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**Class 11**  
**Physics**

**Chapter 3**

# **Handwritten Notes**

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# Chapter #03

## "Translatory motion"

**Motion:-** If position of an object changes with time, we say that the object is in motion

**Types:-** There are three types of motion

Translatory motion

Rotatory motion

Vibratory motion

**Translatory motion:-** Motion of an object without rotation is called translatory motion. In such motion, each particle / joint of moving object follows similar path

There are three types of translatory motion

i). Linear motion / rectilinear motion

ii) Circular motion

iii) Random motion

**Random Linear motion:-** Translatory motion of an object in an irregular path is called random motion

Example: motion of bird in air

**Linear motion:-** Translatory motion of an object in a straight line is called linear motion.

Examples Motion of plane on runway  
Motion of freely falling object.

**Circular motion:** Translatory motion of an object on a circular path is called circular motion.

Examples Motion of earth around Sun  
Motion of stones attached to a string

**Rotatory motion:** Motion of an object about a fixed axis is called rotatory motion

Examples Motion of a fan  
Motion of wheel of any vehicle

**Vibratory motion:** "To and fro" motion or "Back & forth" motion of an object about a fixed point is called vibratory motion.

Examples Motion of a pendulum  
Motion of object attached to a spring

# Equations of

## Motion:

The equations which relate initial velocity, final velocity, time, acceleration and displacement are called "equation of motion".

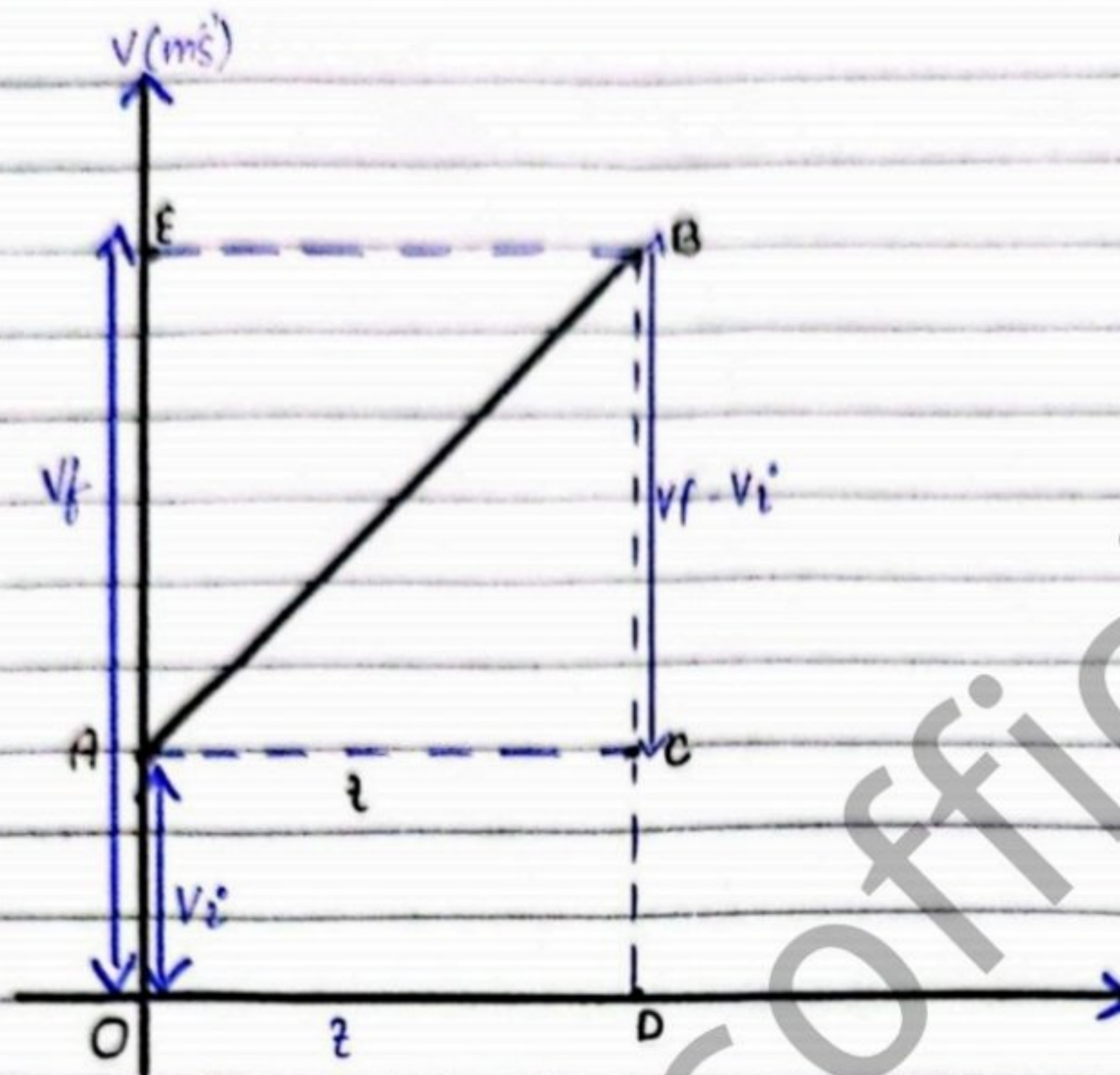
• To derive the equations of motion, we make following assumptions

- Object moves in a straight line
- Acceleration of object remains uniform/constant
- We consider only magnitude of vector quantities such as displacement, velocity and acceleration.
- Initial velocity and all vector quantities in direction of initial velocity are taken as positive and opposite as negative.

### ⇒ First Equation of motion:-

Consider a particle of mass " $m$ " moving with initial velocity " $v_i$ ". A constant acceleration acts on the particle and its velocity changes from " $v_i$ " to " $v_f$ " in time " $t$ ". Displacement travelled during this interval is " $s$ ". Speed-time graph of this particle is given as;

Speed time graph



From the graph:-

$$OA = DC = V_i$$

$$OE = DB = V_f$$

$$OD = AC = t$$

$$CB = DB - DC$$

$$CB = V_f - V_i$$

We know that, shape of speed time graph gives acceleration of particle.

According to:

Acceleration: Slope of line AB

$a$ : Slope of line AB

$$a = \frac{\Delta y}{\Delta x} = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t - 0}$$

$$a = \frac{V_f - V_i}{t - 0} = \frac{V_f - V_i}{t}$$

$$at = V_f - V_i$$

$$at + V_i = V_f$$

$$V_f = V_i + at$$

# Second equation

We know that, area under speed-time graph gives total distance travelled by the particle

Distance = Area of trapezium ODCOA

$$S = \left( \frac{OA + DB}{2} \right) \times AC$$

$$S = \left( \frac{Vi + Vf}{2} \right) \times t$$

$$S = \left( \frac{Vi + Vi + at}{2} \right) \times t$$

$$S = \frac{2Vi}{2} + \frac{at^2}{2}$$

$$S = Vi t + \frac{1}{2} at^2$$

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Friday

## Third equation of motion

We know that, area under speed-time graph gives total distance travelled by the object. Therefore, area under line  $OAB$  is total distance of moving object.

Total object distance: Area of trapezium

$$S = \frac{(\text{Sum of parallel sides})}{2} \times (\text{Separation b/w parallel sides})$$

$$S = \left( \frac{OA + DB}{2} \right) \times AC$$

Multiplying both sides by  $\frac{BC}{AC}$

$$S \times \frac{BC}{AC} = \left( \frac{OA + DB}{2} \right) \times AC \times \frac{BC}{AC}$$

$$S \times \frac{BC}{AC} = \left( \frac{OA + DB}{2} \right) \times (BC)$$

$$S \left( \frac{v_f - v_i}{t} \right) = \left( \frac{v_i + v_f}{2} \right) (v_f - v_i)$$

$$S \times a = \frac{(v_f + v_i)(v_f - v_i)}{2}$$

$$2aS = v_f^2 - v_i^2$$

This is 3<sup>rd</sup> equation of motion.

Example:-

Given Data-

$$v_i = 4 \text{ ms}^{-1}, a = 2 \text{ ms}^{-2}$$

$$t = 5 \text{ s}$$

To find

$V_f = ?$   
Solution:

$$V_f = ?$$

$$V_f = V_i + at$$

$$V_f = (10) + (2)(5)$$

$$V_f = 10 + 10$$

$$V_f = 20 \text{ ms}^{-1}$$

Ex 3.2

Given:

$$V_i = 4 \text{ ms}^{-1}$$

$$a = 1 \text{ ms}^{-2}$$

$$t = 10 \text{ s}$$

To find:

$$s = ?$$

Solution:

$$s = V_i t + \frac{1}{2} a t^2$$

$$s = (4)(10) + \frac{1}{2}(1)(10)^2$$

$$s = 40 + 50$$

$$s = 90 \text{ m}$$

OR

$$2as = V_f^2 - V_i^2$$

$$V_f = 4 + 1(10)$$

$$V_f = 4 + 10$$

$$V_f = 14 \text{ ms}^{-1}$$

$$2as = V_f^2 - V_i^2$$

$$2(1)s = (14)^2 - (4)^2$$

$$2s = 196 - 16$$

$$2s = 180$$

$$s = 180/2$$

$$s = 90 \text{ m}$$

# Motion under gravity

## Free fall motion

Motion of an object under the action of gravity is called free fall motion. It has been observed that during free fall, the acceleration of object remains uniform, with magnitude of  $9.8 \text{ ms}^{-2}$  and is directed towards towards the centre of earth. This acceleration is called gravitational acceleration, and is denoted by " $g$ ". Since, acceleration is uniform equations of motions are applicable with some symbolic changes

$$V_f = V_i + at$$

$$V_f = V_i + gt$$

$$s = V_i t + \frac{1}{2} at^2$$

$$V_f = V_i t + \frac{1}{2} gt^2$$

$$2as = V_f^2 - V_i^2$$

$$2gH = V_f^2 - V_i^2$$

### Example 3.3

Given Data:-

$$H = 2.50 \text{ m}, V_f = 0$$

$$g = -9.8 \text{ ms}^{-2}$$

To find:-

$$V_i = ?$$

$$t = ?$$

Sol

We know that

$$2gH = V_f^2 - V_i^2$$

$$2(-9.8)(2.5) = (0)^2 - (V_i)^2$$

$$\sqrt{49} = \sqrt{V_i^2}$$

$$v_i = 7 \text{ m/s}^{-1}$$

## Assignment 3.2

Given Data:-

$$m = 5 \text{ kg}, v_i = 0$$

$$t = 3.34 \text{ s}, g = 9.8 \text{ m/s}^2$$

To find:

$$H = ?$$

$$v_f = ?$$

Solution:-

We know that

$$H = v_i t + \frac{1}{2} g t^2$$

$$H = (0)(3.34) + \frac{1}{2} (9.8)(3.34)^2$$

$$H = 0 + \frac{1}{2} (9.8)(3.34)^2$$

$$H = 54.6 \text{ m}$$

Now;

$$v_f = v_i + g t$$

$$v_f = 0 + (9.8)(3.34)$$

$$v_f = (9.8)(3.34)$$

$$v_f = 32.73 \text{ m/s}^{-1}$$

# Assignments: 3.1

Q#01).

Given Data:-

$$V_i = 80 \text{ kmh}^{-1} = 22.2 \text{ ms}^{-1}$$

$$V_f = 20 \text{ kmh}^{-1} = 5.6 \text{ ms}^{-1}$$

To find:-

$$t = ?$$

Solution:-

We know that

$$V_f = V_i + at$$

$$at = V_f - V_i$$

$$t = \frac{V_f - V_i}{a}$$

$$t = \frac{5.6 - 22.2}{-2}$$

$$t = 8.3 \text{ s} \quad \text{--- Answer}$$

Q#02).

Given Data

$$V_i = 5 \text{ ms}^{-1}, V_f = 15 \text{ ms}^{-1}$$

$$S = 50 \text{ m}$$

To find:-

$$a = ?$$

$$t = ?$$

Solution:-

We know that

$$2as = v_f^2 - v_i^2$$

$$a = \frac{v_f^2 - v_i^2}{2s}$$

$$a = \frac{(15)^2 - (5)^2}{2(50)}$$

$$a = \frac{225 - 25}{100}$$

$$a = \frac{200}{100}$$

$$a = 2 \text{ ms}^{-2}$$

Now

$$v_f = v_i + at$$

$$at = v_f - v_i$$

$$t = \frac{v_f - v_i}{a}$$

$$t = \frac{15 - 5}{2}$$

$$t = \frac{10}{2}$$

$$t = 5 \text{ s} \text{ - Ans}$$

# Projectile motion

Two dimensional motion of an object thrown into air under action of gravity only is called projection motion. Object in motion is called projectile and path followed by projectile is called Trajectory of projectile.

## Example:-

→ Motion of a cricket ball thrown into the air.

→ Motion of a missile

## Assumptions:-

While studying the projectile motion in our syllabus, we make following assumptions

→ Only gravitational force is acting on the projectile

→ No functional/air drag force is acting on the object/projectile

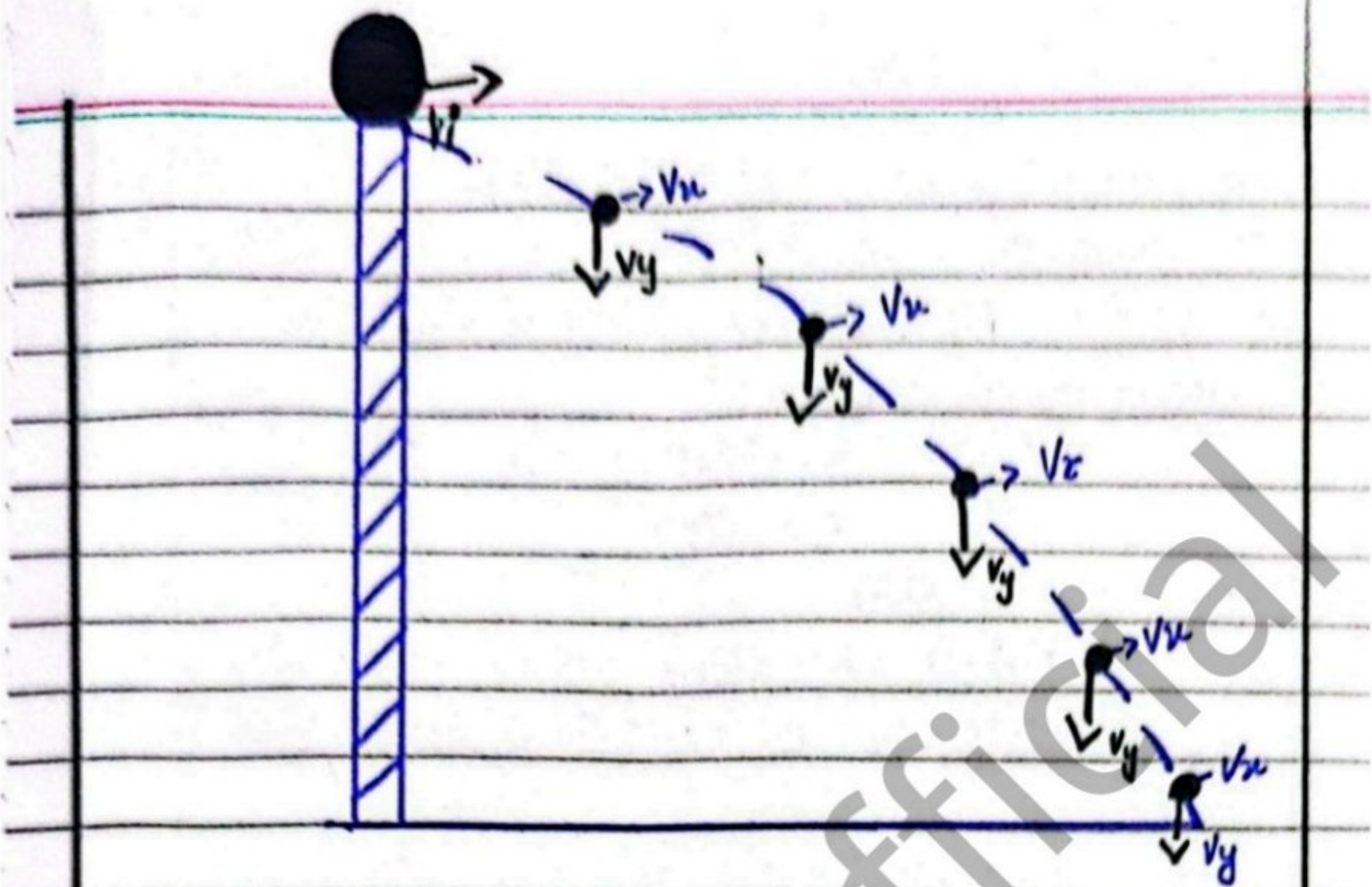
→ No force due to rotation of earth is acting on projectile

## Types:-

There are two types:-

- 1). Projectile from the height in horizontal
- 2). Projectile from the surface of earth at same angle with horizontal.

## Projectile from height in horizontal:-



Consider a projectile of mass " $m$ " thrown at from the top of a height " $h$ " with initial velocity " $V_i$ " along horizontal direction only. The projectile follow a parabolic "2-D" trajectory during its motion. We analyze the motion along  $x$ -axis and  $y$ -axis independently.

#### A. Velocity of projectile:

Along  $x$ -axis

There is no along  $x$ -axis

Therefore,

$$F_x = 0$$

$$a_x = 0$$

$$V_{ix} = V_i$$

$$V_{fx} = V_i$$

Along  $y$ -axis

Gravitational force acts on the projectile along  $y$ -axis

Therefore,

$$F_y = mg$$

$$a_y = g$$

$$V_{iy} = 0$$

$$V_{fy} = gt$$

## b). Displacement of projectile:-

i). Along x-axis:-

Projectile moves along x-axis with uniform velocity, therefore:-

$$S_x = V_x t$$

$$x = V_x t$$

ii). Along y-axis:-

Projectile moves along y-axis with uniformly varying velocity due to gravitational acceleration. Therefore,

$$S_y = V_{iy} t + \frac{1}{2} a_y t^2$$

$$y = (0)t + \frac{1}{2} g t^2$$

$$y = \frac{1}{2} g t^2$$

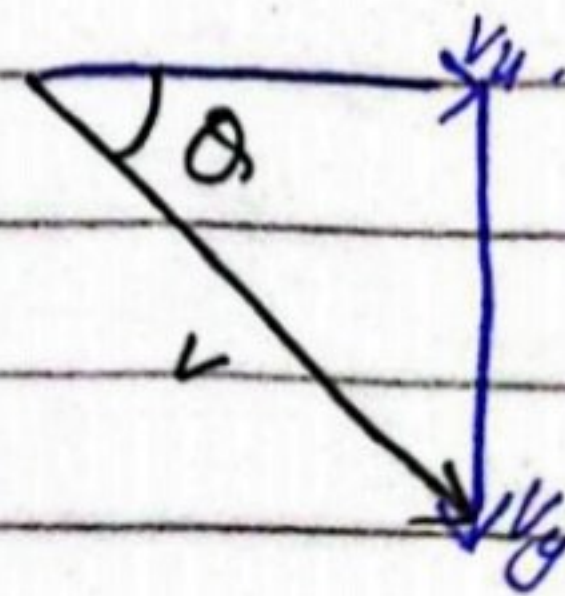
c). Angle made by velocity with horizontal at any instant of time (t):-

From the graph:-

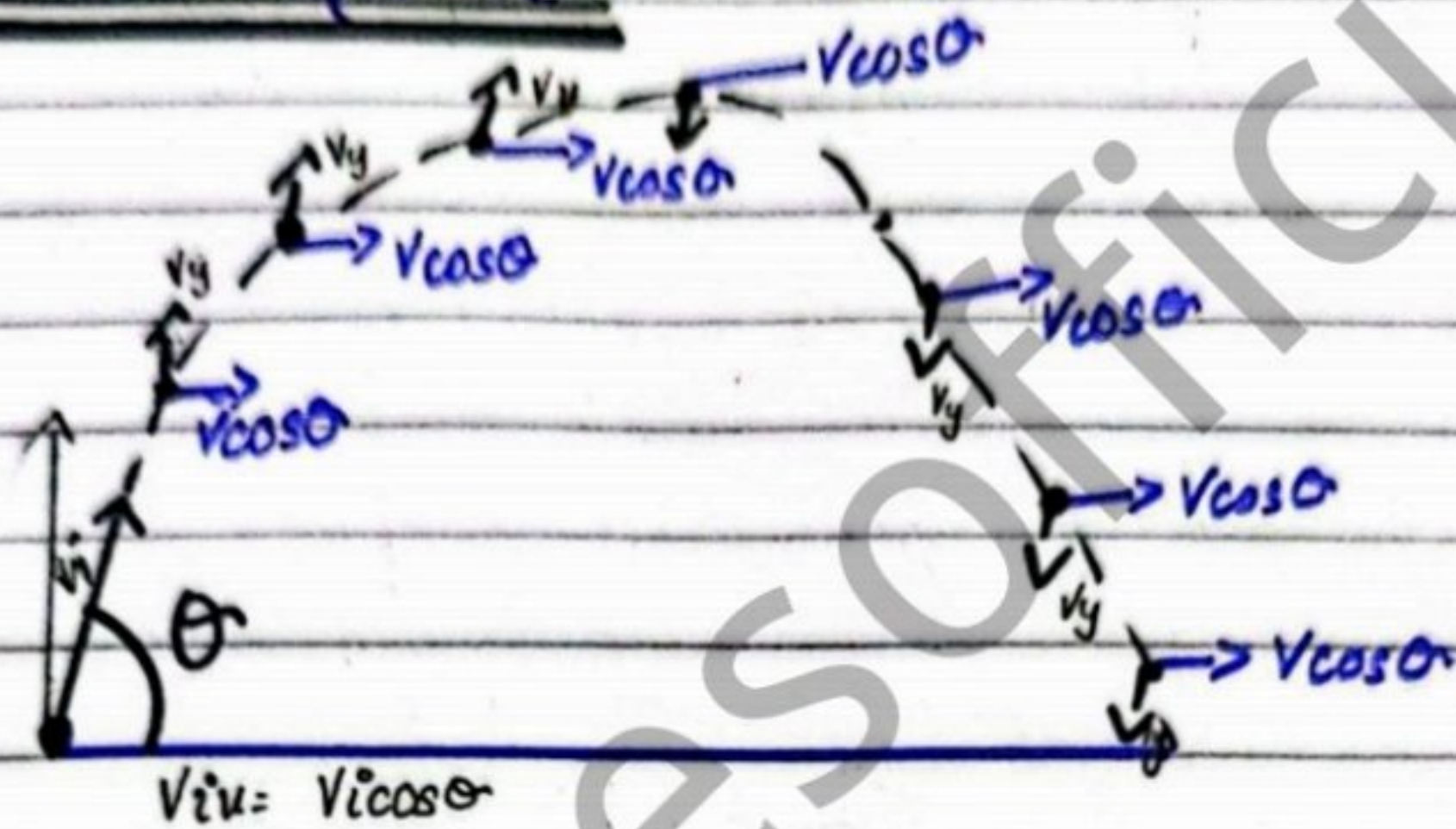
$$\tan \theta = \frac{V_y}{V_x}$$

$$\tan \theta = \frac{gt}{V_x}$$

$$\theta = \tan^{-1} \left( \frac{gt}{V_x} \right)$$



# Projectile Thrown at same angle with horizontal:



Explanation:- Consider a projectile of mass " $m$ " thrown with initial velocity " $V_i$ " at some angle " $\theta$ " with the horizontal. The projectile follows a "2-D" parabolic trajectory during its motion to analyze the motion of projectile, we consider the horizontal and vertical motion of projectile independently.

## Q4. Analysis of Velocity of projectile:-

→ Along x-axis:-

There is no net force acting on projectile along horizontal direction, therefore, there is no acceleration along horizontal direction.

$$F_x = 0$$

$$a_x = 0$$

$$V_{ix} = V_i \cos \theta$$

$$V_x = V_{ix} = V_{fx} = V_i \cos \theta$$

Along y axis:-

$$F_y = mg$$

$$a_y = -g$$

$$V_{iy} = V_i \sin \theta$$

$$V_{fy} = ?$$

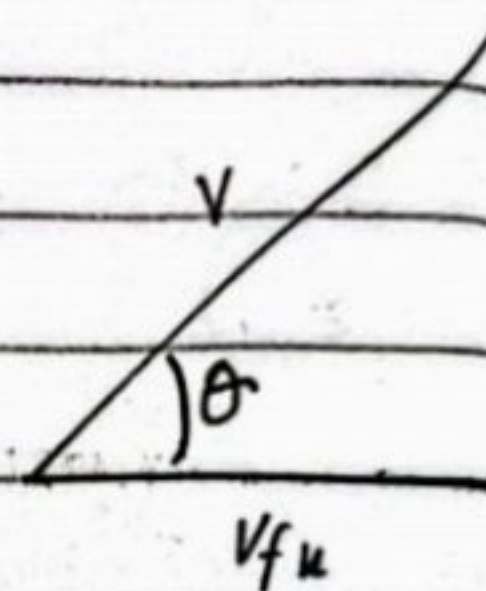
$$V_{fy} = V_{iy} + a_y t$$

$$V_{fy} = V_i \sin \theta - gt$$

Angle made by velocity of projectile with horizontal:-

$$\tan \theta = \frac{V_{fy}}{V_{fx}}$$

$$\tan \theta = \frac{V_i \sin \theta - gt}{V_i \cos \theta}$$



Analysis of displacement of projectile:-

Along x axis:-

Projectile moves with uniform velocity along horizontal direction

Therefore

$$V_x = V_i \cos \theta$$

$$a_x = 0$$

$$S_x = V_x \times t$$

$$S_x = V_i \cos \theta \times t$$

Along y-axis:-

Along vertical direction, reciprocal moves under gravitational acceleration.

$$V_y = V_i \sin \theta$$

$$a_y = -g$$

$$S_y = y$$

Use 2<sup>nd</sup> equation of motion

$$S_y = V_y t + \frac{1}{2} a_y t^2$$

$$y = V_i \sin \theta t + \frac{1}{2} g t^2$$

**Time of flight:** Time of flight is spent by projectile in air during its motion is called time of flight. It is represented by  $T$ . It's SI unit is second.

**Derivation:** To derive the formula for time of flight, we consider vertical motion of projectile

$$v_y = v_i \sin \theta$$

$$a_y = -g$$

$$s_y = 0$$

$$t = T$$

Use 2<sup>nd</sup> equation of motion:

$$s_y = v_y t + \frac{1}{2} a_y t^2$$

$$0 = (v_i \sin \theta) T + \frac{1}{2} (-g) T^2$$

$$0 = v_i \sin \theta T - \frac{1}{2} g T^2$$

$$\frac{g T^2}{2} = v_i \sin \theta T^2$$

$$\frac{g T}{2} = v_i \sin \theta$$

$$T = \frac{2 v_i \sin \theta}{g}$$

This is time of flight of projectile.

**Height of projectile ( $H$ ):** The maximum vertical distance of projectile from horizontal is called height of projectile

It is represented by  $H$ . It's SI unit is meter.

**Derivations.** To derive formula for height of projectile, we consider vertical motion of projectile.

$$V_{iy} = V_i \sin \theta$$

$$a_y = -g$$

$$V_{fy} = 0$$

$$S_y = H$$

Use 3<sup>rd</sup> equation

$$2as = V_f^2 - V_i^2$$

$$2(-g)H = (0)^2 - (V_i \sin \theta)^2$$

$$-2gH = -V_i^2 \sin^2 \theta$$

$$2gH = V_i^2 \sin^2 \theta$$

$$H = \frac{V_i^2 \sin^2 \theta}{2g}$$

$$H = \frac{V_i^2 \sin^2 \theta}{2g}$$

This is height of projectile.

## Range of Projectile

The horizontal displacement of projectile from point of projection to landing point is called range of projectile. It is represented by 'R'. Its unit is meter (m).

**Derivation:** To calculate range of projectile we consider horizontal motion of projectile.

$$V_x = V_i \cos \theta$$

$$t = T$$

$$S_x = R$$

We know that

$$S_x = V_x \times t$$

$$R = V_i \cos \theta \times T$$

$$R = V_i \cos \theta \times \frac{2 V_i \sin \theta}{g}$$

$$R = \frac{2 V_i^2 \sin \theta \cos \theta}{g}$$

$$R = \frac{V_i^2 \sin 2\theta}{g}$$

For maximum range:

Range of projectile is given by relation

$$R = \frac{V_i^2 \sin 2\theta}{g} \rightarrow \text{eq (1)}$$

It will be maximum range only if;

$$\sin 2\theta = 1$$

$$2\theta = \sin^{-1}(+1)$$

$$2\theta = 90$$

$$\theta = 90/2$$

$$\theta = 45^\circ$$

This is condition for maximum range of projectile corresponding maximum range is;

$$R_{\max} = \frac{V_i^2 (1)}{g}$$

$$R_{\max} = \frac{V_i^2}{g}$$

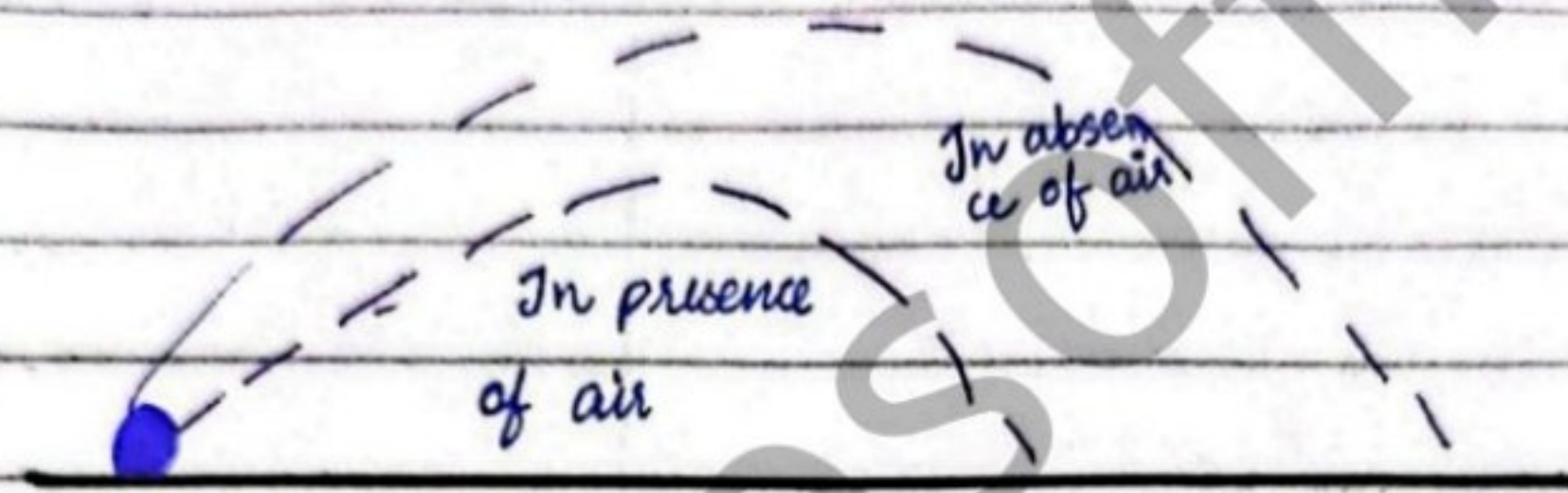
Putting in eq (1) we get,

$$R = \frac{V_i^2 \sin 2\theta}{g}$$

$$R = V_i^2 / g \cdot \sin 2\theta$$

# Effect of air resistance on Projectile

Air resistance on projectile is just like friction. Air resistance decreases the horizontal and vertical component of the projectile consequently the time of flight ( $T$ ), maximum height ( $H$ ) and range of projectile decrease in presence of air friction. This is because, some kinetic energy of projectile is converted into heat, sound and other forms of energy.



**Momentum:** The quantity of an object is called momentum. The product of mass and velocity of object is called momentum.

Momentum is a vector quantity. It is represented by  $\vec{P}$ . It is a vector quantity. Its direction is same as velocity.

**Formula:** By definition of momentum, we have

$$\vec{P} = m \times \vec{v}$$

$$\vec{P} = m\vec{v}$$

**Unit:**

$$P = mv$$

$$= \text{kg m s}^{-2}$$

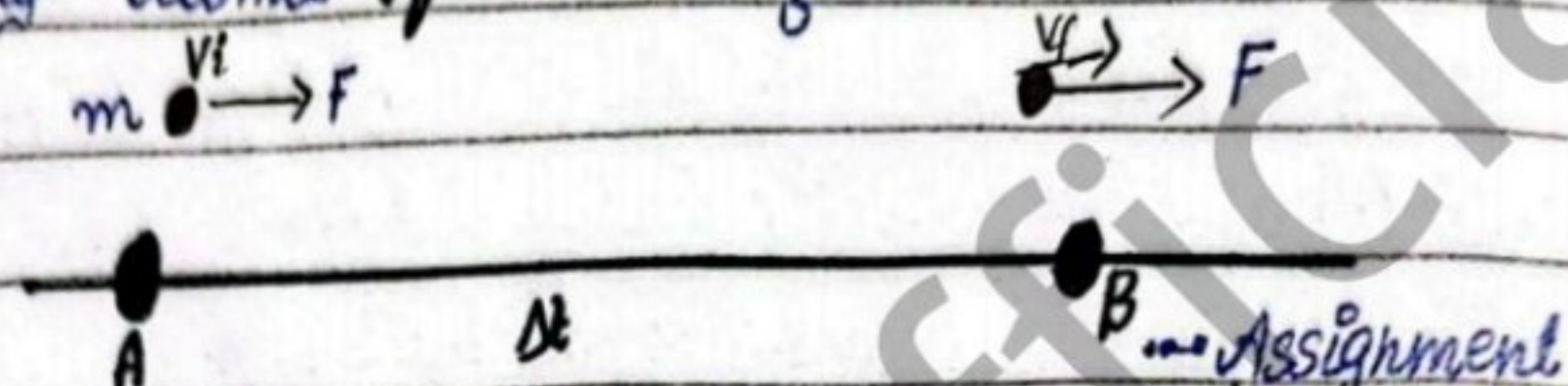
$$= \frac{\text{kg m}}{\text{s}} = \frac{\text{kg m}}{\text{s}} \times \frac{\text{s}}{\text{s}} = \left( \frac{\text{kg m}}{\text{s}^2} \right)$$

$$= (\text{kg m s}^{-2}) \text{s} = \text{Ns}$$

# Relation b/w force and

## momentum

Consider a projectile of mass " $m$ " moving with initial velocity " $V_i$ ". A constant force " $F$ " acts on the projectile for time " $\Delta t$ " and its velocity becomes " $V_f$ ". According to Newton 2<sup>nd</sup> law



$$F = ma$$

but

$$a = \frac{V_f - V_i}{\Delta t}$$

Put eq. (1) in (2)

$$F = ma$$

$$F = m \left( \frac{V_f - V_i}{\Delta t} \right)$$

$$F = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t}$$

$$F = \frac{\Delta \vec{p}}{\Delta t}$$

We know that

$$T = \frac{2V_i \sin \theta}{g}$$

$$T = 3.06 \text{ s}$$

$$H = \frac{V_i^2 \sin^2 \theta}{2g}$$

$$H = 11.476 \text{ m}$$

$$R = \frac{V_i^2 \sin 2\theta}{g}$$

$$R = 79.533 \text{ m}$$

Rate of change of momentum of an object is equal to the net force acting on the object. This statement is called Newton 2<sup>nd</sup> law in terms of momentum.

## Assignment:

DATA:

$$V_i = 30 \text{ ms}^{-1}$$

$$\theta = 30^\circ$$

To find:

$$H = ?$$

$$T = ?$$

## Example 3.2 Given Data:

$$V_{ix} = 12 \text{ ms}^{-1}$$

$$H = 24.6 \text{ m}$$

To find:

$x = ?$   
Calculations:

We know that

$$x = V_{ix} \times T \rightarrow (1)$$

$$y = \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2y}{g}}$$

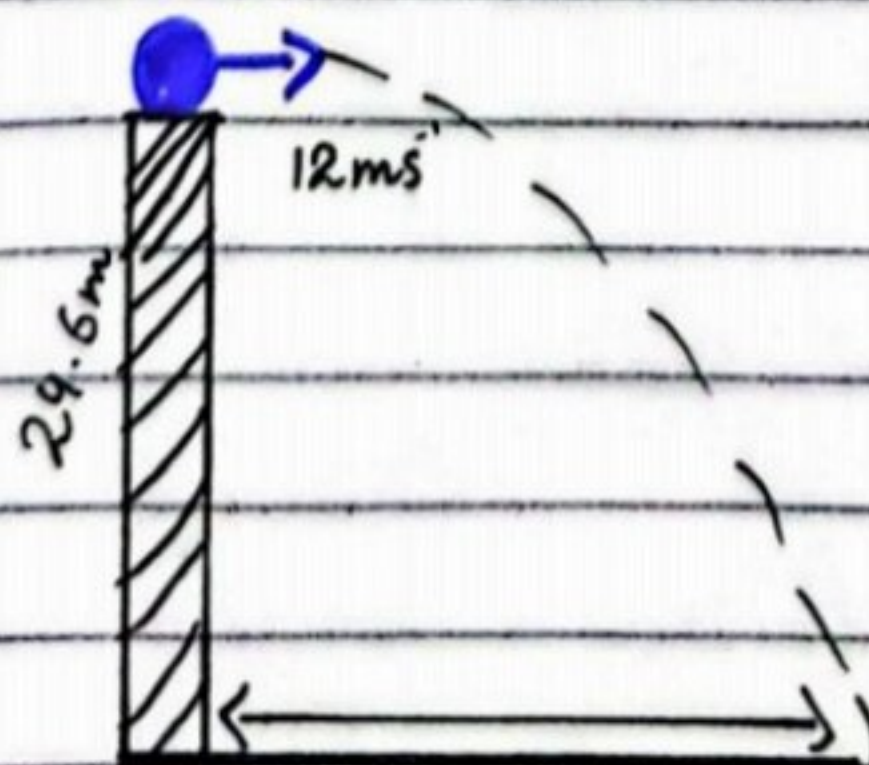
$$t = \sqrt{\frac{2 \times 24.6}{9.8}} = 2.24$$

$$t = 2.24 \text{ s}$$

Put in equation (1)

$$x = 12 \times 2.24$$

$$x = 26.6 \text{ m}$$



## Example 3.3

## Given Data:-

$$V_i = 30 \text{ ms}^{-1}$$

$$\theta = 30$$

To find:-

$$T = ?$$

$$H = ?$$

$$R = ?$$

Calculation:-

We know that

$$\textcircled{1} \quad T = \frac{2V_i \sin \theta}{g}$$

$$T = \frac{2 \times 30 \times \sin 30}{9.8}$$

$$T = 3.06 \text{ s}$$

$$\textcircled{2} \quad H = \frac{V_i^2 \sin^2 \theta}{2g}$$

$$H = \frac{(30)^2 (\sin 30)^2}{2(9.8)}$$

$$H = 11.476 \text{ m}$$

$$\textcircled{3} \quad R = \frac{V_i^2 \sin 2\theta}{g}$$

$$R = \frac{(30)^2 \times \sin 60}{9.8}$$

$$R = 79.533 \text{ m}$$

## Problem 3.2:-

Given data

$$R = \frac{H}{2}$$

To find:-

$$V_u = ?$$

Solution:-

According to given condition

$$R = H$$

$$\frac{2V_i \sin \theta \cos \theta}{g} = \frac{V_i^2 \sin^2 \theta}{2g}$$

$$2 \cos \theta = \frac{\sin \theta}{2}$$

$$4 = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = 4$$

$$\theta = \tan^{-1}(4)$$

$$\theta = 76^\circ$$

# Complementary angles

The angle whose sum is exactly  $90^\circ$  are called complementary angles.

For example:-

$$(30^\circ, 60^\circ)$$

$$(70^\circ, 20^\circ)$$

$$(80^\circ, 10^\circ)$$

Are called complementary angles. Range of projectile at complementary angle remains same range at:

Range at  $30^\circ$ :

$$R = \frac{V_i^2 \sin 2\theta}{g}$$

$$R_{30} = \frac{V_i^2 \sin^2(30)}{g}$$

$$R = \frac{V_i^2 \sin(60)}{g}$$

$$R = \frac{V_i^2 (0.866)}{g} \rightarrow \text{equation (1)}$$

Range at  $60^\circ$ :

$$R_{60} = \frac{V_i^2 \sin^2 \theta}{g}$$

$$R_{60} = \frac{V_i^2 \sin^2(60)}{g}$$

$$R_{60} = \frac{V_i^2 \sin(120)}{g}$$

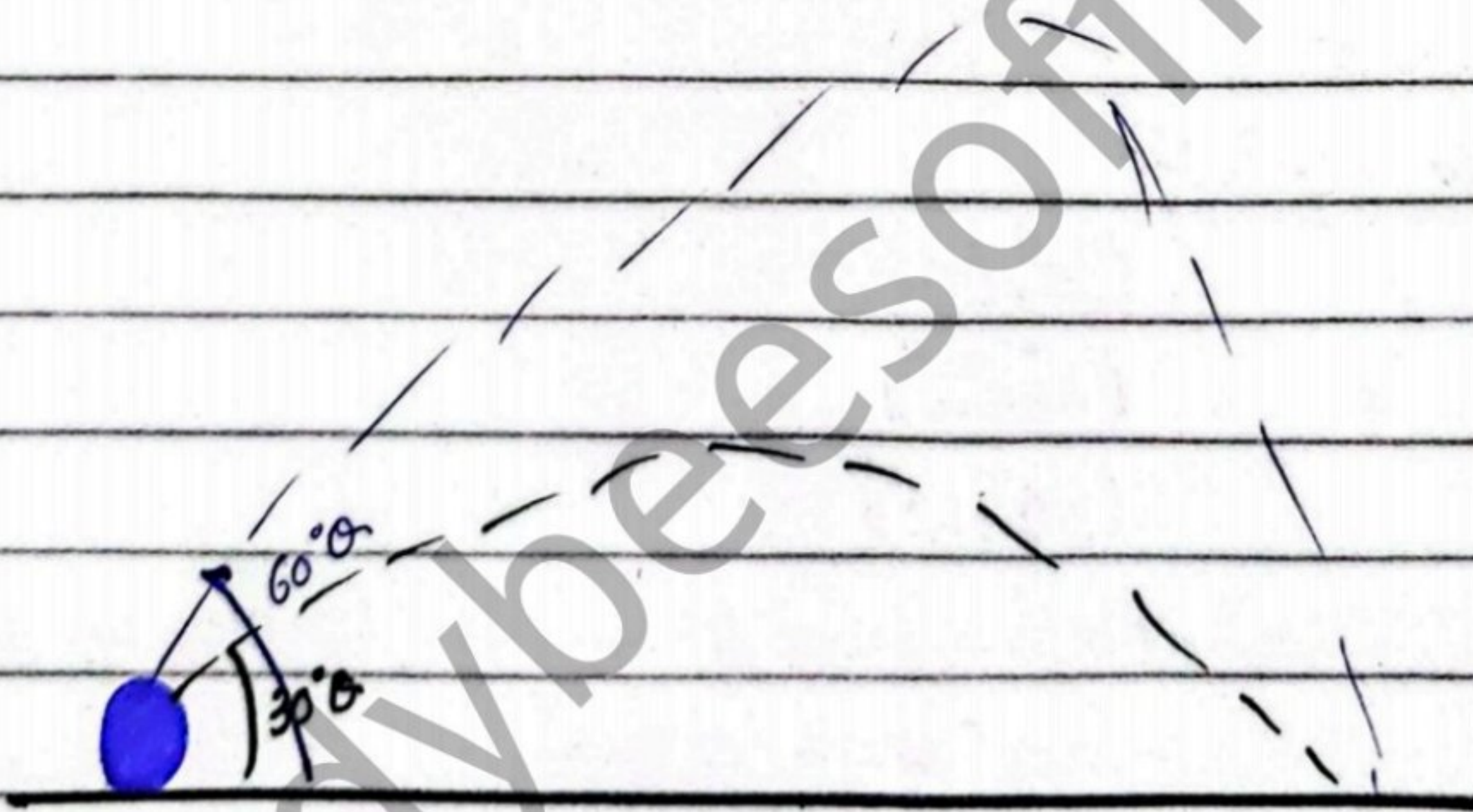
$$R_{60} = \frac{V_i^2 (0.866)}{g}$$

$$R_{60} = 0.866 \frac{V_i^2}{g} \rightarrow \text{equation (2)}$$

Comparing eq ① and ②

$$R_{30} = R_{60}$$

Thus, range of projectile of complementary angles remains same



**System of Particles:** If two or more than two particles are moving dependently or independently, they are collectively known as system of particles.

**Isolated System:** Any system of particles that is not affected by its surroundings is called isolated system. If no external force is acting on system of particles is zero then system is said to be isolated.

## Types of conservation of momentum

According to relationship between force and momentum,

$$\vec{F} = \frac{\Delta \vec{P}}{\Delta t}$$

if  $\vec{F} = 0$

Then,

$$0 = \frac{\Delta \vec{P}}{\Delta t}$$

$$\frac{\Delta \vec{P}}{\Delta t} = 0$$

$$\Delta P = 0$$

$$\vec{P}_f - \vec{P}_i = 0$$

$$P_f = P_i$$

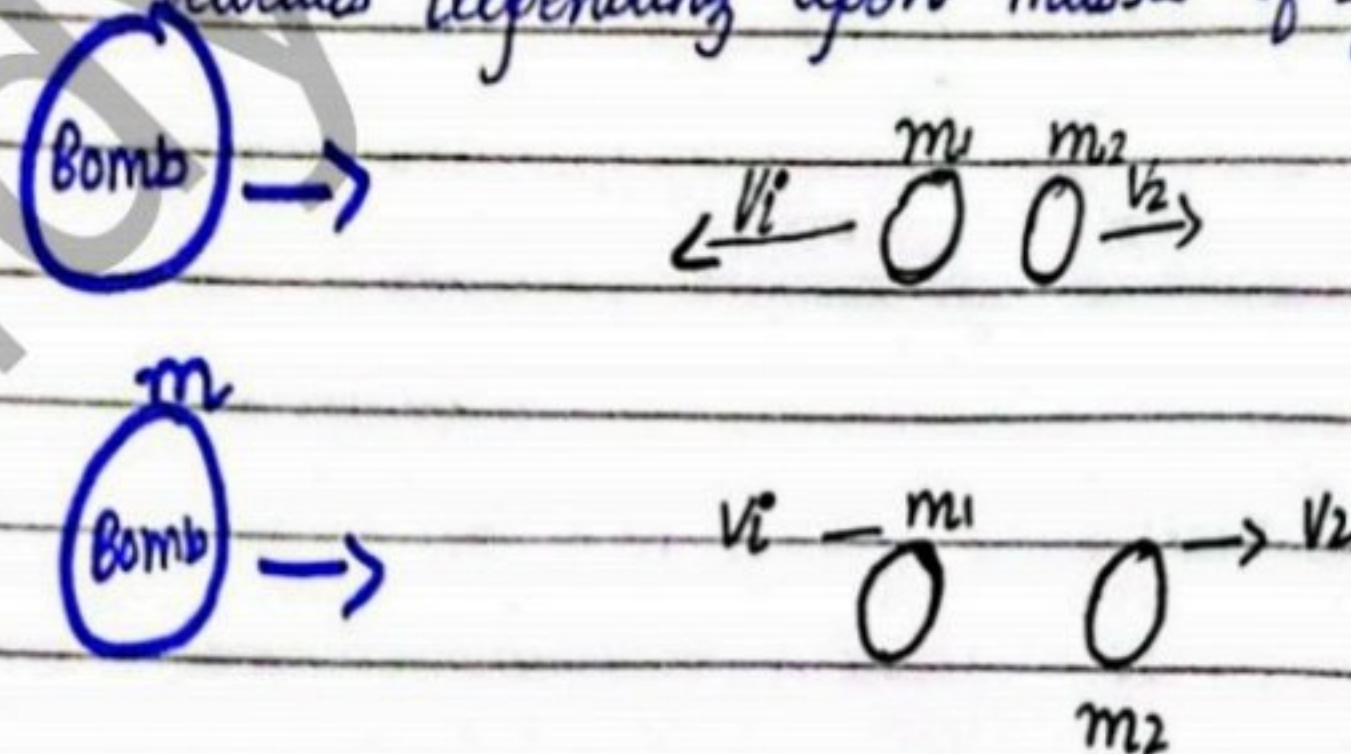
If no external force is acting on a particle or system of particles is zero (i.e. system is isolated), the momentum remains conserved.

Then total momentum of particles or system of particles remains zero:-

## (Assignment) Application:-

### A) Conservation of momentum during explosion:-

During an explosion, chemical energy of system is converted into kinetic energy of the fragments of system. Since, system is isolated (as no external force is involved), total momentum of system after and before explosion must remain constant/same. System is initially at rest ( $P_i = 0$ ) after explosion fragments move in opposite direction with same or different velocities depending upon masses of fragments



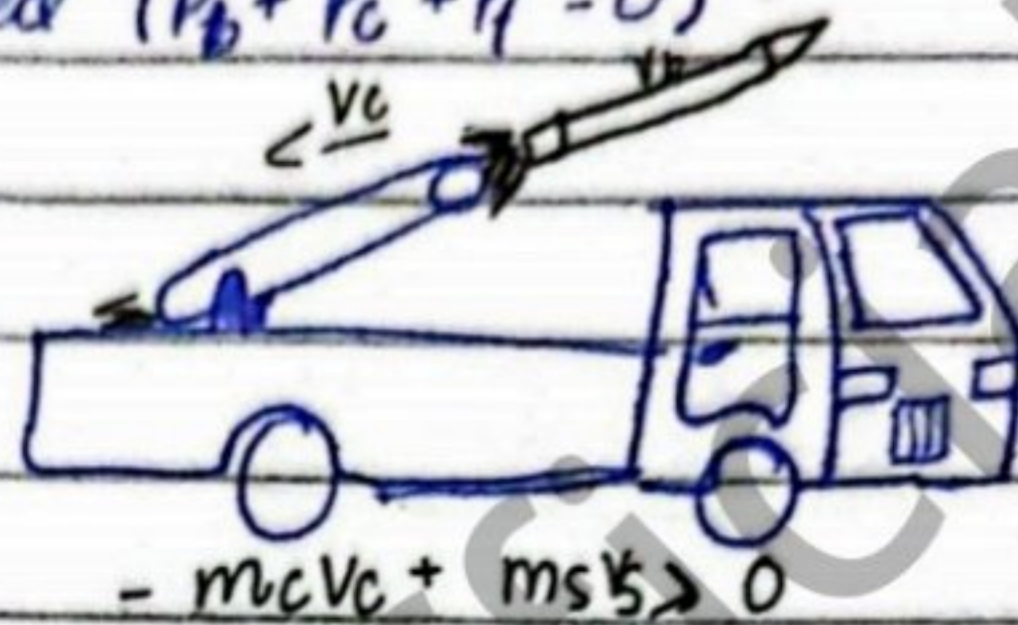
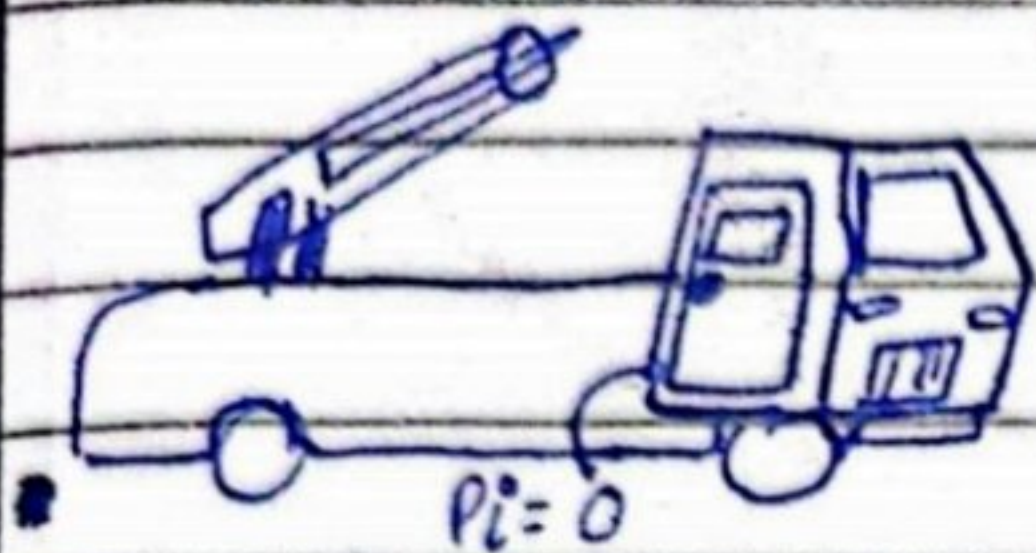
$$m_1 > m_2$$

$$v_1 < v_2$$

### B). Conservation of momentum during firing in canon:-

Common bullet is an isolated system and momentum of this system remains conserved. Before firing both are at rest ( $P_i = 0$ ). After firing both

are at rest ( $P_i = 0$ ). After firing, bullet moves forward ( $P_b$ ) and cannon moves backward i.e. recoils ( $P_c > 0$ ) such that momentum remains conserved ( $P_b + P_c + P = 0$ )



c). Momentum conservation during collision of cars/vehicles

During any collision of system of particle, then momentum of system remains constant. Sum of momentum of particles of system before collision is equal to the sum of momentum of system of particles after collision.

d) Momentum conservation during collision of cricket ball and cricket bat

During the collision between a cricket bat and ball, momentum of system remains conserved/constant/same. Before collision between bat and ball sum of momentum of ball and bat. after collision is same as momentum before collision.

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**Collisions**:- If velocity and momentum of one particle affected by higher / another particle, we can say that there is a collision between particles.

**Types**:- There are two types of collision:

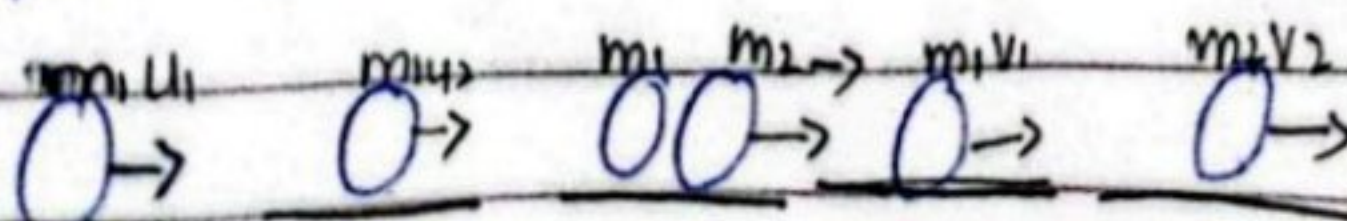
i. Elastic collision

ii. Inelastic collision

**Inelastic collision**:- A type of collision in which kinetic energy of colliding particle does not remain conserved constant / same. A part of kinetic energy of particle is converted into sound, heat, deformation and other types of energy.

**Elastic collision**:- A type of collision in which kinetic energy of colliding particle remains conserved before and after the collision. No part of kinetic energy of colliding particle is converted into heat, sound and other forms of energy. K.E of colliding particles before collision is equal to after collision.

**Explanation**:- Consider a system of two particles of masses  $m_1$  and  $m_2$  moving with initial velocity  $u_1$  and  $u_2$  such that  $u_1 > u_2$ . After collision their velocities become  $v_1$  and  $v_2$ .



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Mathematically:- Since, system of particles under consideration is isolated, therefore momentum of system is conserved;

$$(P_i)_{\text{system}} = (P_f)_{\text{system}}$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 (u_1 + v_2) = m_2 (v_2 - u_2) \quad \text{--- (A)}$$

Since, collision is elastic, K.E of colliding particle is conserved.

$$(K.E)_{\text{system}} = (K.E_f)_{\text{system}}$$

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\frac{1}{2} (m_1 u_1^2 + m_2 u_2^2) = \frac{1}{2} (m_1 v_1^2 + m_2 v_2^2)$$

$$m_1 u_1^2 + m_2 u_2^2 = m_1 v_1^2 + m_2 v_2^2$$

$$m_1 u_1^2 + m_1 v_1^2 = m_2 v_2^2 - m_2 u_2^2$$

$$m_1 (u_1^2 - v_1^2) = m_2 (v_2^2 - u_2^2)$$

$$m_1 (u_1 + v_1) (u_1 - v_1) = m_2 (v_2 + u_2) (v_2 - u_2) \quad \text{--- (b)}$$

$\div$  (b) by (A) we get,

$$\frac{m_1 (u_1 + v_1) (u_1 - v_1)}{m_1 (u_1 - v_1)} = \frac{m_2 (v_2 + u_2) (v_2 - u_2)}{m_2 (v_2 - u_2)}$$

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - v_2 = v_2 - v_1$$

$$u_1 - u_2 = -(v_1 - v_2)$$

∴ This equation says that, velocity of approach of particles is equal to the velocity of separation of particles after collision.

### 2) Determination of velocity after collision:

By using equation (c) we have

$$u_1 + v_1 = v_2 + u_2$$

$$v_1 = u_2 + v_2 - u_1$$

Put in equation (3) we get,

$$m_1 u_1 + m_2 u_2 = m_1 (u_2 + v_2 - u_1) + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 u_2 + m_1 v_1 + m_1 v_2 - m_1 u_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 + m_1 u_1 - m_1 u_2 = v_2 (m_1 + m_2)$$

$$2m_1 u_1 + (m_2 - m_1) u_2 = v_2 (m_1 + m_2)$$

$$v_2 (m_1 + m_2) = 2m_1 u_1 + (m_2 - m_1) u_2$$

$$v_2 = \frac{2m_1 u_1}{m_1 + m_2} - \frac{(m_1 - m_2) u_2}{m_1 + m_2}$$

### 3) Determination of velocity of m<sub>2</sub> after collision

We know that

$$u_1 - u_2 = -(v_1 - v_2)$$

$$u_1 - u_2 = -v_1 + v_2$$

$$u_1 - u_2 + v_1 = v_2$$

Put in equation (1)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 (u_1 - u_2 + v_1)$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 u_1 = (m_1 + m_2) v_1$$

$$m_1 u_1 + m_2 u_1 + 2m_2 u_2 = (m_1 + m_2) v_1$$

$$(m_1 - m_2) u_1 + 2m_2 u_2 = (m_1 + m_2) v_1$$

$$v_1 = \frac{2m_2 u_2}{m_1 + m_2} + \frac{(m_1 - m_2) u_1}{m_1 + m_2} \rightarrow \text{A}$$

### Example / Numerical 3.6

Given data:-

$$m_1 = m \quad m_2 = m$$

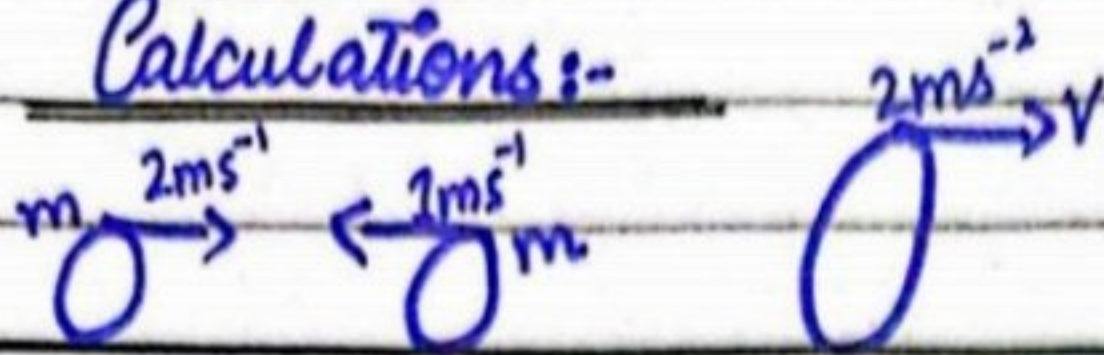
$$u_1 = 2 \text{ ms}^{-1}, \quad u_2 = 2 \text{ ms}^{-1}$$

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To find:-

$$V = ?$$

Calculations:-



$$(P_i) \text{ system} = (P_f) \text{ system}$$

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) V$$

$$V = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

$$V = \frac{2m - m}{2m}$$

$$V = \frac{m}{2m}$$

$$V = \frac{1}{2}$$

$$V = 0.5\text{ms}^{-1} \text{ - Answer....}$$

Numerical 3.3)-

Given data:-

$$V_i = 100\text{ms}^{-1}, V_f = 0, a = -6\text{ms}^{-2}$$

To find:-

$$t = ?, S = 0$$

Calculations:-

$$5\text{ms} \rightarrow 100\text{ms}^{-1} \quad V_f = 0$$

$$t = \frac{V_f^2 - V_i^2}{a}$$

$$t = \frac{0 - 100}{-5} = \frac{100}{5}$$

$$t = 20\text{s}$$

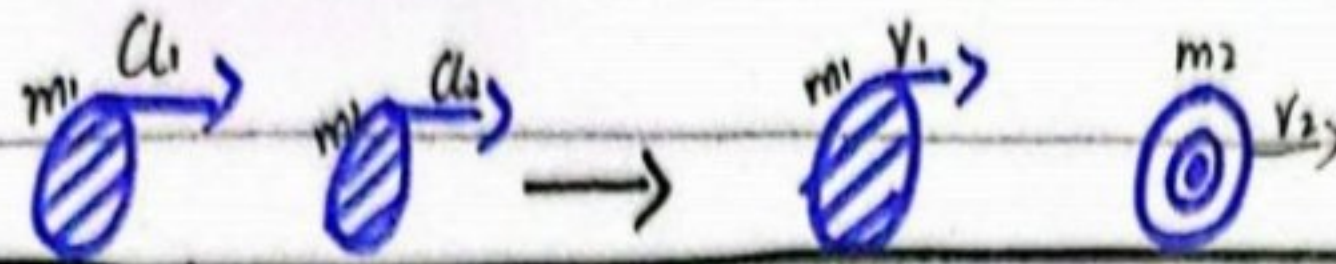
$$S = V_i t + \frac{1}{2} a t^2$$

$$S = (100)(20) - \frac{5}{2} \times 200$$

$$S = 2000 - 1000\text{m}$$

$$S = 1000\text{m}$$

$$S = 1\text{km}$$



$$V_1 = \frac{2m_2 u_2}{m_1 + m_2} + \left( \frac{m_1 - m_2}{m_1 + m_2} \right) u_1$$

$$V_2 = \frac{2m_1 u_1}{m_1 + m_2} + \left( \frac{m_1 - m_2}{m_1 + m_2} \right) u_2$$

Case-01

If  $m_1 = m_2 = m$

$$V_1 = \frac{2m u_2}{m + m} + \frac{(m - m)}{m + m} u_1$$

$$V_1 = \frac{2m u_2}{2m}$$

$$V_1 = u_2$$

$$V_2 = \frac{2m_1 u_1}{m + m} - \left( \frac{m - m}{m + m} \right) u_2$$

$$V_2 = \frac{2m_1 u_1}{2m}$$

$$V_2 = u_1$$

Case 02:-

If  $m_1 = m_2 = m$ ,  $u_2 = 0$

$$V_1 = \frac{2m(0)}{m + m} + \frac{m - m}{m + m} u_1$$

$$V_1 = 0$$

$$V_2 = \frac{2m_1 u_1}{m + m} - \left( \frac{m - m}{m + m} \right) (0)$$

$$V_2 = \frac{2m u_1}{2m}$$

$$V_2 = u_1$$

Case #01:-

$$m_1 = m_2 = m$$

$$V_1 = U_2$$

$$V_2 = U_1$$

Case 02:-

$$m_1 = m_2 = m$$

$$U_2 = 0$$

$$V_1 = 0$$

$$V_2 = U_1$$

Case 03:-

$$m_1 \gg m_2$$

$$U_2 = 0$$

$$V_2 = 0$$

$$U_2 = 0$$

$$V_2 = ?$$

$$V_1 = -U_1$$

$$V_1 = \frac{2m_2(0)}{0+m_2} + \frac{(0-m_2)}{0+m_2} U_1$$

$$V_1 = 0 - \frac{m_2}{m_2} U_1$$

$$V_1 = -U_1$$

$$\text{Now } V_2 = \frac{2(0)(U_1)}{0+m_2} - \left( \frac{0-m_2}{0+m_2} \right) (0)$$

$$V_2 = 0$$

## Case 4:-

$$m_1 \gg m_2$$

$$u_2 = 0$$

$$m_1 + m_2 = m_1$$

$$m_1 - m_2 = m_1$$

$$V_1 = \frac{2m_2(0)}{m_1} + \frac{m_1 u_1}{m_1}$$

$$V_1 = u_1$$

$$V_2 = \frac{2m_1 u_1}{m_1} - \left( \frac{m_1}{m_1} \right) (0)$$

$$V_2 = 2u_1$$