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Where Minds Pollinate

Class 11
Physics

Chapter 2

Handwritten Notes

Chapter # 02
"Vectors"

On the basis of algebra, used to add physical quantities, physical quantities have been grouped into two groups called scalars and vectors

Scalars:- These physical quantities, that have magnitude, unit and obey ordinary rules of algebra are called scalars.

Vectors:- These physical quantities that have magnitude, unit, direction and obey vector basis of algebra are called vectors

Example:- Displacement, velocity and momentum

Representation of Vectors:- Vectors are graphically represented by a straight line, with an arrow head at one side

Direction of arrow points the direction of vector & length of line guides. The magnitude of vector according to scale.

Vectors are given names with letters having an arrow or bar above or below

Then;

Mostly, direction of vectors is given by the angle that vectors, make with +ive x-axis.

Types of Vectors:- There are many types of vectors.

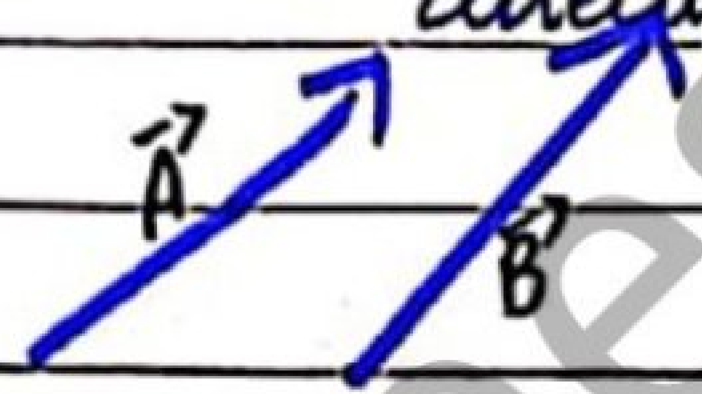
Unit of Vector: A vector whose magnitude is one and it gives direction to a vector is called unit.

$$\vec{A} = A\hat{A}$$

$$A\hat{A} = \vec{A}$$

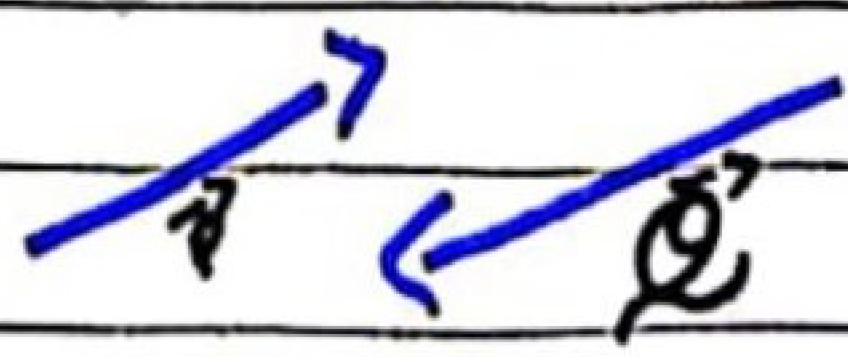
$$\hat{A} = \frac{\vec{A}}{\hat{A}}$$

Parallel Vectors:- Two vectors are said to be parallel if they are directed along same direction



$\vec{A} \parallel \vec{B}$

Anti-Parallel Vectors:- Two vectors are said to be anti-parallel if they are directed opposite to each other



$\vec{P} \parallel \vec{Q}$

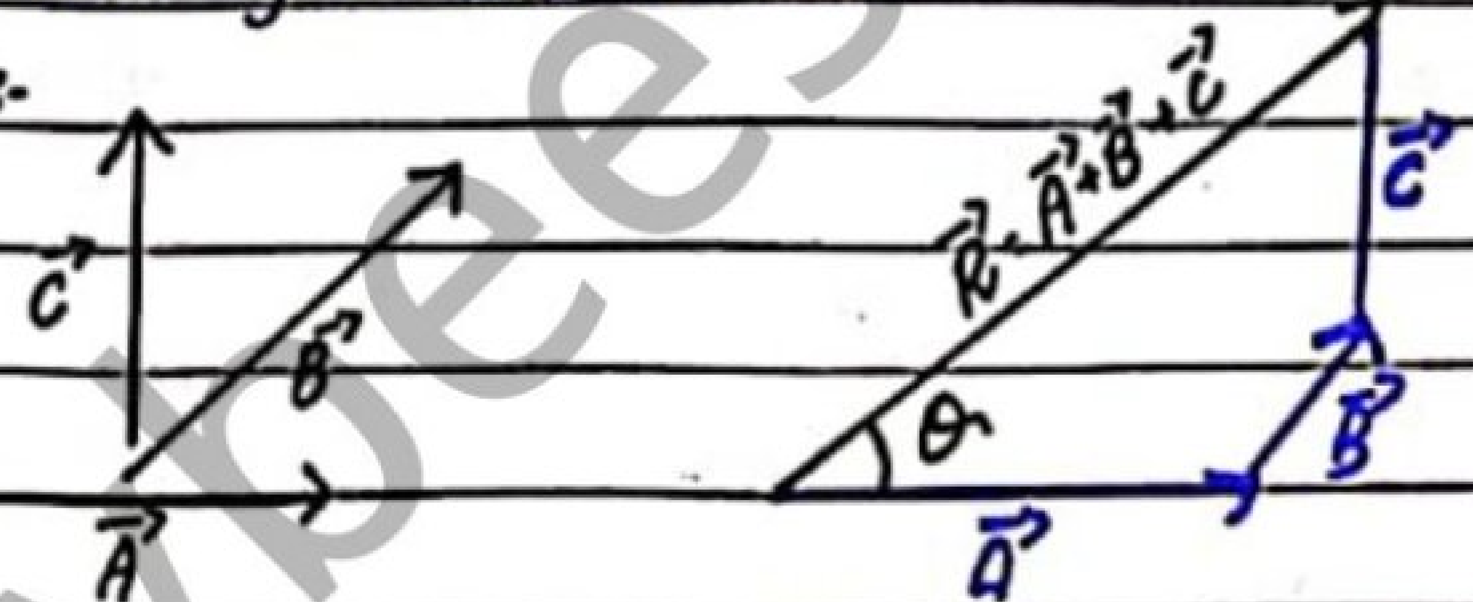
Equal Vectors:- Two or more than two vectors are said to be equal if:-

- They are equal in magnitude
- Directed along same direction
- Have same nature

Addition of Vectors

Head To tail rules:- Vectors are added by head to tail rule graphically. According to this rule, draw the vectors graphically according to the rule scale. Now, join the head of first vector with the tail of other vector with tail of first vector with the tail of 2nd vector and head of 2nd vector with tail of 3rd vector and so on finally, join the tail of first vector with head of last vector. This is resultant of all vectors to be added. Direction of resultant vector is given by angle of resultant and positive x-axis.

Example:-



Resolution of Vectors

Definition:- Splitting or dividing of a vector into two mutually perpendicular parts (compounds) is called resolution of vectors. The two perpendicular components are called rectangular components

Explanation:- Consider a force F making an angle θ with the positive x-axis. Now, we want to calculate the compositions of this force along x & y axis

Diagram on next page

Determination of components of forces:

From ΔAOB

$$\sin\theta = \frac{Perp}{hyp} = \frac{AB}{OB}$$

$$\sin\theta = \frac{AB}{OB} = \frac{F_y}{F}$$

$$F \cdot \sin\theta = F_y$$

Again from ΔAOB

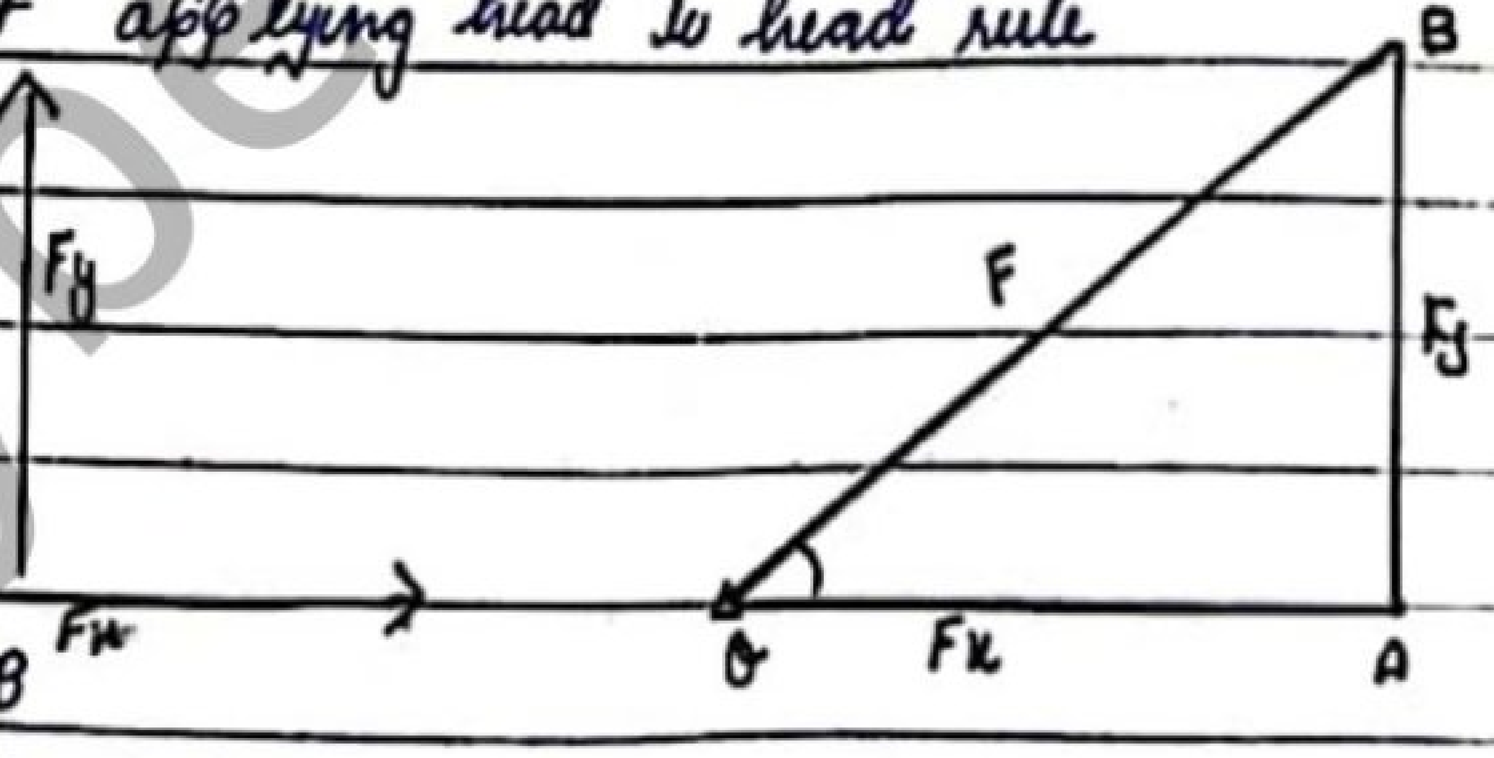
$$\cos\theta = \frac{Base}{hyp} = \frac{OA}{OB}$$

$$\cos\theta = \frac{OA}{OB} = \frac{F_x}{F}$$

$$F_x = F\cos\theta.$$

Determination of a force from its components:

Suppose F_x and F_y are the horizontal and vertical components of a force and we want to calcul this force 'F' applying head to head rule



From ΔAOB

$$(hyp)^2 = (Base)^2 + (perp)^2$$

$$(OB)^2 = (OA)^2 + (AB)^2$$

$$F^2 = F_x^2 + F_y^2$$

Taking $\sqrt{}$ on b/s

$$F = \sqrt{F_x^2 + F_y}$$

Again from ΔAOB

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$\tan\theta = \frac{AB}{OA}$$

$$\tan\theta = F_y/F_x$$

Assingment 2.1:-

Given Data:-

$F_x = 12N$

$F_y = 5N$

To Find:-

$\theta = ?$

Solution:-

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{12^2 + 5^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169}$$

$$F = 13N$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$= \tan\left(\frac{12}{13}\right)$$

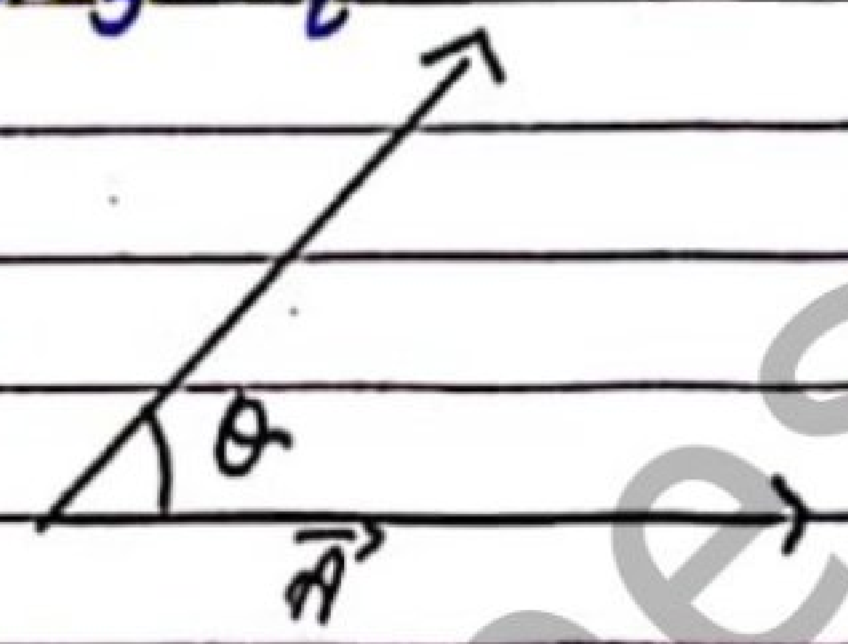
$$\theta = 22.6^\circ$$

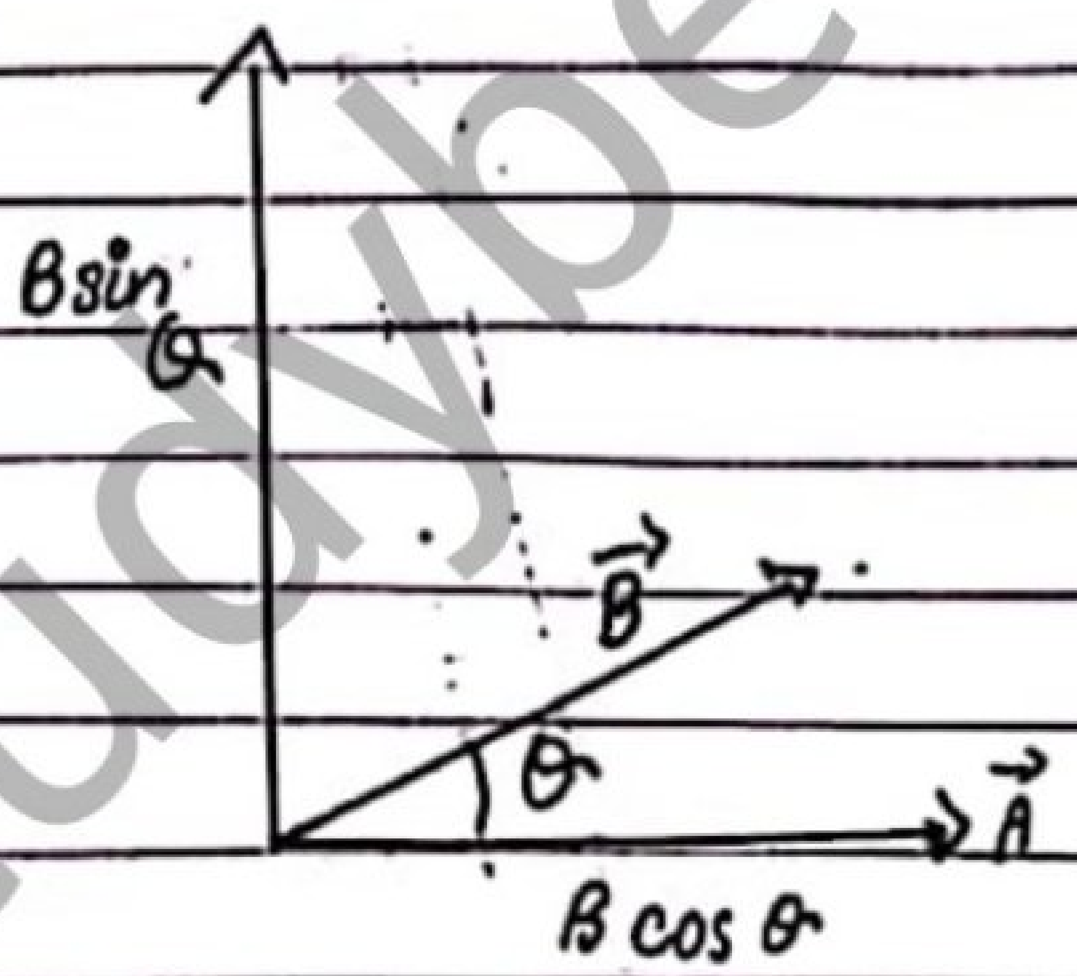
Dot Product of Vectors:-

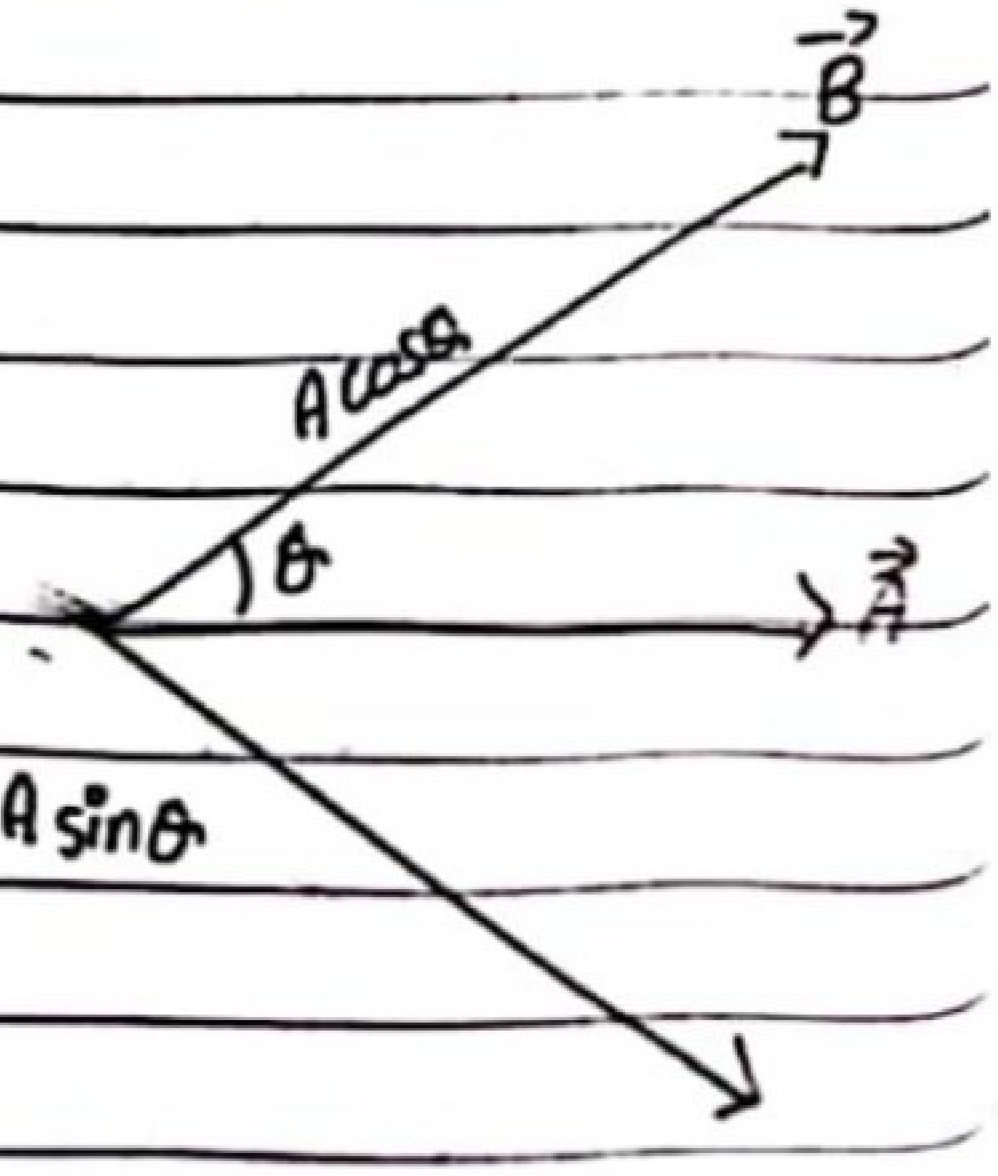
Definition: If product of two vectors is a scalar quantity, then this type of product of vector is called dot product or scalar product.

Representation: If $\vec{A}$ and $\vec{B}$ are two vectors then their dot product is represented as $\vec{A}\cdot\vec{B}$

Diagram:







Explanation: If $\vec{A}$ and $\vec{B}$ are two vectors having angle "θ" between them then their scalar product is defined as:

$$\vec{A}\cdot\vec{B} = |A||B| \cos\theta$$

$$\vec{A}\cdot\vec{B} = AB\cos\theta = A(B\cos\theta)$$

$$= (A\cos\theta)B$$

$\vec{A}\cdot\vec{B}$ = Product of magnitude of $\vec{A}$ and magnitude of component of $\vec{B}$ along $\vec{A}$.

Properties:-

- **Dot Product of Parallel vectors:-** If $\vec{A}$ and $\vec{B}$ are parallel then $\theta = 0°$

Therefore

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$= AB\cos 0$$

$$= AB \times 1$$

$$\vec{A}\cdot\vec{B} = AB$$

- **Dot Product of perpendicular:-**

If $\vec{A}$ and $\vec{B}$ are perpendicular then $\theta = 90°$

Therefore;

$$\vec{A}\cdot B = AB\cos\theta$$

$$= AB\cos 90°$$

$$= AB\cos 90°$$

$$= AB(0)$$

$$\vec{A}\cdot\vec{B} = 0$$

- **Dot product of a vector with itself**

$\theta = 0°$

$\vec{A} \cdot \vec{A} = AA\cos\theta$

$= A^2 \cos\theta$

$= A^2 \cos\theta$

$= A^2(1)$

$\vec{A} \cdot \vec{A} = A^2$

Dot product of orthogonal unit vector:-

Unit vector along x, y and z axis are $\hat{i}$, $\hat{j}$ and $\hat{k}$

Now,

$\hat{i} \cdot \hat{i} = |\hat{i}||\hat{i}| \cos\theta = \hat{i} \times \hat{i} \times \cos\theta = 1 \times 1 \times 1 =$

$\hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = \hat{i} \cdot \hat{i} = 1$

$\hat{i} \cdot \hat{j} = |\hat{i}||\hat{j}| \cos\theta = 1 \times 1 \cos 90 = 1 \times 1 \times 0 = 0$

$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

Dot product of vectors in cartesian form:-

If $\vec{A} = A_x i + A_y j + A_z k$ and

$\vec{B} = B_x i + B_y j + B_z k$

Then,

$\vec{A} \cdot \vec{B} = (A_x i + A_y j + A_z k) \cdot (B_x i + B_y j + B_z k)$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Dot product is commutative

$\vec{A} \cdot \vec{B} = AB\cos\theta \rightarrow eq(1)$

$\vec{B} \cdot \vec{A} = BA\cos(360 - \theta)$

$\vec{B} \cdot \vec{A} = BA\cos\theta$

$\vec{B} \cdot \vec{A} = AB\cos\theta \rightarrow eq(2)$

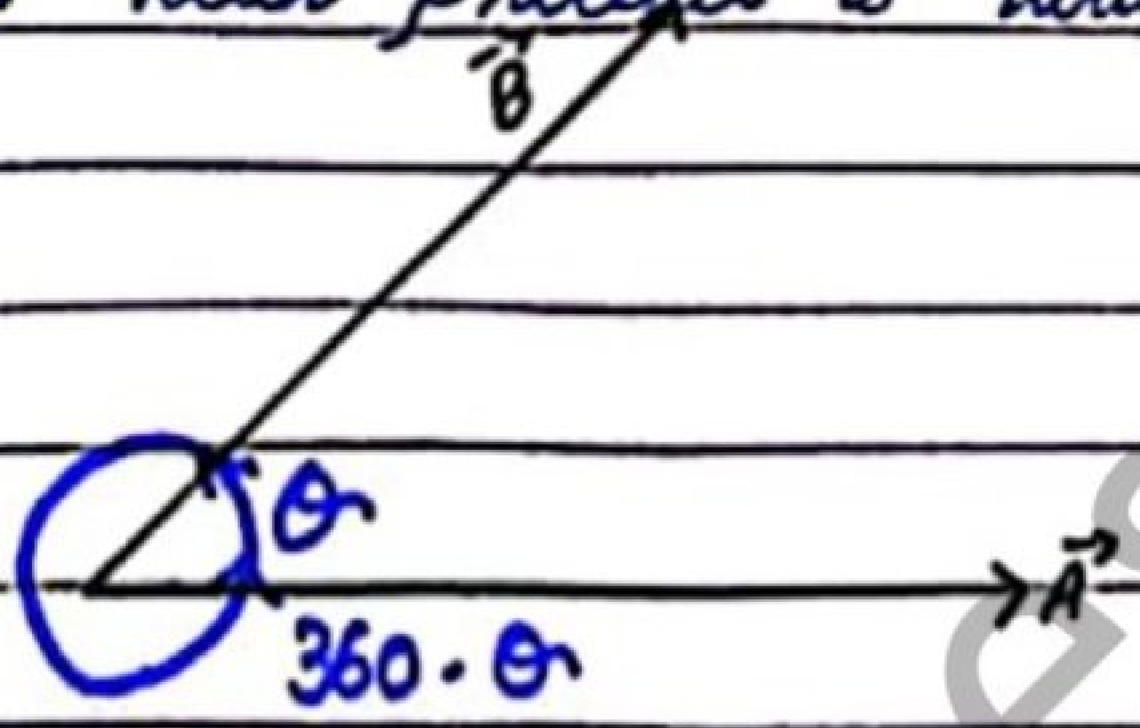
Compare eq (1) and (2)

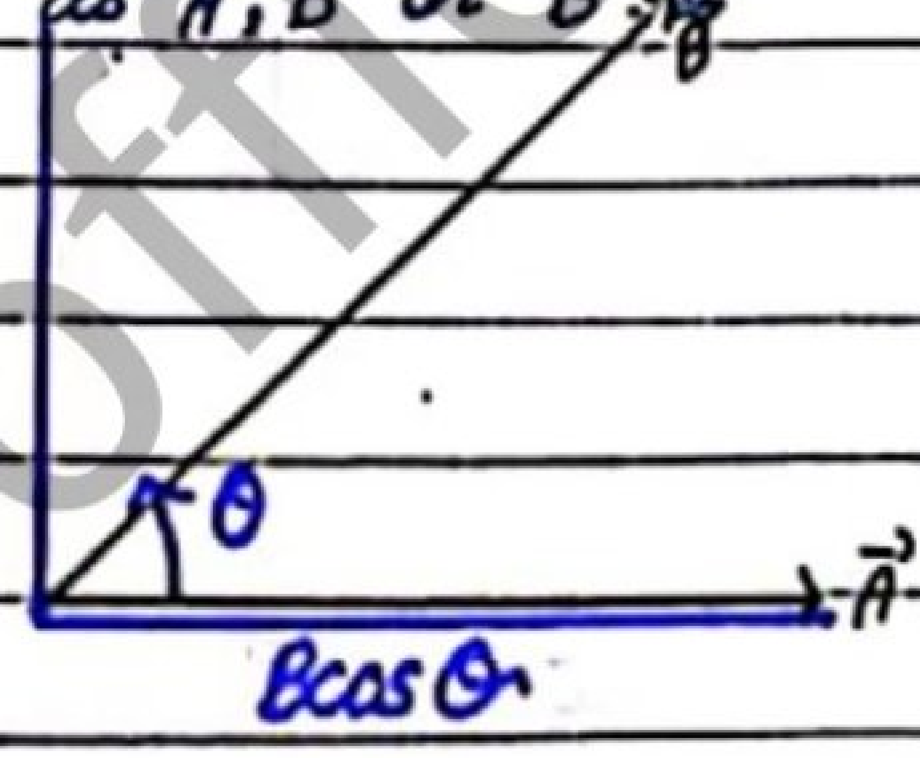
$\vec{B}\vec{A} = \vec{A}\vec{B}$

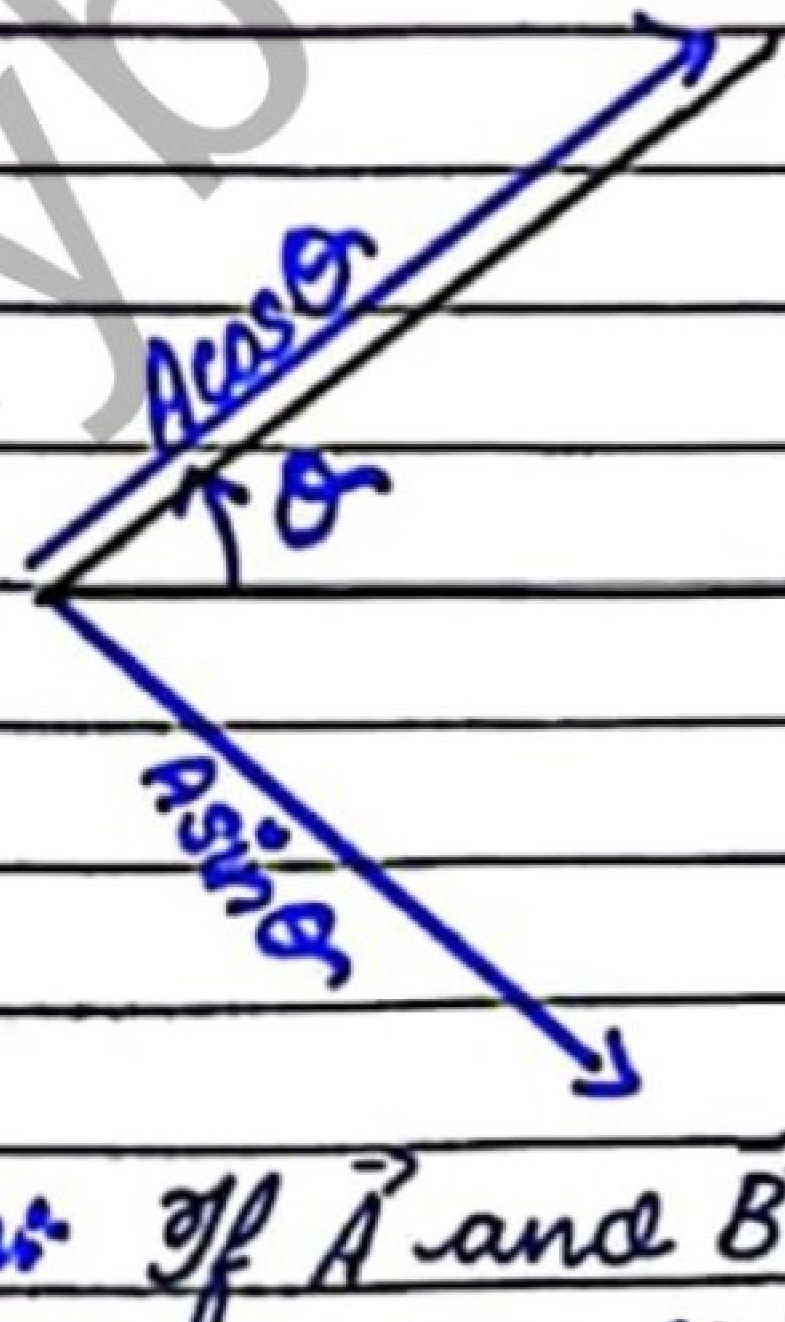
Cross Product of Vectors

Definition:- If products of two vectors gives a vector quantity, this kind of product of vectors is called cross product or vector product.

Representation: If $\vec{A}$ and $\vec{B}$ are two vectors then their vector product is written as $\vec{A} \times \vec{B}$ or $\vec{B} \times \vec{A}$







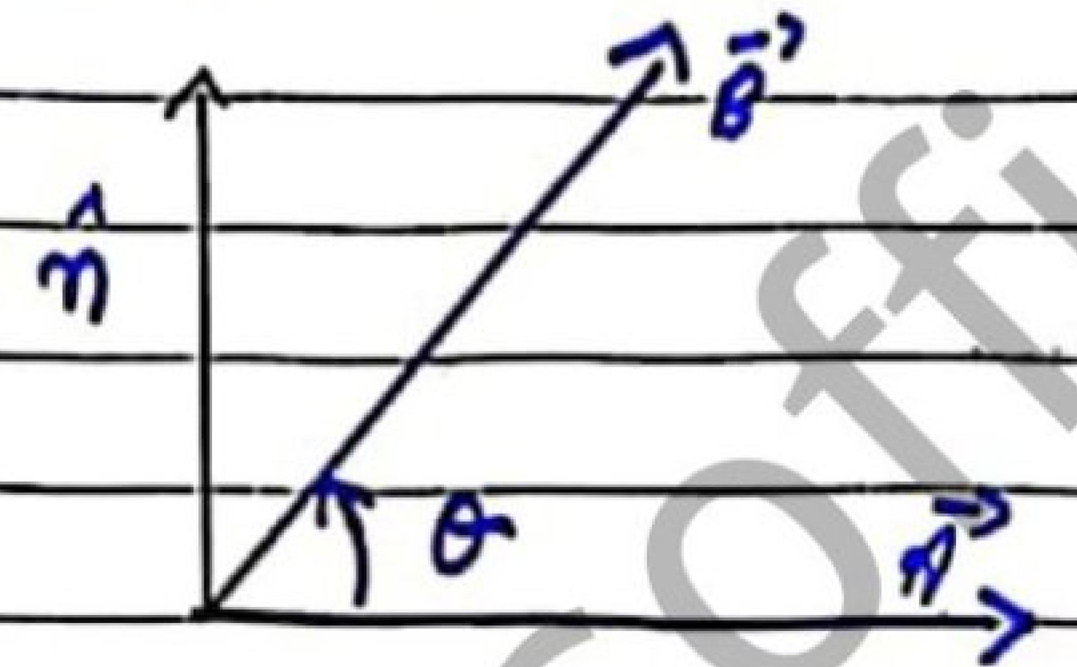
Explanation:- If $\vec{A}$ and $\vec{B}$ are two vectors having angle θ between them, their cross product is mathematically defined as;

$$A \times B = |A|\,|B| \sin\theta\, \hat{n}$$

$\vec{A}\times\vec{B}$: $(AB\sin\theta)\hat{n}$ = $A(B\sin\theta)\hat{n}$: $A(\sin\theta\, B)\hat{n}$

$\vec{A}\times\vec{B}$: Product of magnitude of $\vec{A}$ and magnitude of component of $\vec{B}$ perpendicular to $\vec{A}$.

Here, $\hat{n}$ is the unit vector which gives direction to $\vec{A}\times\vec{B}$



$\hat{n}$ is perpendicular to $\vec{A}$, $\vec{B}$ and to the plane containing $\vec{A}$ and $\vec{B}$

Right Hand Rule:- Right hand rule gives direction of vector product. According to this rule place the fingers of your right hand in direction of first vector. Now rotate/curl the fingers in direction of 2^{nd} vector your erect thumb will point in direction of the resultant. ie $\vec{A}\times\vec{B}$ or $\vec{B}+\vec{A}$

Properties:-

Vector product is not comutative:- If $\vec{A}$ & B. are two vectors with angle from $\vec{A}$ to $\vec{B}$, $\vec{A}\times\vec{B}$

Here;

$\vec{A}\times\vec{B}$ = $(AB\sin\theta)\hat{n}$ → (1)

$\vec{B}\times\vec{A}$ = $BA\sin\theta\,(360-\theta)\hat{n}$

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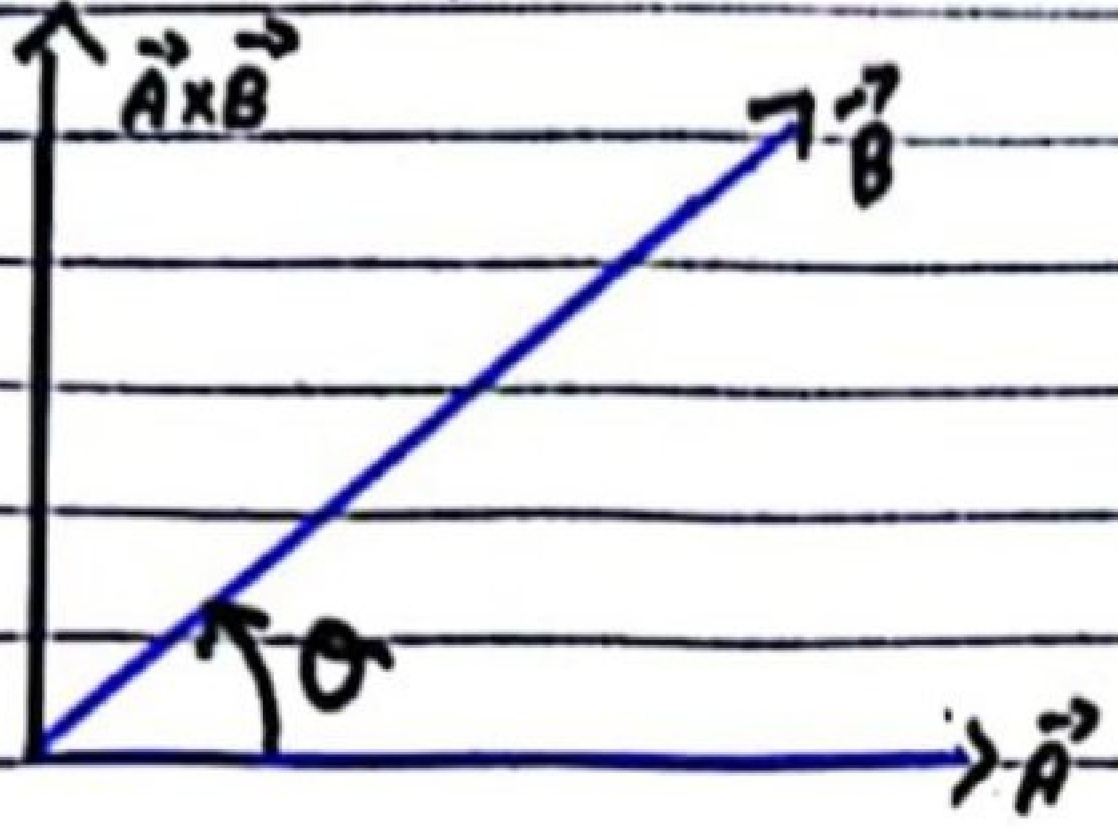
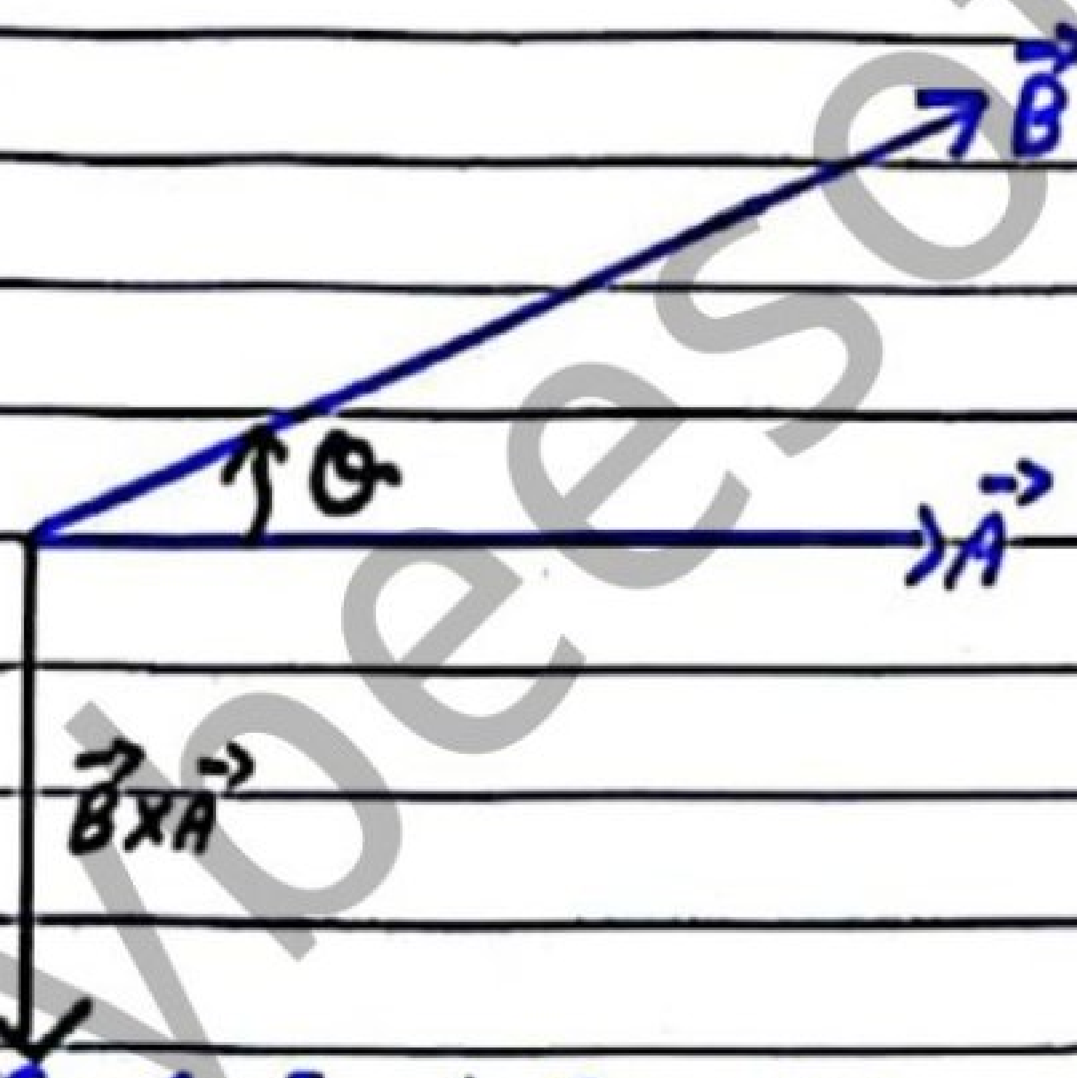
$\vec{B} \times \vec{A} = (AB \sin\theta)\hat{n}$

Compare eq ① and ②

$(\vec{A} \times \vec{B}) = -(\vec{B} \times A)$





ii) Cross product of parallel vectors remain zero

$\vec{A} \times \vec{B} = (AB \sin\theta)\eta$ $\theta = 0$

$\vec{A} \times \vec{B} = (AB \sin 0^\circ)\eta$

$\vec{A} \times \vec{B} = AB(2)\hat{\eta}$

$\vec{A} \times \vec{B} = 0n$

iii). Cross product of perpendicular vectors.

$\vec{A} \times \vec{B} = (AB \sin\theta)\hat{\eta}$ $\theta = 90^\circ$

$\vec{A} \times \vec{B} = (AB \sin 90)\hat{\eta}$

$\vec{A} \times \vec{B} = (AB(01)\hat{\eta}$

$\vec{A} \times \vec{B} = (AB)n$

iv). Vector unit of a vector with itself :-

$$\vec{A} \times \vec{A} = (AA \sin\theta)\hat{n}$$
$$\vec{A} \times \vec{A} = (A^2 \sin\theta)\hat{n}$$
$$\vec{A} \times \vec{A} = A^2 \times 0\hat{n}$$
$$\vec{A} \times \vec{A} = 0\hat{n}$$

v). Cross Product of Orthagonal unit vector

Unit Vetor directed along X-axis, y axis and Z axis are $\hat{i}$, $\hat{j}$ and $\hat{k}$. These unit vectors are perpendicular to each other. That is why, these are called orthagonal unit vector.

$$\hat{i} \times \hat{i} = (|i||i| \sin\theta)\hat{n}$$
$$= (1 \times 1 \times \sin(0))\hat{n}$$
$$= (1 \times 1 \times 0)\hat{n}$$
$$i \times i = 0\hat{n}$$

Similarly;

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0\hat{n}$$

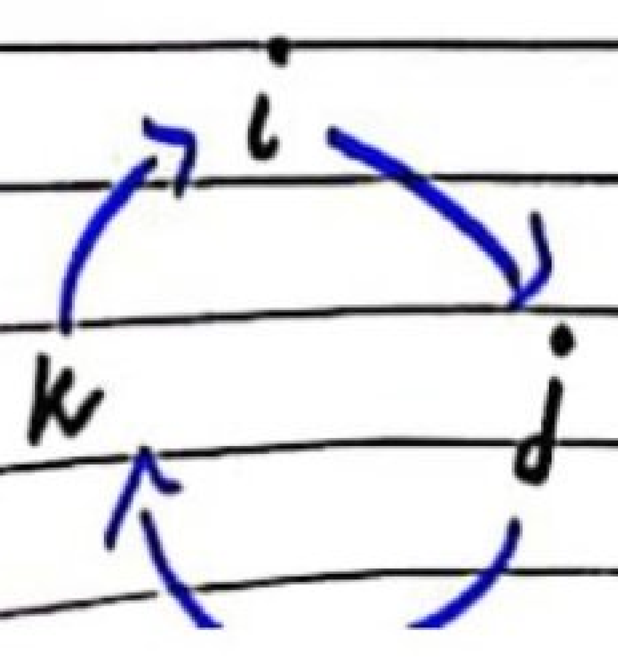
and;

$$\hat{i} \times \hat{j} = (|i| \times |j| (\sin\theta))$$
$$= (1 \times 1 \times \sin 90^\circ)$$
$$= (1 \times 1 \times 1)$$
$$= 1\hat{k}$$

$i \times j = k$ and $j \times i = -k$

$j \times k = i$ and $k \times j = -i$

$k \times j = j$ and $i \times k = -j$



vi). Cross product of vectors in cartesian form:

Suppose, $\vec{A}$ and $\vec{B}$ are two vectors such, that:

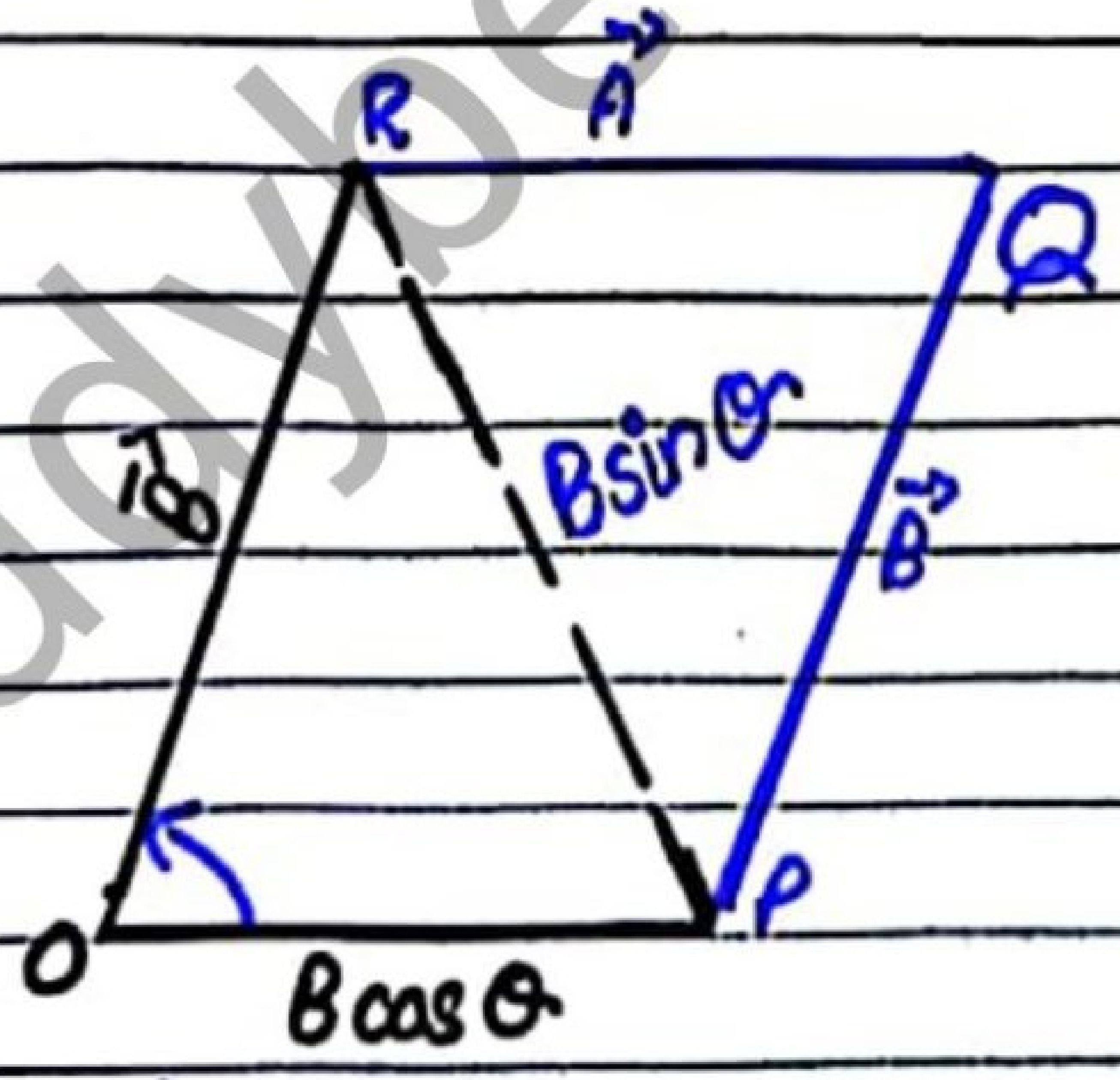
$$\vec{A} = A\hat{i} + A_y\hat{j} + A\hat{k}$$
$$B = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A} \times \vec{B} = (A_x i + A_y j + A_z k) \times (B_x i + B_y j + B_z k)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i}(A_y B_z - B_y A_z) - \hat{j}(A_z B_z - A_x B_x) + \hat{k}(A_x B_y - A_y B_x)$$

Physical Significance of Cross-product by parallelogram:



If $\vec{A}$ and $\vec{B}$ are two vectors that represent adjacent sides of a parallelogram then magnitude of $\vec{A} \times \vec{B}$ gives magnitude and area of parallelogram

Area of parallelogram OPQR $= |\vec{A} \times \vec{B}|$

Assignment 2.3

Given Data:-

$|A| = 3.2$

$|B| = 5.2$

$\theta = 60^\circ$

We know that:-

$\vec{A} \times \vec{B} = |A||B| \sin\theta$

$|\vec{A} \times \vec{B}| = |A|\,|B| \sin\theta$

$= 3.2 \times 5.2 \times \sin(60)$

$|\vec{A} \times \vec{B}| = 14.4$ Answer.