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Where Minds Pollinate

Class 11
Mathematics

Chapter 9
Exercise 9.1
Notes

National Book Foundation

Unit # 9

Trigonometric Functions

Exercise:- 9.1

Q1. Find the maximum and minimum values of the following functions.

i) $y = 2 - 2\cos\theta$

Sol:-

Given function, $y = 2 - 2\cos\theta$

Here, $a = 2$ and $b = -2$

For Maximum value $= a + |b|$

$= 2 + |-2|$

$= 4$

Now for Minimum value $= a - |b|$

$= 2 - |-2|$

$= 0$

ii) $y = \frac{2}{3} - \frac{1}{2}\sin\theta$

Sol:-

Given function, $y = \frac{2}{3} - \frac{1}{2}\sin\theta$

Here $a = \frac{2}{3}$ and $b = -\frac{1}{2}$

For Maximum value $= a + |b|$

$= \frac{2}{3} + \left|-\frac{1}{2}\right|$

$= \frac{7}{6}$

Now, Minimum value $= a - |b|$

$= \frac{2}{3} - \left|-\frac{1}{2}\right|$

$= \frac{1}{6}$

iii) $y = \frac{1}{5} - 2\sin(3\theta - 7)$

Sol:-

Given function, $y = \frac{1}{5} - 2\sin(3\theta - 7)$

Here, $a = \frac{1}{5}$, $b = -2$

For Maximum value $= a + |b|$

$= \frac{1}{5} + |-2|$

$= \frac{11}{5}$

Now, Minimum value $= a - |b|$

$$= \frac{1}{5} - |-2|$$

$$= -\frac{9}{5}$$

iv) $y = 7 + \frac{3}{5}\cos(2\theta - 1)$

Sol:-

Given function, $y = 7 + \frac{3}{5}\cos(2\theta - 1)$

Here, $a = 7$, $b = \frac{3}{5}$

For Maximum value $= a + |b|$

$$= 7 + \left|\frac{3}{5}\right|$$

$$= \frac{38}{5}$$

Now, Minimum value $= a - |b|$

$$= 7 - \left|\frac{3}{5}\right|$$

$$= \frac{32}{5}$$

Q.2 Find the maximum and minimum values of the following reciprocal functions:-

i) $y = \frac{1}{4 + 3\sin\theta}$

Sol:-

Given function, $y = \dfrac{1}{4+3\sin\theta}$

We know that,

$$-1 \leq \sin\theta \leq 1$$

Multiply "3" on b/s

$$\therefore \quad -3 \leq 3\sin\theta \leq 3$$

Add "4" on b/s

$$\therefore \quad 4-3 \leq 4+3\sin\theta \leq 3+4$$

$$1 \leq 4+3\sin\theta \leq 7$$

Now take reciprocal.

$$\therefore \quad 1 \geq \frac{1}{4+3\sin\theta} \geq \frac{1}{7}$$

So, maximum value of y is 1 and minimum value is 1/7

ii) $y = \dfrac{1}{\frac{1}{2} - 5\cos\theta}$

Sol:-

Given function, $y = \dfrac{1}{\frac{1}{2} - 5\cos\theta}$

we know that

$$-1 \leq \cos\theta \leq 1$$

Multiply "-5" on b/s

$$\therefore \quad 5 \geq -5\cos\theta \geq -5$$

Add "$1/2$" on b/s

$$\therefore \quad 1/2 + 5 \geq 1/2 - 5\cos\theta \geq -5 + 1/2$$

$$\frac{11}{2} \geq \frac{1}{2} - 5\cos\theta \geq \frac{-9}{2}$$

Take reciprocal.

$$\therefore \quad \frac{2}{11} \leq \frac{1}{\frac{1}{2} - 5\cos\theta} \leq -\frac{2}{9}$$

So, maximum value of y is $\frac{2}{11}$ and minimum value is $-\frac{2}{9}$

iii) $y = \dfrac{1}{\frac{1}{3} - 4\sin(2\theta - 5)}$

Sol:-

Given function, $y = \dfrac{1}{\frac{1}{3} - 4\sin(2\theta - 5)}$

We know that,

$$-1 \le \sin\theta \le 1$$

Multiply "-4" on b/s

$$\therefore \quad 4 \ge -4\sin\theta \ge -4$$

Add "$1/3$" on b/s

$$\therefore \quad 1/3 + 4 \ge 1/3 - 4\sin\theta \ge -4 + 1/3$$

$$\frac{13}{3} \ge \frac{1}{3} - 4\sin\theta \ge \frac{-11}{3} \quad -(i)$$

Now changing angle (θ) will have no effect on eq.(i)

$$\therefore \quad \frac{13}{3} \ge \frac{1}{3} - 4\sin(2\theta - 5) \ge \frac{-11}{3}$$

Take reciprocal

$$\frac{3}{13} \le \frac{1}{\frac{1}{3} - 4\sin(2\theta-5)} \le \frac{-3}{11}$$

So, maximum value of y is $\frac{3}{13}$ and minimum value is $\frac{-3}{11}$.

iv) $y = \dfrac{1}{3 + \frac{2}{5}\sin(5\theta - 7)}$

Sol:-

Given function, $y = \dfrac{1}{3 + \frac{2}{5}\sin(5\theta - 7)}$

We know that,

$$-1 \leq \sin\varphi \leq 1$$

Changing φ will not effect the inequality. Put $\varphi = 5\theta - 7$

$$\therefore \quad -1 \leq \sin(5\theta - 7) \leq 1$$

Multiply "$\frac{2}{5}$" on b/s

$$\therefore \quad -\frac{2}{5} \leq \frac{2}{5}\sin(5\theta - 7) \leq \frac{2}{5}$$

Add "3" on b/s

$$\therefore \quad 3 - 2/5 \leq 3 + 2/5\sin(5\theta - 7) \leq 2/5 + 3$$

$$\frac{13}{5} \leq 3 + \frac{2}{5}\sin(5\theta - 7) \leq \frac{17}{5}$$

Take reciprocal

$$\therefore \quad \frac{5}{13} \geq \frac{1}{3 + \frac{2}{5}\sin(5\theta - 7)} \geq \frac{5}{17}$$

So, maximum value of y is $\frac{5}{13}$ and minimum value is $\frac{5}{17}$.

Q3. Find domain and range of the following.

i) $y = 7\cos 4x$

Sol:-

Given function, $y = 7\cos 4x$

Dom. of $\cos x = [-\infty, \infty]$
Dom. of $\cos 4x = [-\infty, \infty]$
Dom. of $7\cos 4x = 7[-\infty, \infty]$
Dom. of $7\cos 4x = [-\infty, \infty]$

Range of $\cos x = [-1, 1]$
Range of $\cos 4x = [-1, 1]$
Range of $7\cos 4x = [-7, 7]$

ii) $y = \cos \frac{x}{3}$

Sol:-

Given function, $y = \cos \frac{x}{3}$

Dom. of $\cos x = [-\infty, \infty]$
Dom. of $\cos \frac{x}{3} = [-\infty, \infty]$

Range of $\cos x = [-1, 1]$
Range of $\cos \frac{x}{3} = [-1, 1]$

iii) $y = \sin \frac{2x}{3}$

Sol:-

Given function, $y = \sin\frac{2x}{3}$

Dom. of $\sin x = [-\infty, \infty]$

Dom. of $\sin\frac{2x}{3} = [-\infty, \infty]$

Range of $\sin x = [-1, 1]$

Range of $\sin\frac{2x}{3} = [-1, 1]$

iv) $y = 7\cot\frac{\pi}{2}x$

Sol:-

Given function, $y = 7\cot\frac{\pi}{2}x$

Dom. of $\cot x = R - \{\pi n\}$ $n \in Z$

~~Dom. of cot $\frac{\pi}{2}x$~~

It means angle of "cot" in undefined when $x = \pi n$, $n \in Z$

$\therefore$ for $\cot\frac{\pi}{2}x$

$\frac{\cancel{\pi}}{2}x = \cancel{\pi}n \Rightarrow x = 2n$, $n \in Z$

So, at $x = 2n$ "$\cot\frac{\pi}{2}x$" is undefined

$\therefore$ Dom. of $7\cot\frac{\pi}{2}x = R - \{2n\}$ $n \in Z$

Range of $\cot x = [-\infty, \infty]$
Range of $7\cot\frac{\pi}{2}x = [-\infty, \infty]$

v) $y = 4\tan\pi x$

Sol:-

Dom. of $\tan x = R - \left\{(2n+1)\frac{\pi}{2}\right\}$ $n \in Z$

It means angle(x) of function $\tan x$ is undefinied when $x = (2n+1)\frac{\pi}{2}$, $n \in Z$

$\therefore$ for $\tan\pi x$

$\Rightarrow \cancel{\pi}x = (2n+1)\frac{\cancel{\pi}}{2} \Rightarrow x = \frac{(2n+1)}{2}$

So, $\tan\pi x$ is undefinied when $x = \frac{(2n+1)}{2}$.

$\therefore$ Dom. of $4\tan\pi x = R - \left\{\frac{(2n+1)}{2}\right\}$, $n \in Z$

Range of $\tan x = [-\infty, \infty]$
Range of $4\tan\pi x = [-\infty, \infty]$

vi) $y = \text{cosec } 4x$

Sol:-

Domain of $\text{cosec } x = R - \{\pi n\}$ $n \in Z$

It means when $x = \pi n$ then $\text{cosec } x$ is undefinied

$\therefore$ for $\text{cosec } 4x$

$\Rightarrow \quad 4x = \pi n \quad \Rightarrow \quad x = \dfrac{\pi n}{4}$, $n \in Z$

$\therefore$ Domain of $\text{cosec } 4x = R - \left\{\dfrac{\pi n}{4}\right\}$ $n \in$

Range of $\text{cosec } x = (-\infty, -1] \cup [+1, +\infty$

Range of $\text{cosec } 4x = (-\infty, -1] \cup [+1, +\infty$

Q4. Check whether the following function are odd or even

i) $y = \sin x + x.\cos x$

Sol:-

Let $f(x) = y = \sin x + x.\cos x$

$f(x) = \sin x + x.\cos x$

Replace "x" with "$-x$"

$\therefore f(-x) = \sin(-x) + (-x)\cos(-x)$

$f(-x) = -\sin x - x\cos x$

$f(-x) = -[\sin x + x.\cos x]$

$f(-x) = -f(x)$

$\therefore$ Given function is odd.

$y = x^3.\sin x.\cos x$

Sol:-

Let $f(x) = y = x^3 \sin x.\cos x$

$f(x) = x^3 \sin x.\cos x$

Replace "x" with "$-x$"

$\therefore f(-x) = (-x)^3 \sin(-x)\cos(-x)$

$f(-x) = -x^3.-\sin x\cos x$

$f(-x) = x^3 \sin x\cos x$

$f(-x) = f(x)$

$\therefore$ Given function is even.

iii) $y = \dfrac{x^2 . \tan x}{x + \sin x}$

Sol:-

Let $f(x) = y = \dfrac{x^2 . \tan x}{x + \sin x}$

$f(x) = \dfrac{x^2 . \tan x}{x + \sin x}$

Replace "x" with "$-x$".

$\therefore f(-x) = \dfrac{(-x)^2 . \tan(-x)}{-x + \sin(-x)}$

$f(-x) = \dfrac{x^2 . -\tan x}{-x - \sin x}$

$f(-x) = \dfrac{\cancel{-}(x^2 . \tan x)}{\cancel{-}(x + \sin x)}$

$f(-x) = f(x)$

So, Given function is even.

iv) $y = x^3 . \sin x \cos^2 x$

Sol:-

Let $f(x) = x^3 . \sin x \cos^2 x$

$f(x) = x^3 . \sin x . \cos x . \cos x$

Replace "x" with "$-x$"

$\therefore$ $f(-x) = (-x)^3 \cdot \sin(-x) \cdot \cos(-x)\cos(-x)$

$f(-x) = -x^3 \cdot -\sin x \cdot \cos x \cdot \cos x$

$f(-x) = x^3 \cdot \sin x \cos^2 x$

$f(-x) = f(x)$

$\therefore$ Given function is even.

v) $y = \dfrac{\sin^2 x}{x + \operatorname{Tan} x}$

Sol:-

Let $f(x) = \dfrac{\sin^2 x}{x + \operatorname{Tan} x}$

$f(x) = \dfrac{\sin x \cdot \sin x}{x + \operatorname{Tan} x}$

Replace "x" with "$-x$"

$f(-x) = \dfrac{\sin(-x) \cdot \sin(-x)}{-x + \operatorname{Tan}(-x)}$

$f(-x) = \dfrac{-\sin x \cdot -\sin x}{-x - \operatorname{Tan} x}$

$f(-x) = \dfrac{\sin^2 x}{-(x + \operatorname{Tan} x)}$

$f(-x) = -f(x)$

$\therefore$ Given function is odd.

vi) $y = \dfrac{\tan x - \sin x}{\sin^3 x}$

Sol:-

Let $f(x) = \dfrac{\tan x - \sin x}{\sin^3 x}$

Replace "x" with "$-x$"

$\therefore f(-x) = \dfrac{\tan(-x) - \sin(-x)}{[\sin(-x)]^3}$

$f(-x) = \dfrac{-\tan x + \sin x}{(-\sin x)^3}$

$f(-x) = \dfrac{\cancel{-}(\tan x - \sin x)}{\cancel{-}\sin^3 x}$

$f(-x) = f(x)$

$\therefore$ Given function is even.

vii) $y = \dfrac{\sec x}{x + \tan x}$

Sol:-

Let $f(x) = \dfrac{\sec x}{x + \tan x}$

Replace "x" with "$-x$"

$\therefore f(-x) = \dfrac{\sec(-x)}{-x + \tan(-x)}$

$$f(-x) = \frac{+\sec x}{-x-\tan x}$$

$$f(-x) = \frac{-\sec x}{+(x+\tan x)}$$

$$f(-x) = -f(x)$$

$\therefore$ Given function is odd.

viii) $y = x^2 . \sin x - \cot x$

Sol:-

Let $f(x) = x^2 . \sin x - \cot x$

Replace "x" with "$-x$"

$\therefore$ $f(-x) = (-x)^2 . \sin(-x) - \cot(-x)$

$f(-x) = x^2 . -\sin x + \cot x$

$f(-x) = -(x^2 . \sin x - \cot x)$

$f(-x) = -f(+x)$

$\therefore$ Given function is odd.

Q6. Find periods of the following.

i) $y = 6\sec(2x-3)$

Sol:-

Given function, $y = 6\sec(2x-3)$

Period of $\sec x = 2\pi$

Period of $\sec(2x-3) = \cancel{2}\pi/\cancel{2}$

Period of $6\sec(2x-3) = \pi$

ii) $y = \cos(5x+4)$

Sol:-

Given function $y = \cos(5x+4)$

Period of $\cos x = 2\pi$

Period of $\cos(5x+4) = 2\pi/5$

iii) $y = \cot 4x + \sin \frac{5x}{2}$

Sol:-

Given function; $y = \cot 4x + \text{si}$

Period of $\cot x = \pi$

Period of $\cot 4x = \pi/4$

Now, Period of $\sin x = 2\pi$

Period of $\sin \frac{5x}{2} = \frac{2}{5} \times 2\pi$

Period of $\sin \frac{5x}{2} = 4\pi$

Period of $\cot 4x + \sin \frac{5x}{2} = \frac{\text{L.C.M of } \pi \text{ and } 4\pi}{\text{H.C.F of 1 and 5}}$

$= \frac{4\pi}{1}$

Period of $\cot 4x + \sin \frac{5x}{2} = 4\pi$

iv) $y = 7\sin(3x+3)$

Sol:-

Given function, $y = 7\sin(3x+3)$

Period of $\sin x = 2\pi$

Period of $\sin(3x+3) = 2\pi/3$

Period of $7\sin(3x+3) = \frac{2\pi}{3}$

v) $y = 5\sin(2x+3)$

Sol:-

Given function, $y = 5\sin(2x+3)$

Period of $\sin x = 2\pi$

Period of $\sin(2x+3) = 2\pi/\cancel{2}$

Period of $5\sin(2x+3) = \pi$

vi) $y = 2\tan 3x + 7\cos 5x$

Sol:-

Given function, $y = 2\tan 3x + 7\cos 5x$

Period of $\tan x = \pi$

Period of $\tan 3x = \pi/3$

Period of $2\tan 3x = \frac{\pi}{3}$

Now, Period of $\cos x = 2\pi$

Period of $\cos 5x = 2\pi/5$

Period of $7\cos 5x = \frac{2\pi}{5}$

Period of $y = \frac{\text{L.C.M of } 2\pi \text{ and } \pi}{\text{H.C.F of } 3 \text{ and } 5}$

$= \frac{2\pi}{1}$

Period of $y = 2\pi$

Q10. Alternating current cyclically reverses direction. The maximum voltage is about 180 volts when the standard frequency is 56 Hz (56 cycles per second). The voltage can be modeled as

$$V(t) = a \sin[k(t-d)] + c$$

a. Period of 56 Hz AC.

Sol:-

Period of 56Hz AC = $T = \frac{1}{f}$

$$T = \frac{1}{56} \qquad \{f = 56Hz\}$$

$$T = 0.018\ s$$

b. The value of k.

Sol:-

Value of k is releated to the frequency i.e

$$k = 2\pi f$$

$$k = 2\pi(56) \qquad \{f = 56Hz\}$$

$$k = 112\pi$$

c. The amplitude of the voltage function.

Sol:-

In given voltage function "a" is amplitude i.e maximum voltage

$\therefore$ Amplitude $= a = 180$ vots.

d. Model the voltage with an appropriate transformed sine function.

Sol:-

Given,

Amplitude $= a = 180$ V

Value of $k = 112\pi$

Frequency $= 56$ Hz

Since, no specific phase shift (d) is mentioned, we assume $d = 0$.

$$\therefore V(t) = 180 \sin\left[112\pi(t-0)\right] + C$$

$$V(t) = 180 \sin(112\pi t) + C$$

Q.7 Draw the graphs of $y = \sin x$ and $y = \sin 2x$ in $[0, 2\pi]$ on the same scale.

Sol:-

Given function, $y = \sin x$

The table for $y = \sin x$ in $[0, 2\pi]$ is:-

x	0	30°	60°	90°	120°	150°
$y = \sin x$	0	0.5	0.86	1	0.86	0.5

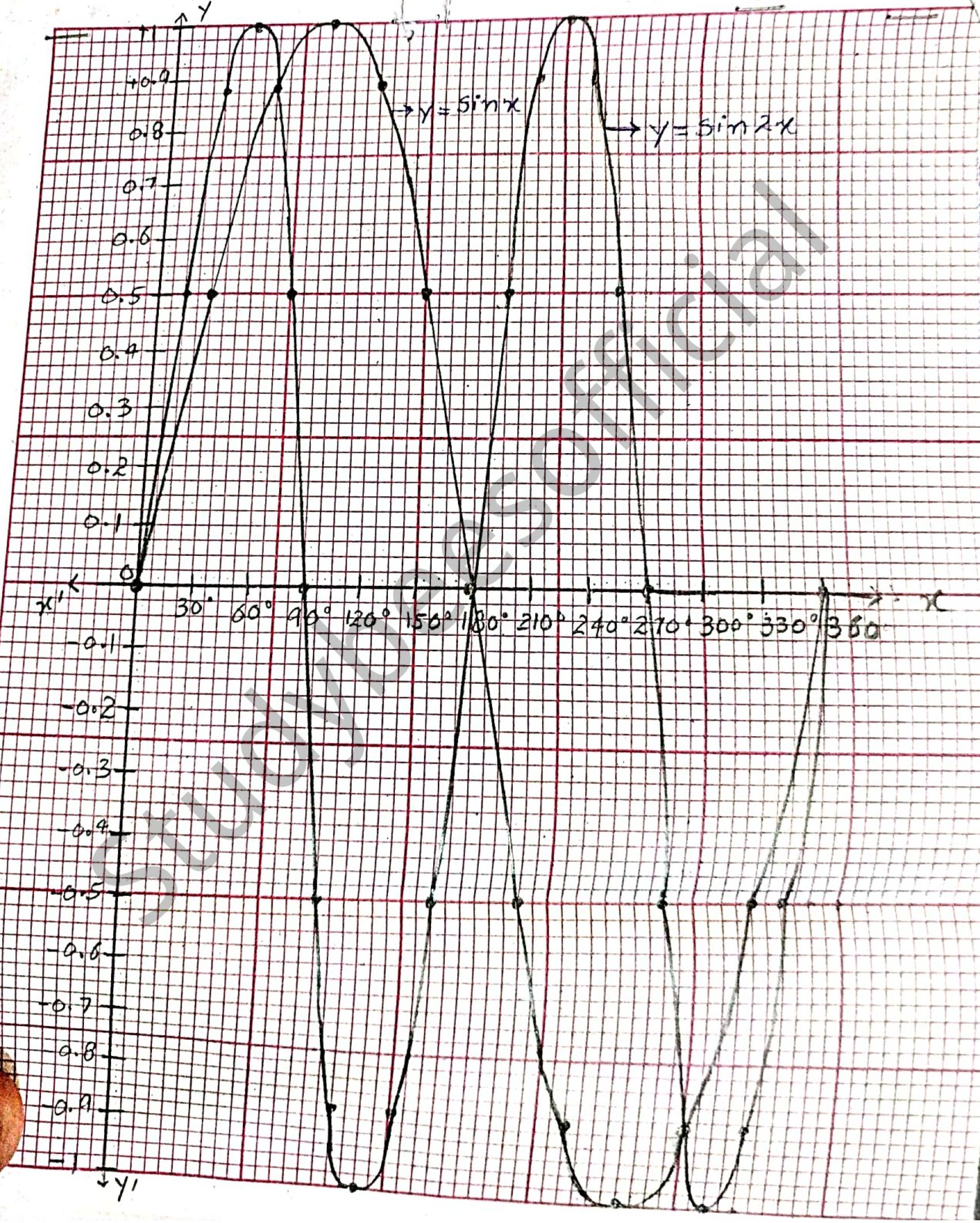
180°	210°	240°	270°	300°	330°	360
0	−0.5	−0.86	−1	−0.86	−0.5	0

Now, for $y = \sin 2x$ table on $[0, 2\pi]$

x	0°	15°	30°	45°	60°	75°
$y = \sin 2x$	0	0.5	0.86	1	0.86	0.5

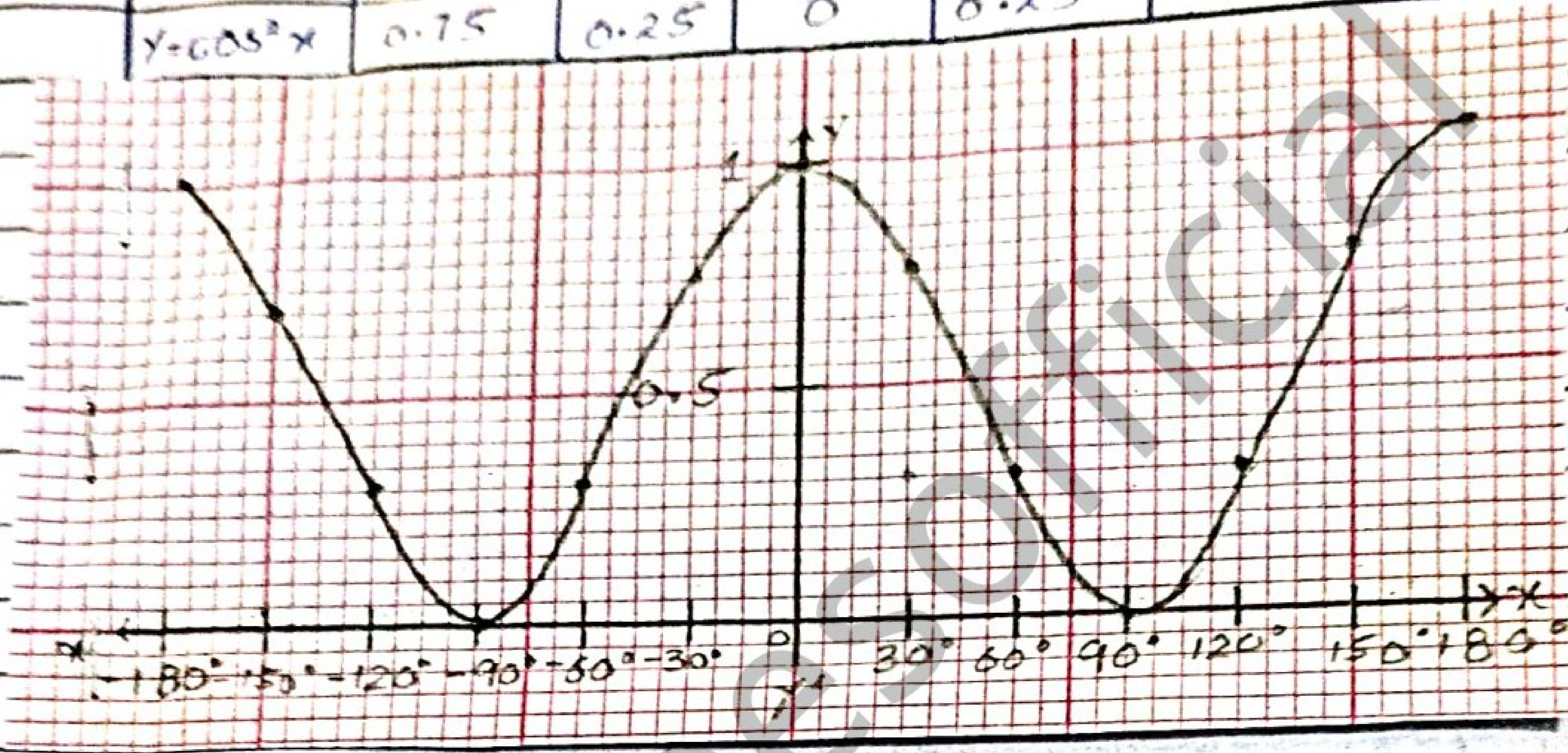
90°	105°	120°	135°	150°	165°	180°
0	−0.5	−0.86	−1	−0.86	−0.5	0

195°	210°	225°	240°	255°	270°	285°	300°
0.5	0.86	1	0.86	0.5	0	−0.5	−0.86



Y
X
X'
Y'
O
y = sin x
y = sin 2x
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
360°

x	30°	60°	90°	120°	150°	180°
$y = \cos^2 x$	0.75	0.25	0	0.25	0.75	1



vi) $y = 3\cos x$

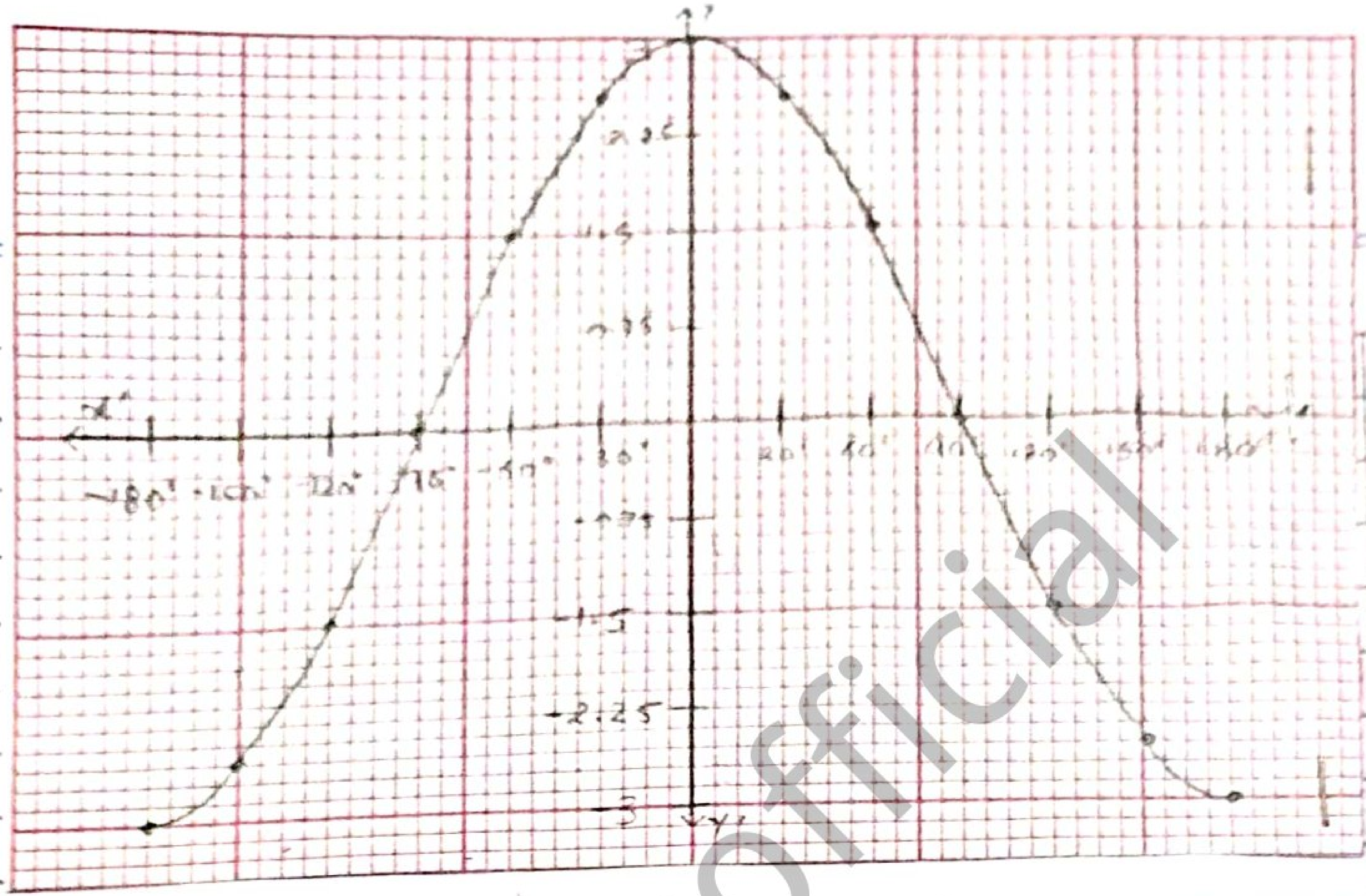
Sol:-

Given function :- $y = 3\cos x$

Period of $\cos x = 2\pi$
Period of $3\cos x = 2\pi$

x	-180°	-150°	-120°	-90°	-60°	-30°	0°
$y = 3\cos x$	-3	-2.59	-1.5	0	1.5	2.59	3

x	30°	60°	90°	120°	150°	180°
$y = 3\cos x$	2.59	1.5	0	-1.5	-2.59	-3

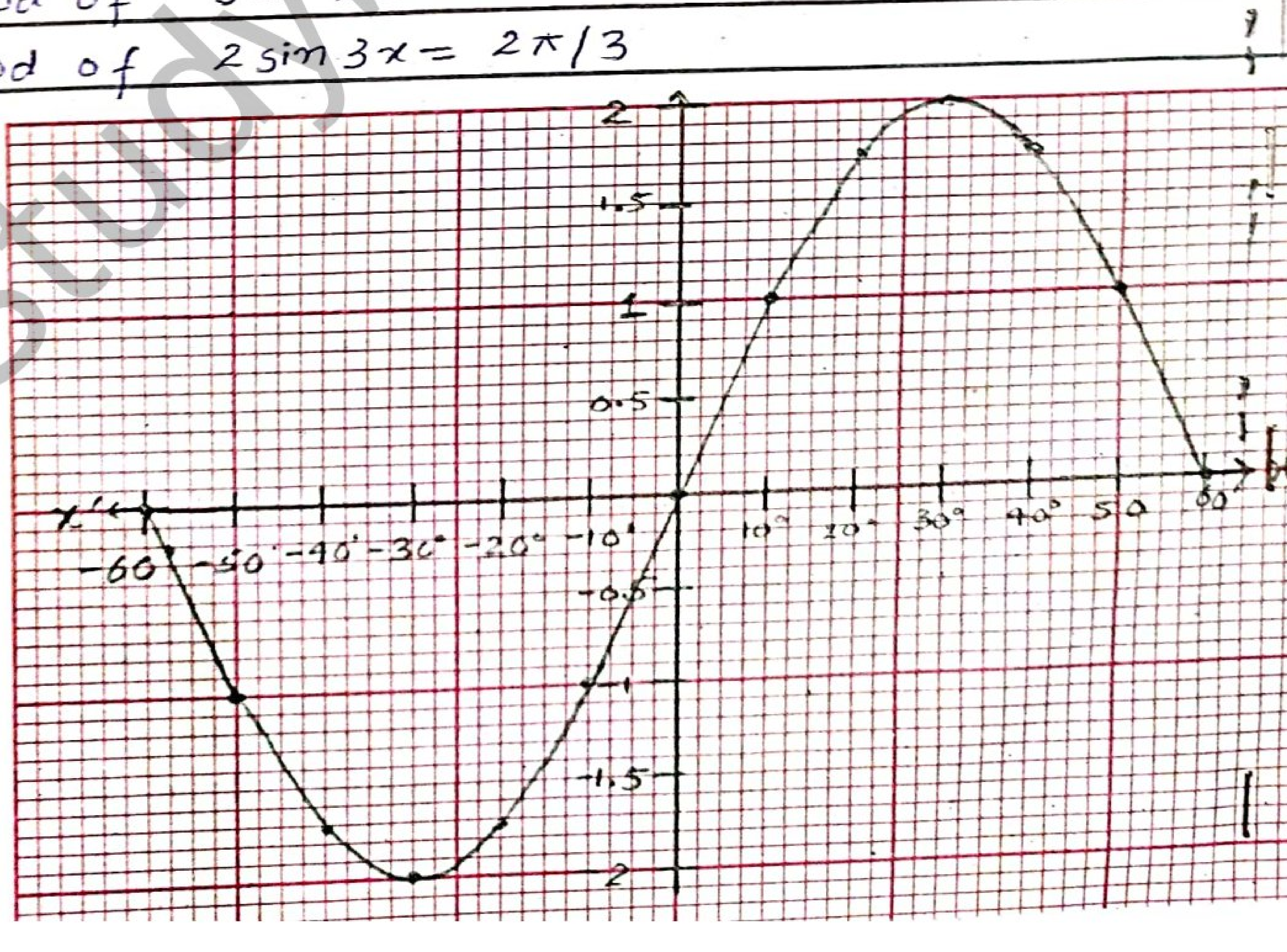


v) $Y = 2\sin 3x$

Sol:- Given function :- $y = 2\sin 3x$

Period of $\sin x = 2\pi$

Period of $2\sin 3x = 2\pi/3$



iv) $y = \cos\frac{x}{2}$

Sol:-

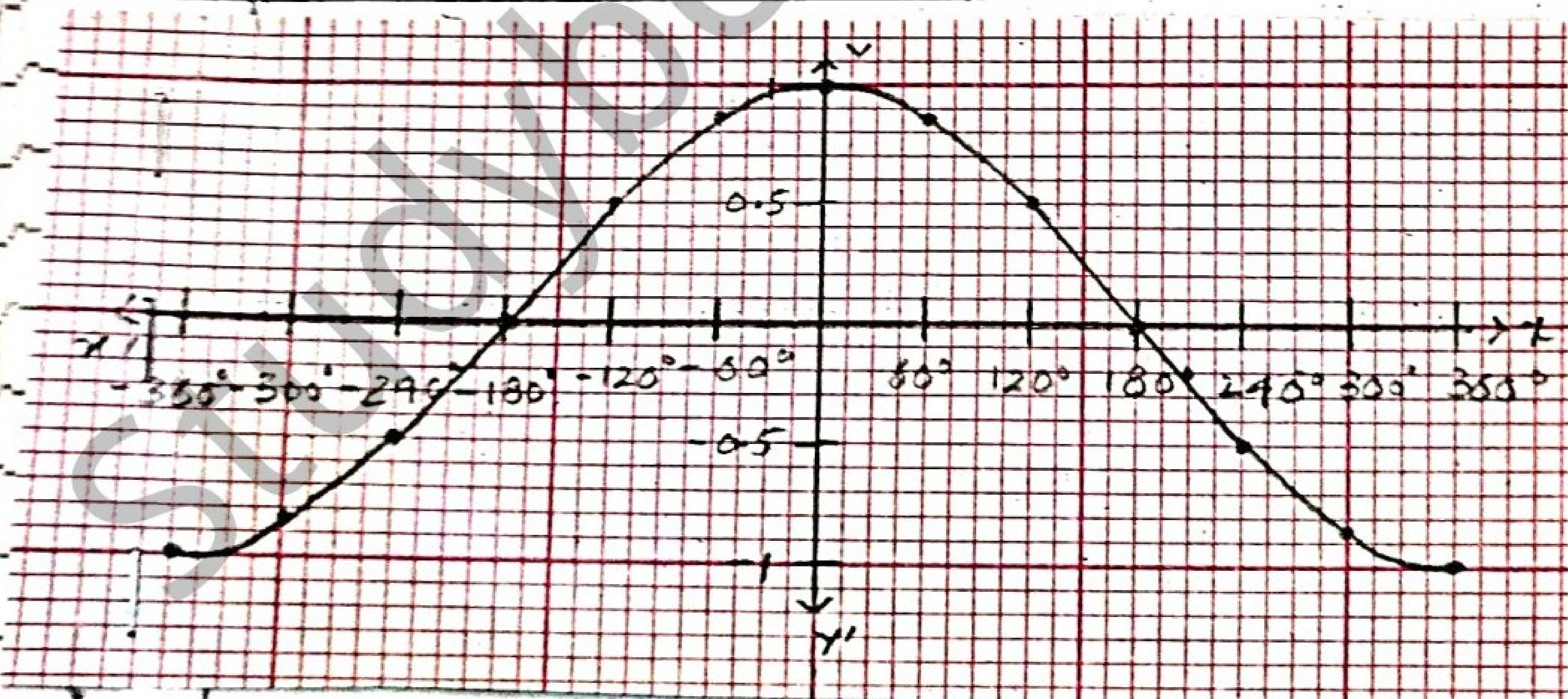
Given function:- $y = \cos\frac{x}{2}$

Period of $\cos x = 2\pi$

Period of $\cos\frac{x}{2} = 4\pi$

x	$-360°$	$-300°$	$-240°$	$-180°$	$-120°$	$-60°$	$0°$
$y=\cos\frac{x}{2}$	-1	-0.86	-0.5	0	0.5	0.86	1

x	$60°$	$120°$	$180°$	$240°$	$300°$	$360°$
$y=\cos\frac{x}{2}$	0.86	0.5	0	-0.5	-0.86	-1



iii) $y = 2\tan 2x$

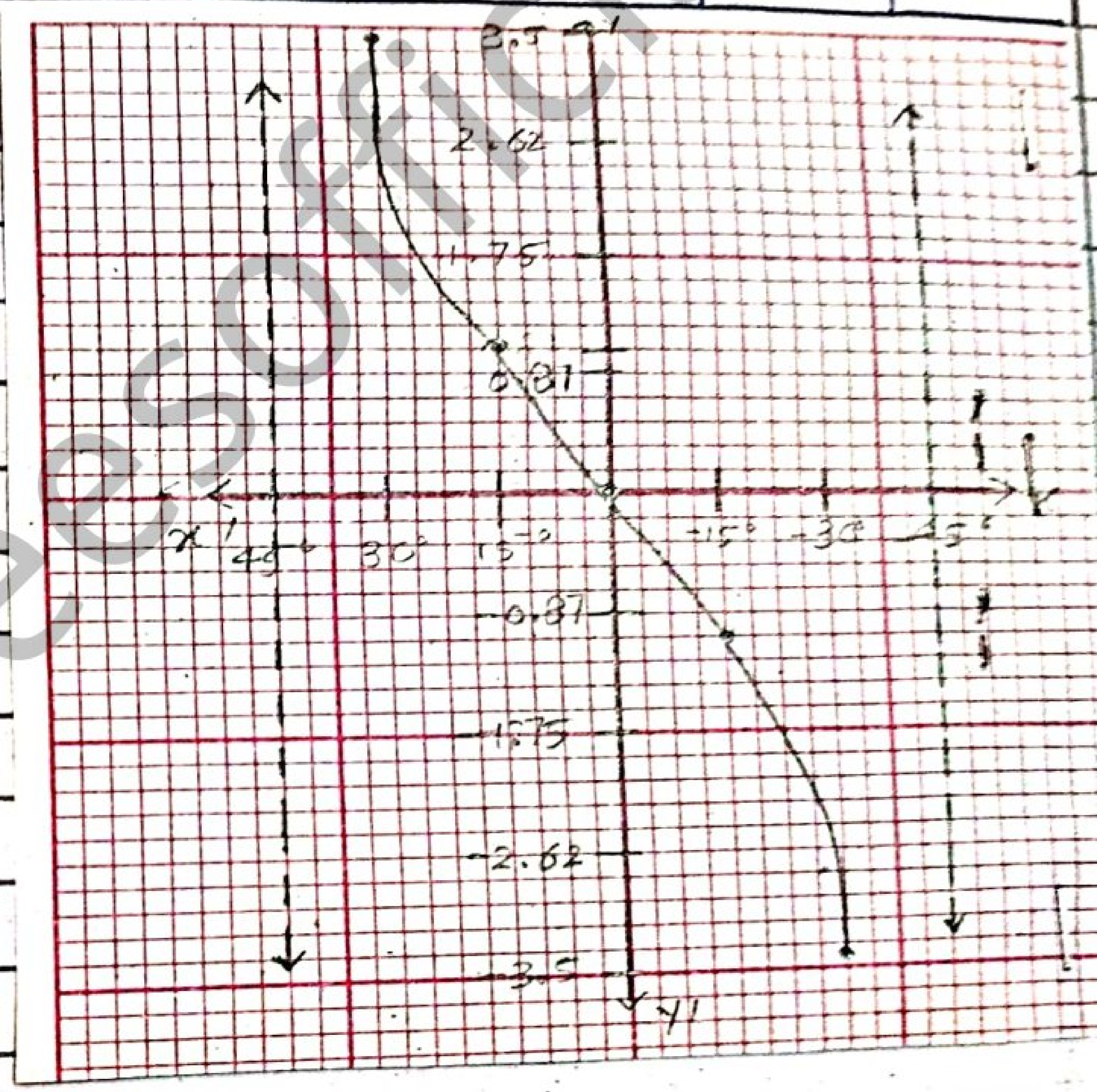
Sol:-

Given function:- $y = 2\tan 2x$

Period of $\tan x = \pi$

Period of $2\tan 2x = \pi/2$

x	-45°	-30°	-15°	0°	15°	30°	45°
$2\tan 2x$	∞	-3.46	-1.15	0	1.54	3.46	∞



ii) $y = 2\cos 3x$

Sol:-

Given function :- $y = 2\cos 3x$

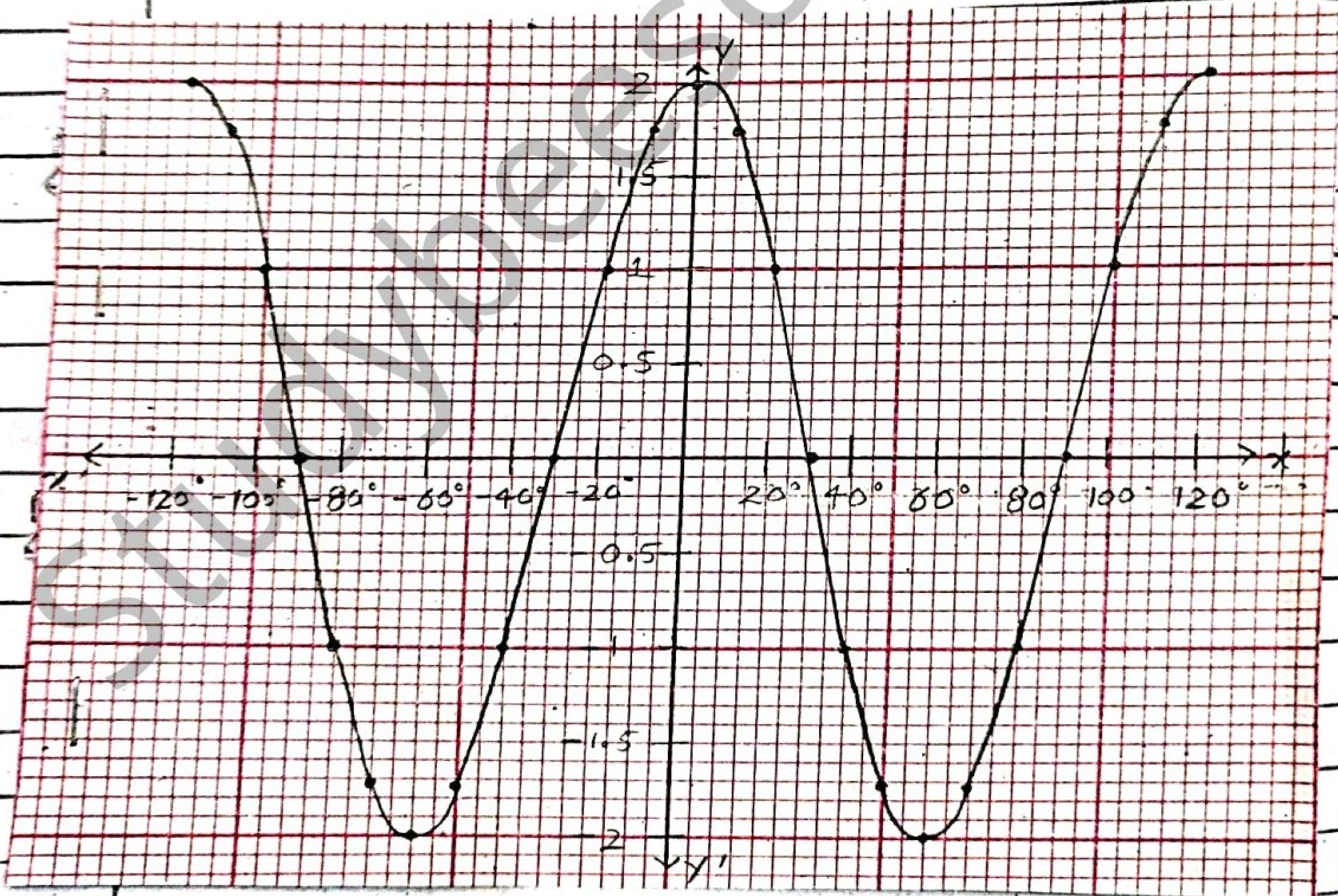
Period of $\cos x = 2\pi$

Period of $2\cos 3x = 2\pi/3$

x	-120°	-110°	-100°	-90°	-80°	-70°	-60°	-50°
2cos2x	2	1.73	1	0	-1	-1.73	-2	-1.73

x	-40°	-30°	-20°	-10°	0°	10°	20°	30°
2cos3x	-1	0	1	1.73	2	1.73	1	0

x	40°	50°	60°	70°	80°	90°	100°	110°	120°
2cos3x	-1	-1.73	-2	-1.73	-1	0	1	1.73	2



i) $y = 2\sin x$

Sol:-

Given function:- $y = 2\sin x$

Period of $2\sin x = 2\pi$

x	$-180°$	$-150°$	$-120°$	$-90°$	$-60°$	$-30°$
$y=2\sin x$	0	-1	-1.73	-2	-1.73	-1

x	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$
$y=2\sin x$	0	1	1.73	2	1.73	1	0