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Where Minds Pollinate

Class 11
Mathematics

Chapter 9
Review Exercise 9
Notes

National Book Foundation

Review Exercise 9.

(12) $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

~~cos~~ squaring b/s

$(\cos\theta - \sin\theta)^2 = (\sqrt{2}\sin\theta)^2$

$\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta = 2\sin^2\theta$

$\cos^2\theta - 2\sin\theta\cos\theta = 2\sin^2\theta - \sin^2\theta$

adding $\cos^2\theta$ on b/s

$\cos^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = \sin^2\theta + \cos^2\theta$

$2\cos^2\theta = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$\sqrt{2\cos^2\theta} = \sqrt{(\sin\theta + \cos\theta)^2}$

$\sqrt{2}\cos\theta = \sin\theta + \cos\theta$

Q3) (i) $\dfrac{\tan x - \cot x}{\sin x \cos x} = \sec^2 x - \operatorname{cosec}^2 x$

$\dfrac{\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}}{\sin x \cos x} = \dfrac{\frac{\sin^2 x - \cos^2 x}{\sin x \cos x}}{\sin x \cos x}$ or $\dfrac{\sin^2 x - \cos^2 x}{\sin x \cos x} \cdot \dfrac{1}{\sin x \cos x}$

$= \dfrac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}$

$= \dfrac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}\cos^2 x} - \dfrac{\cancel{\cos^2 x}}{\sin^2 x \cancel{\cos^2 x}}$

$= \dfrac{1}{\cos^2 x} - \dfrac{1}{\sin^2 x}$

$= \sec^2 x - \operatorname{cosec}^2 x$

(ii) $\dfrac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x} = \sec^2 x - \tan^2 x$

$\dfrac{(\sec^2 x)^2 - (\tan^2 x)^2}{\sec^2 x + \tan^2 x}$

$\dfrac{\cancel{(\sec^2 x + \tan^2 x)}(\sec^2 x - \tan^2 x)}{\cancel{\sec^2 x + \tan^2 x}}$

$= \sec^2 x - \tan^2 x$

(c) $\frac{\sin t}{1-\cos t} - \frac{\sin t \cos t}{1+\cos t} = \operatorname{cosec} t(1+\cos^2)$

$$\frac{\sin t(1+\cos t) - \sin t\cos t(1-\cos t)}{(1-\cos t)(1+\cos t)}$$

$$= \frac{\sin t + \sin t\cos t - \sin t\cos t + \sin t\cos^2 t}{1^2 - (\cos t)^2}$$

$$= \frac{\sin t + \sin t\cos^2 t}{1-\cos^2 t}$$

$$= \frac{\cancel{\sin t}(1+\cos^2 t)}{\sin^{\cancel{2}} t}$$

$$= \frac{1+\cos^2 t}{\sin t}$$

$$= \frac{1}{\sin t} + \frac{\cos^2 t}{\sin t}$$

$= \operatorname{cosec} t(1+\cos^2 t)$ RHS

Q4) (a) $\frac{\tan(\alpha+\beta)-\tan\beta}{1+\tan(\alpha+\beta)\tan\beta} = \tan\alpha.$

$\tan(\alpha+\beta-\beta)$

$\tan\alpha$ RHS

(b) $\frac{1+\sin 2\theta - \cos 2\theta}{1+\sin 2\theta + \cos 2\theta} = \tan\theta.$ Do from end

$$= \frac{1+2\sin\theta\cos\theta - (\cos^2\theta - \sin^2\theta)}{1+2\sin\theta\cos\theta + \cos^2\theta - \sin^2\theta}$$

$$= \frac{1+2\sin\theta\cos\theta - \cos^2\theta + \sin^2\theta}{1+2\sin\theta\cos\theta + \cos^2\theta - (1-\cos^2\theta)}$$

$$= \frac{1+2\sin\theta\cos\theta - (1-\sin^2\theta) + \sin^2\theta}{1+2\sin\theta\cos\theta + \cos^2\theta - 1 + \cos^2\theta}$$

$$= \frac{2\sin\theta\cos\theta + 2\sin^2\theta}{2\sin\theta\cos\theta + 2\cos^2\theta}$$

$$= \frac{\cancel{2}\sin\theta\,\cancel{[\cos\theta+\sin\theta]}}{\cancel{2}\cos\theta\,\cancel{[\sin\theta+\cos\theta]}}$$

$= \boxed{\tan\theta} = RH$

Do from left

Q5) $\cos\alpha + \cos\beta + \cos\gamma = 0$ prove that

$\cos\alpha \cdot \cos\beta \cdot \cos\gamma = \frac{1}{12}(\cos 3\alpha + \cos 3\beta + \cos\gamma)$

$\cos\alpha + \cos\beta + \cos\gamma = 0$

multiply by 3

$3\cos\alpha + 3\cos\beta + 3\cos\gamma = 0$ —— (1)

Start from here

again) $\cos\alpha + \cos\beta = -\cos\gamma$

Taking cube on b/s

$(\cos\alpha + \cos\beta)^3 = (-\cos\gamma)^3$

$\cos^3\alpha + \cos^3\beta + 3\cos\alpha\cos\beta(\cos\alpha + \cos\beta) = -\cos^3\gamma$

$\cos^3\alpha + \cos^3\beta + 3\cos\alpha\cos\beta(-\cos\gamma) = -\cos^3\gamma$

$\cos^3\alpha + \cos^3\beta - 3\cos\alpha\cos\beta\cos\gamma = -\cos^3\gamma$

multiplying by -4 on b/s.

$$-4\cos^3\alpha - 4\cos^3\beta + 12\cos\alpha\cos\beta\cos\gamma = 4\cos^3\gamma$$

$$12\cos\alpha\cos\beta\cos\gamma = 4\cos^3\alpha + 4\cos^3\gamma + 4\cos^3\beta \quad \text{—(2)}$$

$-$ing (2) & (1)

$$12\cos\alpha\cos\beta\cos\gamma - 0 = 4\cos^3\alpha + 4\cos^3\beta + 4\cos^3\gamma - 3\cos\alpha - 3\cos\beta - 3\cos\gamma$$

$$12\cos\alpha\cos\beta\cos\gamma = 4\cos^3\alpha - 3\cos\alpha + 4\cos^3\beta - 3\cos\beta + 4\cos^3\gamma - 3\cos\gamma$$

$$12\cos\alpha\cos\beta\cos\gamma = \cos 3\alpha + \cos 3\beta + \cos 3\gamma$$

$$\cos\alpha\cos\beta\cos\gamma = \frac{1}{12}(\cos 3\alpha + \cos 3\beta + \cos 3\gamma)$$

Q6 $\dfrac{\cos 7\alpha - \cos 8\alpha}{1 + 2\cos 5\alpha} = \dfrac{\sin 7\alpha - \sin 8\alpha}{1 + 2\sin 5\alpha} = \cos 2\alpha - \ldots$

Do it from END

$$\frac{\cos(5\alpha + 2\alpha) - \cos(5\alpha + 3\alpha)}{1 + 2\cos 5\alpha}$$

$$= \frac{\cos 5\alpha\cos 2\alpha - \sin 5\alpha\sin 2\alpha - \cos 5\alpha\cos 3\alpha + \ldots}{1 + 2\cos 5\alpha}$$

$$= \frac{\cos 5\alpha(\cos 2\alpha - \cos 3\alpha) + \sin 5\alpha(\sin 3\alpha - \sin 2\alpha)}{1 + 2\cos 5\alpha}$$

$$= \frac{\cos 5\alpha(\cos 2\alpha - \cos 3\alpha) + \sin(3\alpha + 2\alpha)(\sin \ldots}{1 + 2\cos 5\alpha}$$

$$\frac{\cos 5\alpha(\cos 2\alpha-\cos 3\alpha)+(\sin 3\alpha\cos 2\alpha+\cos 3\alpha\sin 2\alpha)(\sin 3\alpha-\sin 2\alpha)}{1+2\cos 5\alpha}$$

$$=\frac{\cos 5\alpha(\cos 2\alpha-\cos 3\alpha)+\sin^2 3\alpha\cos 2\alpha-\sin 3\alpha\cos 2\alpha\sin 2\alpha+\cos 3\alpha\sin 2\alpha\sin 3\alpha-\cos 3\alpha\sin^2 2\alpha}{1+2\cos 5\alpha}$$

$$=\frac{\cos 5\alpha(\cos 2\alpha-\cos 3\alpha)+(1-\cos^2 3\alpha)\cos 2\alpha-\sin 3\alpha\cos 2\alpha\sin 2\alpha+\cos 3\alpha\sin 2\alpha\sin 3\alpha-\cos 3\alpha(1-\cos^2 2\alpha)}{1+2\cos 5\alpha}$$

$$=\frac{\cos 5\alpha(\cos 2\alpha-\cos 3\alpha)+\cos 2\alpha-\cos^2 3\alpha\cos 2\alpha-\sin 3\alpha\cos 2\alpha+\cos 3\alpha\sin 2\alpha\sin 3\alpha-\cos 3\alpha+\cos 3\alpha\cos^2 2\alpha}{(1+2\cos 5\alpha)}$$

$$=\frac{\cos 5\alpha(\cos 2\alpha-\cos 3\alpha)+(\cos 2\alpha-\cos 3\alpha)+\sin 3\alpha\sin 2\alpha(\cos 2\alpha-\cos 3\alpha)+\cos 3\alpha\cos 2\alpha(\cos 2\alpha-\cos 3\alpha)}{1+2\cos 5\alpha}$$

$$=\frac{(\cos 2\alpha-\cos 3\alpha)[\cos 5\alpha+1+\sin 3\alpha\sin 2\alpha+\cos 3\alpha\cos 2\alpha]}{1+2\cos 5\alpha}$$

$$=\frac{(\cos 2\alpha-\cos 3\alpha)[\cos 5\alpha+1+\cos(3\alpha+2\alpha)]}{1+2\cos 5\alpha}$$

$$= \frac{2(\cos 2\alpha - \cos 3\alpha)(1 + 2\cos 5\alpha)}{1 + 2\cos 5\alpha}$$

$$= \cos 2\alpha - \cos 3\alpha \quad RHS.$$

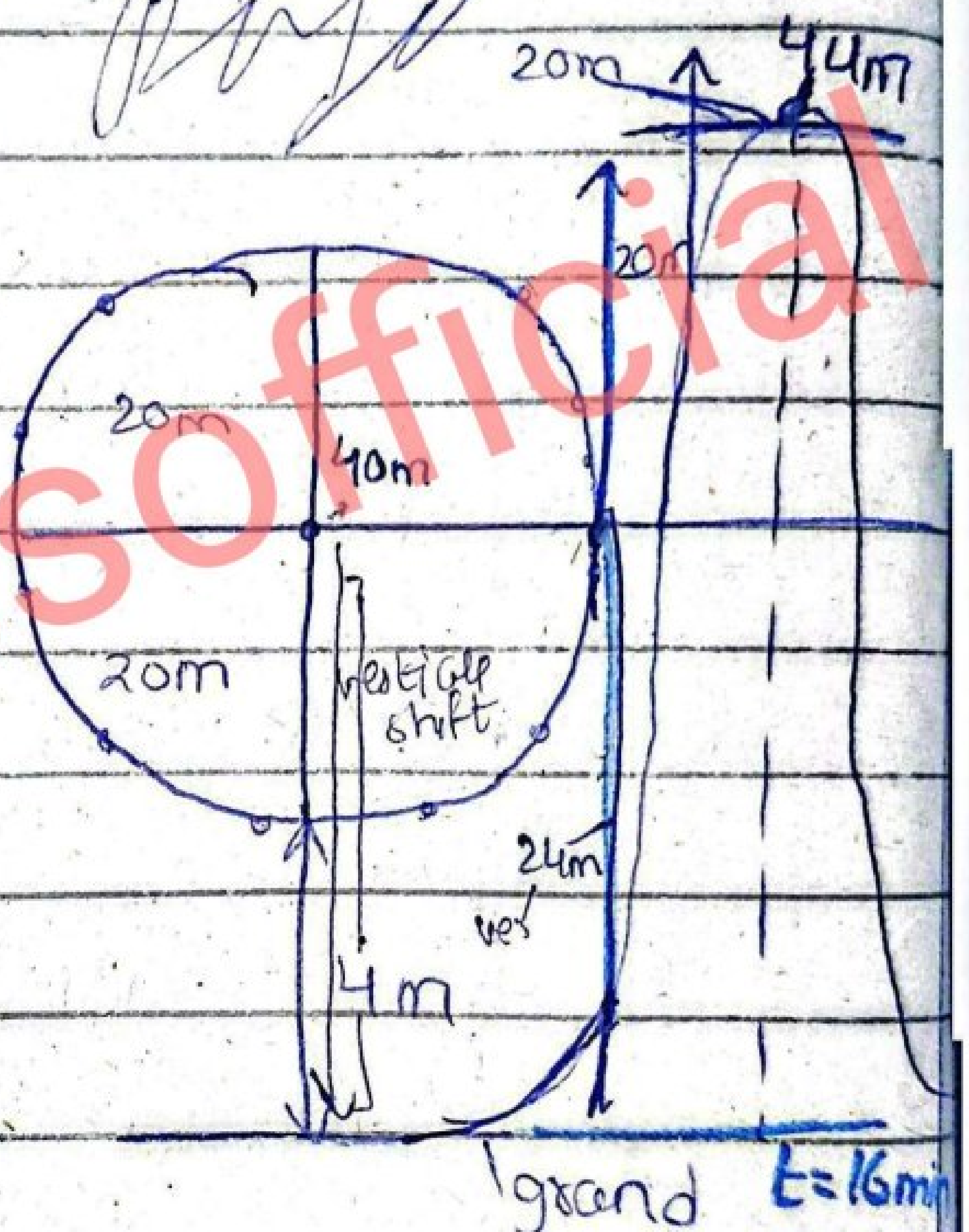
(7) $d = 40m$

Ferris wheel 4m above.

1 Revalution = 16 minuites.

$h(t) = ?$

$h(t) = \underbrace{A}_{\text{amplitude}}\cos(\underbrace{B}_{\text{frequency}}t - \underbrace{D}_{\text{horizental shift}}) + \underbrace{C}_{\text{verticale shift}}$



(a) $|A| = 20$

$\boxed{A_{max} = -20}$ -ve becau cos

Time period = $\boxed{16 \text{ minuttes}}$

Verticle shift = $20 + 4 = \boxed{24}$

(b) $h(t) = -20\cos(Bt - \underbrace{0}_{\text{becau it doent move horizontally}}) + 24$

$B = \frac{2\pi}{t} = \frac{2\pi}{16} = \boxed{\frac{\pi}{8}}$

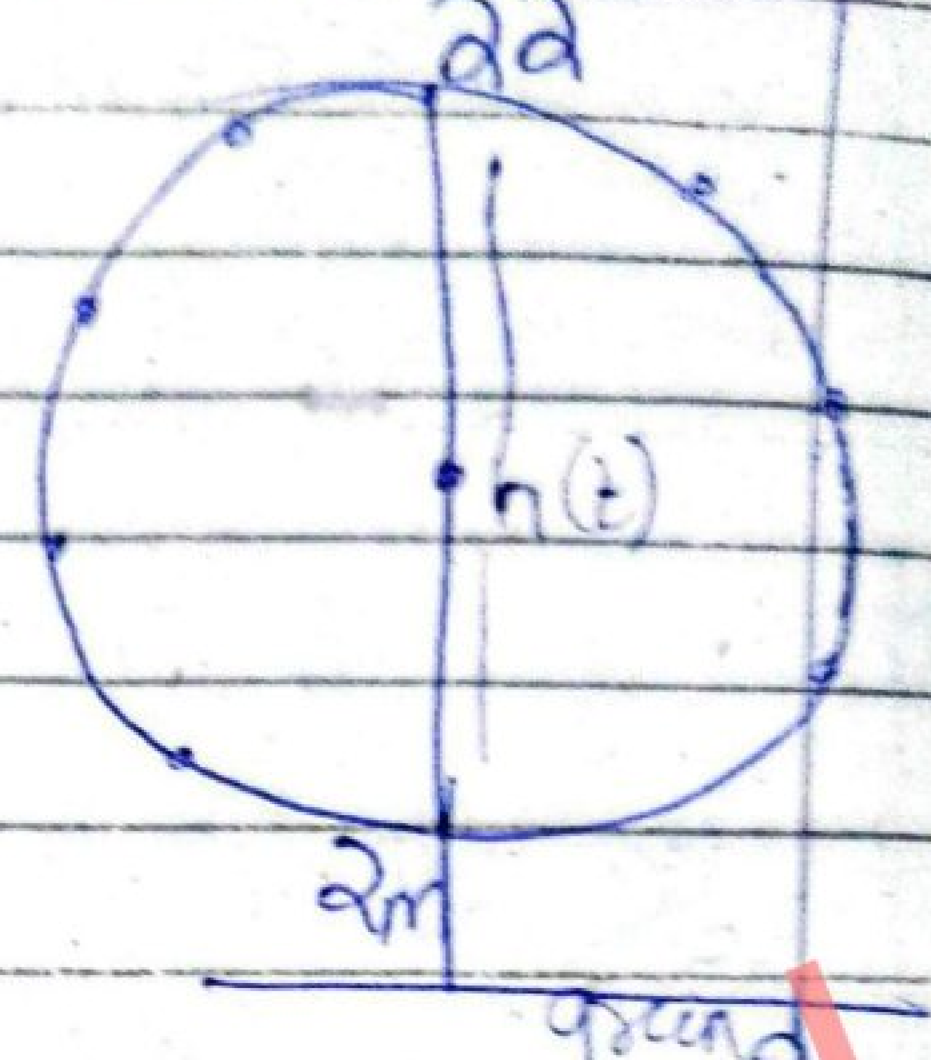
$h(t) = -20\cos\left(\frac{\pi}{8}t\right) + 24$

(c) $h(t) = -20\cos\left(\frac{\pi}{8}t\right) + 24$

$h(5) = -20\cos\left(\frac{180\times5}{8}\right)+24$

$= 7.653 + 24$

$= 32m$. Dobanbak



⑧ $h(t) = 10\sin\{3(t-30)\}+12$

Verticle shift $= 12 = B$

Amphlitude $= A = 10$.

a) Max height $= 12+10(1) = 22$.

Min height $= 12-10(-1) = 2$

$-1 \leq \sin[3(t-30)] \leq 1$

b) $h(30) = 10\sin 3(30-30)+12$

$h(30) = 12m$

© $h(t) = A\sin(Bt-C)+D$

$= 10\sin(Bt-30)+12$

as $\boxed{B=3}$

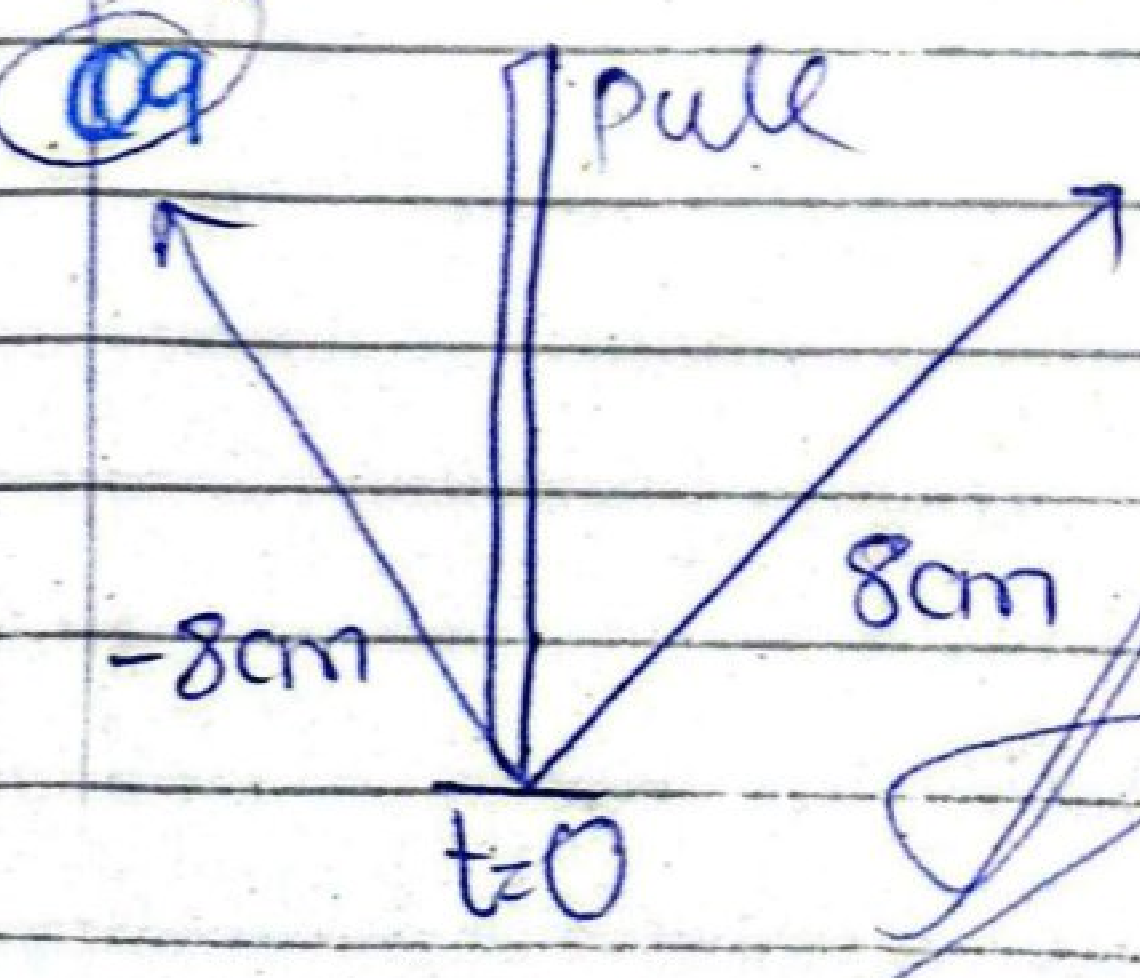
so $B = \frac{2\pi}{T}$

$T = \frac{2\pi}{B} = \frac{360}{3} = \boxed{120s}$

put $h(t) = 0$

$10\sin[3t-90] = -10$

Q9



1min = 260 times

60sec = 260 times

$\frac{60}{260}$ = 1 times

$\boxed{1\text{time} = 0.23sec}$

$T = 0.23sec \approx 0.23sec$

no horizontal & v. shift

$y = A\sin(Bt - C) + D$

$y = A\sin Bt$ —①

$B = \frac{2\pi}{T} = \frac{2\pi}{0.25} = \frac{360}{0.25} = \boxed{1440}$

ⓐ $\boxed{\text{so } y = 8\sin 1440t^\circ}$

ⓑ Domain = Real no

Range = $-8 \le y \le 8$.

⑩ ⓘ $7\sin\frac{\pi}{4}x$

Domain = $(-\infty, \infty)$

Range = $-7 \le y \le 7$.

Period = $\frac{2\pi}{\pi/4} = \boxed{8}$

ⓘⓘ $2\sin\frac{x}{3}$

Domain = $(-\infty, \infty)$

Range = $-2 \le y \le 2$

Period = $\frac{2\pi}{1/3} = 6\pi$

iii $\frac{3}{2}\operatorname{cosec} x$

Domain = $R - (x \ne n\pi, x \in R)$

Period = 2π

Range = $y \ge 3/2$ or $y \le -3/2$

(iv) $5\sin 3x$

Domain = Real no $(-\infty, \infty)$

Range = $-5 \leq y \leq 5$

period = $\frac{2\pi}{3}$

(v) $\frac{1}{2} \sin \frac{2x}{3}$

Domain = Real no

Range = $-\frac{1}{2} \leq y \leq \frac{1}{2}$

Period = $\frac{2\pi}{2/3} = \frac{6\pi}{2} = 3\pi$

(vi) $\frac{5}{3} \cos \frac{4x}{3}$

Domain: Real number

Range = $-\frac{5}{3} \leq y \leq \frac{5}{3}$

Time period = $\frac{2\pi}{4/3} = \frac{3\pi}{2}$.

(vii) $3\sin\pi x$

Domain = Real no $(-\infty, \infty)$

Range = $-3 \leq y \leq 3$

Period = $\frac{2\pi}{\pi} = 2$

(viii) $7\sin 5x$

Domain = Real no

Range = $-7 \leq y \leq 7$

Period = $\frac{2\pi}{5}$

$R - [\frac{\pi}{4}x = (2n+1)(\frac{\pi}{2})]$

$\boxed{R - [x = (2n+1)2 : n \in Z]}$

(ix) $\frac{1}{3}\sec\frac{\pi}{4}x$

Domain = ~~Sec $x \neq (2n+1)\pi/2$, $x \in R$~~

~~$\frac{\pi}{4}x \neq (2n+1)\frac{\pi}{2}$~~

Domain = $x \neq (2n+1)2$

Range = $y \geq 1/3$ $y \leq -1/3$

Period = $\frac{2\pi}{\pi/4} = \boxed{8}$

(x) $\frac{3}{2}\cot\frac{2\pi}{3}x$

$R - [\frac{2\pi}{3}x \neq n\pi]$

$R - [x \neq \frac{3n}{2}]$ Domain

Range = Real no

Period = $\frac{\pi}{2\pi/3} = 3/2$

(xi) $9\cos(3x-2)$

Domain = Real no

Range = $-9 \leq y \leq 9$

Period = $\frac{2\pi}{3}$

$-4 \le y \le 4$

(xii) $8-\cancel{4}\cos 4x$

Domain = Real $(-\infty, \infty)$

Range = $a+|b| = 8+|-1| = 9$

$8-|-1| = 7$

$\cancel{a \pm |b|}$ Range = $7 \le y \le 9$

period = $2\pi/4 = \pi/2$.

(xiii) $7+5\sin(2x-\pi/6)$

Domain = Real number

Max = 12

Min = 2.

Range = $2 \le y \le 12$

Period = $\frac{2\pi}{2} = \pi$.

XIV $6+4\cos(2x+\pi/3)$

Domain = Real no.

Max = 10

Min = 2

Range = $2 \le y \le 10$.

Period = $\frac{2\pi}{2} = \pi$

(xv) $\frac{1}{1-\sin x}$

Domain = $R-\{\frac{\pi}{2}+2n\pi\}$

Rang $-1 \le \sin x \le 1$

$+1 \ge -\sin x \ge -1$

$-1 \le -\sin x \le 1$

$1-1 \le 1-\sin x \le 1+1$

$0 \le 1-\sin x \le 2$

$\frac{1}{0} \ge \frac{1}{1-\sin x} \ge \frac{1}{2}$

Range = $\infty \ge \frac{1}{1-\sin x} \ge \frac{1}{2}$

$R-\{x \in R \mid x = \frac{\pi}{2}+2n\pi$

(xv) $\frac{1}{1-\sin x}$ → period = 2π Domain: $R-\{x \in R$ and $x=\frac{\pi}{2}+2n\pi\}$.

Self Domain = $\frac{1}{1-\sin 90^\circ} = \frac{1}{1-1} = \frac{1}{0}$ ~~Und~~ Undefined

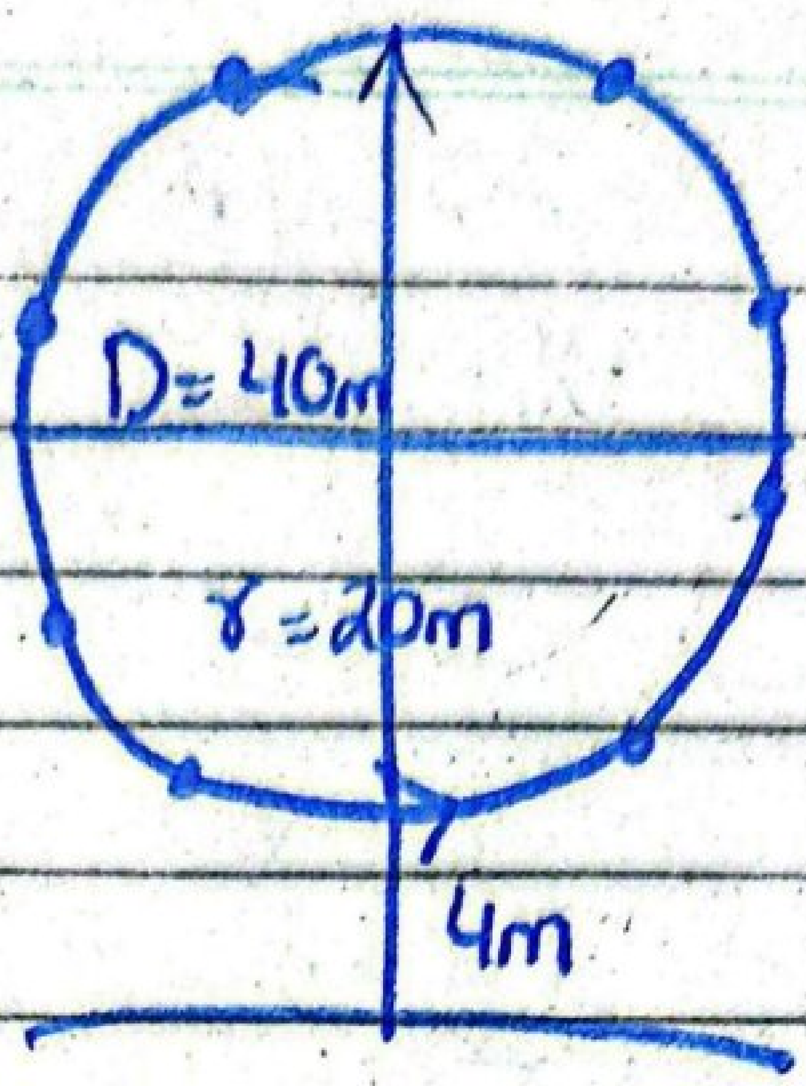
Range: $-1 < \sin x \le 1$

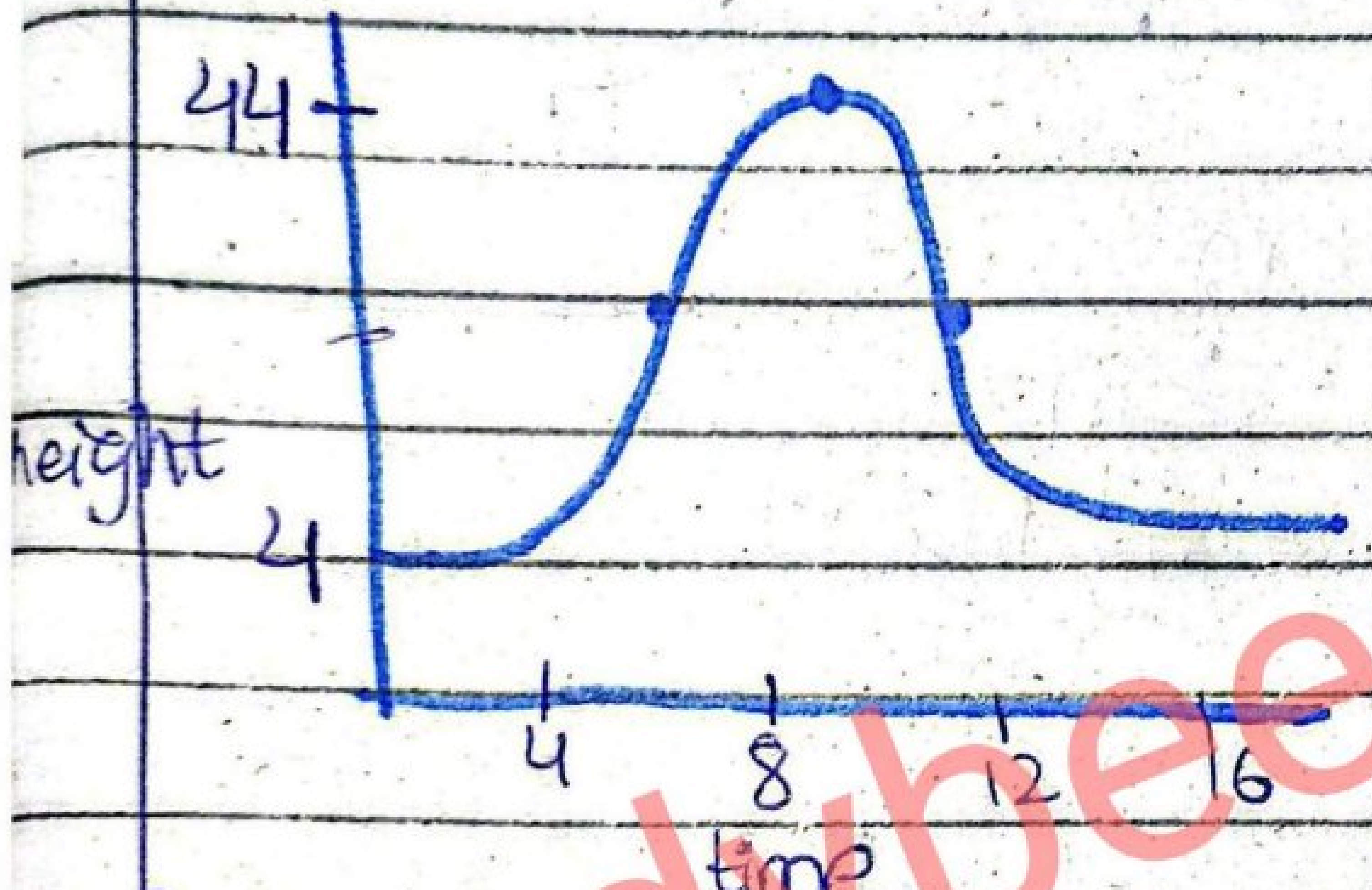
$-1 \le -\sin x \le 1$

$\ldots < 1-\sin x < 2 \Rightarrow 0 < 1-\sin \le 2$ Range: $\frac{1}{1-\sin x} \ge 1/2$

Review 9

Q7)





Sol: $y = h(t) = A\cos(Bt - C) + D$

horizontal shift, vertical shift, function of time, amplitude, period

a) $A_{(amplitude)} = \dfrac{\text{max height} - \text{min height}}{2} = \dfrac{44-4}{2} = \dfrac{40}{2}$

$A = \boxed{-20m}$

$B_{(period)} = \dfrac{2\pi}{16} = \dfrac{\pi}{8}$ the period

$C = 0$ (because it is not moving horizontally)

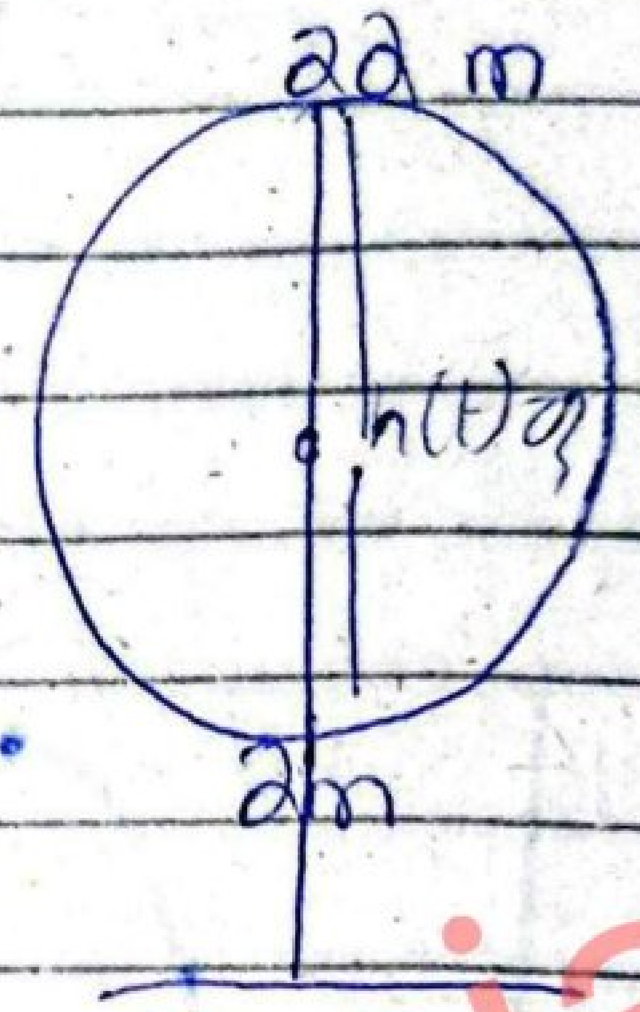
$D = \dfrac{h(max) + h(min)}{2} = \dfrac{44+4}{2} = \boxed{24m}$

b) $h(t) = -20\cos\left(\frac{\pi}{8}t\right) \cancel{- 0} + 24$

(c) $t = 5 \Rightarrow h(5) = -20\cos\left(\frac{\pi}{5}(5)\right) + 24$

$\boxed{h(5) = 31.65}$ Answer

Q8.



(a) as we know that

$-1 \leq \sin\theta \leq 1$

$-1 \leq \sin[3(t-30)] \leq 1$

$max = 10(1) + 12 = 22m$

$min = 10(-1) + 12 = 2m$

(b) $h(t) = 10\sin[3(t-30)] + 12$

Given $t = 30$

$h(30) = 10\sin[3(30-30)] + 12$

$= 10\sin(0) + 12$

$= 0 + 12$

$\boxed{h(30) = 12}$

(c) $h(t) = 0$

because to reach to ground lvl

$10\sin[3(t-30)] + 12 - 2 = 0$

$10\sin[3t-90] = -10$

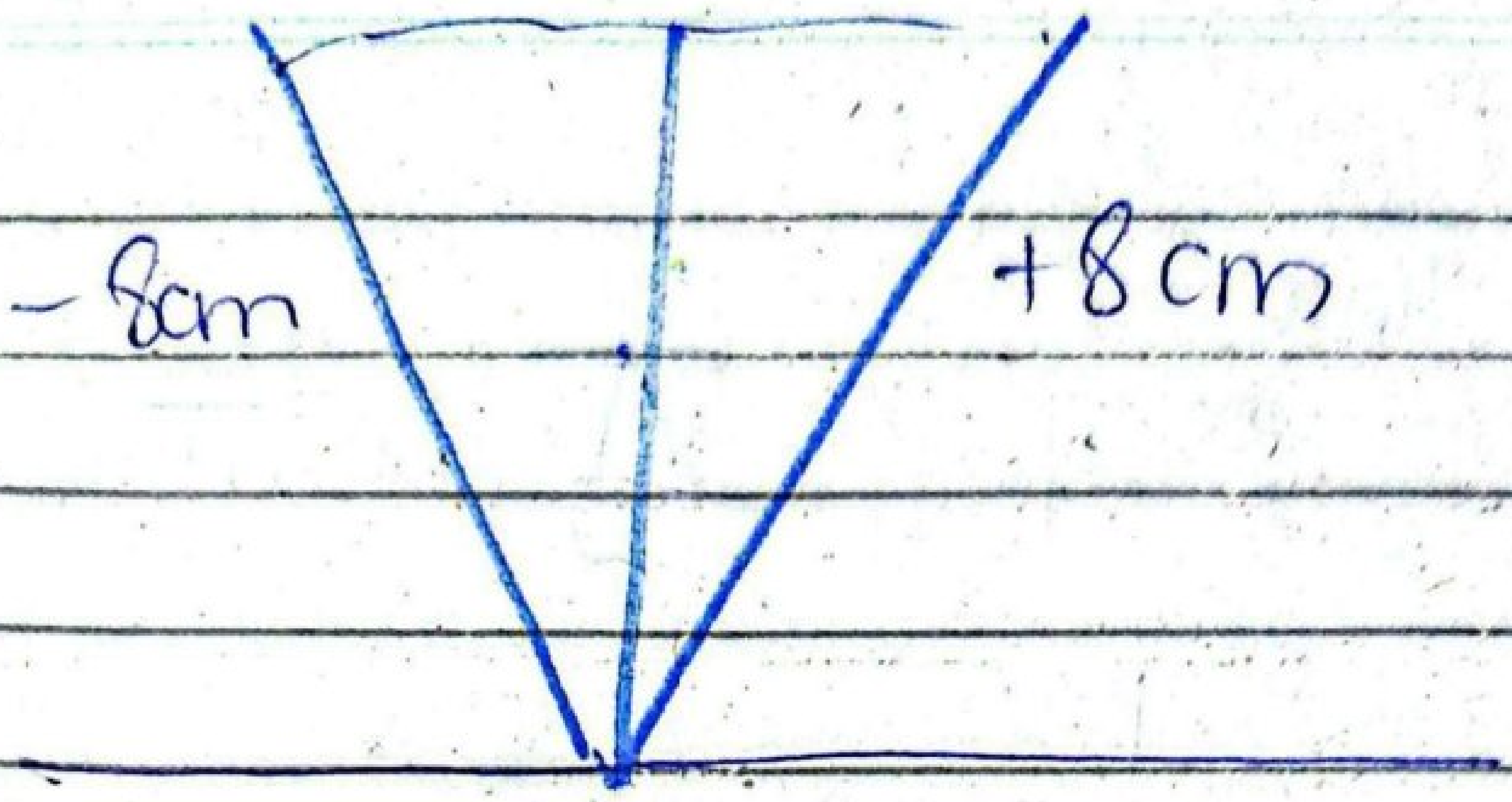
$\sin[3t-90] = -\frac{10}{10}$

$3t - 90 = \sin^{-1}(-1)$

$3t - 90 = 270$

$3t = 360$ $\boxed{t = 120 sec}$

Q.9.



260 times = 1min

260 times = 60sec

1 times = $\frac{60}{260}$ sec

$\boxed{1 \text{time} = 0.23 \text{ sec}}$

(a) $Y(t) = A\sin(Bt - C) + D$

$A = \frac{max - min}{2} = \frac{+8-(-8)}{2} = \frac{8+8}{2} = 8cm$

B = time period = $\frac{2\pi}{0.23} = \boxed{1565.21}$

C = 0 , D = 0

$\boxed{Y(t) = 8\sin(1565.21t)}$

(b) Domain: $(-\infty, +\infty) = \mathbb{R}$

Range: $-8 \le y \le 8$

Exercise 9. Q5(i) $\cos\alpha \cdot \cos\beta \cdot \cos\gamma = \frac{1}{12}(\cos 3\alpha + \cos 3\beta + \cos 3\gamma)$

Solution: Given

$$\cos\alpha + \cos\beta + \cos\gamma = 0 \quad \text{--- (1)}$$

$$\cos\alpha + \cos\beta = -\cos\gamma$$

Taking cube on b/s

$$(\cos\alpha + \cos\beta)^3 = (-\cos\gamma)^3$$

$$\cos^3\alpha + \cos^3\beta + 3(\cos\alpha\cos\beta)(\cos\alpha + \cos\beta) = -\cos^3\gamma$$

$$\cos^3\alpha + \cos^3\beta + 3\cos\alpha\cos\beta(-\cos\gamma) = -\cos^3\gamma$$

$$\cos^3\alpha + \cos^3\beta - 3\cos\alpha\cos\beta\cos\gamma = -\cos^3\gamma$$

$$\cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$$

multiply with 4 on b/s

$$4\cos^3\alpha + 4\cos^3\beta + 4\cos^3\gamma = 12\cos\alpha\cos\beta\cos\gamma$$

now as multiply eq (1) with 3

$$3\cos\alpha + 3\cos\beta + 3\cos\gamma = 0 \quad \text{--- (ii)}$$

subtracting eq (ii) with eq (i)

$$\underline{4\cos^3\alpha - 3\cos\alpha} + 4\cos^3\beta - 3\cos\beta + 4\cos^3\gamma - 3\cos\gamma = 12\cos\alpha\cos\beta\cos\gamma - 0$$

$$\frac{1}{12}(\cos 3\alpha + \cos 3\beta + \cos 3\gamma) = \cos\alpha\cos\beta\cos\gamma.$$