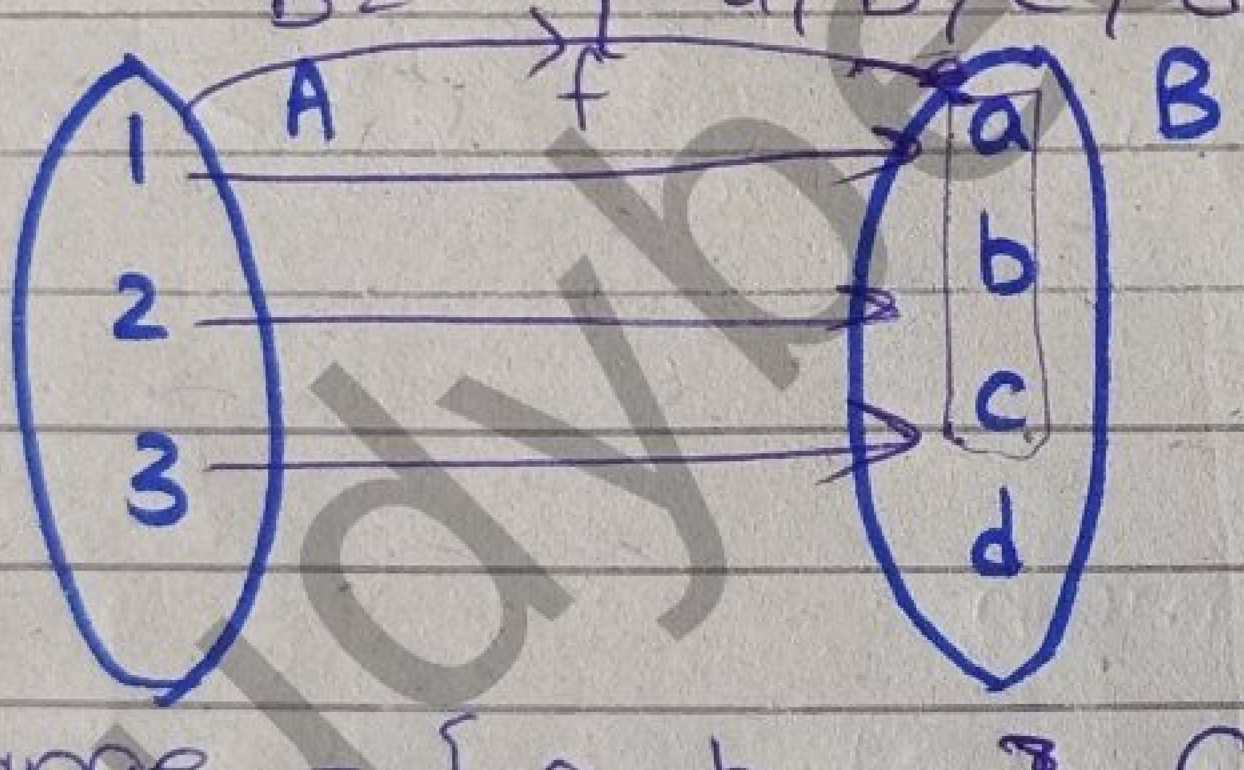
Distinct element have distinct image

$\therefore f(x_1) = f(x_2)$

$x_1 = x_2$

4) Into and one-one

let $A = \{1, 2, 3\}$
$B = \{a, b, c, d\}$.

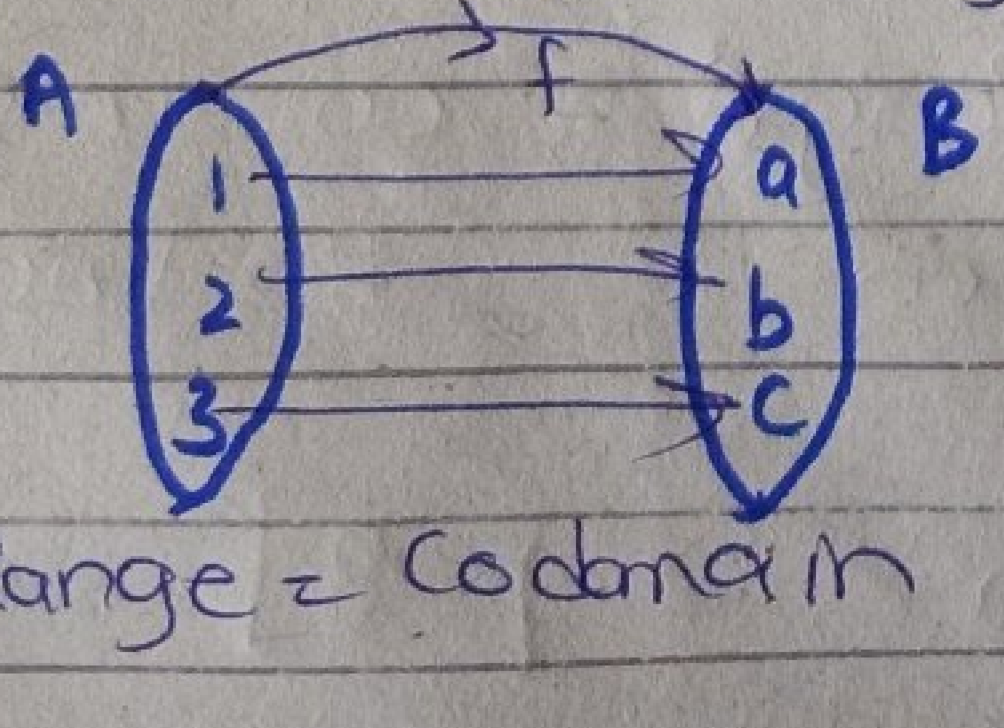


Range $= \{a, b, c\} \subset$ Co-domain

Into and one-one is called **Injective**

(5) **On-to & one-one** .

let $A = \{1, 2, 3\}$; $B = \{a, b, c\}$.

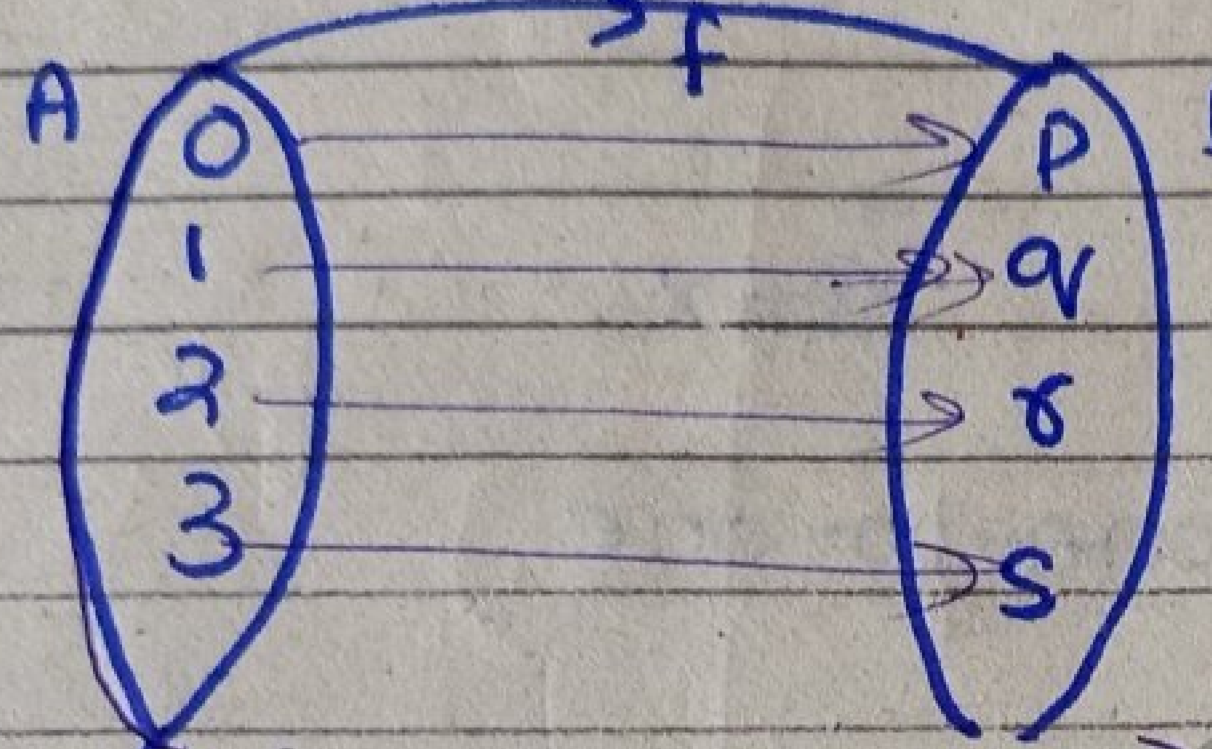


Range = Codomain

one-one & onto then it is called **bijective function** .

Q3 $A = \{0, 1, 2, 3\}$ $B = \{p, q, r, s\}$ and function is one to one and/or into.

Sol



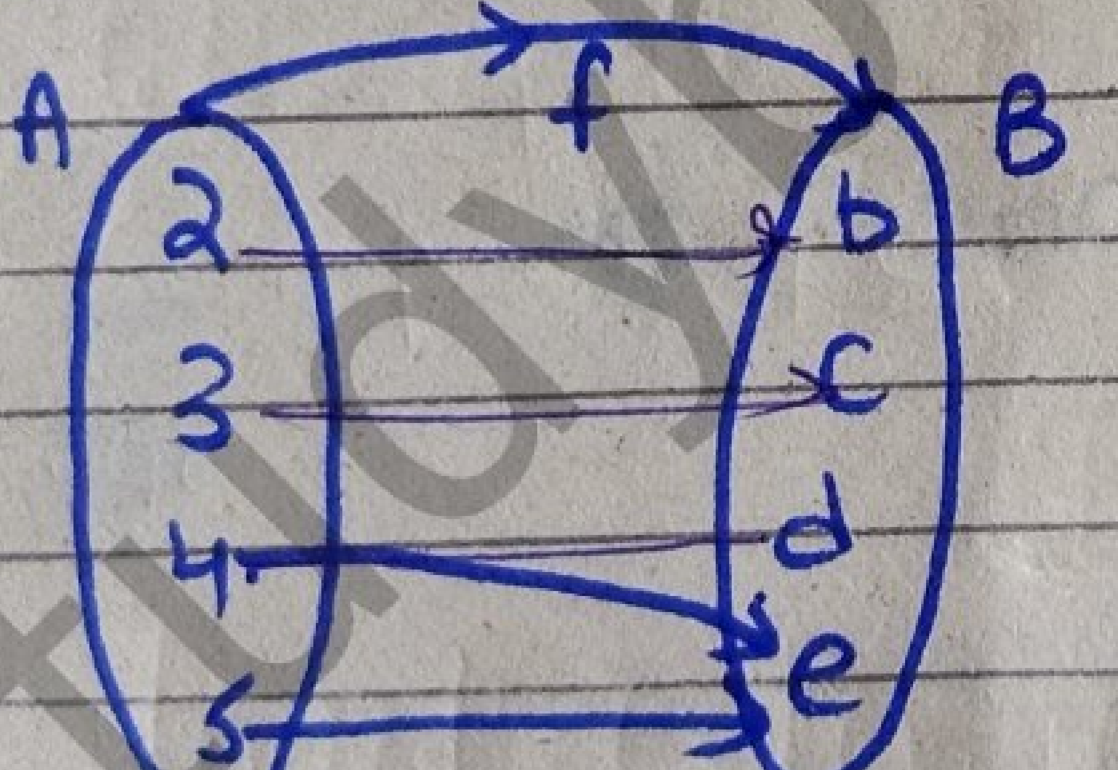
Range $= \{p, q, r, s\} =$ Co-domain.

→ Onto

→ One-one also called bijective function.

Q4 $A = \{2, 3, 4, 5\}$, $B = \{b, c, d, e\}$ and function is one to one, onto or into

Sol



~~$f = \{(0, p), (1, q), (2, r),$~~

$f = \{(2, b), (3, c), (4, e), ($

Range $= \{b, c, e\} \subseteq$ co-domain

→ Into-function

Q5 (i) $f(x) = 4x - 7$

Sol, $f(x_1) = f(x_2)$

$4x_1 - 7 = 4x_2 - 7$

$4x_1 \cancel{-7} \cancel{+7} = 4x_2$

$4x_1 = 4x_2$

$x_1 = x_2$

One-to-one.

(ii) $f(x) = 6x^2+2$

Sol let $f(x_1) = f(x_2)$

$6x_1^2+2 = 6x_2^2+2$

$6x_1^2+\cancel{2}-\cancel{2} = 6x_2^2$

$\sqrt{x_1^2} = \sqrt{x_2^2}$

$\pm x_1 = \pm x_2$ not one-to-one

$x_1 \neq x_2$ for every value of x

(iii) $f(x) = \dfrac{x^3-1}{2}$

Sol $f(x_1) = f(x_2)$

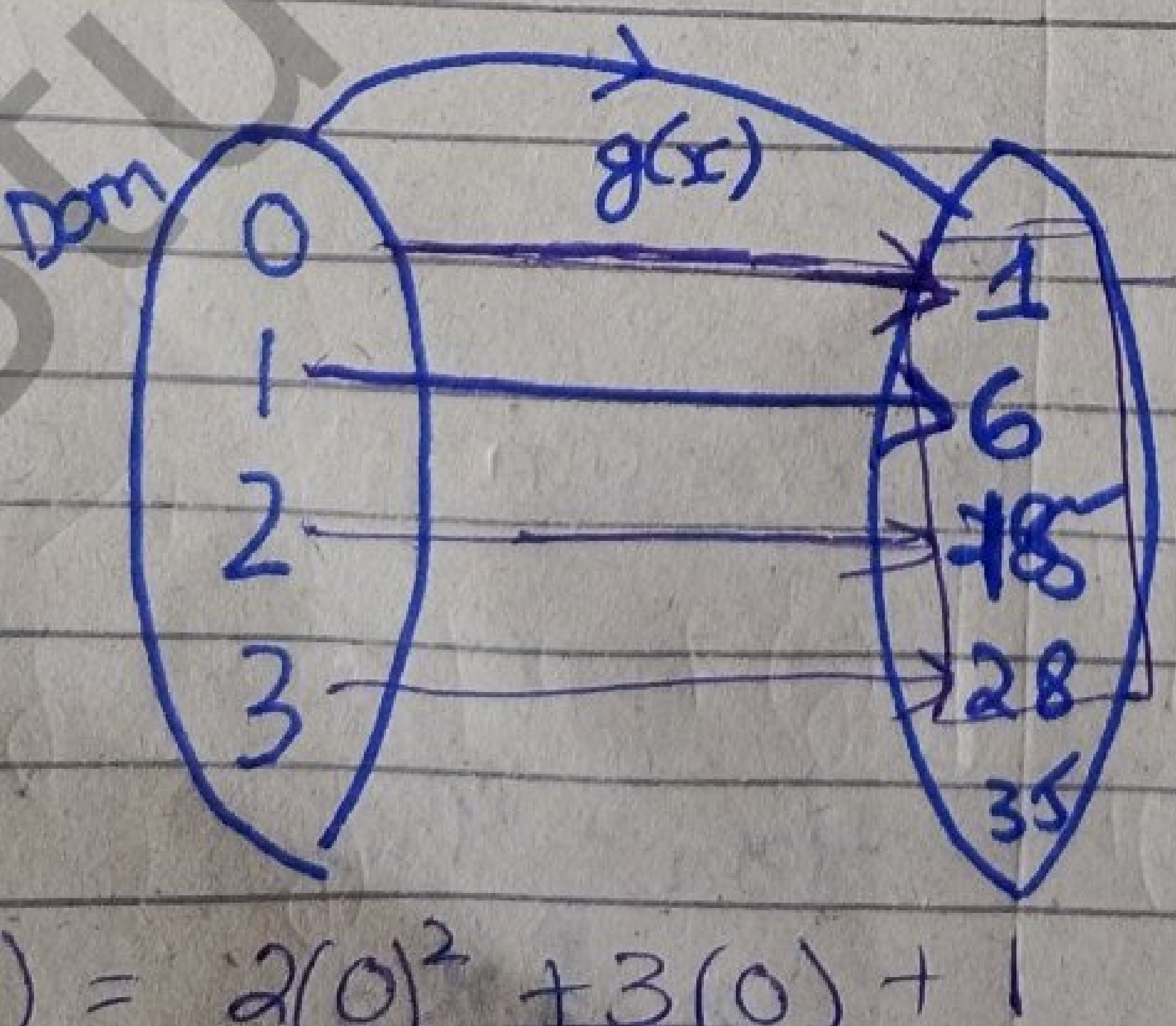
$\dfrac{x_1^3-1}{\cancel{2}} = \dfrac{x_2^3-1}{\cancel{2}}$

$x_1^3-\cancel{1} = x_2^3-\cancel{1}$

$\sqrt[3]{x_1^3} = \sqrt[3]{x_2^3}$

$\boxed{x_1 = x_2}$ One-to-One An

Q6)



$g(0) = 2(0)^2+3(0)+1$

$\boxed{g(0) = 1}$

$g(1) = 2+3+1 = 6$

$g(2) = 2(4)+3(2)+1 = 15$

$g(3) = 2(9)+3(3)+1 = 28$

Range = $\{1,6,15,28\} \subset$ Co-domain

injective function → Into function

→ 1-1 function

Q 7) Ans) Dom = R = real no
Co-domain : R = real no

$h(x) = 2x + 1$

let $h(x_1) = h(x_2)$

$2x_1 + \cancel{1} = 2x_2 + \cancel{1}$

$\Rightarrow \quad x_1 = x_2$

→ One-to-one

now range = ?

let $y = 2x + 1$

$y - 1 = 2x$

$$\boxed{x = \frac{y-1}{2}}$$

Range = R = co-domain On-to function.

On-to function
+ one-one function
= bijective function

Q8 Sol:- A ⟶ Domain → $R - \{3\}$

B ⟶ co-domain → $R - \{1\}$.

$f(x) = \frac{x-2}{x-3}$

check

→ One-to-one

$f(x_1) = f(x_2)$

$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\cancel{x_1 x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1 x_2} - 3x_2 - 2x_1 + \cancel{6}$

$-3x_1 - 2x_2 = -3x_2 - 2x_1$

$-2x_2 + 3x_2 = -2x_1 + 3x_1$

$\boxed{x_2 = x_1}$

This shows that function is one-to-one

now Range = ?.

$y = \frac{x-2}{x-3}$; $x = ?$

$y(x-3) = x-2$

$xy - 3y = x-2$

$xy - x = -2+3y$

$x(y-1) = 3y-2$

$\boxed{x = \frac{3y-2}{y-1}}$ $y \neq 1$

Range = $R - \{1\}$ = co-domain

This shows Onto-function

Hence proved it is bijective function

Q9 (i) $f(x) = 4x-3$

Dom f = $\mathbb{R} = (-\infty, \infty)$ = Range f^{-1}

Range f = $\mathbb{R} = (-\infty, \infty)$ = Dom f^{-1}

let $\boxed{f(x) = y}$

$y = 4x-3$

$y+3 = 4x$

$x = \boxed{y+3/4}$

as $f(x) = y$ then $\boxed{x = f^{-1}(y)}$

$\frac{y+3}{4} = f^{-1}(y)$

Replace y with x

$$\boxed{f^{-1}(x) = \frac{x+3}{4}}$$

Dom $f^{-1} = (-\infty, +\infty)$

Range $f^{-1} = (-\infty, +\infty)$.

Proof

$f[f^{-1}(x)] = f^{-1}[f(x)]$

$f\left[\frac{x+3}{4}\right] = f^{-1}[4x-3]$

$4\left(\frac{x+3}{4}\right) - 3 = \frac{(4x-3)+3}{4}$

$x+3-3 = \frac{4x}{4}$

$x = x$

Proved.

(ii) $f(x) = \dfrac{x}{x-5}$

Sol:- Dom $f(x)$ = $R - \{5\}$ = Range f^{-1}

Range $f(x)$ = $R - \{1\}$ = Dom f^{-1}

let $y = \dfrac{x}{x-5}$ $y = f(x)$

$y(x-5) = x$

$xy - 5y = x$

$x(y-1) = 5y$

$\boxed{x = \dfrac{5y}{y-1}}$

$f^{-1}(y) = \dfrac{5y}{y-1}$

$\boxed{f^{-1}(x) = \dfrac{5x}{x-1}}$ ∴ replace y with x, $x \neq 1$

$f[f^{-1}(x)] = f^{-1}[f(x)]$

$f\left[\dfrac{5x}{x-1}\right] = f^{-1}\left[\dfrac{x}{x-5}\right]$

$\dfrac{\left(\dfrac{5x}{x-1}\right)}{\left(\dfrac{5x}{x-1}\right) - 5} = \dfrac{5\left[\dfrac{x}{x-5}\right]}{\left(\dfrac{x}{x-5}\right) - 1}$

$\dfrac{\dfrac{5x}{\cancel{x-1}}}{\dfrac{5x - 5(x-1)}{\cancel{x-1}}} = \dfrac{\dfrac{5x}{\cancel{x-5}}}{\dfrac{x-1(x-5)}{\cancel{x-5}}}$

$\dfrac{\cancel{5x}}{5x - 5x + 5} = \dfrac{5x}{x - x + 5}$

$x = x$ proved

(iii) $f(x) = \dfrac{x+2}{x-1}$

Sol:- Dom = $R - \{1\}$ = Range f^{-1}

Range f = $R - [1]$ = Dom f^{-1}

let $y = f(x)$

$y = \dfrac{x+2}{x-1}$

$xy - x = x + 2$

$x(y-1) = x + 2$

$\boxed{x = \dfrac{x+2}{y-1}}$ $y \neq 1$

$f^{-1}(y) = \dfrac{y+2}{y-1}$

replace y with x

$\boxed{f^{-1}(x) = \dfrac{x+2}{x-1}}$

$f[f^{-1}(x)] = f^{-1}[f(x)]$

$f\left[\dfrac{x+2}{x-1}\right] = f^{-1}\left[\dfrac{x+2}{x-1}\right]$

L.H.S = $\dfrac{\left(\dfrac{x+2}{x-1}\right) + 2}{\left(\dfrac{x+2}{x-1}\right) - 1}$

$$= \frac{\frac{x+2+2x-2}{x-1}}{\frac{x+2-x+1}{x-1}} = x$$

RHS $= f^{-1}[f(x)]$

$= f^{-1}\left[\frac{x+2}{x-1}\right]$

$$= \frac{\left(\frac{x+2}{x-1}\right)+2}{\left(\frac{x+2}{x-1}\right)-1}$$

$= x$ LHS = RHS Proved.

4) $f(x) = \sqrt{x+2}$

$x+2 \geq 0$

$\boxed{x \geq -2}$

Dom $f = [-2, \infty) =$ Range f^{-1}

Range $= [0, \infty) =$ Dom f^{-1}

let $y = f(x) \Rightarrow x = f^{-1}(y)$

$(y)^2 = (\sqrt{x+2})^2$

$y^2 = x+2$

$\boxed{y^2 - 2 = x}$

$f^{-1}(y) = y^2 - 2$

replace $y = x$

$\boxed{f^{-1}(x) = x^2 - 2}$

LHS $= f[f^{-1}(x)]$

$f[x^2 - 2]$

$f = \sqrt{(x^2-2)+2}$

$= \sqrt{x^2} = x$

RHS $= f^{-1}[f(x)]$

$= f^{-1}(\sqrt{x+2})$

$= (\sqrt{(x+2)})^2 - 2$

$= x$

LHS = RHS

5) $f(x) = x^2 + 6$ $\therefore x^2 \geq 0$

Dom $f = [0, \infty) =$ Range f^{-1}

Range $= [6, \infty) =$ Dom f^{-1}

$\because \underbrace{x^2+6}_{y} \geq 6$

let $y = f(x)$

$y = x^2 + 6$

$\sqrt{y-6} = \sqrt{x^2}$

$x = \pm\sqrt{y-6}$

$x = +\sqrt{y-6}$ $\therefore x^2 \geq 0$

$f^{-1}(y) = \sqrt{y-6}$

replace y with x

$f^{-1}(x) = \sqrt{x-6}$

$x - 6 \geq 0$

$\longrightarrow \boxed{x \geq 6}$

$L.H.S = f[f^{-1}(x)]$

$= f[\sqrt{x-6}]$

$= (\sqrt{x-6})^2 + 6$

$= x$

$RHS = f^{-1}[f(x)]$

$= f^{-1}[x^2+6]$

$= \sqrt{(x^2+6)-6}$

$= \sqrt{x^2}$

$= x$ L.HS = R.HS proved.

(vi) $f(x) = \dfrac{2x-1}{x+4}$

Sol:- $Dom f = R - \{-4\} = Rang f^{-1}$

$Range f = R - \{2\} = Dom f^{-1}$

let $y = f(x) \Rightarrow x = f^{-1}(y)$

$y = \dfrac{2x-1}{x+4}$

$y(x+4) = 2x - 1$

$xy + 4y - 2x = -1$

$x(y-2) = -1 - 4y$

$x = \dfrac{-1-4y}{y-2}$

$x = \dfrac{-(4y+1)}{-(2-y)}$

$x = \dfrac{4y+1}{2-y}$

$$\boxed{f^{-1}(y) = \frac{4y+1}{2-y}}$$

$$\boxed{f^{-1}(x) = \frac{4x+1}{2-x}}$$

$LHS = f[f^{-1}(x)]$

$= f\left[\dfrac{4x+1}{2-x}\right]$

$= \dfrac{2\left(\frac{4x+1}{2-x}\right) - 1}{\left(\frac{4x+1}{2-x}\right) + 4}$

$= \dfrac{\frac{8x+2-2+x}{(2-x)}}{\frac{4x+1+8-4x}{(2-x)}}$

$= x$

$RHS = f^{-1}[f(x)]$

$= f^{-1}\left(\dfrac{2x-1}{x+4}\right)$

$= \dfrac{4\left(\frac{2x-1}{x+4}\right)+1}{2 - \left(\frac{2x-1}{x+4}\right)}$

$= \dfrac{9x}{9} = \boxed{x}$

L.H.S = RHS Proved