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Where Minds Pollinate

Class 11
Mathematics

Chapter 8
Exercise 8.3
Notes

National Book Foundation

Exercise # 8.5

Q.1 Use product-to-sum formula to change the following to sum or difference.

i) $4\sin 16x \cos 10x$

Sol:-

$$= 4\sin 16x \cos 10x$$
$$= 2[2\sin 16x \cos 10x]$$

$$\because 2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$

$$\therefore = 2[\sin(16x+10x) + \sin(16x-10x)]$$

$$= 2[\sin 26x + \sin 6x]$$

ii) $10\cos 10y \cos 6y$

Sol:-

$$= 10\cos 10y \cos 6y$$
$$= 5[2\cos 10y \cos 6y]$$

$$\because +2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$\therefore \; = 5\left[\cos(10y+6y)+\cos(10y-6y)\right]$

$= 5\left[\cos 16y + \cos 4y\right]$

iii) $2\cos 5t \sin 3t$

Sol:-

$= 2\cos 5t \sin 3t$

$\because \; 2\cos\alpha \sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

$= \sin(5t+3t) - \sin(5t-3t)$

$= \sin 8t - \sin 2t$

iv) $6\cos 5x \sin 10x$

Sol:-

$= 6\cos 5x \sin 10x$

$= 3\left[2\sin 10x \cos 5x\right]$

$\because \; 2\sin\alpha \cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

$= 3\left[\sin(10x+5x) + \sin(10x-5x)\right]$

$= 3\left[\sin 15x + \sin 5x\right]$

v $\sin(-u)\sin 5u$

Sol:-

$$\sin(-u)\sin 5u$$
$$= -\sin u \sin 5u$$
$$= -\sin 5u \sin u$$
$$= -2\sin 5u \sin u / 2$$

$\because -2\sin\alpha \sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$$\therefore = \frac{1}{2}\left[\cos(5u+u) - \cos(5u-u)\right]$$

$$= \frac{1}{2}\left[\cancel{\cos 6u - \cos 4u}\right]$$

vi) $-2\sin(-100^\circ)\sin(-20^\circ)$

Sol:-

$$= -2\sin(-100^\circ)\sin(-20^\circ)$$
$$= +2\sin(100^\circ)\sin(-20^\circ)$$
$$= -2\sin(100^\circ)\sin(20^\circ)$$
$$= \cos(100^\circ + 20^\circ) - \cos(100^\circ - 20^\circ)$$
$$= \cos 120^\circ - \cos 80^\circ$$

vii) $\cos 23^\circ \sin 17^\circ$

Sol:-

$$= \cos 23^\circ \sin 17^\circ$$

$$= \frac{2\cos 23^\circ \sin 17^\circ}{2}$$

$$= \frac{\sin(23^\circ+17^\circ) - \sin(23^\circ-17^\circ)}{2}$$

$$= \frac{1}{2}\left[\sin 40^\circ - \sin 6^\circ\right]$$

viii) $2\cos 56^\circ \sin 48^\circ$

Sol:-

$$= 2\cos 56^\circ \sin 48^\circ$$
$$= \sin(56^\circ+48^\circ) - \sin(56^\circ-48^\circ)$$
$$= \sin 104^\circ - \sin 8^\circ$$

ix) $2\sin 75^\circ \sin 15^\circ$

Sol:-

$$= -\left[-2\sin 75^\circ \sin 15^\circ\right]$$
$$= -\left[\cos(75^\circ+15^\circ) \pm \cos(75^\circ-15^\circ)\right]$$
$$= -\left[\cos 90^\circ \pm \cos 60^\circ\right]$$

$$= \cos 60^\circ \pm \cos 90^\circ$$

x) $4 \sin \frac{u+v}{2} \cos \frac{u-v}{2}$

Sol:-

$$= 4 \sin \frac{u+v}{2} \cos \frac{u-v}{2}$$

$$= 2\left[2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)\right]$$

$$= 2\left[\sin\left(\frac{u+v}{2} + \frac{u-v}{2}\right) + \sin\left(\frac{u+v}{2} - \frac{u-v}{2}\right)\right]$$

$$= 2\left[\sin\left(\frac{u+\cancel{v}+u-\cancel{v}}{2}\right) + \sin\left(\frac{u+v-\cancel{u}+v}{2}\right)\right]$$

$$= 2\left[\sin u + \sin v\right]$$

xi) $2 \cos \frac{2u+2v}{2} \sin \frac{2u-2v}{2}$

Sol:-

$$= 2 \cos\left(\frac{\cancel{2}(u+v)}{\cancel{2}}\right) \sin\left(\frac{\cancel{2}(u-v)}{\cancel{2}}\right)$$

$$= 2 \cos(u+v) \sin(u-v)$$

$$= \sin(u+\cancel{v}+u-\cancel{v}) - \sin(\cancel{u}+v-\cancel{u}+v)$$

$$= \sin 2u - \sin 2v$$

Q2. Rewrite the sum or difference as a product of two functions.

i) $\sin 70^\circ + \sin 30^\circ$

Sol:-

$$= \sin 70^\circ + \sin 30^\circ$$

$$\because \sin\alpha + \sin\varphi = 2\sin\left(\frac{\alpha+\varphi}{2}\right)\cos\left(\frac{\alpha+\varphi}{2}\right)$$

$$\therefore 2\sin\left(\frac{70^\circ+30^\circ}{2}\right)\cos\left(\frac{70^\circ-30^\circ}{2}\right)$$

$$= 2\sin 50^\circ \cos 20^\circ$$

ii) $\sin 76^\circ - \sin 14^\circ$

Sol:-

$$= \sin 76^\circ - \sin 14^\circ$$

$$\because \sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\therefore = 2\cos\left(\frac{76^\circ+14^\circ}{2}\right)\sin\left(\frac{76^\circ-14^\circ}{2}\right)$$

$$= 2\cos 45^\circ \sin 31^\circ$$

iii) $\cos 58^\circ + \cos 12^\circ$

Sol:-

$$= \cos 58^\circ + \cos 12^\circ$$

$$\because \cos\varphi + \cos\alpha = 2\cos\left(\frac{\varphi+\alpha}{2}\right)\cos\left(\frac{\varphi-\alpha}{2}\right)$$

$$\therefore 2\cos\left(\frac{58^\circ+12^\circ}{2}\right)\cos\left(\frac{58^\circ-12^\circ}{2}\right)$$

$$= 2\cos 35^\circ \cos 23^\circ$$

iv) $\cos\frac{p-q}{2} + \cos\frac{p+q}{2}$

Sol:-

$$= \cos\frac{p-q}{2} + \cos\frac{p+q}{2}$$

$$= 2\cos\left[\frac{p+q}{2}+\frac{p-q}{2}\right]\cos\left[\frac{p+q}{2} \quad \frac{p-q}{2}\right]$$

$$= 2\cos\left[\frac{p+\cancel{q}+p-\cancel{q}}{2\times 2}\right]\cos\left[\frac{\cancel{p}+q-\cancel{p}+q}{2\times 2}\right]$$

$$= 2\cos\left(\cancel{2}p/_{\cancel{2}\times 2}\right)\cos\left(\cancel{2}q/_{\cancel{2}\times 2}\right)$$

$$= 2\cos\frac{p}{2}\cos\frac{q}{2}$$

v) $\sin(-10^\circ) + \sin(-20^\circ)$

Sol:-

$$= \sin(-10^\circ) + \sin(-20^\circ)$$
$$= -\sin 10^\circ - \sin 20^\circ$$
$$= -(\sin 20^\circ + \sin 10^\circ)$$

$$= +2\sin\left(\frac{20^\circ + 10^\circ}{2}\right)\cos\left(\frac{20^\circ - 10^\circ}{2}\right)$$

$$= +2\sin 15^\circ \cos 5^\circ$$

Q3. Prove the following :-

i) $\dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)} = \dfrac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}$

Proof:-

L.H.S $\Rightarrow \dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$

$$= \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

$$OR = \frac{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$= \frac{1 - \tan\alpha\tan\beta}{1 + \tan\alpha\tan\beta}$$

$$= R.H.S$$

ii) $$\frac{6\cos 8u \sin 2u}{\sin(-6u)} = \frac{-3\sin 10}{\sin 6u} + 3$$

Proof:-

$$L.H.S \Rightarrow \frac{6\cos 8u \sin 2u}{\sin(-6u)}$$

$$= \frac{3[2\cos 8u \sin 2u]}{-\sin 6u}$$

$$= \frac{3[\sin(8u+2u) - \sin(8u-2u)]}{-\sin 6u}$$

$$= \frac{-3[\sin 10u - \sin 6u]}{-\sin 6u}$$

$$= \frac{3\sin 10u - 3\sin 6u}{-\sin 6u}$$

$$= \frac{3\sin 10u}{-\sin 6u} - \frac{3\sin 6u}{-\sin 6u}$$

$$= -\frac{3\sin 10u}{\sin 6u} + 3 \Rightarrow R.H.S$$

iii) $4\cos 4v \sin 3v = 2(\sin 7v - \sin v)$

Proof:-

$$= 4\cos 4v \sin 3v$$
$$= 2.2\cos 4v \sin 3v$$
$$= 2[\sin(4v+3v) - \sin(4-3v)]$$

$$= 2(\sin 7v - \sin v)$$
$$= R.H.S.$$

iv) $\sin 3\theta + \sin\theta = 4\cos^2\theta \sin\theta$

Proof:-

$L.H.S \Rightarrow \sin 3\theta + \sin\theta$

$$= 2\sin\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)$$

$$= 2\sin(2\theta)\cos(\theta)$$
$$\because 2\sin 2\theta \cos\theta = \sin 2\theta$$
$$= 2(2\sin\theta\cos\theta)\cos\theta$$

$= 4\sin\theta\cos^2\theta$

$= R.H.S.$

v) $\cos 3x + \cos x = 2\cos x(\cos 2x)$

Proof:-

$L.H.S \Rightarrow \cos 3x + \cos x$

$= 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)$

$= 2\cos 2x \cos x$

$= 2(\cos 2x \cos x)$

$= 2\cos x(\cos 2x)$

$= R.H.S.$

vi) $2\tan y \cos 3y = \sec y(\sin 4y - \sin 2y)$

Proof:-

$L.H.S \Rightarrow 2\tan y \cos 3y$

$= 2\frac{\sin y}{\cos y}\cos 3y$

$= \frac{1}{\cos y}\left[2\sin y \cos 3y\right]$

$= \sec y \left[\sin(3y+y) - \sin(3y-y)\right]$

$= \sec y(\sin 4y - \sin 2y)$

$=$ R.H.S.

vii) $\dfrac{\sin 6\beta + \sin 4\beta}{\sin 6\beta - \sin 4\beta} = \tan 10\beta \cot 2\beta$

Proof:-

L.H.S $\Rightarrow \dfrac{\sin 6\beta + \sin 4\beta}{\sin 6\beta - \sin 4\beta}$

$$= \frac{2\sin\left(\frac{6\beta+4\beta}{2}\right)\cos\left(\frac{6\beta-4\beta}{2}\right)}{2\cos\left(\frac{6\beta+4\beta}{2}\right)\sin\left(\frac{6\beta-4\beta}{2}\right)}$$

$$= \frac{\cancel{2}\sin 10\beta \cos 2\beta}{\cancel{2}\cos 10\beta \sin 2\beta}$$

$= \tan 10\beta \cot 2\beta$

$=$ R.H.S.

viii) $\dfrac{\cos 3\theta + \cos\theta}{\cos 3\theta - \cos\theta} = -\cot 2\theta \cot\theta$

Proof:-

L.H.S $\Rightarrow$ $\dfrac{\cos 3\theta + \cos\theta}{\cos 3\theta - \cos\theta}$

$$= \frac{\cancel{2}\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)}{-\cancel{2}\sin\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)}$$

$$= \frac{\cos 2\theta \cos\theta}{-\sin 2\theta \sin\theta}$$

$$= -\cot 2\theta \cot\theta$$

$=$ R.H.S.

ix) $\dfrac{\cos 6x + \cos 8x}{\sin 6x - \sin 4x} = \cot x \cos 7x \sec 5x$

Proof:-

L.H.S $\Rightarrow$ $\dfrac{\cos 6x + \cos 8x}{\sin 6x - \sin 4x}$

$$= \frac{\cancel{2}\cos\left(\frac{8x+6x}{2}\right)\cos\left(\frac{8x-6x}{2}\right)}{\cancel{2}\cos\left(\frac{6x+4x}{2}\right)\cancel{\cos}\ \sin\left(\frac{6x-4x}{2}\right)}$$

$$= \frac{\cos 7x \cos x}{\cos 5x\ \cancel{\cos}\ \sin x}$$

$= \cos 7x \sec 5x \cot x$

$= \cot x \cos 7x \sec 5x$

$=$ R.H.S

x) $\frac{\cos 2\alpha - \cos 4\alpha}{\sin 2\alpha + \sin 4\alpha} = \tan\alpha$

Proof:-

L.H.S $\Rightarrow \frac{\cos 2\alpha - \cos 4\alpha}{\sin 2\alpha + \sin 4\alpha}$

$$= \frac{-(\cos 4\alpha - \cos 2\alpha)}{\sin 4\alpha + \sin 2\alpha}$$

$$= \frac{-\left\{-\cancel{2}\sin\left(\frac{4\alpha+2\alpha}{2}\right)\sin\left(\frac{4\alpha-2\alpha}{2}\right)\right.}{\cancel{2}\sin\left(\frac{4\alpha+2\alpha}{2}\right)\cos\left(\frac{4\alpha-2\alpha}{2}\right)}$$

$$= \frac{\sin 3\alpha \sin\alpha}{\sin 3\alpha \cos\alpha}$$

$$= \tan\alpha$$

$$= R.H.S.$$

xi) $2\cos 2u \cos u + \sin 2u \sin u = 2\cos^3 u$

Proof:-

L.H.S $\Rightarrow 2\cos 2u \cos u + \sin 2u \sin u$

$$= \cos(2u+u) + \cos(2u-u) + \frac{\cos(2u+u) - \cos(2u-}{-2}$$

$$= \cos 3u + \cos u - \frac{\cos 3u - \cos u}{2}$$

$$= \frac{2\cos 3u + 2\cos u - \cos 3u + \cos u}{2}$$

$$= \frac{\cos 3u + 3\cos u}{2}$$

$$= \frac{\overset{2}{\cancel{4}}\cos^3 u - \cancel{3\cos u} + \cancel{3\cos u}}{\cancel{2}\,1}$$

$$= 2\cos^3 u \Rightarrow R.H.S$$

xii) $2\sin 2y \sin 3y = \cos y - \cos 5y$

Proof:-

L.H.S $\Rightarrow 2\sin 2y \sin 3y$

$= -\left[\cos(2y+3y) - \cos(3y-2y)\right]$

$= -\left[\cos 5y - \cos y\right]$

$= \cos y - \cos 5y$

$=$ R.H.S.

xiii) $\dfrac{\cos 10x + \cos 6x}{\cos 6x - \cos 10x} = \cot 2x \cot 8x$

Proof:-

L.H.S $\Rightarrow \dfrac{\cos 10x + \cos 6x}{\cos 6x - \cos 10x}$

$= \dfrac{\cos 10x + \cos 6x}{-(\cos 10x - \cos 6x)}$

$= \dfrac{\cancel{2}\cos\left(\frac{10x+6x}{2}\right)\cos\left(\frac{10x-6x}{2}\right)}{-\left[\cancel{2}\sin\left(\frac{10x+6x}{2}\right)\sin\left(\frac{10x-6x}{2}\right)\right]}$

$= \frac{\cos 8x \cos 2x}{\sin 8x \sin 2x}$

$= \cot 2x \cot 8x$

$= R.H.S.$

Q. Prove that.

i) $\cos 80^\circ . \cos 60^\circ . \cos 40^\circ . \cos 20^\circ = \frac{1}{16}$

Proof:-

L.H.S $\Rightarrow \cos 80^\circ \cos 60^\circ \cos 40^\circ \cos 20^\circ$

$= \cos 80^\circ \left(\frac{1}{2}\right) \cos 40^\circ \cos 20^\circ$

$= \frac{1}{2} \cos 80^\circ \left[\cos 40^\circ \cos 20^\circ\right]$

$= \frac{1}{2} \cos 80^\circ \left\{\frac{\cos(40^\circ+20^\circ) + \cos(40^\circ-20^\circ)}{2}\right\}$

$= \frac{1}{2} \cos 80^\circ \left[\frac{\cos 60^\circ + \cos 20^\circ}{2}\right]$

$= \frac{1}{2} \cos 80^\circ \left[\frac{1/2 + \cos 20^\circ}{2}\right]$

$= \frac{1}{2\times 2\times 2} \cos 80^\circ + \frac{1 \cos 20^\circ \cos 80^\circ}{2 \quad 2\times 2}$

$= \frac{1}{8}\cos 80° + \frac{1}{4}\cos 20° \cos 80°$

$= \frac{1}{8}\cos 80° + \frac{1}{4\times 2}[\cos(20°+80°) + \cos(80°-20°)]$

$= \frac{1}{8}\cos 80° + \frac{1}{8}[\cos 100° + \cos 60°]$

$= \frac{1}{8}\cos 80° + \frac{1}{8}[\cos 100° + \frac{1}{2}]$

$= \frac{1}{8}\left[\cos 80° + \cos 100° + \frac{1}{2}\right]$

$= \frac{1}{8}\left[2\cos\left(\frac{100°+80°}{2}\right)\cos\left(\frac{100°-80°}{2}\right) + \frac{1}{2}\right]$

$= \frac{1}{8}\left[2\cos 90° \cos 10° + \frac{1}{2}\right]$

$= \frac{1}{8}\left(2(0)\cos 10° + \frac{1}{2}\right)$

$= \frac{1}{8}\left(\frac{1}{2}\right)$

$= \frac{1}{16}$

$=$ R.H.S

ii) $\sin 70^\circ \sin 50^\circ \sin 30^\circ \sin 10^\circ = \frac{1}{16}$

Proof:-

L.H.S $\Rightarrow \sin 70^\circ \sin 50^\circ \sin 30^\circ \sin 10^\circ$

$= \sin 70^\circ \sin 50^\circ \left(\frac{1}{2}\right) \sin 10^\circ$

$= \frac{1}{2} \sin 70^\circ \sin 50^\circ \sin 10^\circ$

$= \frac{1}{-2\times 2} \left\{\cos(70^\circ+50^\circ) - \cos(70^\circ-50^\circ)\right] \sin 10^\circ$

$= -\frac{1}{4} \left[\cos 120^\circ - \cos 20^\circ\right] \sin 10^\circ$

$= -\frac{1}{4} \left[-\frac{1}{2} - \cos 20^\circ\right] \sin 10^\circ$

$= \left[\frac{1}{4\times 2} + \frac{1}{4} \cos 20^\circ\right] \sin 10^\circ$

$= \frac{1}{8} \sin 10^\circ + \frac{1}{4} \sin 10^\circ \cos 20^\circ$

$= \frac{1}{8} \sin 10^\circ + \frac{1}{4\times 2} \left[2 \cos 20^\circ \sin 10^\circ\right]$

$= \frac{1}{8} \sin 10^\circ + \frac{1}{8} \left[\sin(10^\circ+20^\circ) - \sin(20^\circ-10^\circ)\right]$

$= \frac{1}{8} \sin 10^\circ + \frac{1}{8}\left[\sin 30^\circ - \sin 10^\circ\right]$

$= \frac{1}{8} \sin 10^\circ + \frac{1}{8}\left[\frac{1}{2} - \sin 10^\circ\right]$

$= \cancel{\frac{1}{8} \sin 10^\circ} - \cancel{\frac{1}{8} \sin 10^\circ} + \frac{1}{16}$

$= \frac{1}{16} \Rightarrow$ R.H.S