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Where Minds Pollinate

Class 11
Mathematics

Chapter 8
Review Exercise 8
Notes

National Book Foundation

$$\frac{\sqrt{3}}{-8}\cancel{\sin 80^\circ} + \frac{\sqrt{3}}{8}\cancel{\sin 80^\circ} + \frac{(\sqrt{3})^2}{16}$$

$$= \frac{3}{16} = R.H.S$$

Review Exercise

Qno 2. $\sin\theta = \frac{3}{5}$ obtuse QII, $\sin\phi = \frac{5}{13}$ QI

(i)

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$$

$$\frac{9}{25} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{9}{25}$$

$$\cos\theta = -\frac{4}{5} \quad QII$$

$$\sin^2\phi + \cos^2\phi = 1$$

$$\left(\frac{5}{3}\right)^2 + \cos^2\phi = 1$$

$$\cos^2\phi = 1 - \left(\frac{5}{3}\right)^2$$

$$\cos\phi = +\frac{12}{13} \quad QI \text{ as given in Q}$$

$$\cos(\theta - \phi) = \left(\frac{-4}{5}\right) \times \frac{12}{13} + \frac{3}{5} \times \frac{5}{13}$$

$$= \frac{-33}{65}$$

(i) $\sin(\theta - \varphi) = \sin\theta\cos\varphi - \cos\theta\sin\varphi$

$= \frac{3}{5} \times \frac{12}{3} - \left(-\frac{4}{5}\right) \times \frac{5}{13}$

$= \frac{36}{65} + \frac{20}{65}$

$= \frac{56}{65} = \sin(\theta - \varphi)$

(ii) $\tan(\theta - \varphi) = \frac{\sin(\theta - \varphi)}{\cos(\theta - \varphi)}$

$= \cancel{\text{Sin}}$

$= \frac{\frac{56}{\cancel{65}}}{\frac{-33}{\cancel{65}}} = \frac{56}{-33}$

(iii) $\tan(\theta + \varphi) = \frac{\sin(\theta + \varphi)}{\cos(\theta + \varphi)}$

$\sin(\theta + \varphi) = \left(\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(-\frac{4}{5}\right)\left(\frac{5}{13}\right)$

$= \frac{36}{65} - \frac{20}{65} = \frac{16}{65} = \sin(\theta + \varphi)$

$\cos(\theta + \varphi) = \left(-\frac{4}{5}\right) \times \frac{12}{13} - \left(\frac{3}{5}\right) \times \frac{5}{12}$

$= -\frac{48}{65} - \frac{15}{65} = -\frac{63}{65}$ ✓

$\tan(\theta + \varphi) = \frac{\sin(\theta + \varphi)}{\cos(\theta + \varphi)} = \frac{\frac{16}{\cancel{65}}}{\frac{-63}{\cancel{65}}} = -\frac{16}{63}$ ✓

③ (i) $\frac{1}{\sqrt{2}}(\sin\beta + \cos\beta)$.

$= \frac{1}{\sqrt{2}}\sin\beta + \frac{1}{\sqrt{2}}\cos\beta$

$= \cos 45^\circ \sin\beta + \sin 45^\circ \cos\beta$

$= \sin 45^\circ \cos\beta + \cos 45^\circ \sin\beta$

$= \sin(45^\circ + \beta)$ A

(ii) $\frac{1}{\sqrt{2}}\sin 75^\circ + \frac{1}{\sqrt{2}}\cos 75^\circ$

$= \cos 45^\circ \sin 75^\circ + \sin 45^\circ \cos 75^\circ$

$= \sin 45^\circ \cos 75^\circ + \cos 45^\circ \sin 75^\circ$

$= \sin(45^\circ + 75^\circ)$

$= \sin\{120^\circ\}$ A

$= \frac{\sqrt{3}}{2}$ An

Q4) (i) $\frac{1 + \tan 15^\circ}{1 - \tan 15^\circ} = \frac{1 + \tan 15^\circ}{1 - 1\tan 15^\circ}$

$= \frac{\tan 45^\circ + \tan 15^\circ}{1 - \tan 45^\circ \tan 15^\circ}$

$= \tan(45^\circ + 15^\circ) = \tan 60^\circ$ An

(ii) $\cos 70^\circ \cos 20^\circ - \sin 70^\circ \sin 20^\circ$

$= \cos(70^\circ + 20^\circ)$

$= \cos 90^\circ$ OR

$= 0$ An

Q5. $\tan(\theta - 45^\circ) = \frac{1}{3}$

Sol: $\frac{\tan\theta - \tan 45^\circ}{1 + \tan\theta \tan 45^\circ} = \frac{1}{3}$

$\frac{\tan\theta - 1}{1 + \tan\theta} = \frac{1}{3}$

$3\tan\theta - 3 = 1 + \tan\theta$

$3\tan\theta - \tan\theta = 1 + 3$

$2\tan\theta = 4$

$\tan\theta = \cancel{4}/\cancel{2}$

$\boxed{\tan\theta = 2}$

Q.6(i) $\sin(\alpha+\theta) = 2\cos(\alpha-\theta)$ prove

$\tan\alpha = \frac{2 - \tan\theta}{1 - 2\tan\theta}$

Proof $\sin\alpha\cos\theta + \cos\alpha\sin\theta = 2[\cos\alpha\cos\theta + \sin\alpha\sin\theta]$

$\sin\alpha\cos\theta + \cos\alpha\sin\theta = 2\cos\alpha\cos\theta + 2\sin\alpha\sin\theta$

$\sin\alpha\cos\theta - 2\sin\alpha\sin\theta = 2\cos\alpha\cos\theta - \cos\alpha\sin\theta$

$\sin\alpha[\cos\theta - 2\sin\theta] = \cos\alpha[2\cos\theta - \sin\theta]$

$\frac{\sin\alpha}{\cos\alpha} = \frac{2\cos\theta - \sin\theta}{\cos\theta - 2\sin\theta}$

$\tan\alpha = \frac{\frac{2\cancel{\cos\theta}}{\cancel{\cos\theta}} - \frac{\sin\theta}{\cos\theta}}{\frac{\cancel{\cos\theta}}{\cancel{\cos\theta}} - \frac{2\sin\theta}{\cos\theta}}$ $\times$ & $\div \cos\theta$

$$\tan\alpha = \frac{2-\tan\theta}{1-2\tan\theta}$$ Ans

(ii) $\sin(\alpha-\theta) = \cos(\alpha+\theta)$ prove $\tan\alpha = 1$

Proof:

$\sin\alpha\cos\theta - \cos\alpha\sin\theta = \cos\alpha\cos\theta - \sin\alpha\sin\theta$

$\sin\alpha\cos\theta + \sin\alpha\sin\theta = \cos\alpha\cos\theta + \cos\alpha\sin\theta$

$\sin\alpha(\cos\theta + \sin\theta) = \cos\alpha(\cos\theta + \sin\theta)$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{\cancel{\cos\theta + \sin\theta}}{\cancel{\cos\theta + \sin\theta}}$$

$\tan\alpha = 1$ Ans.

Q7) (i) $\frac{4\sin^2\theta\cos\theta}{\cos3\theta + \cos\theta} = \tan2\theta\tan\theta$

$$\text{LHS} = \frac{4\sin^2\theta\cos\theta}{\cos3\theta + \cos\theta}$$

$$= \frac{4\sin\theta\sin\theta\cos\theta}{2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)}$$

$$= \frac{\cancel{2}(2\sin\theta\cos\theta)\sin\theta}{\cancel{2}\cos2\theta\cos\theta}$$

$$= \frac{\sin2\theta\sin\theta}{\cos2\theta\cos\theta}$$

$= \tan2\theta\tan\theta =$ RHS

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$

(ii) $\dfrac{\sin 10\theta - \sin 4\theta}{\sin 4\theta + \sin 2\theta} = \cos 7\theta \sec\theta$

Sol: L.H.S

$= \dfrac{\cancel{2}\cos\left(\frac{10\theta+4\theta}{2}\right)\sin\left(\frac{10\theta-4\theta}{2}\right)}{\cancel{2}\sin\left(\frac{4\theta+2\theta}{2}\right)\cos\left(\frac{4\theta-2\theta}{2}\right)}$

$= \dfrac{\cos 7\theta \, \cancel{\sin 3\theta}}{\cancel{\sin 3\theta} \, \cos\theta}$

$= \dfrac{\cos 7\theta}{\cos\theta}$

$= \cos 7\theta \cdot \dfrac{1}{\cos\theta} = \cos 7\theta \sec\theta$

(iii) (i) $\sqrt{\dfrac{\cos(90^\circ+x)\sec(-x)\tan(180^\circ-x)}{\sec(360^\circ-x)\sin(180^\circ+x)\cot(90^\circ-x)}} = i$

Sol Concept:

$\left(n\frac{\pi}{2} \pm \theta\right) \rightarrow$ change $n = \text{odd}$

L.H.S. $\sqrt{\dfrac{(-\cancel{\sin x})(\cancel{\sec x})(-\cancel{\tan x})}{(\cancel{\sec x})(-\cancel{\sin x})(\cancel{\tan x})}}$

$= \dfrac{\sqrt{+1}}{-1} = \sqrt{-1} = i$ Proved

$3\pi/2 = 3\times 90$ — odd so tan change

(ii) $$\frac{\tan^2\left(\frac{3\pi}{2}-x\right)\sin^2(\pi+x)\sin(2\pi-x)}{\cos^2(\pi-x)\cot\left(\frac{3\pi}{2}+x\right)} = \cos x$$

Sol. $$= \frac{\cot^2 x\cdot(-\sin^2 x)\cdot(-\sin x)}{(-\cos^2 x)\cdot(-\tan x)}$$

$$= \frac{\frac{\cos^2 x}{\cancel{\sin^2 x}}\cdot(-\cancel{\sin^2 x})\cdot(-\sin x)}{(-\cos^2 x)\left(-\frac{\sin x}{\cos x}\right)}$$

$$= \frac{\cancel{\cos^2 x}(\sin x)}{\cancel{\cos^2 x}\left(\frac{\sin x}{\cos x}\right)}$$

$$= \frac{\cancel{\sin x}\times\cos x}{\cancel{\sin x}} = \cos x$$

OR

$$= \frac{[\cot x]^2\cdot[-\sin x]^2\cdot[-\sin x]}{[-\cos x]^2\cdot[-\tan x]}$$

$$= \frac{\frac{\cos^2 x}{\sin^2 x}\cdot\sin^2 x\cdot\sin x}{-\cos^2 x\cdot\frac{\sin x}{\cos x}}$$

$$= \frac{\cos^2 x\cdot\sin x}{\cos x\cdot\sin x}$$

RHS $= \cos x$

(iii) $$\sqrt{\frac{[1-\tan^2 x\cos(-x)\cos(360^\circ-x)]\tan 45^\circ}{[\sin 90^\circ-\sin(180^\circ+x)][\sin 90^\circ-\cos(90^\circ-x)]}}$$

Sol $$\sqrt{\frac{\left[1-\frac{\sin^2 x}{\cancel{\cos^2 x}}\cancel{\cos x}(\cancel{\cos x})\right]1}{[1-(-\sin x)][1-\sin x]}}$$

$$= \sqrt{\frac{1-\sin^2 x}{(1+\sin x)(1-\sin x)}}$$

$$= \sqrt{\frac{1-\sin^2 x}{1-\sin^2 x}} \Rightarrow \sqrt{1} = 1 \text{ RH}$$

Q10) $\sin(16x) = 16\sin x \cos x \cos 2x \cos 4x \cos 8x$

Sol: R.H.S

$= 8(2\sin x \cos x)\cos 2x \cos 4x \cos 8x$

$= 8(\sin 2x)\cos 2x \cos 4x \cos 8x$

$= 4(2\sin 2x \cos 2x)\cos 4x \cos 8x$

$= 4(\sin 2(2x))\cos 4x \cos 8x$

$= 2(2\sin 4x \cos 4x)\cos 8x$

$= 2\sin(4x)\cos 8x$

$= 2\sin 8x \cos 8x$

$= \sin 2(8x) = \sin 16x = L.H.S$ proved

(ii) $\frac{1+\cos 2\theta}{\sin 2\theta - \cos\theta} = \frac{2\cos\theta}{2\sin\theta - 1}$

Sol: L.H.S

$= \frac{1+\cos 2\theta}{2\sin\theta\cos\theta - \cos\theta}$

$= \frac{\cancel{1}+2\cos^2\theta-\cancel{1}}{\cos\theta(2\sin\theta-1)}$

$= \frac{2\cos^2\theta}{\cancel{\cos\theta}(2\sin\theta-1)}$

$= \frac{2\cos\theta}{2\sin\theta-1}$ RH

(iii) $\dfrac{\cos 3\theta - \cos\theta}{\cos 3\theta + \cos\theta} = \dfrac{2\tan^2\theta}{\tan^2\theta - 1} = \dfrac{-2\tan^2\theta}{\sec^2\theta - 2\tan^2\theta}$

& LHS

$\dfrac{-2\left(\frac{4\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)}{2\cos\left(\frac{4\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)}$

$= -\dfrac{\sin 2\theta \sin\theta}{\cos 2\theta \cos\theta}$

$= \dfrac{-2\sin\cos\theta\sin\theta}{(\cos^2\theta - \sin^2\theta)\cos\theta}$

$= \dfrac{-2\sin^2\theta}{\cos^2\theta - \sin^2\theta}$

$= \dfrac{\cancel{-}2\sin^2\theta}{\cancel{-}(\sin^2\theta - \cos^2\theta)}$

$= \dfrac{\frac{2\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta} - \frac{\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}}}$ $\quad \cos^2\theta \times \div$

$= \dfrac{2\tan^2\theta}{\tan^2\theta - 1} = RHS$

now $\dfrac{2\tan^2\theta}{\tan^2\theta - 1}$

$= \dfrac{2\tan^2\theta}{\tan^2\theta - (\sec^2\theta - \tan^2\theta)}$

$= \dfrac{2\tan^2\theta}{\tan^2\theta - \sec^2\theta + \tan^2\theta}$

$= \dfrac{2\tan^2\theta}{2\tan^2\theta - \sec^2\theta}$

$= \dfrac{-2\tan^2\theta}{\sec^2\theta - 2\tan^2\theta}$